ODU
Introduction to Accelerator Physics
Old Dominion University

Magnets and Magnet Technology

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Outline

- Back to Maxwell
  - Parameterizing fields in accelerator magnets
  - Symmetries, comments about magnet construction
- Relating currents and fields
  - Equipotentials and contours, dipoles and quadrupoles
  - Thin magnet kicks and that ubiquitous rigidity
  - Complications: hysteresis, end fields
- More details about dipoles
  - Sector and rectangular bends; edge focusing
- Superconducting magnets
  - RHIC, LHC, future

$$\frac{p}{q} = B\rho$$

Other References

- Magnet design and a construction is a specialized field all its own
  - Electric, Magnetic, Electromagnetic modeling
    - 2D, 3D, static vs dynamic
  - Materials science and engineering
    - Conductors, superconductors, ferrites, superferrites
  - Measurements and mapping
    - e.g. g-2 experiment: 1 PPM field uniformity, 14m superconducting dipole
- Entire US Particle Accelerator School courses have been given on just superconducting magnet design
  - http://www.bnl.gov/magnets/staff/gupta/scmag-course/

    (Ramesh Gupta and Animesh Jain, BNL)

EM/Maxwell Review I

- Recall our relativistic Lorentz force

$$\frac{d(\gamma m\vec{v})}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

  - For large γ common in accelerators, magnetic fields are much more effective for changing particle momenta
  - Can mostly separate E (RF) and B (DC magnets)
    - Some places you can't, e.g. plasma wakefields, betatrons

- Easiest/simplest: magnets with **constant** B field
  - Constant-strength optics
    - Even varying B field accelerator magnets change field so slowly that E fields are negligible
    - Consistent with assumptions for "standard canonical coordinates", p 49 Conte and MacKay

EM/Maxwell Review II

- Maxwell's Equations for B,H and magetization M are

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \vec{j} \qquad \vec{H} \equiv \vec{B}/\mu - \vec{M}$$

- Magnetic vector potential A

$$\vec{B} = \vec{\nabla} \times \vec{A} \qquad \text{since } \vec{\nabla} \cdot \vec{\nabla} \times \vec{A} = 0$$

- 2D (B_s=H_s=0), using paraxial approximation ($p_{x,y}$<<p_0)
- Our beam is away from magnet coils (j=0, M=0)
  - Unlike the Lithium lens homework!
  - Simple homogeneous differential field equations

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \qquad \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} = 0$$

Parameterizing Solutions

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \qquad \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} = 0$$

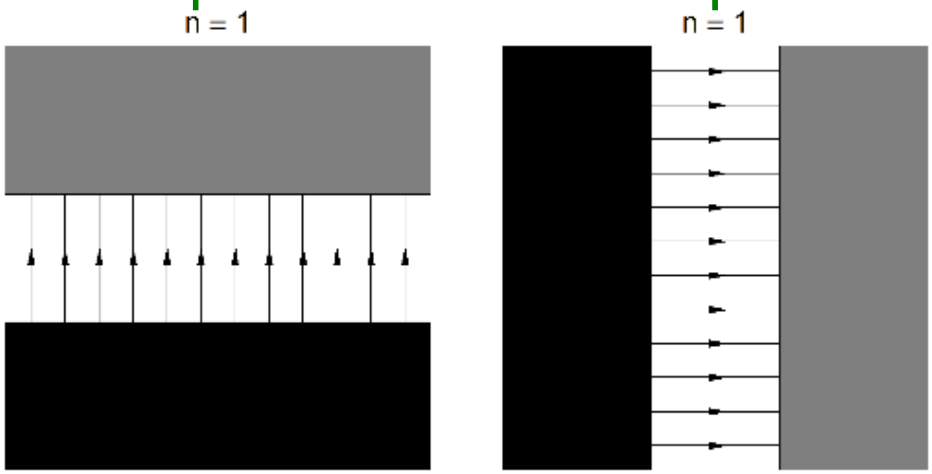
- What are solutions to these equations?
  - Constant field: $\vec{B} = B_x x^0 \hat{x} + B_y y^0 \hat{y}$
    - Dipole fields, usually either only B_x or B_y
    - 360 degree (2 π) rotational "symmetry"
  - First order field: $\vec{B} = (B_{xx} x + B_{xy} y)\hat{x} + (B_{yx} x + B_{yy} y)\hat{y}$
    - Maxwell gives $B_n = B_{xx} = -B_{yy}$ and $B_s = B_{xy} = B_{yx}$

$$\vec{B} = B_n (x\hat{x} - y\hat{y}) + B_s (x\hat{y} + y\hat{x})$$

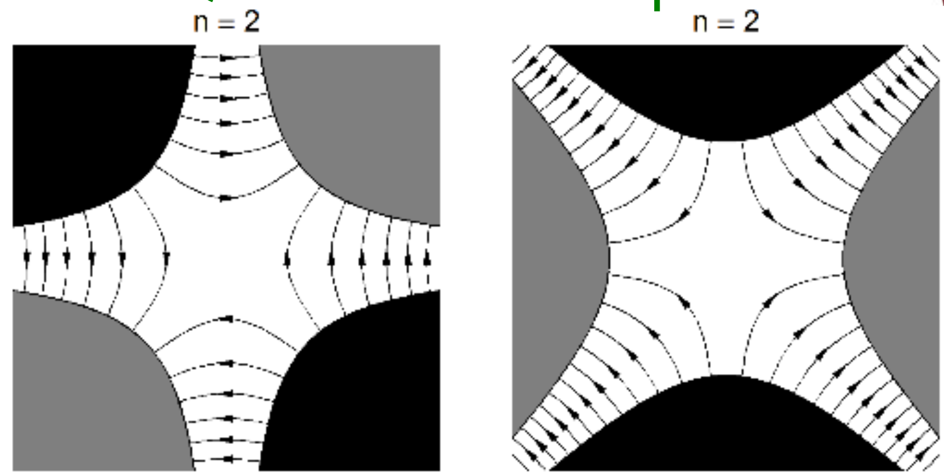
    - Quadrupole fields, either normal B_n or skew B_s
    - 180 degree (π) rotational symmetry
    - 90 degree rotation interchanges normal/skew
  - Higher order…

Visualizing Fields I

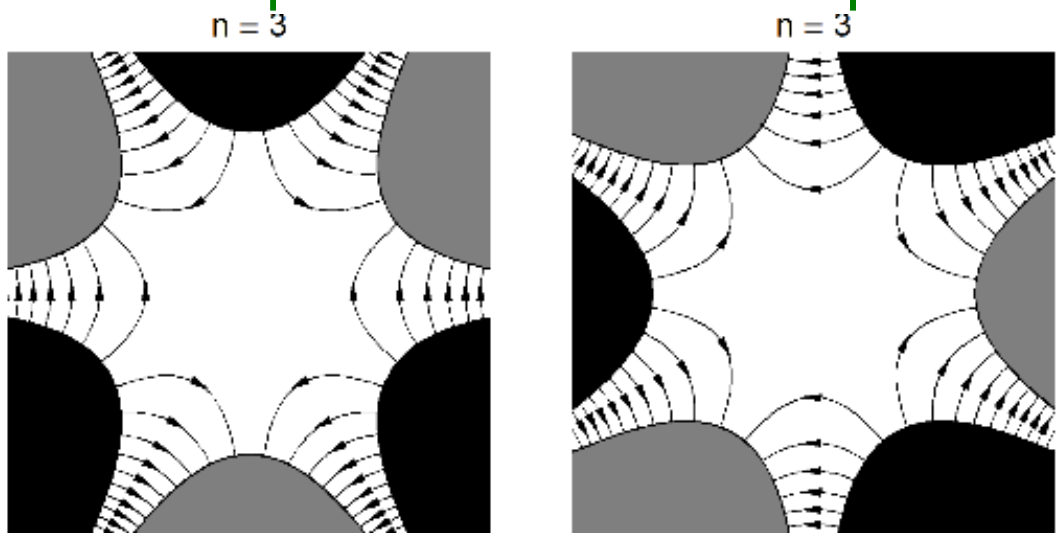
Dipole and “skew” dipole



Quad and skew quad

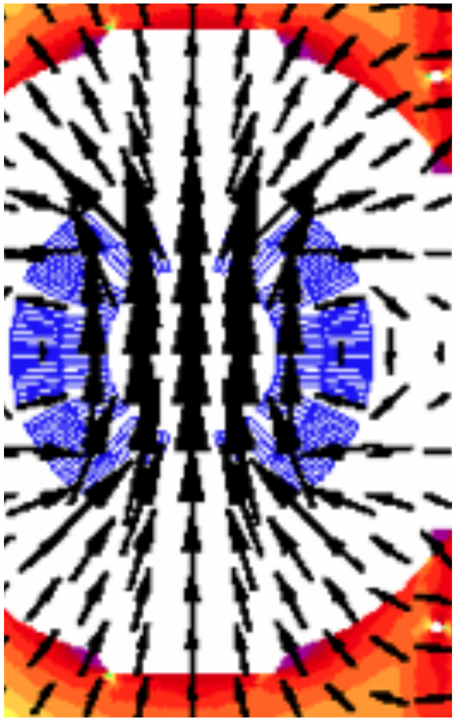


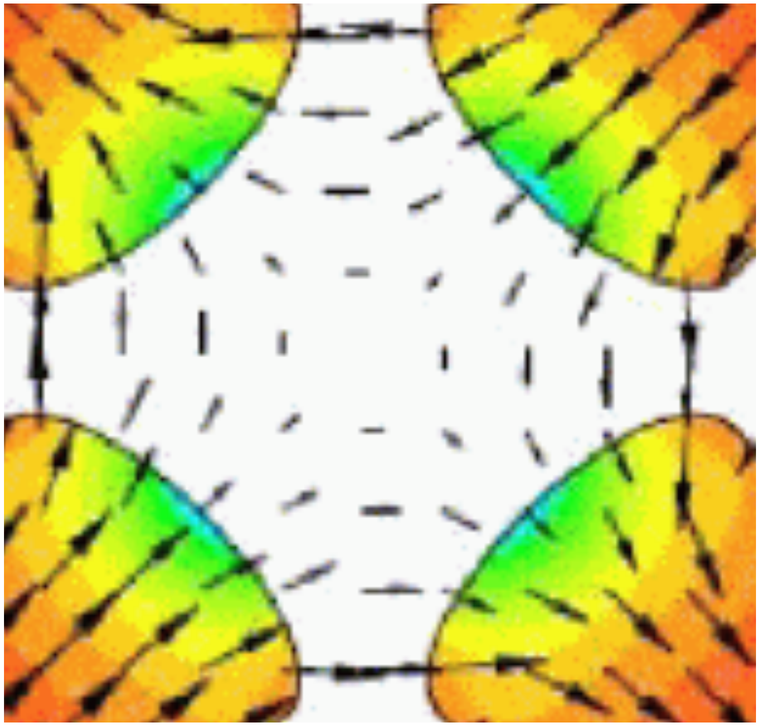
Sextupole and skew sextupole



Visualizing Dipole and Quadrupole Fields II

LHC dipole field





LEP quadrupole field

- LHC dipole: B_y gives horizontal bending
- LEP quadrupole: B_y on x axis, B_x on y axis
  - Horizontal focusing=vertical defocusing or vice-versa
  - No coupling between horizontal/vertical motion
    - Note the nice “harmonic” field symmetries

General Multipole Field Expansions

- Rotational symmetries, cylindrical coordinates
  - Power series in radius r with angular harmonics in θ

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$B_y = B_0 \sum_{n=0}^{\infty} \left(\frac{r}{a}\right)^n (b_n \cos n\theta - a_n \sin n\theta)$$

$$B_x = B_0 \sum_{n=0}^{\infty} \left(\frac{r}{a}\right)^n (a_n \cos n\theta + b_n \sin n\theta)$$

  - Need “reference radius” a (to get units right)
  - (b_n, a_n) are called (normal,skew) **multipole coefficients**
  - We can also write this succinctly using de Moivre as

$$B_x - iB_y = B_0 \sum_{n=0}^{\infty} (a_n - ib_n) \left(\frac{x+iy}{a}\right)^n$$

But Do These Equations Solve Maxwell?

- Yes ☺ Convert Maxwell's eqns to cylindrical coords

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \qquad \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} = 0$$

$$\frac{\partial(\rho B_\rho)}{\partial \rho} + \frac{\partial B_\theta}{\partial \theta} = 0 \qquad \frac{\partial(\rho B_\theta)}{\partial \rho} - \frac{\partial B_\rho}{\partial \theta} = 0$$

  - Aligning r along the x-axis it's easy enough to see

$$\frac{\partial}{\partial x} \Rightarrow \frac{\partial}{\partial r} \qquad \frac{\partial}{\partial y} \Rightarrow \frac{1}{r}\frac{\partial}{\partial \theta}$$

  - In general it's (much, much) more tedious but it works

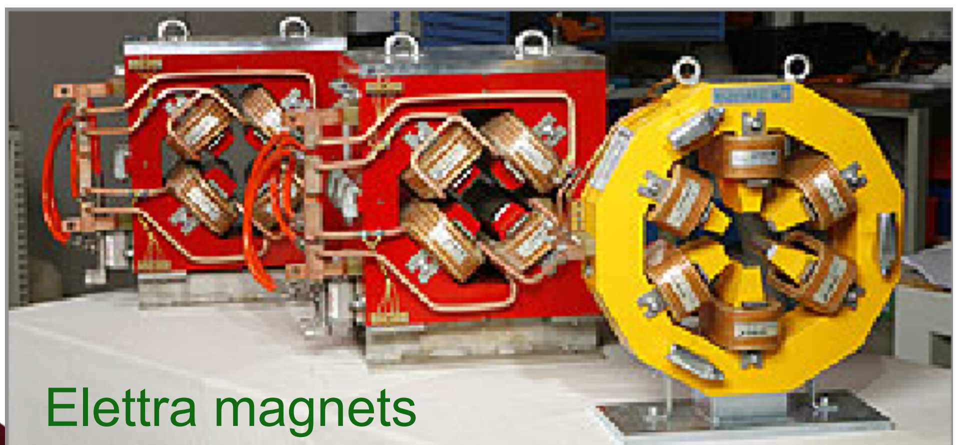
$$\frac{\partial r}{\partial x} = \frac{1}{\cos\theta}, \quad \frac{\partial \theta}{\partial x} = \frac{-1}{r\sin\theta}, \quad \frac{\partial r}{\partial y} = \frac{1}{\sin\theta}, \quad \frac{\partial \theta}{\partial y} = \frac{1}{r\cos\theta}$$

$$\frac{\partial B_x}{\partial x} = \frac{\partial B_x}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial B_x}{\partial \theta}\frac{\partial \theta}{\partial x} \ldots$$

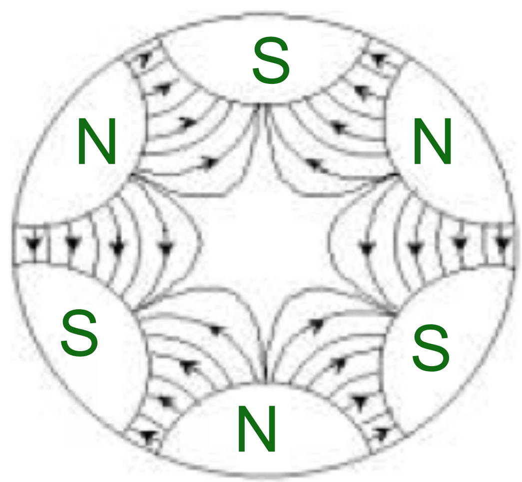
Multipoles

$(b,a)_n$ "unit" is 10^{-4} (natural scale) $(b,a)_n$ (US) = $(b,a)_{n+1}$ (European)!

coefficient	multipole	field	notes
b_0	normal dipole	$B_y = B_0 b_0$	horz. bending
a_0	skew dipole	$B_x = B_0 a_0$	vert. bending
b_1	normal quadrupole	$B_x = B_0 \left(\frac{r}{a}\right) b_1 \sin\theta = B_0 \left(\frac{y}{a}\right) b_1$ $B_y = B_0 \left(\frac{r}{a}\right) b_1 \cos\theta = B_0 \left(\frac{x}{a}\right) b_1$	focusing defocusing
a_1	skew quadrupole	$B_x = B_0 \left(\frac{r}{a}\right) a_1 \cos\theta = B_0 \left(\frac{x}{a}\right) a_1$ $B_y = -B_0 \left(\frac{r}{a}\right) a_1 \sin\theta = -B_0 \left(\frac{y}{a}\right) a_1$	coupling
b_2	normal sextupole	$B_x = B_0 \left(\frac{r}{a}\right)^2 b_2 \sin(2\theta)$ $B_y = B_0 \left(\frac{r}{a}\right)^2 b_1 \cos(2\theta)$	nonlinear!



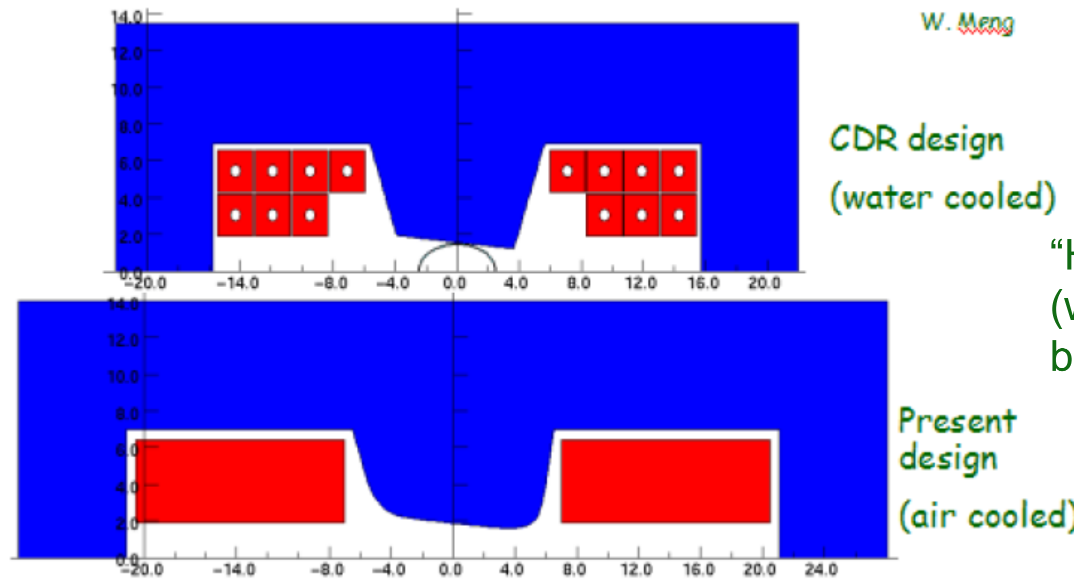
Elettra magnets



Multipole Symmetries

- Dipole has 2π rotation symmetry (or π upon current reversal)
- Quad has π rotation symmetry (or $\pi/2$ upon current reversal)
- k-pole has $2\pi/k$ rotation symmetry upon current reversal
- We try to enforce symmetries in design/construction
  - Limits permissible magnet errors
  - Higher order fields that obey main field symmetry are called allowed multipoles

RCMS half-dipole laminations (W. Meng, BNL)



"H style dipoles" (with focusing like betatron)

Multipole Symmetries II

- So a dipole (n=0, 2 poles) has allowed multipoles:
  - Sextupole (n=2, 6 poles), Decapole (n=4, 10 poles)...
- A quadrupole (n=1, 4 poles) has allowed multipoles:
  - Dodecapole (n=5, 12 poles), Twenty-pole (n=9, 20 poles)…
- General allowed multipoles: (2k+1)(n+1)-1
  - Or, more conceptually, (3,5,7,…) times number of poles
- Other multipoles are forbidden by symmetries
  - Smaller than allowed multipoles, but no magnets are perfect
    - Large measured forbidden multipoles mean fabrication or fundamental design problems!
- Better magnet pole face quality with punched laminations
- Dynamics are usually dominated by lower-order multipoles

Equipotentials and Contours

- Let's get around to building some magnets
  - Conductors on outside, field on inside
  - Use high-permeability iron to shape fields: **iron-dominated**
    - Pole faces are nearly equipotentials, ⊥ B,H field
    - We work with a magnetostatic scaler potential Ψ
    - B,H field lines are ⊥ to equipotential lines of Ψ

$$\vec{H} = \vec{\nabla}\Psi$$

$$\Psi = \sum_{n=0}^{\infty} \frac{a}{n+1} \left(\frac{r}{a}\right)^{n+1} [F_n \cos((n+1)\theta) + G_n \sin((n+1)\theta)]$$

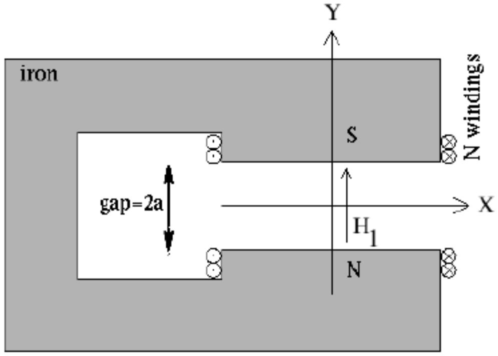
$$\text{where } G_n \equiv B_0 b_n/\mu_0, F_n \equiv B_0 a_n/\mu_0$$

This comes from integrating our B field expansion.
Let's look at normal multipoles G_n and pole faces…

Equipotentials and Contours II

- For general G_n normal multipoles (i.e. for b_n)

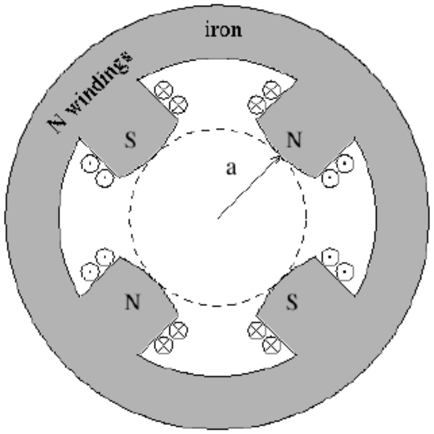
$$\Psi(\text{equipotential for } b_n) \propto r^{n+1} \sin[(n+1)\theta] = \text{constant}$$



- Dipole (n=0): $\Psi(\text{dipole}) \propto r \sin\theta = y$
  - Normal dipole pole faces are y=constant
- Quadrupole (n=1):

$$\Psi(\text{quadrupole}) \propto r^2 \sin(2\theta) = 2xy$$

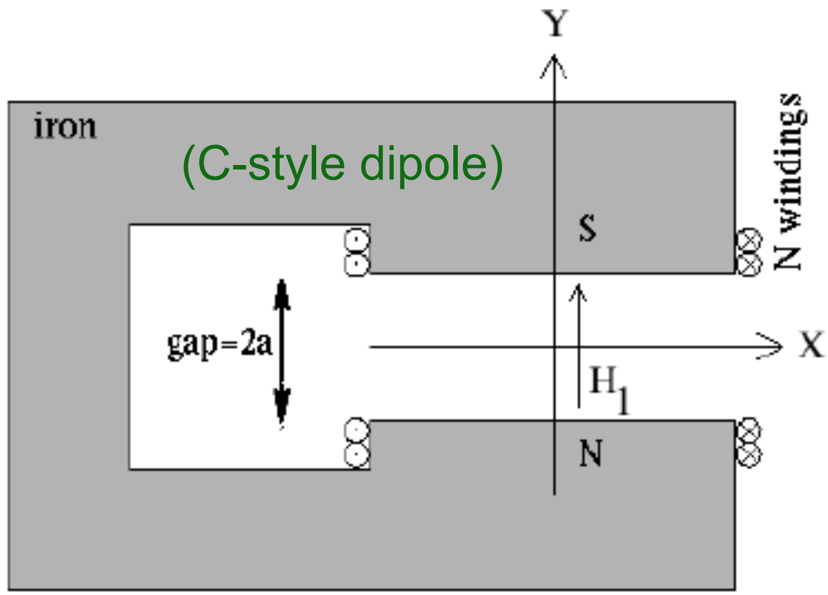
  - Normal quadrupole pole faces are xy=constant (hyperbolic)



- So what conductors and currents are needed to generate these fields?

Dipole Field/Current

- Use Ampere's law to calculate field in gap
  - N "turns" of conductor around each pole
  - Each conductor carries current I



- Field integral is through N-S poles and (highly permeable) iron (including return path)

$$2NI = \oint \vec{H} \cdot d\vec{l} = 2aH \quad \Rightarrow \quad H = \frac{NI}{a}, \; B = \frac{\mu_0 NI}{a}$$

- NI is in "Amp-turns", μ_0=1.2 cm-G/A
  - So a=2cm, B=600 G requires NI~1000 Amp-turns

$$\Delta x' = \frac{BL}{(B\rho)}$$

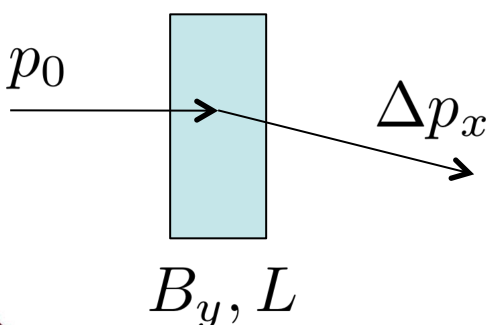
Wait, What's That Δx' Equation?

$$\Delta x' = \frac{BL}{(B\rho)}$$

← Field, length: Properties of **magnet**

← **Rigidity**: property of **beam** (really p/q!)

- This is the angular transverse kick from a thin dipole, like a dipole corrector
  - Really a change in p_x but paraxial approximation applies
  - The B in (Bρ) is not necessarily the main dipole B
  - The ρ in (Bρ) is not necessarily the ring circumference/2π
  - And neither is related to this particular dipole kick!

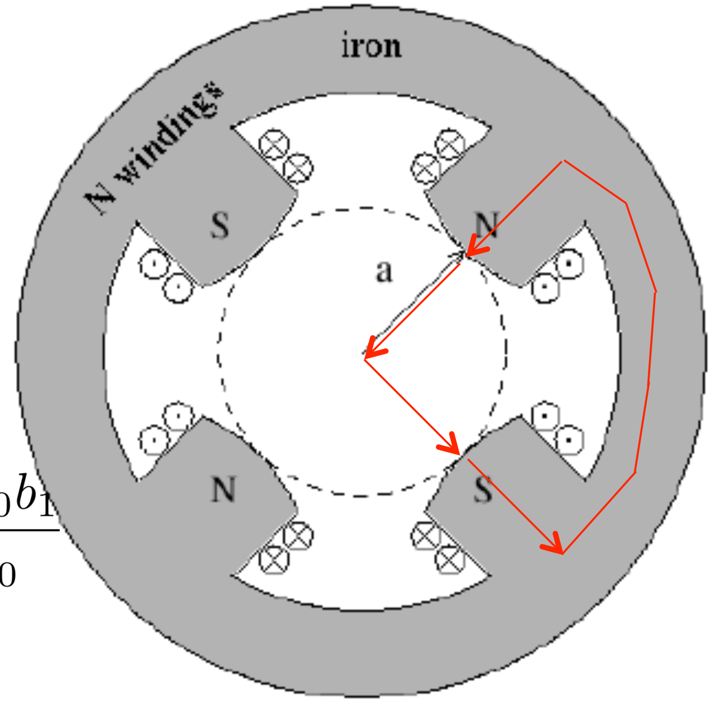


$$F_x = \frac{\Delta p_x}{\Delta t} = q(\beta c) B_y \quad \Delta t = L/(\beta c)$$

$$\Delta p_x = qLB_y$$

$$\Delta x' \approx \frac{\Delta p_x}{p} = \frac{q}{p} LB_y = \frac{B_y L}{(B\rho)}$$

Quadrupole Field/Current



- Use Ampere's law again
  - Easiest to do with magnetic potential Ψ, encloses 2NI

$$\Psi(a,\theta) = \frac{a}{2}\frac{B_0 b_1}{\mu_0}\sin(2\theta)$$

$$2NI = \oint \vec{H}\cdot d\vec{l} = \Psi(a,\pi/4) - \Psi(a,-\pi/4) = \frac{aB_0 b_1}{\mu_0}$$

$$\Psi = NI\sin(2\theta) = \frac{2NI}{a^2}xy$$

$$\vec{H} = \nabla\Psi = \frac{2NI}{a^2}(y\hat{x} + x\hat{y}) \qquad \vec{B} = \frac{2\mu_0 NI}{a^2}(y\hat{x} + x\hat{y})$$

- Quad strengths are expressed as gradients

$$B' \equiv \frac{\partial B_y}{\partial x}\bigg|_{y=0} = \frac{\partial B_x}{\partial y} = \frac{2\mu_0 NI}{a^2} \qquad \Delta x' = \frac{B'L}{(B\rho)}x$$

Higher Orders

- We can follow the full expansion for 2(n+1)-pole:

$$\Psi_n = NI \left(\frac{r}{a}\right)^{n+1} \sin((n+1)\theta)$$

$$H_x = (n+1)\frac{NI}{a}\left(\frac{r}{a}\right)^n \sin n\theta \qquad H_y = (n+1)\frac{NI}{a}\left(\frac{r}{a}\right)^n \cos n\theta$$

- For the sextupole (n=2) we find the nonlinear field as

$$\vec{B} = \frac{3\mu_0 NI}{a^3}[2xy\hat{x} + (x^2+y^2)\hat{y}]$$

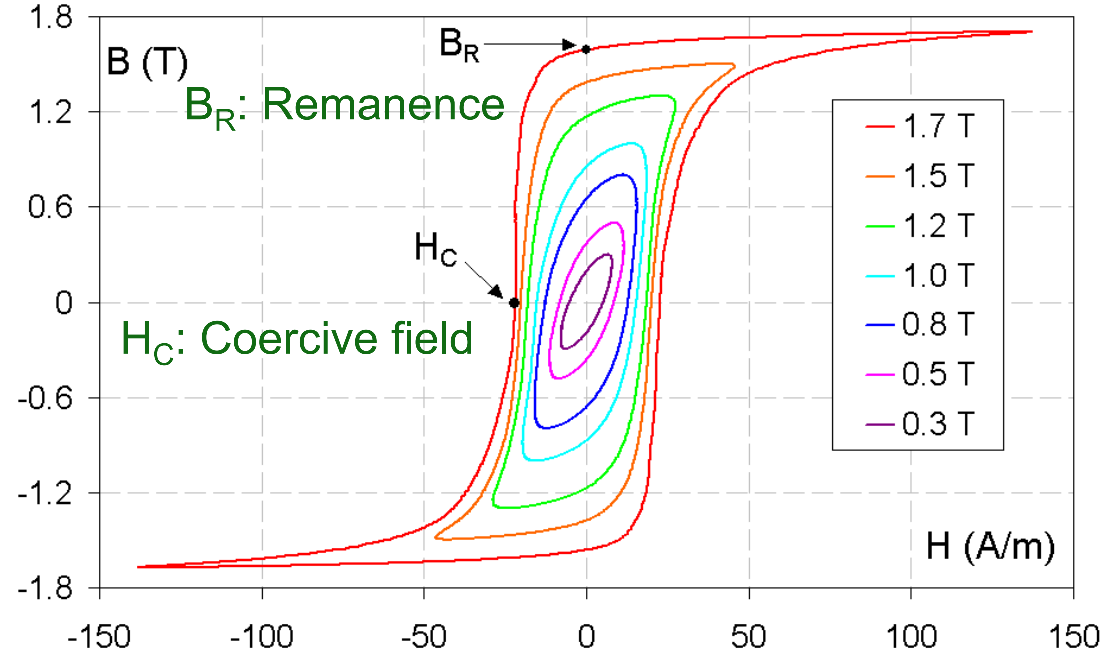
- Now define a strength as an n^{th} derivative

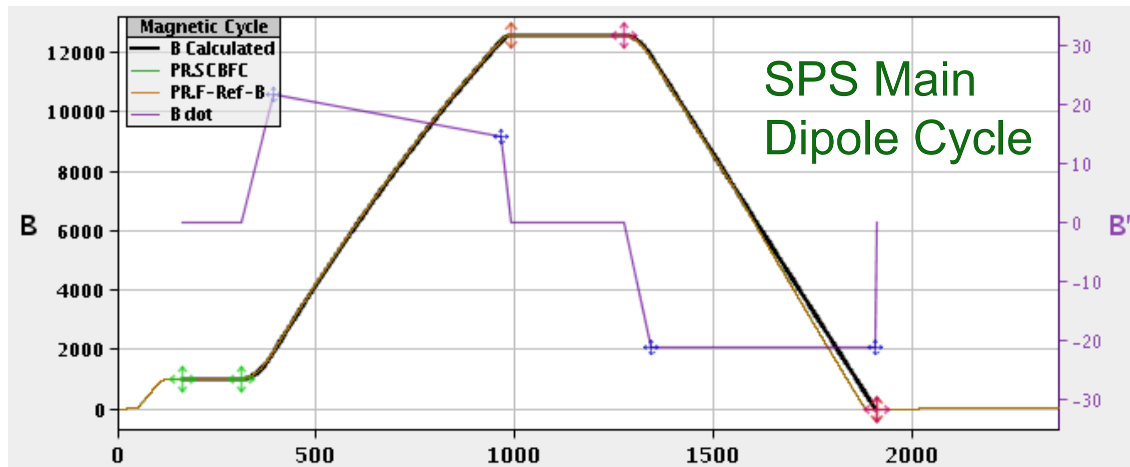
$$B'' \equiv \frac{\partial^2 B_y}{\partial x^2}\Big|_{y=0} = \frac{6\mu_0 NI}{a^3} \qquad \Delta x' = \frac{1}{2}\frac{B''L}{(B\rho)}(x^2+y^2)$$

Hysteresis

- Magnets that are ramped carry "memory"
  - Hysteresis is quite important in iron-dominated magnets
- Usually try to run magnets "on hysteresis"
  - e.g. always rising on hysteresis loop
  - Large spread at large field (1.7 T): saturation

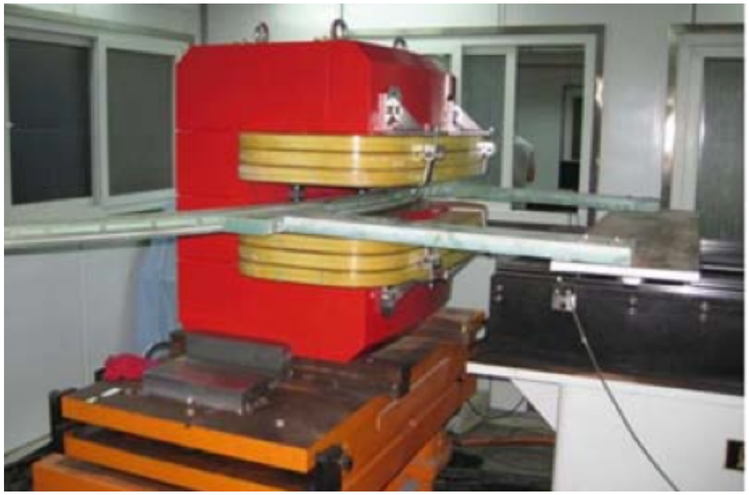
Degaussing





End Fields

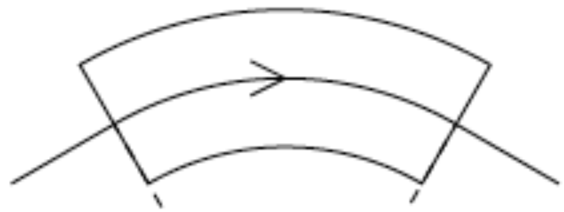
- Magnets are not infinitely long: ends are important!
  - Conductors: where coils usually come in and turn around
  - Longitudinal symmetries break down
  - Sharp corners on iron are first areas to saturate
  - Usually a concern over distances of ±1-2 x magnet gap
    - A big deal for short, large-aperture magnets; ends dominate!
- Solution: simulate… a lot
  - Test prototypes too
  - Quadratic end chamfer eases sextupoles from ends (first allowed harmonic of dipole)
- More on dipole end focusing…



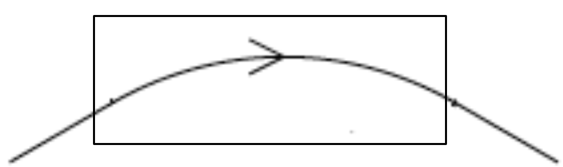
PEFP prototype magnet (Korea)
9 cm gap,1.4m long

Dipoles: Sector and Rectangular Bends

- Sector bend (sbend)
  - Beam design entry/exit angles are ⊥ to end faces

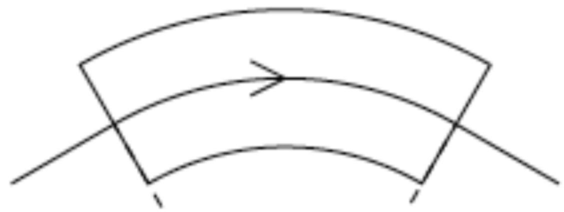


  - Simpler to conceptualize, but harder to build
- Rectangular bend (rbend)
  - Beam design entry/exit angles are half of bend angle



  - Easier to build, but must include effects of edge focusing

Sector Bend Transport Matrix

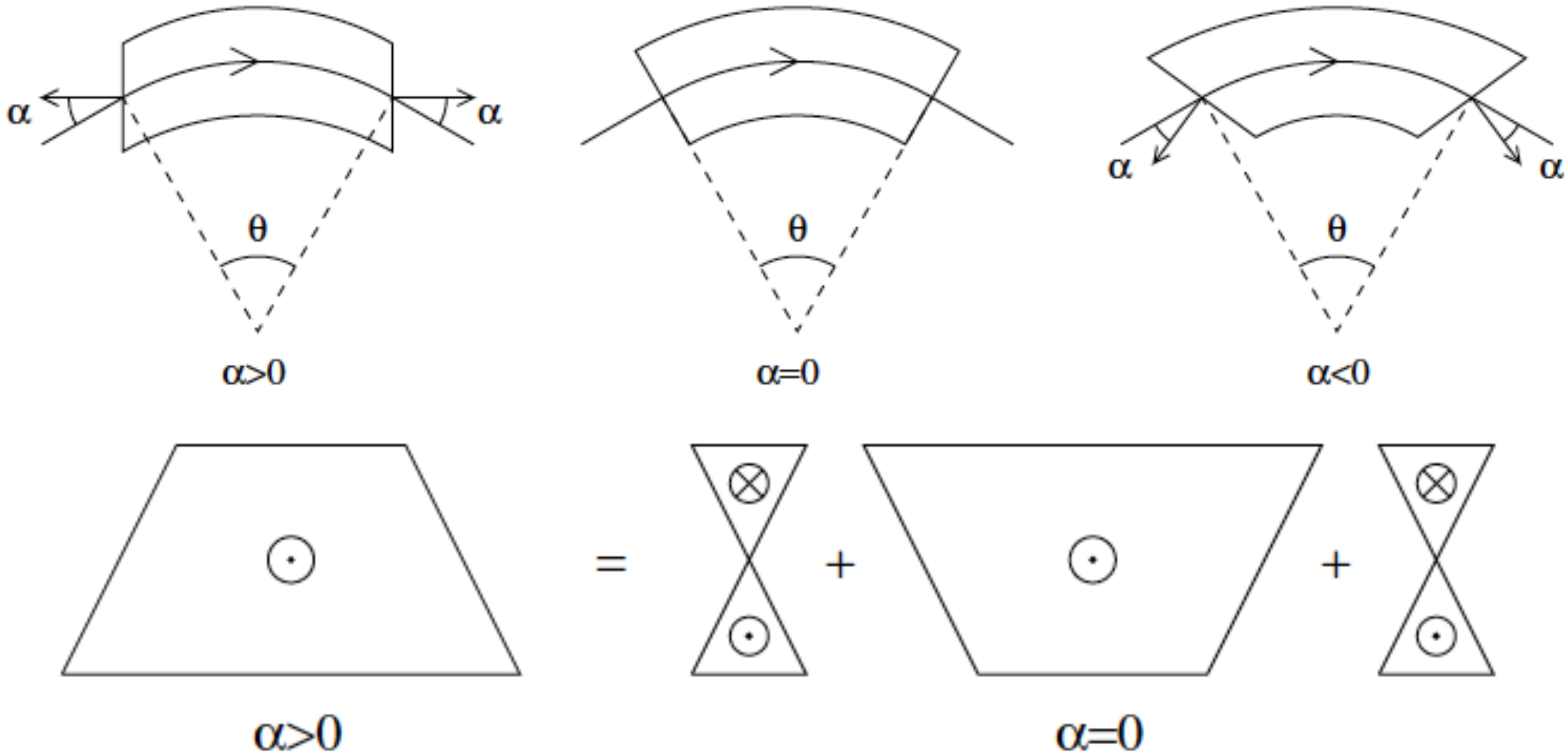


- Eqn 3.102 of Conte and MacKay: (x,x',y,y',z,δ)

$$M_{\text{sector dipole}} = \begin{pmatrix} \cos\theta & \rho\sin\theta & 0 & 0 & 0 & \rho(1-\cos\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & 0 & 0 & \sin\theta & \\ 0 & 0 & 1 & \rho\theta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\sin\theta & -\rho(1-\cos(\theta)) & 0 & 0 & 1 & -\rho(\theta-\sin\theta) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- Has all the "right" behaviors
  - But what about "rectangular" bends?

Dipole End Angles



- We treat general case of symmetric dipole end angles
  - Superposition: looks like wedges on end of sector dipole
  - Rectangular bends are a special case

Kick from a Thin Wedge

- The edge focusing calculation requires the kick from a thin wedge

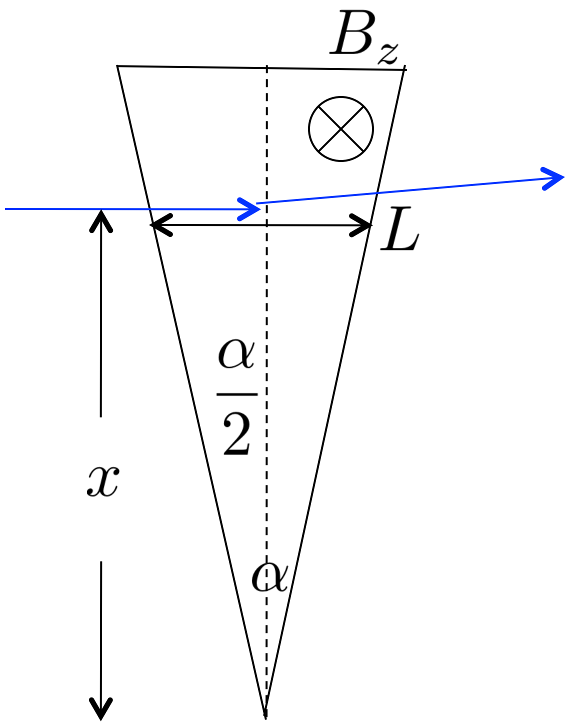
$$\Delta x' = \frac{B_z L}{(B\rho)}$$

What is L? (distance in wedge)

$$\tan\left(\frac{\alpha}{2}\right) = \frac{L/2}{x}$$

$$L = 2x\tan\left(\frac{\alpha}{2}\right) \approx x\tan\alpha$$

$$\text{So}\ \ \Delta x' = \frac{B_z \tan\alpha}{(B\rho)} x = \frac{\tan\alpha}{\rho} x$$



Here ρ is the curvature for a particle of this momentum!!

Dipole Matrix with Ends

- The matrix of a dipole with thick ends is then

$$M_{\text{sector dipole}}(x, x', \delta) = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{pmatrix}$$

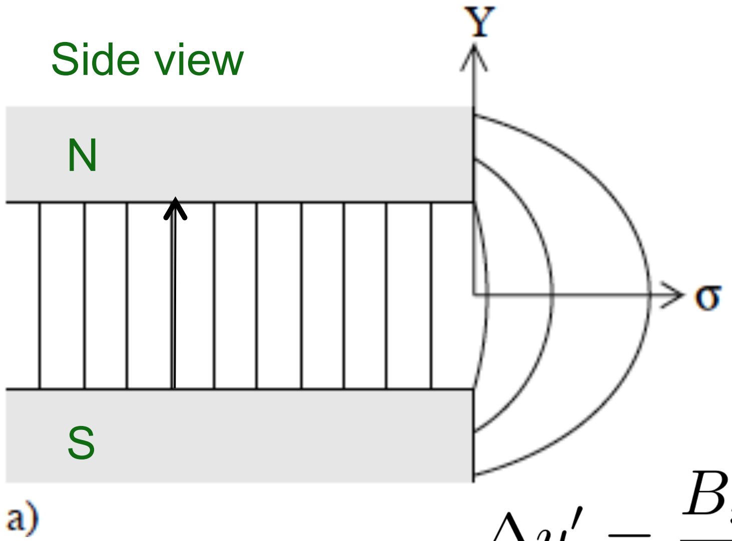
$$M_{\text{end lens}} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{\tan\alpha}{\rho} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

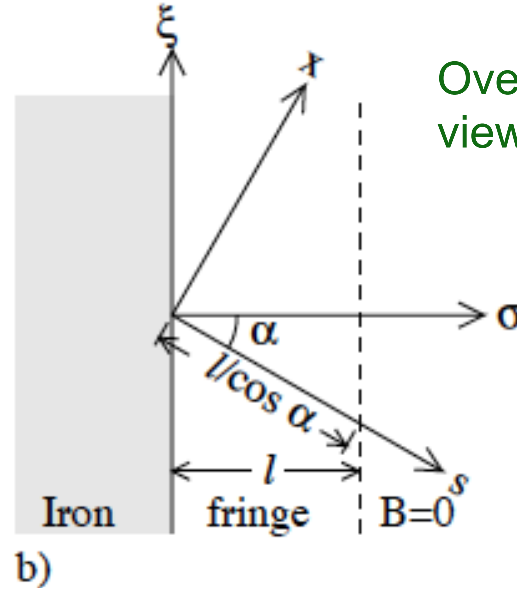
$$M_{\text{edge}-\text{focused dipole}} = M_{\text{end lens}} M_{\text{sector dipole}} M_{\text{end lens}}$$

$$M_{\text{edge}-\text{focused dipole}} = \begin{pmatrix} \frac{\cos(\theta-\alpha)}{\cos\alpha} & \rho\sin\theta & \rho(1-\cos\theta) \\ -\frac{\sin(\theta-2\alpha)}{\rho\cos^2\alpha} & \frac{\cos(\theta-\alpha)}{\cos\alpha} & \frac{\sin(\theta-\alpha)+\sin\alpha}{\cos\alpha} \\ 0 & 0 & 1 \end{pmatrix}$$

- Rectangular bend is special case where $\alpha=\theta/2$

End Field Example (from book)

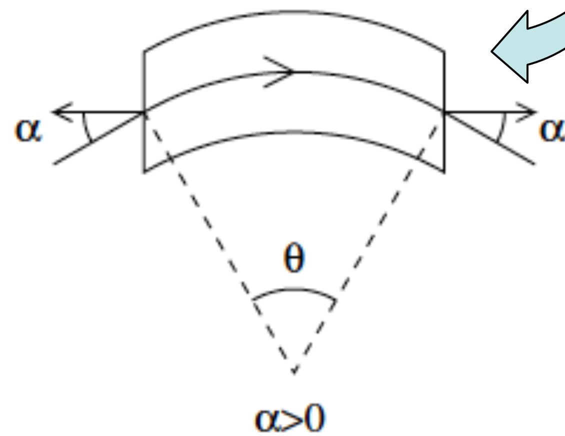




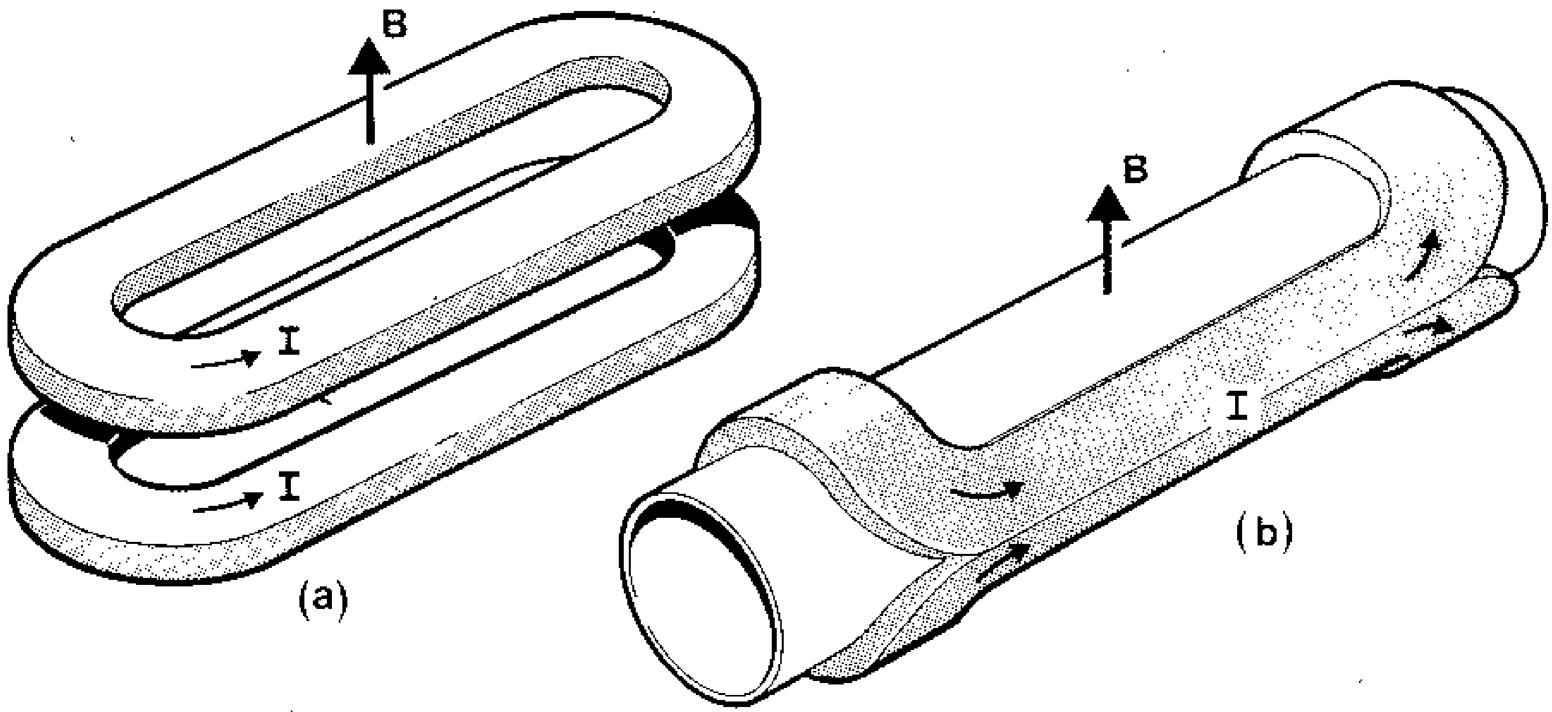
Overhead view, $\alpha>0$

$$\Delta y' = \frac{B_x l_{\text{fringe}}}{(B\rho)}$$

- p. 85 of text
- Field lines go from –y to +y for a positively charged particle
  - B_x <0 for y>0; B_x>0 for y<0
    - Net focusing!
  - Field goes like sin(α)
    - get cos(α) from integral length

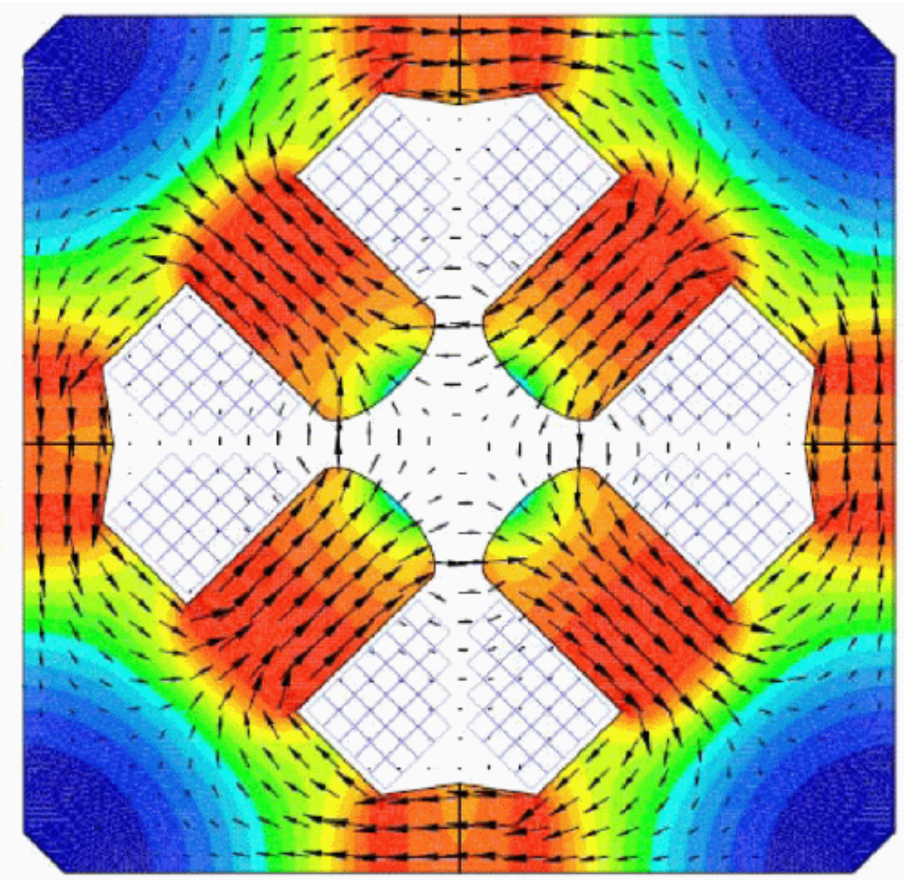


Other Familiar Dipoles

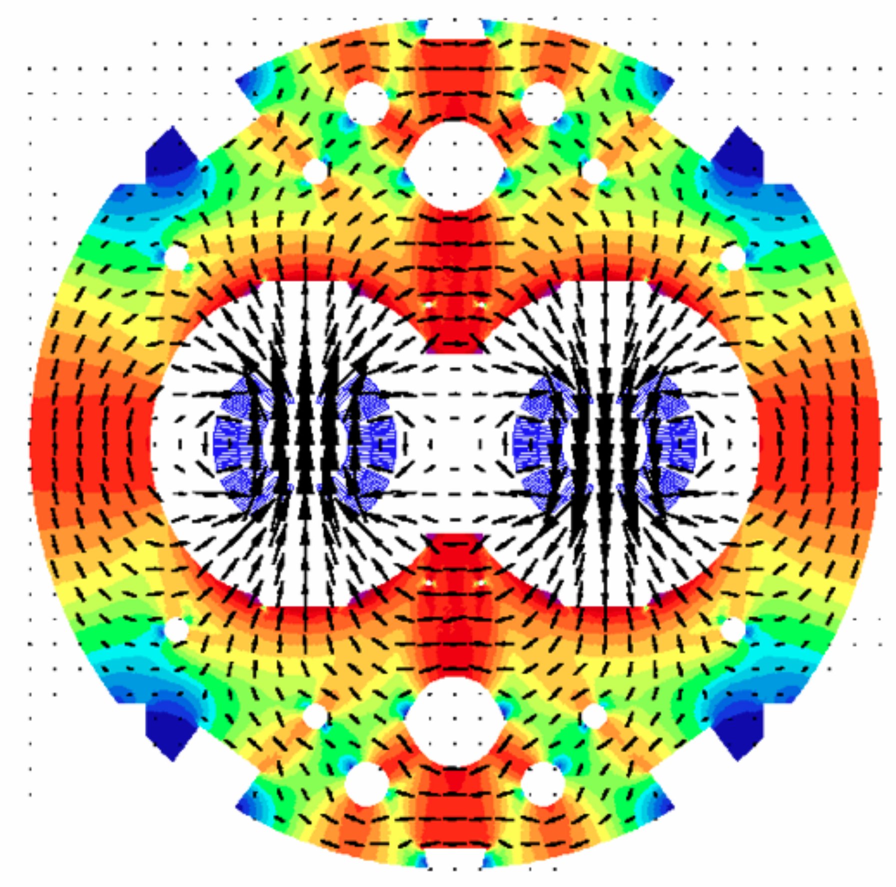


- Weaker, cheaper dipoles can be made by conforming coils to a beam-pipe (no iron)
- Very inexpensive, but you get what you pay for
  - Field quality on the order of percent

Normal vs Superconducting Magnets

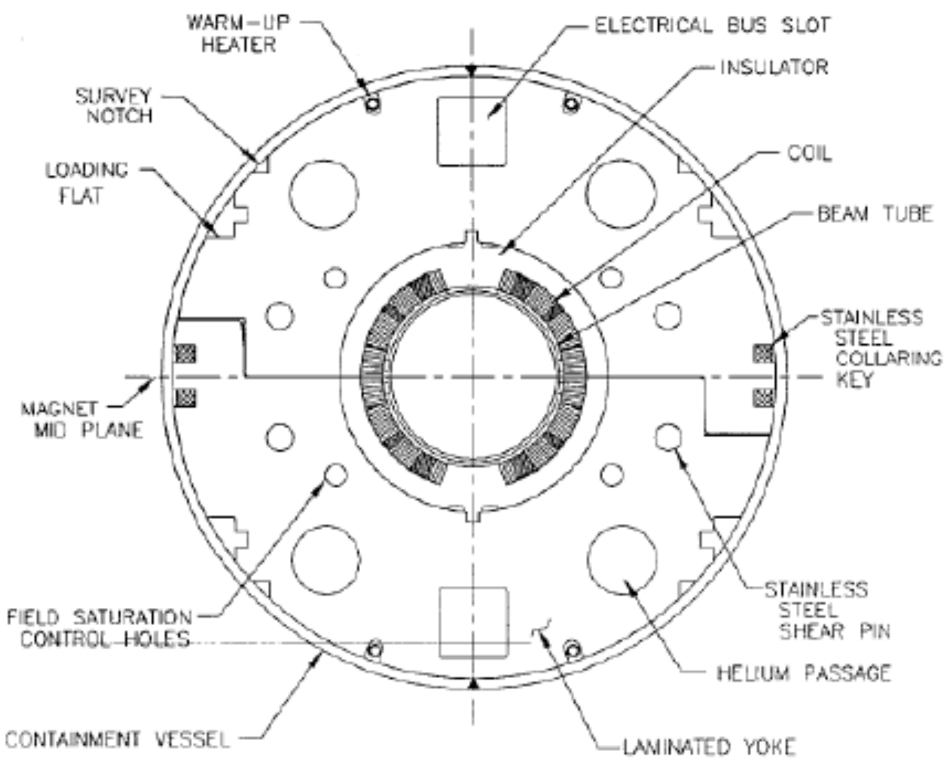


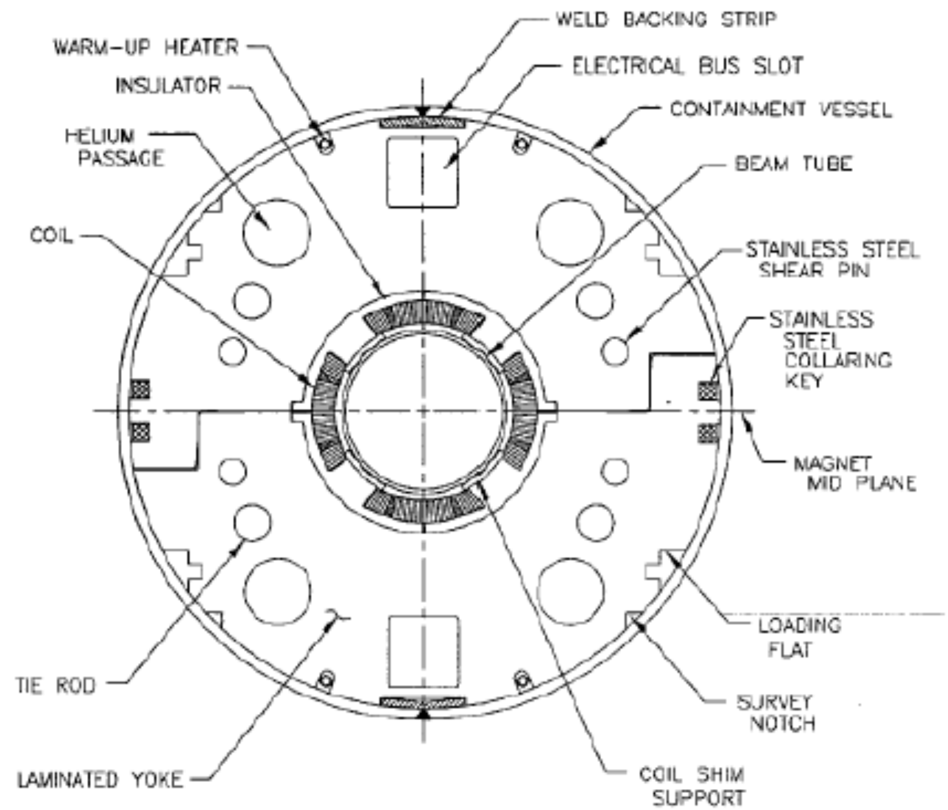
LEP quadrupole magnet (NC)



LHC dipole magnets (SC)

RHIC Dipole/Quadrupole Cross Sections

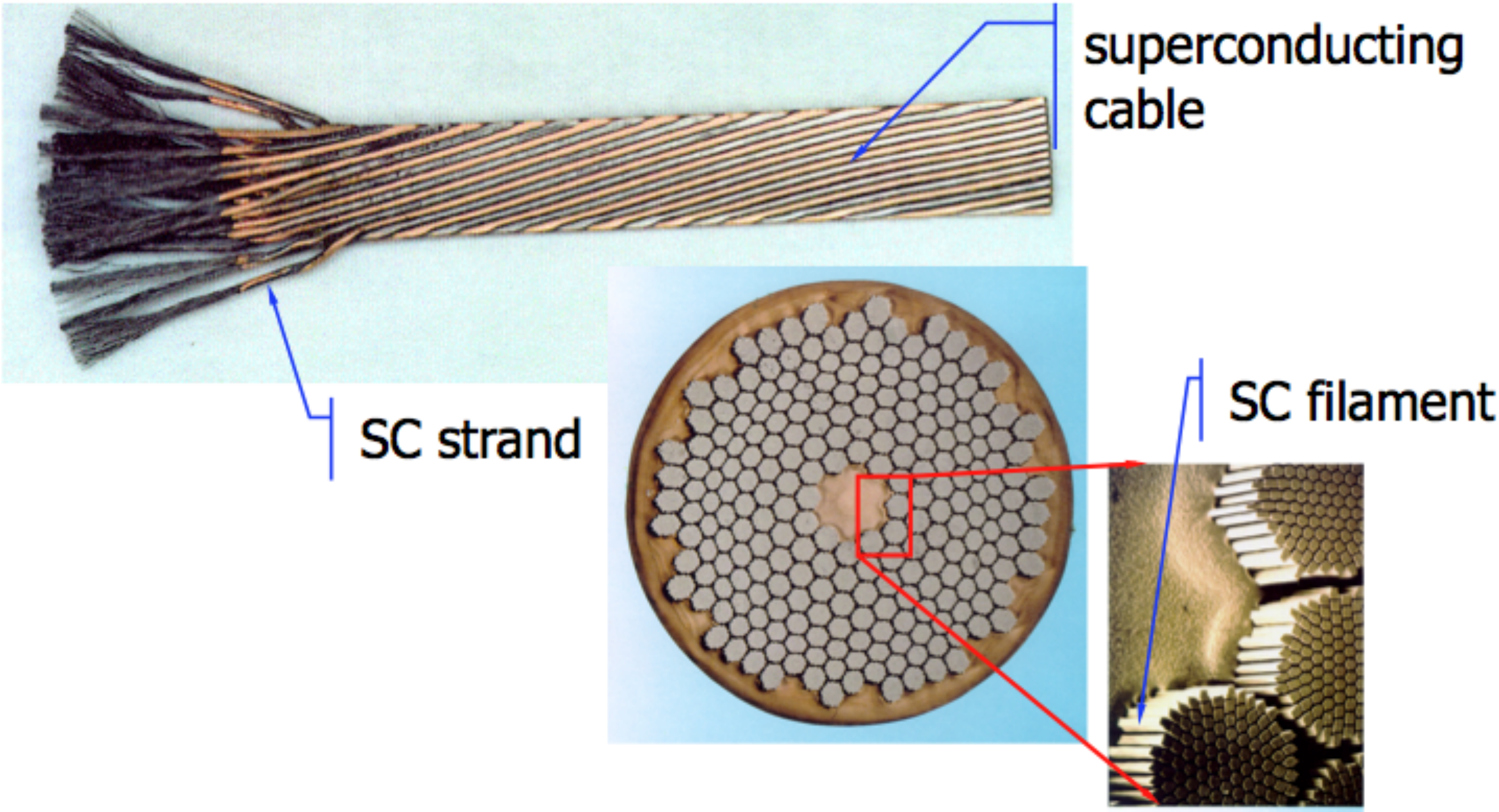




RHIC cos(θ)-style superconducting magnets and yokes

- NbTi in Cu stabilizer, iron yokes, saturation holes
- Full field design strength is up to 20 Mpa (3 kpsi)

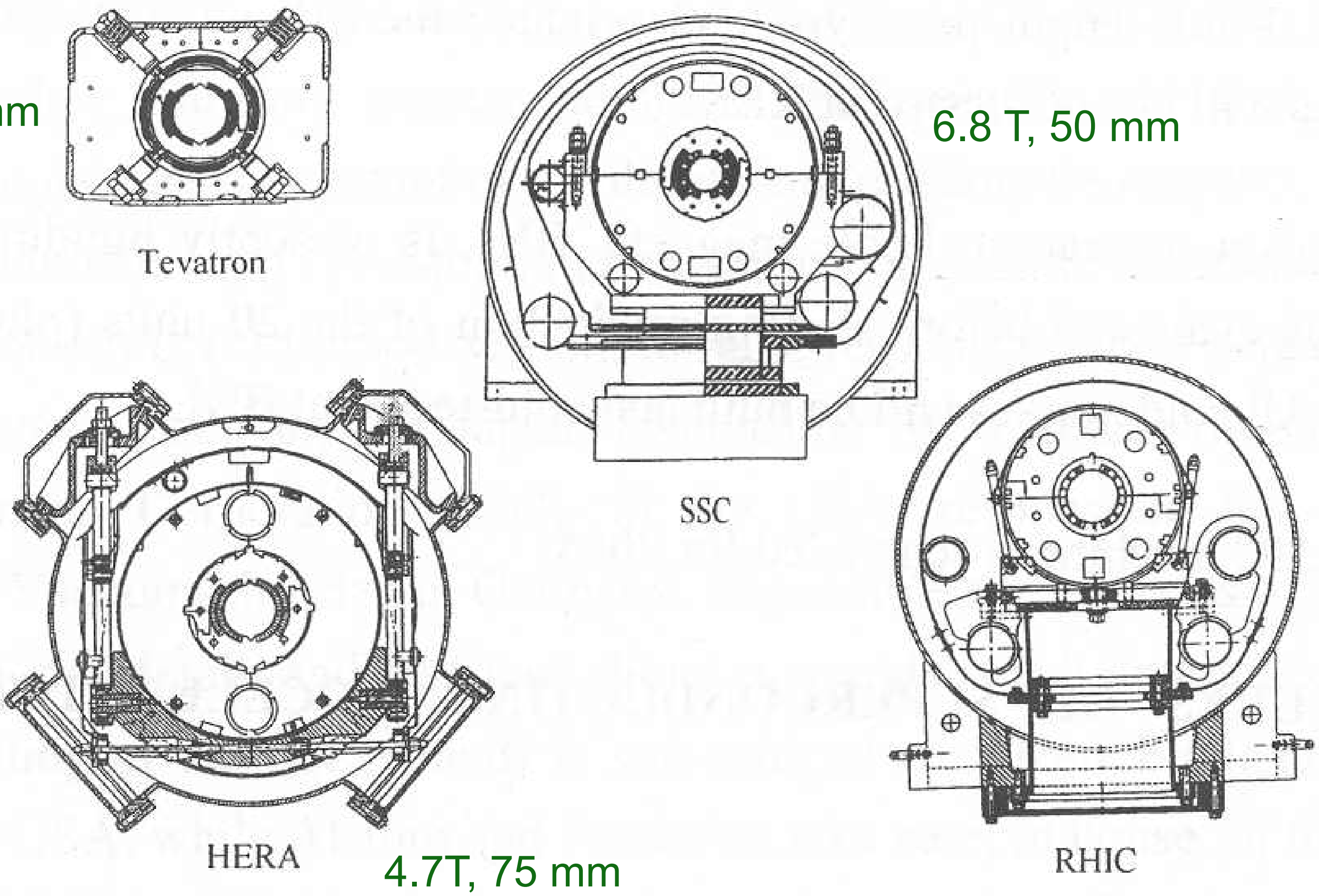
Rutherford Cable



SC Dipole Magnet Comparison

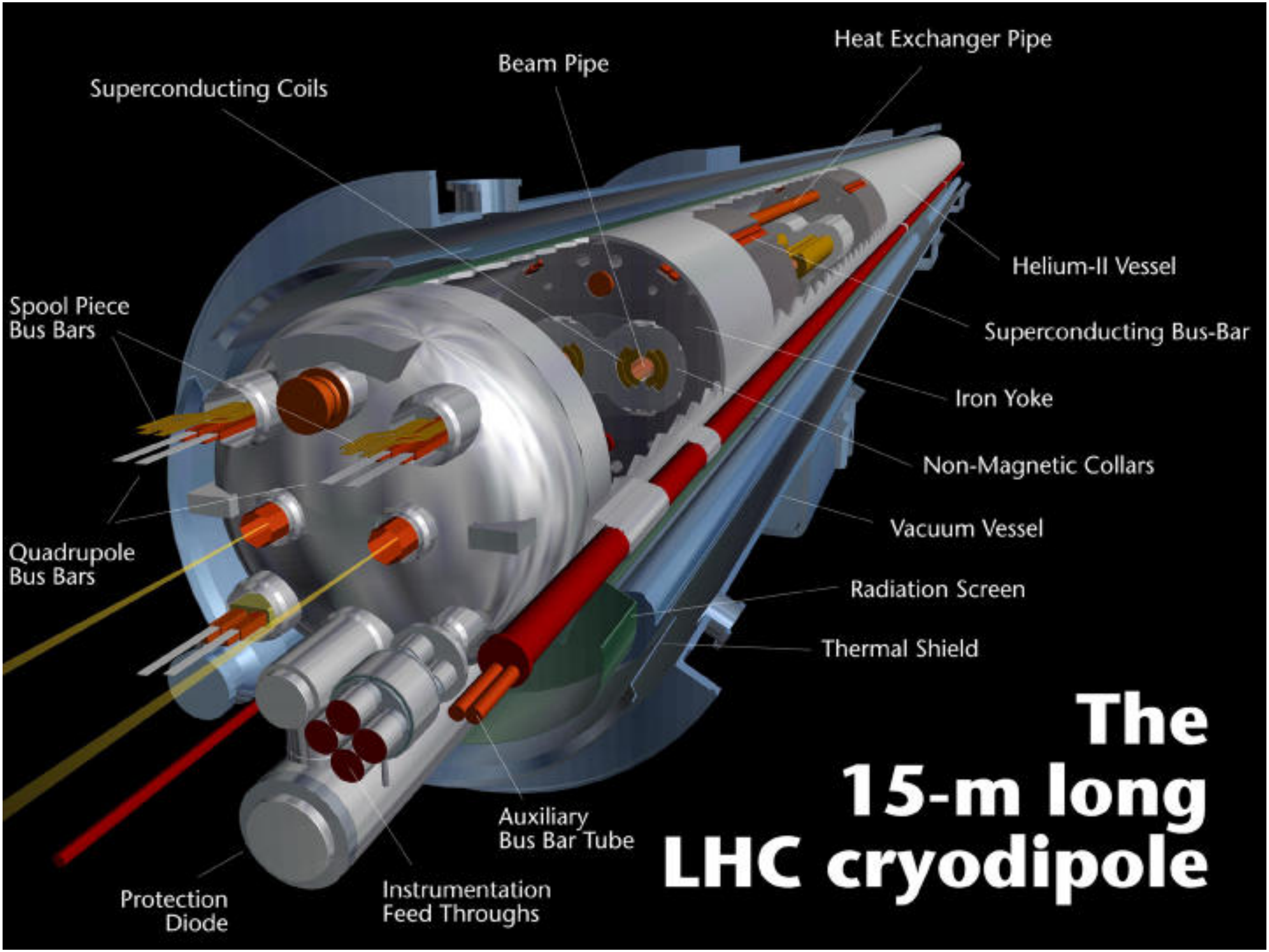
4T, 90mm

6.8 T, 50 mm



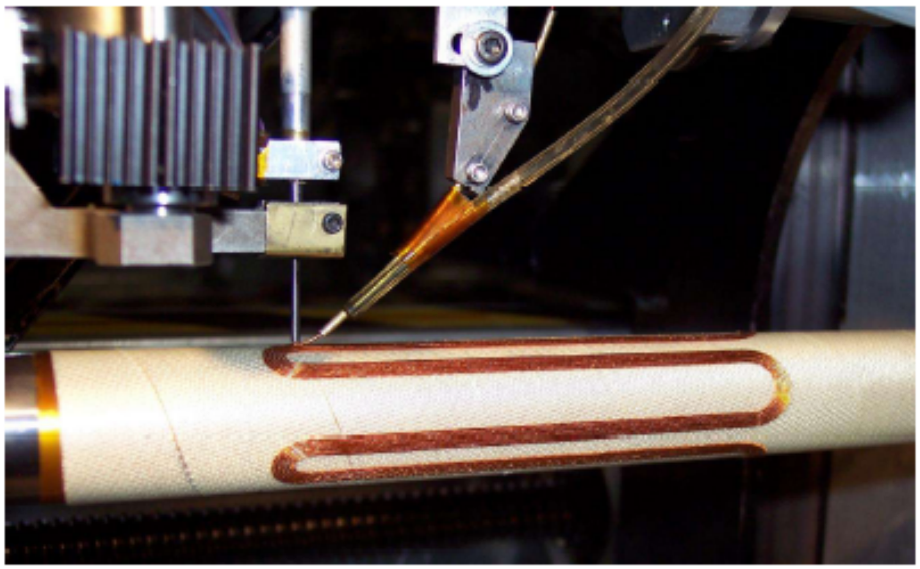
4.7T, 75 mm

3.4T, 80mm

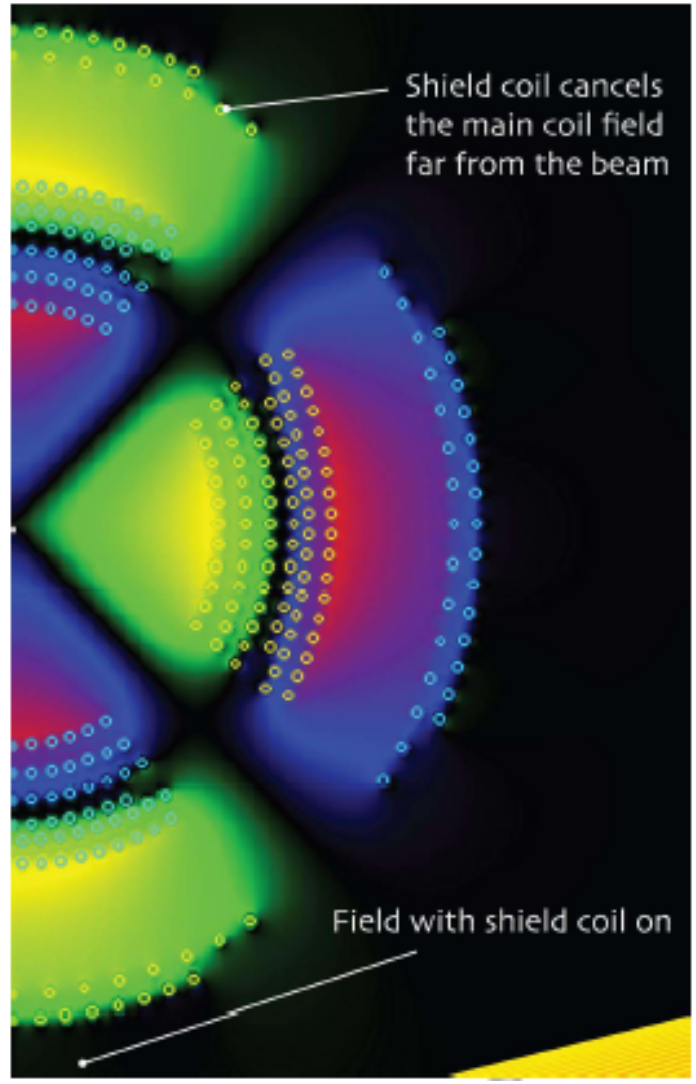

Heat Exchanger Pipe
Beam Pipe
Superconducting Coils
Helium-II Vessel
Spool Piece
Bus Bars
Superconducting Bus-Bar
Iron Yoke
Non-Magnetic Collars
Quadrupole
Bus Bars
Vacuum Vessel
Radiation Screen
Thermal Shield
Auxiliary
Bus Bar Tube
Protection
Diode
Instrumentation
Feed Throughs
The
15-m long
LHC cryodipole

Direct-Wind Superconducting Magnets (BNL)

- 6T Iron-free (superconducting)
- Solid state coolers (no Helium)
- Field containment (LC magnet)
- “Direct-wind” construction

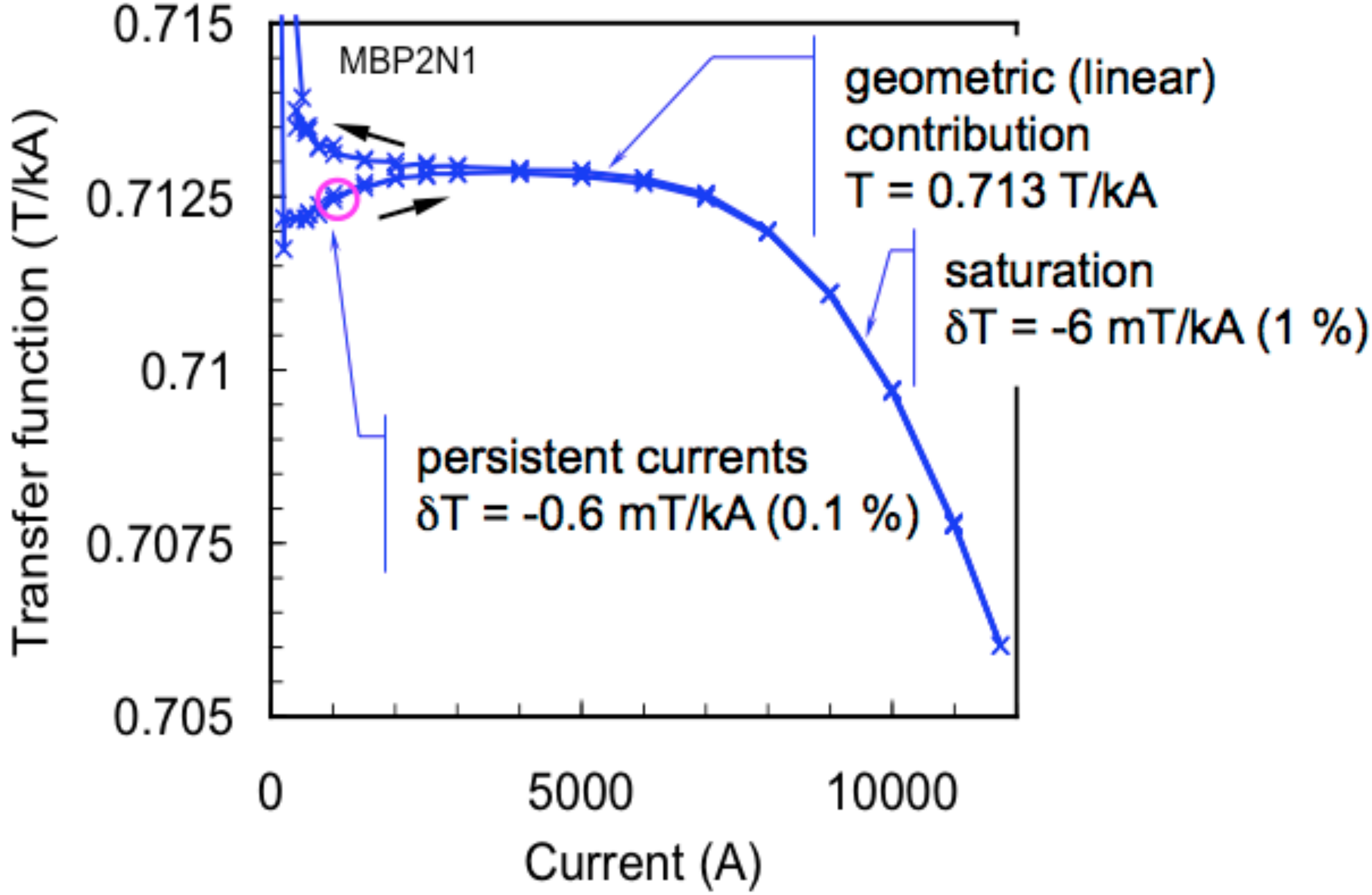


World’s first “direct wind” coil machine at BNL



Linear Collider magnet

SC Magnet Transfer Function



- Transfer function: relationship between current/field

Luca Bottura, CERN