USPAS Graduate Accelerator Physics Homework 1

Due date: Tuesday June 14, 2011

1 RHIC energy and current

Gold ions $^{197}Au^{+77}$ (A=197, Z=77) are injected into the Brookhaven Alternating Gradient Synchrotron (AGS) with a kinetic energy of 1.03 GeV/nucleon. We are using a magnet cycle the same magnet cycle that would extract protons at a kinetic energy of 22.9 GeV for use in the Relativistic Heavy Ion Collider (RHIC). The circumference of the AGS is 807.1 m, and the rest mass of a gold ($^{197}Au^{+77}$) ion is 183.434 GeV/c^2.

(a) (4 points) What is the velocity of the injected gold ions?

(b) (4 points) What is the corresponding kinetic energy for $^{197}Au^{+77}$ ions extracted from the AGS for RHIC using this magnet cycle?

(c) (2 points) Why does the beam current increase even though the circulating charge stays constant during acceleration?

2 Basic collision kinematics (Conte-Mackay 1-4 and then some)

(a) (3 points) Show that the total energy for a head-on collision of two particles, each with center of mass energy $\gamma_{\rm cm}mc^2$, is equal to the total energy of a fixed-target collision, where one particle is at rest and the other has energy $\gamma_{\rm fixed}mc^2$, where

$$\gamma_{\rm fixed} = 2\gamma_{\rm cm}^2 - 1 \tag{2.1}$$

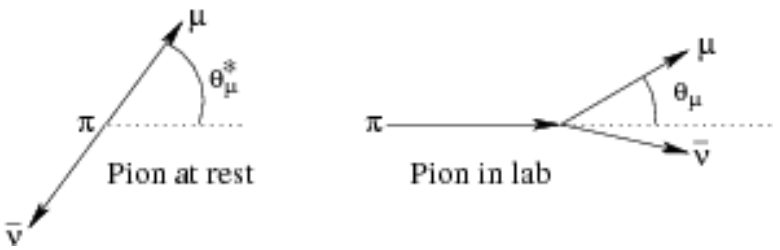
Consider a charged pion decaying into a muon plus an antineutrino:

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \tag{2.2}$$

Use $M_{\pi^\pm} = 140{\rm MeV/c}^2$, $m_\mu = 106{\rm MeV/c}^2$, and $m_{\bar{\nu}} = 0$.

(b) (3 points) In the rest system of the pion, what are the energies and momenta of the muon and antineutrino?

(c) (4 points) For a moving pion with total energy $U_\pi = \gamma M_\pi c^2$ find an expression for the direction, θ_μ of the muon relative to the pion in the lab in terms of the angle θ_μ^* in the in the pion's rest system.



3 Lithium lens (Conte-Mackay 4-1; yes, you can do it)

(10 points) A Lithium lens of length l and radius a has a current I flowing through its end caps with uniform current density as pictured in Fig. 1. Consider a beam of antiprotons with momentum p. What is the focal length of this lens for the antiprotons? Does the Lithium lens current need to flow from left to right or right to left for the lens to focus antiprotons? Recall that the focal length for a focusing lens is defined as the distance at which incoming paraxial rays converge.

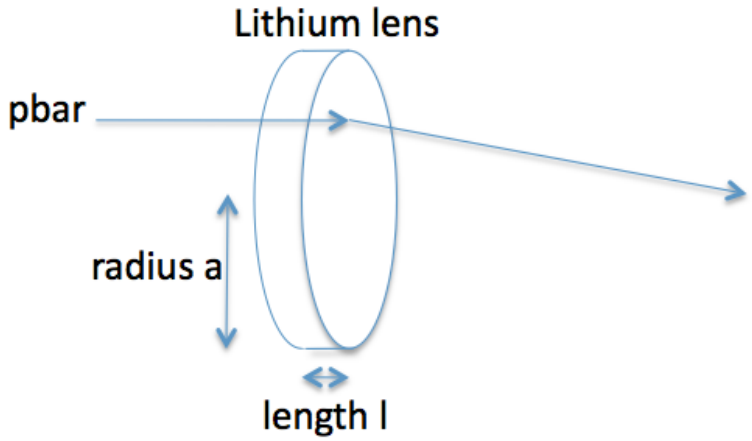


Figure 1: Lithium lens diagram. A uniform current I is applied through the end caps of the Lithium cylinder to create a focusing lens.