USPAS Graduate Accelerator Physics Homework 8

Due date: Thursday June 23, 2011

1 Dodecapole Resonance Driving

For a one-dimensional single dodecapole kick Hamiltonian,

$$H = 2\pi Q_x J_x + V_6(x) \tag{1.1}$$

where the perturbative nonlinear term is related to a dodecapole strength $\tilde{b}_5$ by

$$V_6 = \tilde{b}_5 x^6 \tag{1.2}$$

The derivative of V_6 in Hamilton's equations produces a kick that is proportional to x^5.

(a) The Hamiltonian is in "mixed" coordinates since V is written as a function of x. Using Eq. (3.6) from the handout, convert V to action-angle coordinates (ϕ_x, J_x).

(b) Expand the cosine term that you found in (a) to terms that depend on trigonometric functions of $\cos(k\phi_x(s))$. For example, there will be a term that depends on $\cos(6\phi_x(s))$.

(c) You should have found four terms. Which one-dimensional resonances do they drive? What is the physical meaning of the term that does not depend on $\phi_x(s)$?

2 Octupoles and Fourth Order Resonances

Consider an otherwise linear lattice with a single octupolar nonlinearity:

$$H = Q_x J_x + Q_y J_y + V_4(x, y; s) \tag{2.1}$$

where $V_4(x, y; s) = \frac{1}{24} O(s)(x^4 - 4x^2y^2 + y^4)$ and $O(s) = -b_3(s)/B\rho$.

(a) Expand V_4 in action-angle coordinates. What resonances does the octupole drive?

(b) Expand into Fourier harmonics and assume that the tune is near the $4Q_x = l$ resonance, $Q_x = l/4 + \delta Q$ where $\delta Q \ll 1$. Find the fixed points of this resonance where $\Delta J_x = \Delta\phi_x = 0$ as functions of the octupole strength $O(s)$ and δQ.

(c) **Bonus:** Which of these fixed points are elliptical (locally stable) and which are hyperbolic (locally unstable)?

3 Beam-beam for Unequal Species

Modify the equation for the beam-beam tune shift, Eq. (CM:11.34),

$$\Delta Q_{\mathrm{bb}} = -\frac{N_{\mathrm{IP}} N r_0 \beta_V^\star}{2\pi\gamma\sigma_V(\sigma_V + \sigma_H)} \tag{3.1}$$

to deal with beams of unequal species, such as for RHIC[1] with fully stripped gold ions ($^{197}Au^{+79}$) in one ring and deuterons ($^{2}H^{+}$) in the other ring.

(1) T. Satogata, "Commissioning of RHIC Deuteron-Gold Collisions", Proc. of PAC2003, Portland, OR, 1706 (2003).