# Physics 696
# Topics in Advanced Accelerator Design I
# Monday, September 10 2012

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Happy Birthday to Arthur Compton (1927 Nobel), Joey Votto, Colin Firth, and the LHC!
Happy Swap Ideas Day, International Make-Up Day, and Cheap Advice Day!

# According to the Syllabus

TAAD1 Course Outline

| Session | Topic | Instructor |
|---|---|---|
| | | |
| 8/27/2011 9:00-10:15 | Introduction to Accelerators | Krafft |
| 8/27/2011 10:30-11:45 | Electromagnetism | Krafft |
| 9/10/2011 11:30-12:45 | Cyclotrons/Betatrons/Medical Accelerators | Satogata |
| 9/10/2011 1:00-2:15 | Electromagnetism | Satogata |
| 9/17/2011 9:00-10:15 | Mechanics | Krafft |
| 9/17/2011 10:30-11:45 | Mechanics | Krafft |
| 9/24/2011 9:00-10:15 | RF Fundamental (TE,TM,TEM) | Delayen |

But first, reminders and review of relativistic motion

# Relativity Review (Again)

- Accelerators: applied special relativity
- Relativistic parameters:

$$\beta \equiv \frac{v}{c} \qquad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \sqrt{1 - 1/\gamma^2}$$

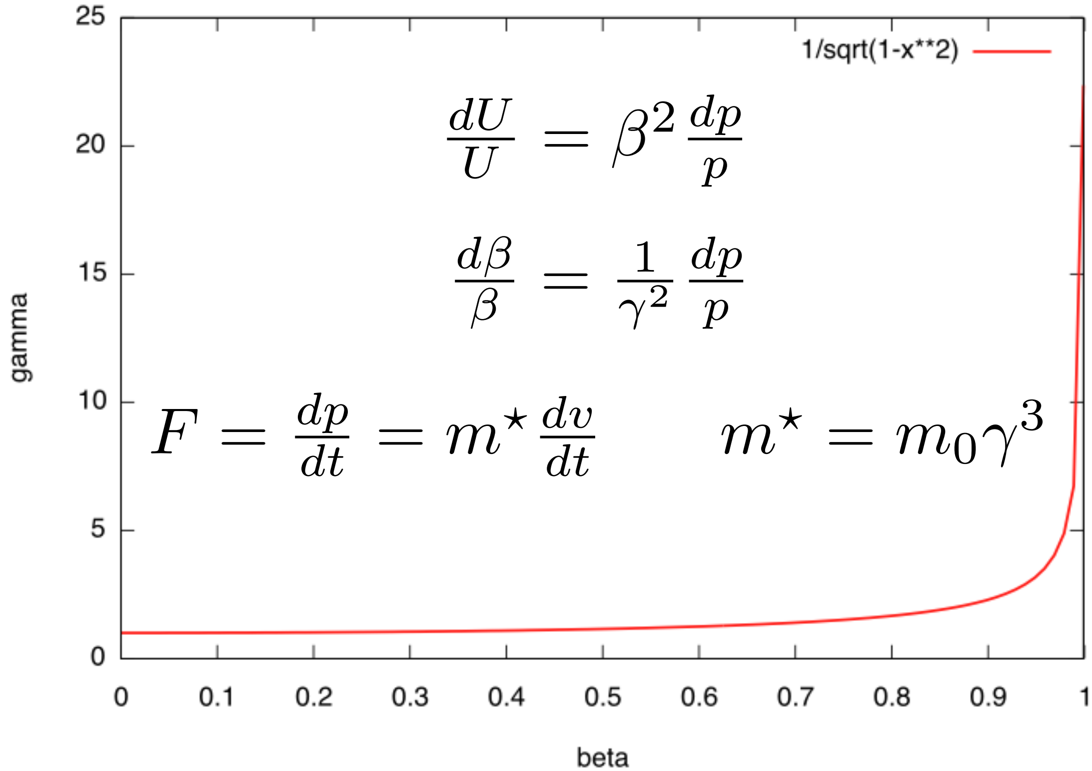
  - After this lecture, will try to use $\beta_r$ and $\gamma_r$ to avoid confusion with other lattice parameters
  - $\gamma$=1 (classical mechanics) to ~$2.05 \times 10^5$ (LEP, not LHC)

- Total energy U, momentum p, and kinetic energy W

$$U = \gamma mc^2 \qquad p = (\beta\gamma)mc = \beta\left(\frac{U}{c}\right) \qquad W = (\gamma - 1)\, mc^2$$

J.D. Jackson, Classical Electrodynamics 2nd Ed, Chapter 11

# Convenient Relativity Relations




- All derived in the text, hold for all $\gamma$
- In highly relativistic limit $\beta \approx 1$
  - Usually must be careful below $\gamma \approx 5$ or $U \approx 5\ mc^2$
  - Many accelerator physics phenomena scale with $\gamma^k$ or $(\beta\gamma)^k$

# Frames and Lorentz Transformations

- The lab frame will dominate most of our discussions
  - But not always (synchrotron radiation, space charge…)

- Invariance of space-time interval (Minkowski)

$$(ct')^2 - x'^2 - y'^2 - z'^2 = (ct)^2 - x^2 - y^2 - z^2$$

- Lorentz transformation of four-vectors
  - For example, time/space coordinates in z velocity boost

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

# Four-Velocity and Four-Momentum

- The proper time interval $d\tau = dt/\gamma$ is Lorentz invariant
- So we can make a velocity 4-vector

$$cu^\alpha \equiv \left( \frac{dct}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau} \right) = c\gamma(1, \beta_x, \beta_y, \beta_z)$$

$$\text{Metric } \; g^{\mu\nu} = g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$

- We can also make a 4-momentum

$$p^\alpha \equiv mcu^\alpha = mc\gamma(1, \beta_x, \beta_y, \beta_z)$$

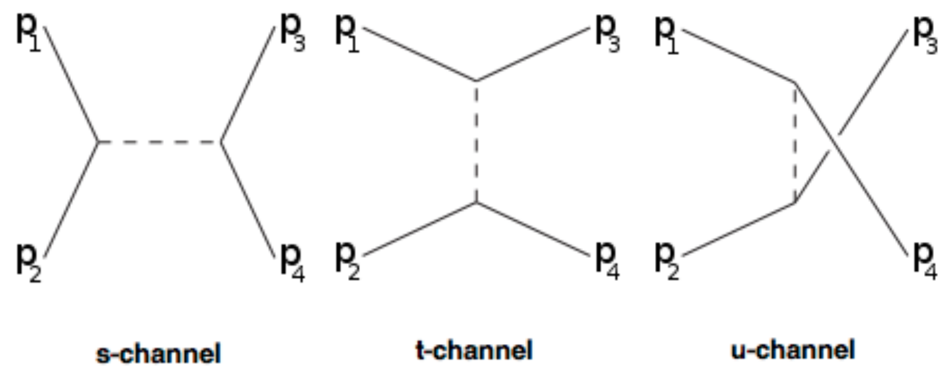
- Double-check that Minkowski norms are invariant

$$u^\alpha u_\alpha = u^\alpha g_{\alpha\beta} u^\beta = \gamma^2(1 - \beta^2) = 1$$

$$p^\alpha p_\alpha = m^2c^2 u^\alpha u_\alpha = m^2c^2$$

# Mandelstam Variables

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$
$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$$
$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2$$




$$s + t + u = (m_1^2 + m_2^2 + m_3^2 + m_4^2)c^2$$

- Lorentz-invariant two-body kinematic variables
  - $p_{1\text{-}4}$ are four-momenta
- √s is the total available center of mass energy
  - Often quoted for colliders
- Used in calculations of other two-body scattering processes
  - Moller scattering (e-e), **Compton scattering** (e-γ)

# Relativistic Newton

$$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$$

- But now we can define a four-vector force in terms of four-momenta and proper time:

$$F^\alpha \equiv \frac{dp^\alpha}{d\tau}$$

- We are primarily concerned with electrodynamics so now we must make the classical electromagnetic force obey Lorentz transformations

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

# Relativistic Electromagnetism

- Classical electromagnetic potentials can be shown to combine to a four-potential (with c=1):

$$A^\alpha \equiv (\Phi, \vec{A})$$

- The field-strength tensor is related to the four-potential

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

  - E/B fields Lorentz transform with factors of $\gamma$, $(\beta\gamma)$

# (Lorentz Lie Group Generators)

- Lorentz transformations can be described by a Lie group where a general Lorentz transformation is

$$A = e^L \qquad \det A = e^{\mathrm{Tr}\, L} = +1$$

where L is 4x4, real, and traceless. With metric g, the matrix gL is also antisymmetric, so L has the general six-parameter form

$$L = \begin{pmatrix} 0 & L_{01} & L_{02} & L_{03} \\ L_{01} & 0 & L_{12} & L_{13} \\ L_{02} & -L_{12} & 0 & L_{23} \\ L_{03} & -L_{13} & -L_{23} & 0 \end{pmatrix}$$

Deep and profound connection to EM tensor $F^{\alpha\beta}$

J.D. Jackson, Classical Electrodynamics 2nd Ed, Section 11.7

# Relativistic Electromagnetism II

- The relativistic electromagnetic force equation becomes

$$\frac{dp^\alpha}{d\tau} = m\frac{du^\alpha}{d\tau} = \frac{q}{c}F^{\alpha\beta}u_\beta$$

  - Thankfully we can write this in somewhat simpler terms

$$\frac{d(\gamma m\vec{v})}{dt} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

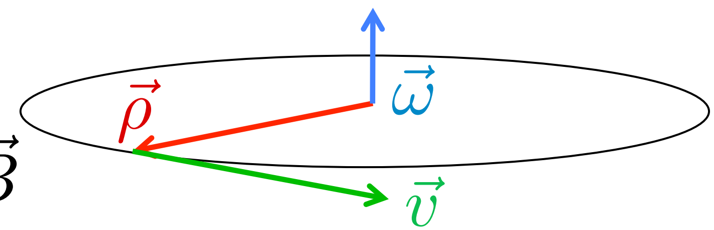
  - That is, "classical" E&M force equations hold if we treat the momentum as relativistic.
  - If we dot in the velocity, we get energy transfer

$$\frac{d\gamma}{dt} = \frac{q\vec{E}\cdot\vec{v}}{mc^2}$$

  - Unsurprisingly, we can only get energy changes from electric fields, not magnetic fields

# Constant Magnetic Field

- In a constant magnetic field, charged particles move in circular arcs of radius ρ with constant angular velocity ω:




$$\vec{F} = \frac{d}{dt}(\gamma m \vec{v}) = \gamma m \frac{d\vec{v}}{dt} = q\vec{v} \times \vec{B}$$

$$\vec{v} = \vec{\omega} \times \vec{\rho} \qquad \Rightarrow \qquad q\vec{v} \times \vec{B} = \gamma m \vec{\omega} \times \frac{d\vec{\rho}}{dt} = \gamma m \vec{\omega} \times \vec{v}$$

  - For $\vec{B} \perp \vec{v}$ we then have

$$qvB = \frac{\gamma m v^2}{\rho} \qquad p = \gamma m(\beta c) = q(B\rho) \qquad \boxed{\frac{p}{q} = (B\rho)}$$

$$\boxed{\omega = \frac{v}{\rho} = \frac{qB}{\gamma m}}$$

# Rigidity: Bending Radius vs Momentum

Beam

$$\frac{p}{q} = (B\rho)$$

Accelerator (magnets, geometry)

- This is such a useful expression in accelerator physics that it has its own name: **rigidity**
- Ratio of momentum to charge
  - How hard (or easy) is a particle to deflect?
  - Often expressed in [T-m] (easy to calculate B)
  - Be careful when q≠e!!
- A veeeeeery useful expression

$$p[\mathrm{GeV/c}] \approx 0.3\, B[\mathrm{T}]\, \rho[\mathrm{m}]$$

## Cyclotron Frequency

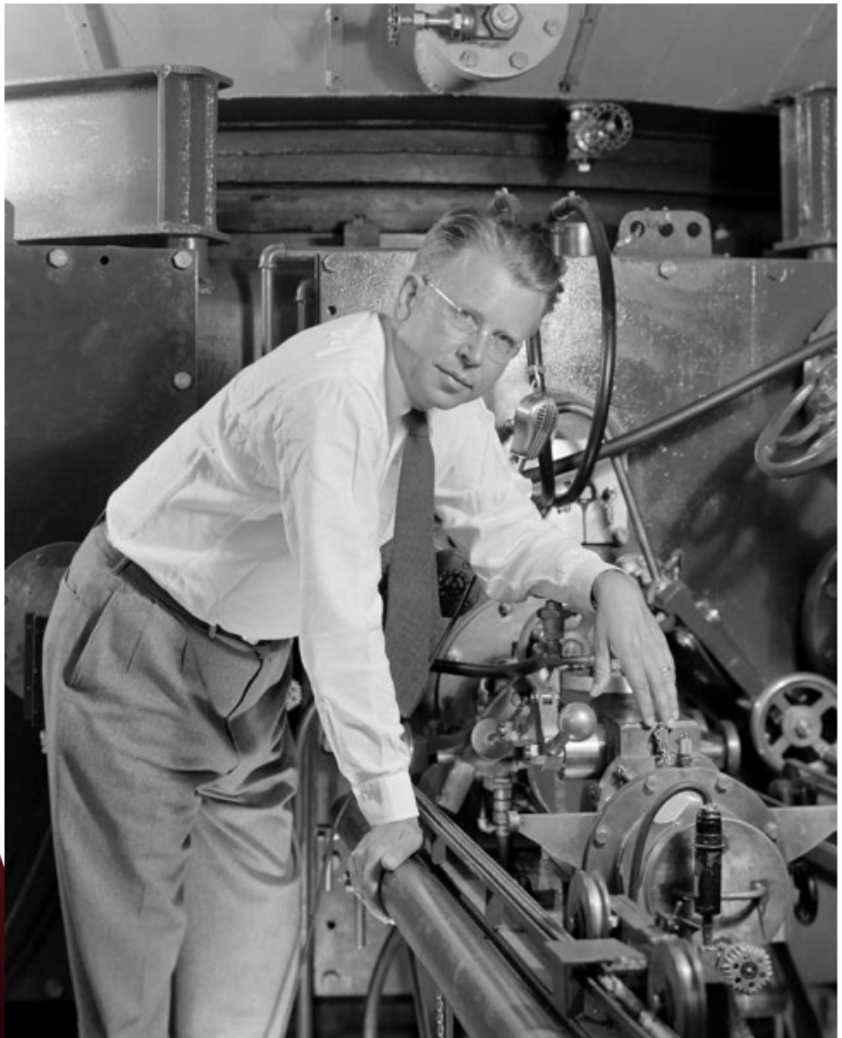
$$\omega = \frac{v}{\rho} = \frac{qB}{\gamma m}$$

- Another very useful expression for particle angular frequency in a constant field: **cyclotron frequency**
- In the nonrelativistic approximation

$$\omega_{\text{nonrelativistic}} \approx \frac{qB}{m}$$

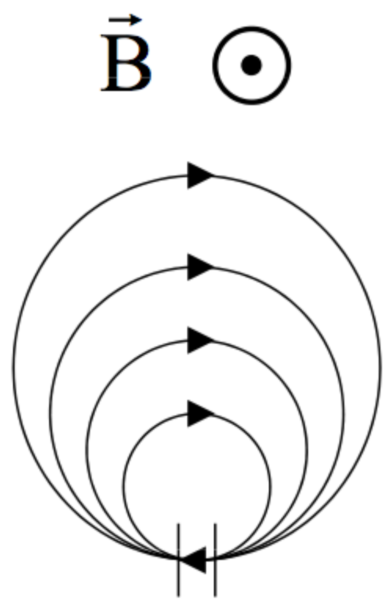
Revolution frequency is independent of radius or energy!

# Lawrence and the Cyclotron



Ernest Orlando Lawrence

- Can we repeatedly spiral and accelerate particles through the same potential gap?




Accelerating gap $\Delta\Phi$

# Cyclotron Frequency Again

- Recall that for a constant B field

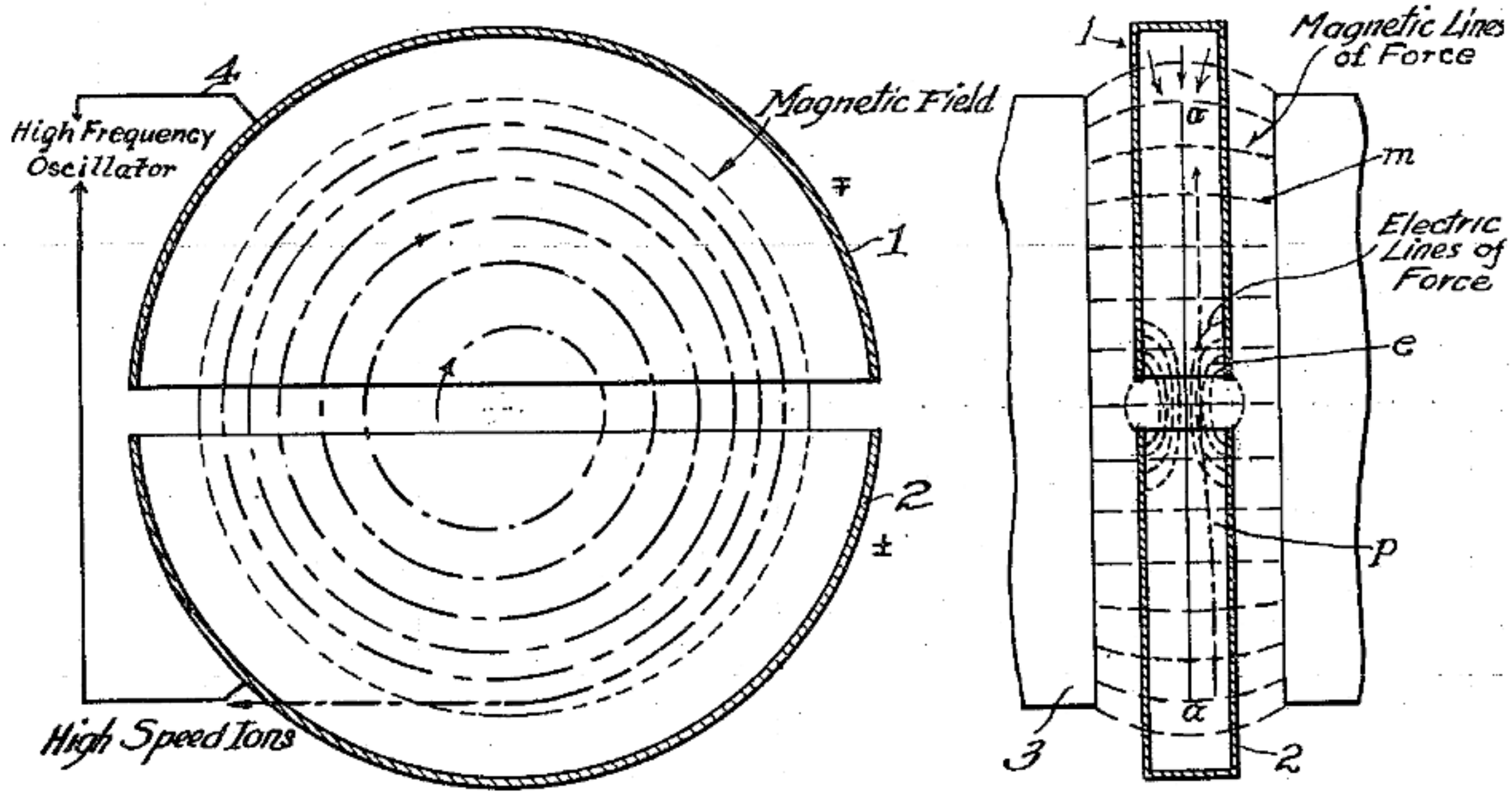
$$p = \gamma m v = q(B\rho) \quad \Rightarrow \quad \rho = \left(\frac{\gamma m}{qB}\right) v$$

- Radius/circumference of orbit scale with velocity
  - Circulation time (and frequency) are **independent** of v
- Apply AC electric field in the gap at frequency $f_{rf}$
  - Particles accelerate until they drop out of resonance

$$\omega = \frac{v}{\rho} = \frac{qB}{\gamma m} \qquad f_{\rm rf} = \frac{\omega}{2\pi} = \frac{qB}{2\pi\gamma m}$$

  - Note a first appearance of "bunches", not DC beam
  - Works "best" with heavy particles (hadrons, not electrons)

# A Patentable Idea



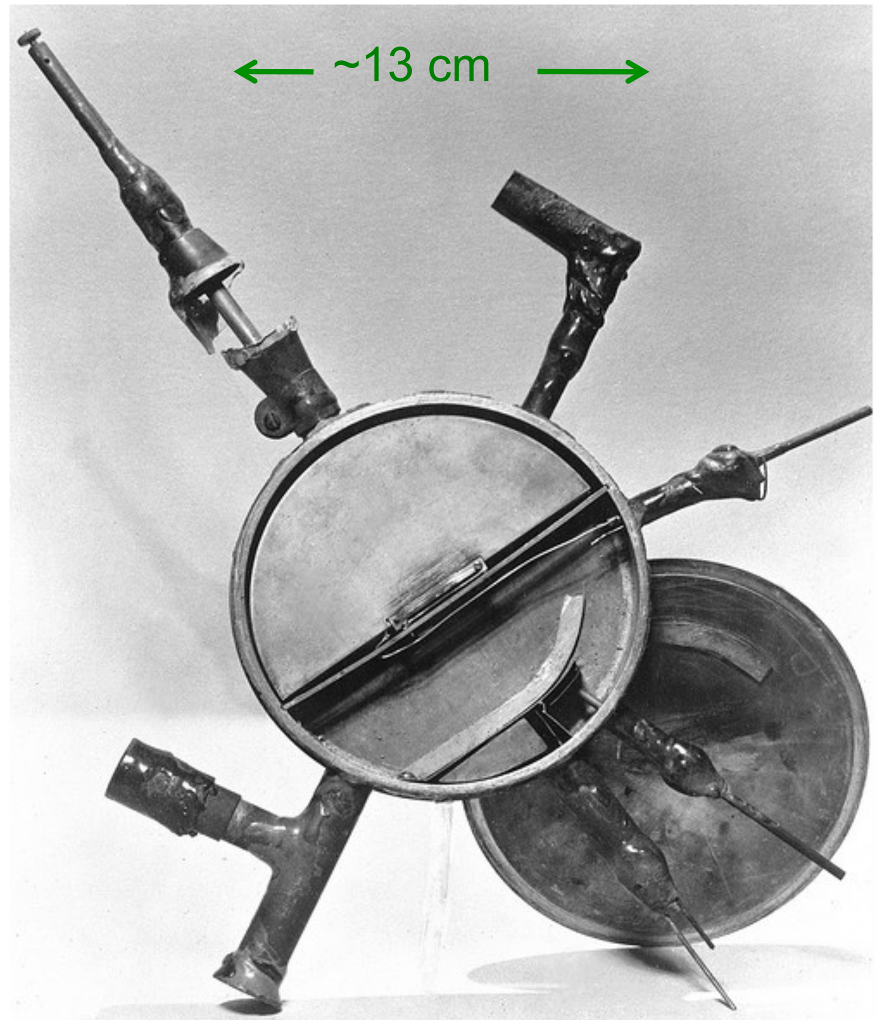

- 1934 patent 1948384
  - Two accelerating gaps per turn!

# All The Fundamentals of an Accelerator

- Large static magnetic fields for guiding (~1T)
- HV RF electric fields for accelerating
  - (Phase focusing)
- p/H source, injection, extraction, vacuum

- 13 cm: 80 keV
- 28 cm: 1 MeV
- 69 cm: ~5 MeV
- … 223 cm: ~55 MeV

(Berkeley)



# Livingston, Lawrence, 27”/69 cm Cyclotron


M.S. Livingston and E.O. Lawrence, 1934

# The Joy of Physics

- Describing the events of January 9, 1932, Livingston is quoted saying:

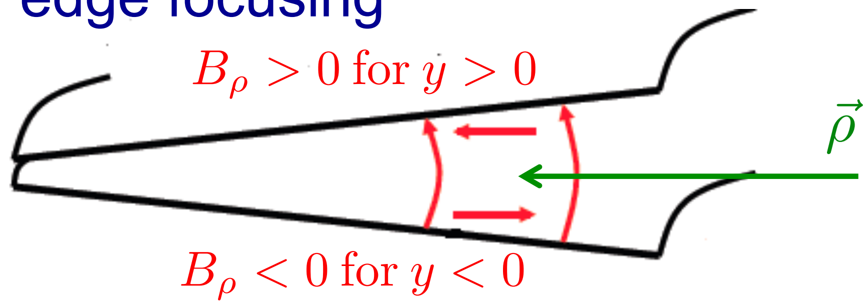
"I recall the day when **I had adjusted the oscillator** to a new high frequency, and, with Lawrence looking over my shoulder, tuned the magnet through resonance. As the galvanometer spot swung across the scale, indicating that protons of 1-MeV energy were reaching the collector, **Lawrence literally danced** around the room with glee. The news quickly spread through the Berkeley laboratory, and we were busy all that day demonstrating million-volt protons to eager viewers."

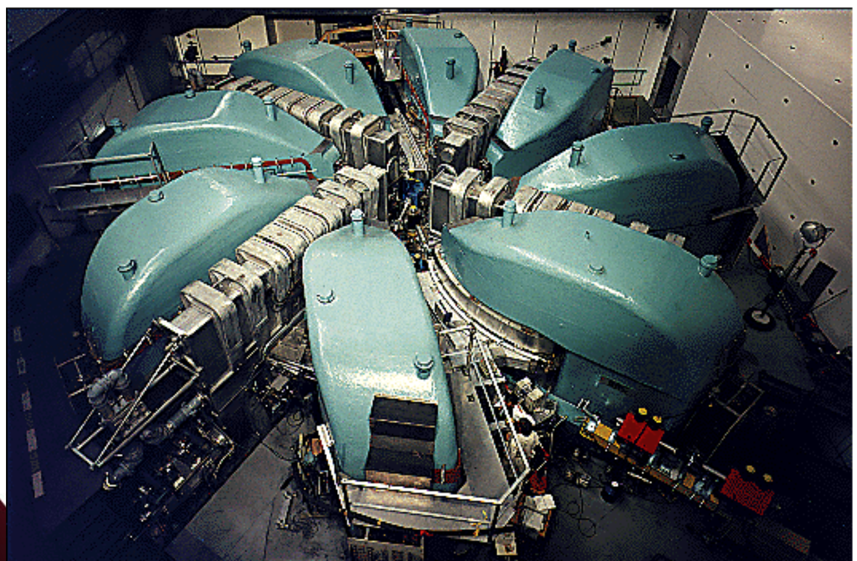
APS Physics History, "Ernest Lawrence and M. Stanley Livingston

# Modern Isochronous Cyclotrons

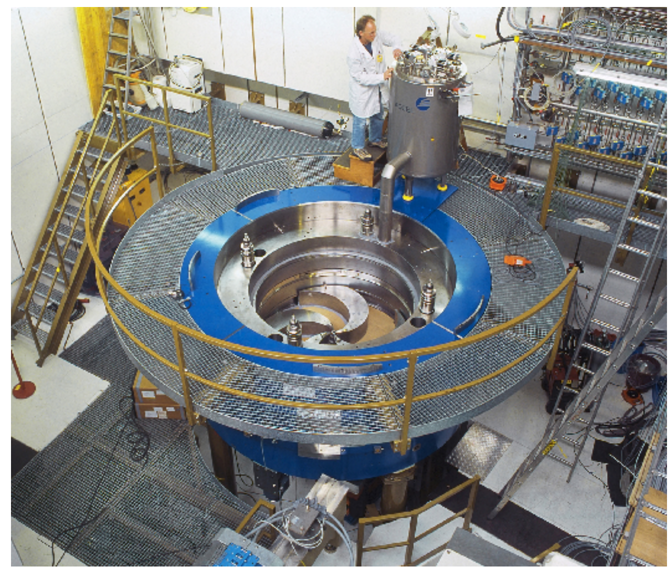
- Higher bending field at higher energies
  - But also introduces vertical defocusing
  - Use bending magnet "edge focusing"

$$f_{\mathrm{rf}} = \frac{qB(\rho)}{2\pi\gamma(\rho)m}$$



590 MeV PSI Isochronous Cyclotron (1974)



250 MeV PSI Isochronous Cyclotron (2004)

# Cyclotrons Today

- Cyclotrons continue to evolve
  - Many contemporary developments
    - Superconducting cyclotrons (higher B, faster $\omega$)
    - Synchrocyclotrons (FM modulated RF)
    - Isochronous/Alternating Vertical Focusing (AVF)
    - FFAGs (Fixed Field Alternating Gradient)
  - Versatile with many applications even below ~500 MeV
    - High power (>1MW) neutron production
    - Reliable (medical isotope production, ion radiotherapy)
    - Power+reliability: ~5 MW p beam for accelerator driven subcritical reactors (e.g. burning Thorium for power)

# Even Cyclotron RF Has Evolved

## The Structure of a GANIL Double Gap $\lambda/2$ Resonator:

Schematic drawing from the previous page – illustration of physical details

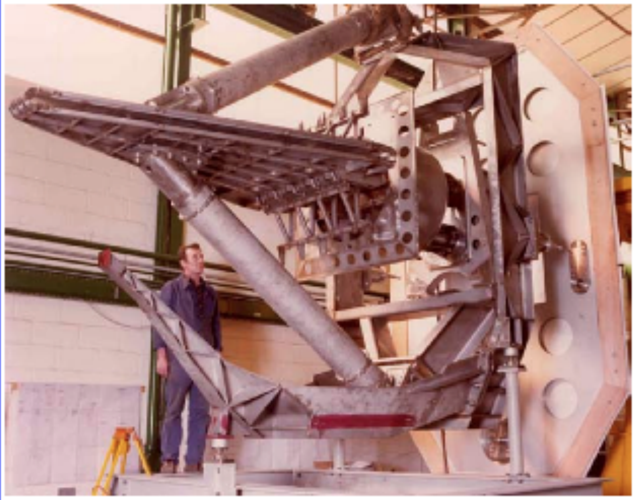


*Fig. 24: Stainless steel support frame, beam plane is visible*

*Fig. 25: Copper skinned inner conductors with 'Dee' (inner electrode)*

*Fig. 26: Outer shell of resonator, with support frame and beam slit*

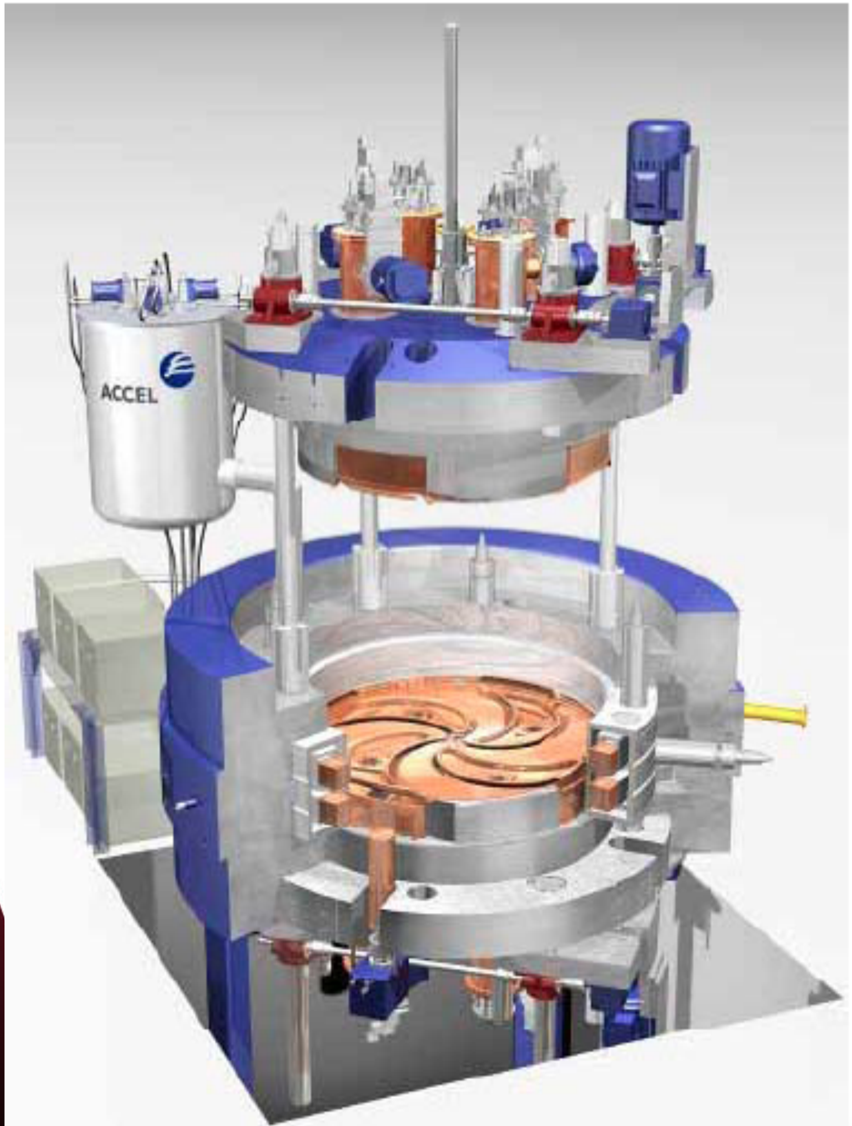
Gap transit time issues come later…

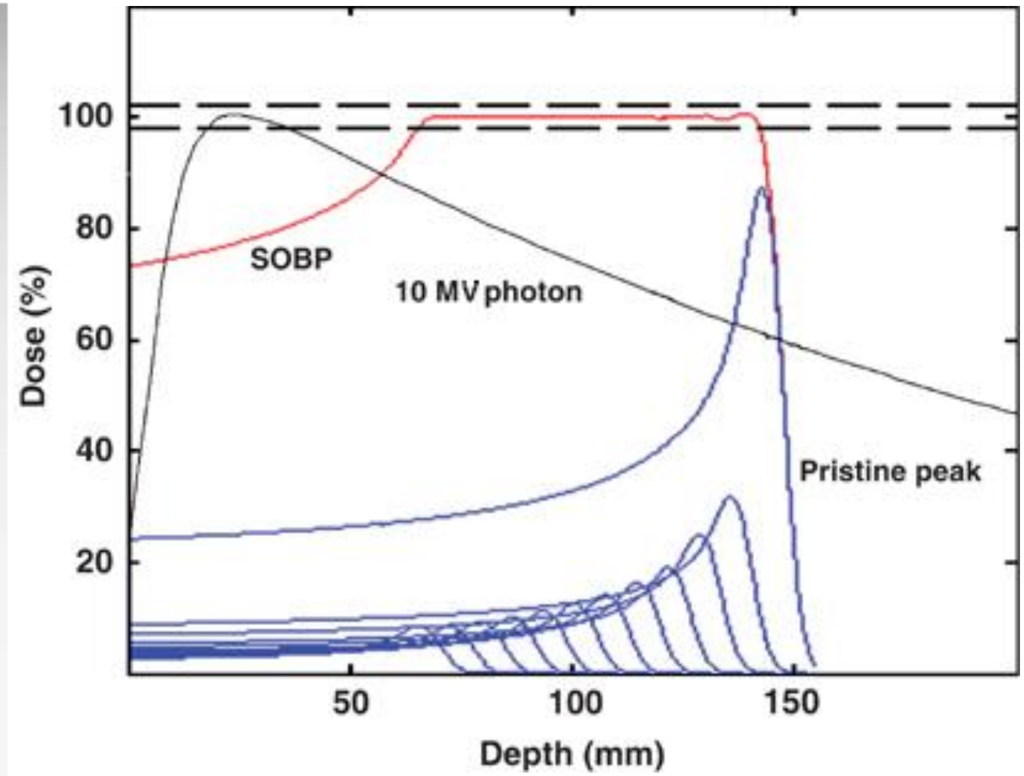
http://cas.web.cern.ch/cas/Holland/PDF-lectures/Sigg/CyclotronRFfinal.pdf

P. K. Sigg/PSI/May2005 CAS/RF for Cyclotrons 30

Peter Sigg

# Accel Radiotherapy Cyclotron



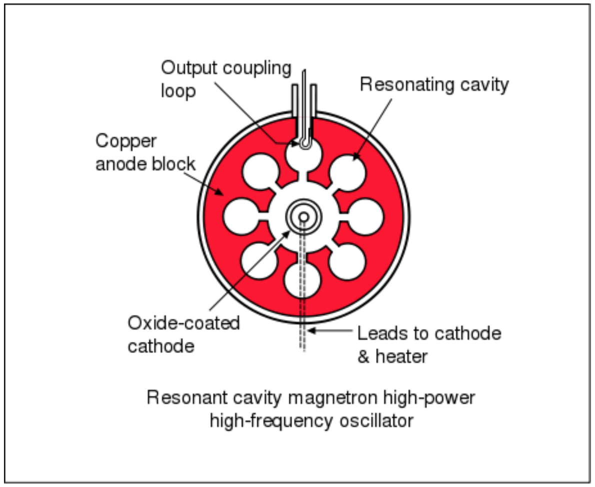



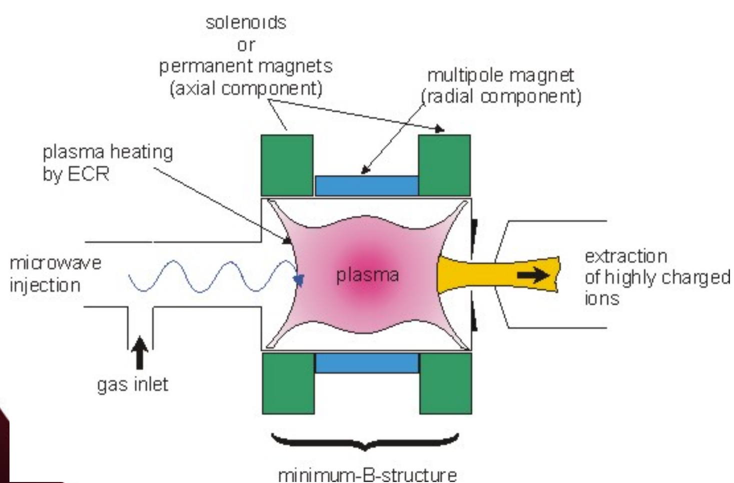

Distinct dose localization advantage for hadrons over X-rays

Also present work on proton and carbon radiotherapy fast-cycling synchrotrons

# Electrons, Magnetrons, ECRs

Radar/microwave magnetron



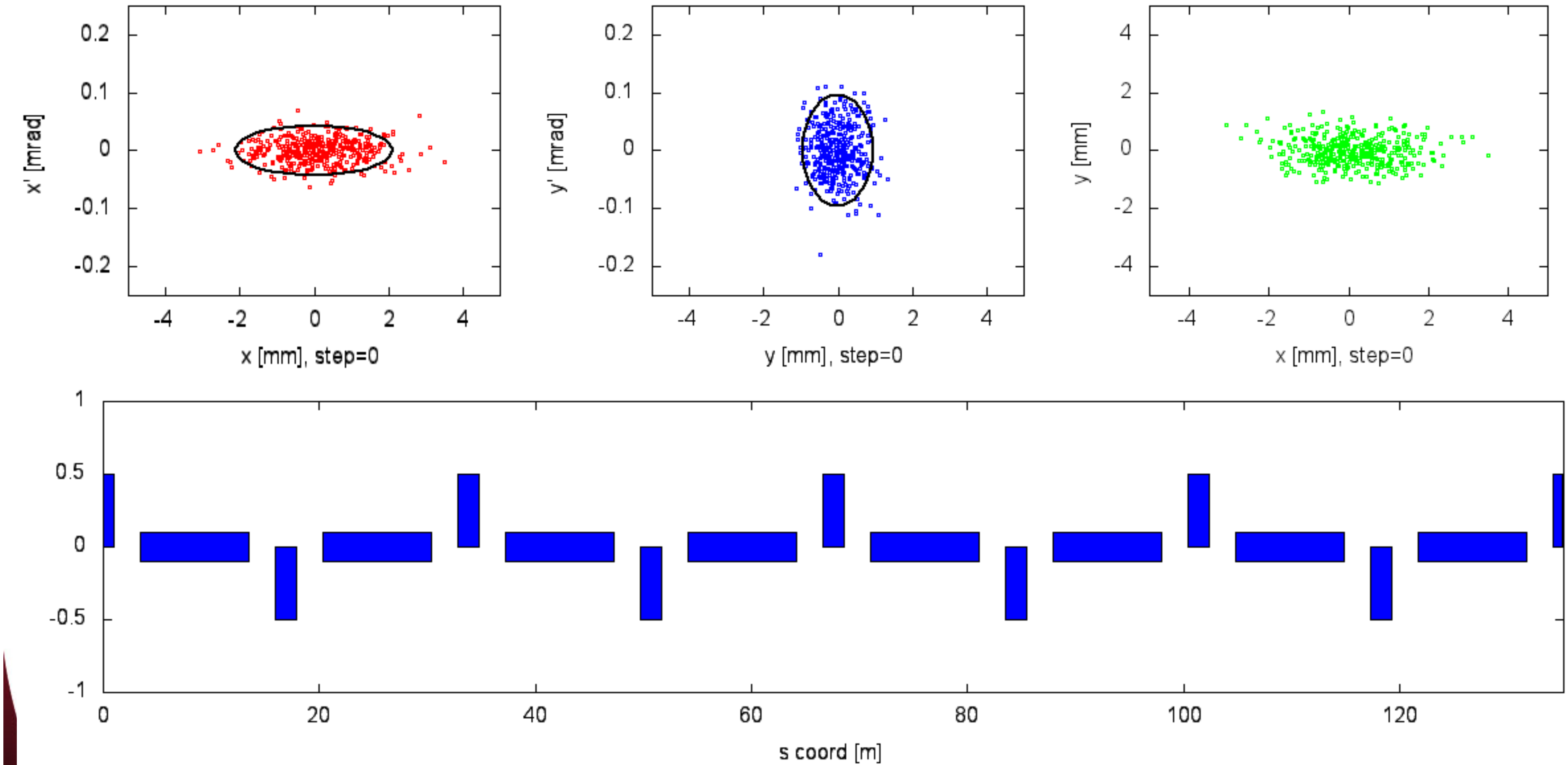




ECR plasma/ion source

- Cyclotrons aren't very good for accelerating electrons
  - $\gamma$ changes too quickly!
- But narrow-band response has advantages and uses
  - Microtrons

    generate high-power microwaves from circulating electron current
  - ECRs
    - generate high-intensity ion beams and plasmas by resonantly stripping electrons with microwaves

# =========== Break ===========

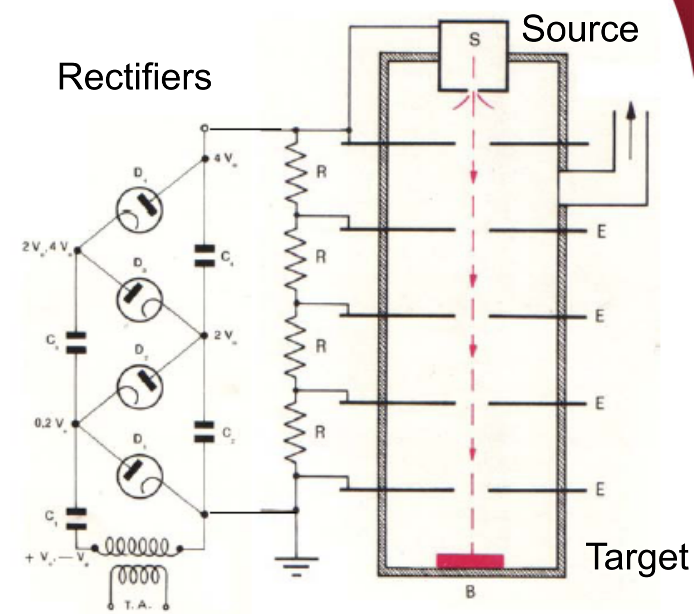


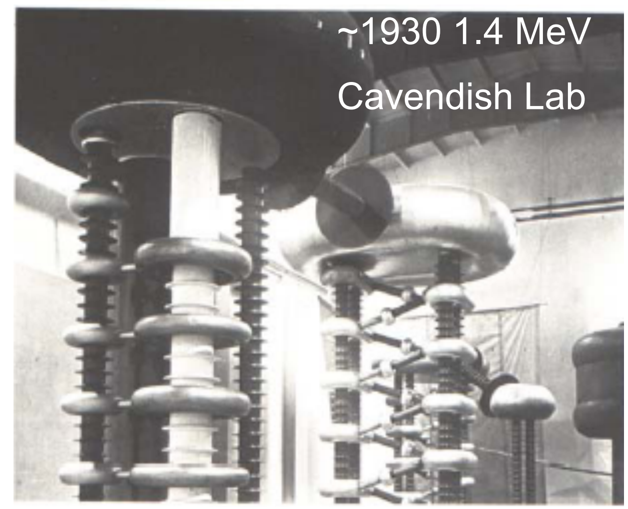
# (Too Brief) Survey of Accelerator Concepts

- Producing accelerating gaps and fields (DC/AC)
- Microtrons and their descendants
- Betatrons (and betatron motion)
- Synchrotrons
  - Fixed Target Experiments
  - Colliders and Luminosity (Livingston Plots)
  - Light Sources (FELs, Compton Sources)
- Medical Applications…
- Spallation Sources (ESS, Mats Lindroos)
- Power Production (ADS)…

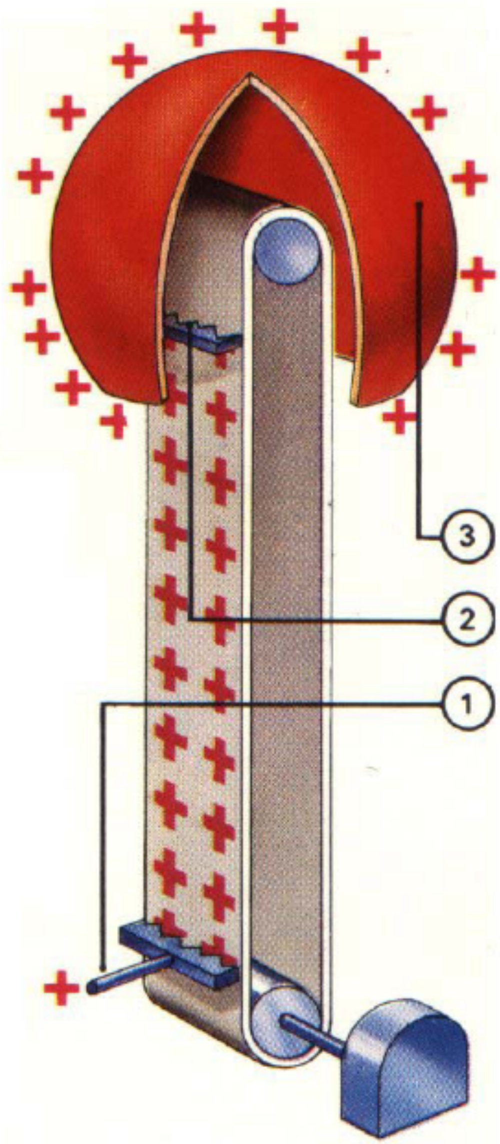
# DC Accelerating Gaps: Cockcroft-Walton

- Accelerates ions through successive electrostatic voltages
  - First to get protons to >MeV
  - Continuous HV applied through intermediate electrodes
  - Rectifier-multipliers (voltage dividers)
  - Limited by HV sparking/breakdown
  - FNAL still uses a 750 kV C-W
- Also example of early ion source
  - H gas ionized with HV current
  - Provides high current DC beam



# DC Accelerating Gaps: Van de Graaff

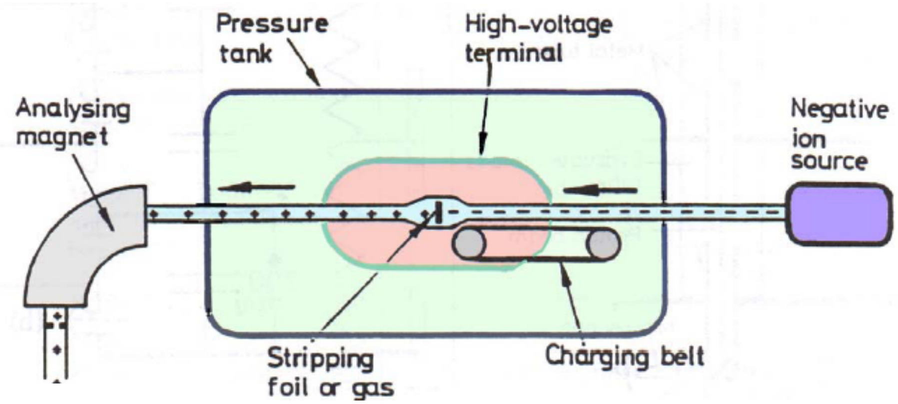



- How to increase voltage?
  - R.J. Van de Graaff: charge transport
  - Electrode (1) sprays HV charge onto insulated belt
  - Carried up to spherical Faraday cage
  - Removed by second electrode and distributed over sphere
- Limited by discharge breakdown
  - ~2MV in air
  - Up to 20+ MV in $SF_6$!
  - Pelletrons (chains)/Laddertrons (stripes)

# Van de Graaff Popularity

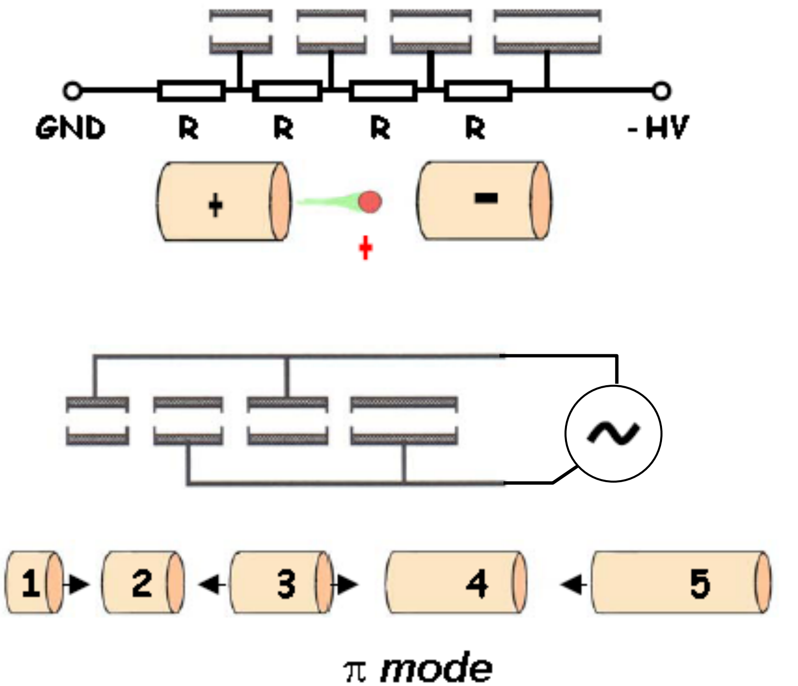
# DC Accelerating Gaps: Tandem Van de Graaff

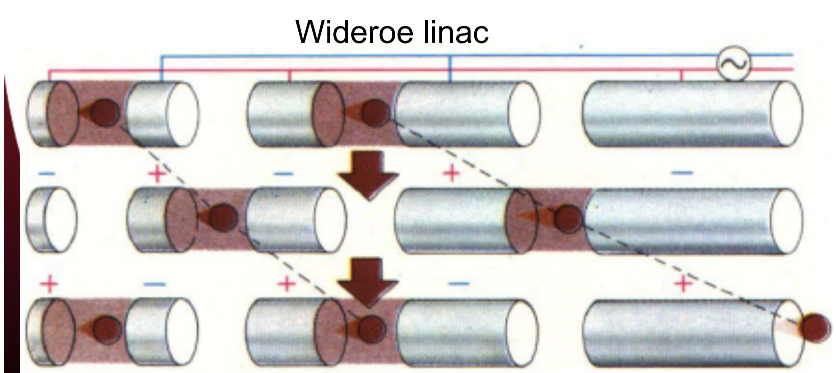
- Reverse ion charge state in middle of Van de Graaff allows over twice the energy gain
  - Source is at ground




- This only works for negative ions
- However, stripping need not be symmetric
  - Second stage accelerates more efficiently
- BNL: two Tandems (1970, 14 MV, 24m)
  - $Au^{-1}$ to $Au^{+10}/Au^{+11}/Au^{+12}$ to $Au^{+32}$ for RHIC
  - About a total of 0.85 MeV/nucleon total energy

# From Electrostatic to RF Acceleration



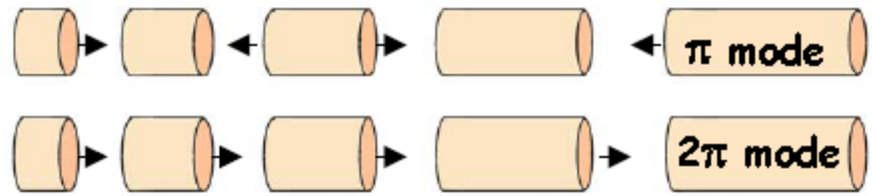




Pagani and Mueller 2002

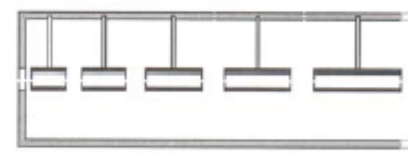
- Cockcroft-Waltons and Van de Graaffs have DC voltages, E fields
- What about putting on AC voltage?
  - Attach consecutive electrodes to opposite polarities of ACV generator
  - Electric fields between successive electrodes vary sinusoidally
  - Consecutive electrodes are 180 degrees out of phase ($\pi$ mode)
- At the right drive frequency, particles are accelerated in each gap
  - While polarity change occurs, particles are shielded in drift tubes
  - To stay in phase with the RF, drift tube length or RF frequency must increase at higher energies

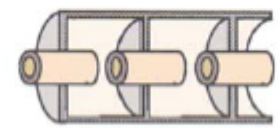
# Resonant Linac Structures

- Wideroe linac: $\pi$ mode
- Alvarez linac: $2\pi$ mode




- Need to minimize excess RF power (heating)
  - Make drift tubes/gaps resonant to RF frequency
  - In $2\pi$ mode, currents in walls separating two subsequent cavities cancel; tubes are passive
  - We'll cover RF and longitudinal motion next week…





Wideroe linac

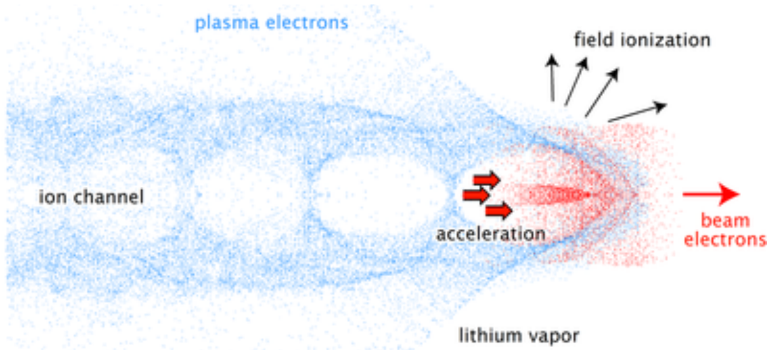
ALICE HI injector, IPN Orsay

Drift tube linac

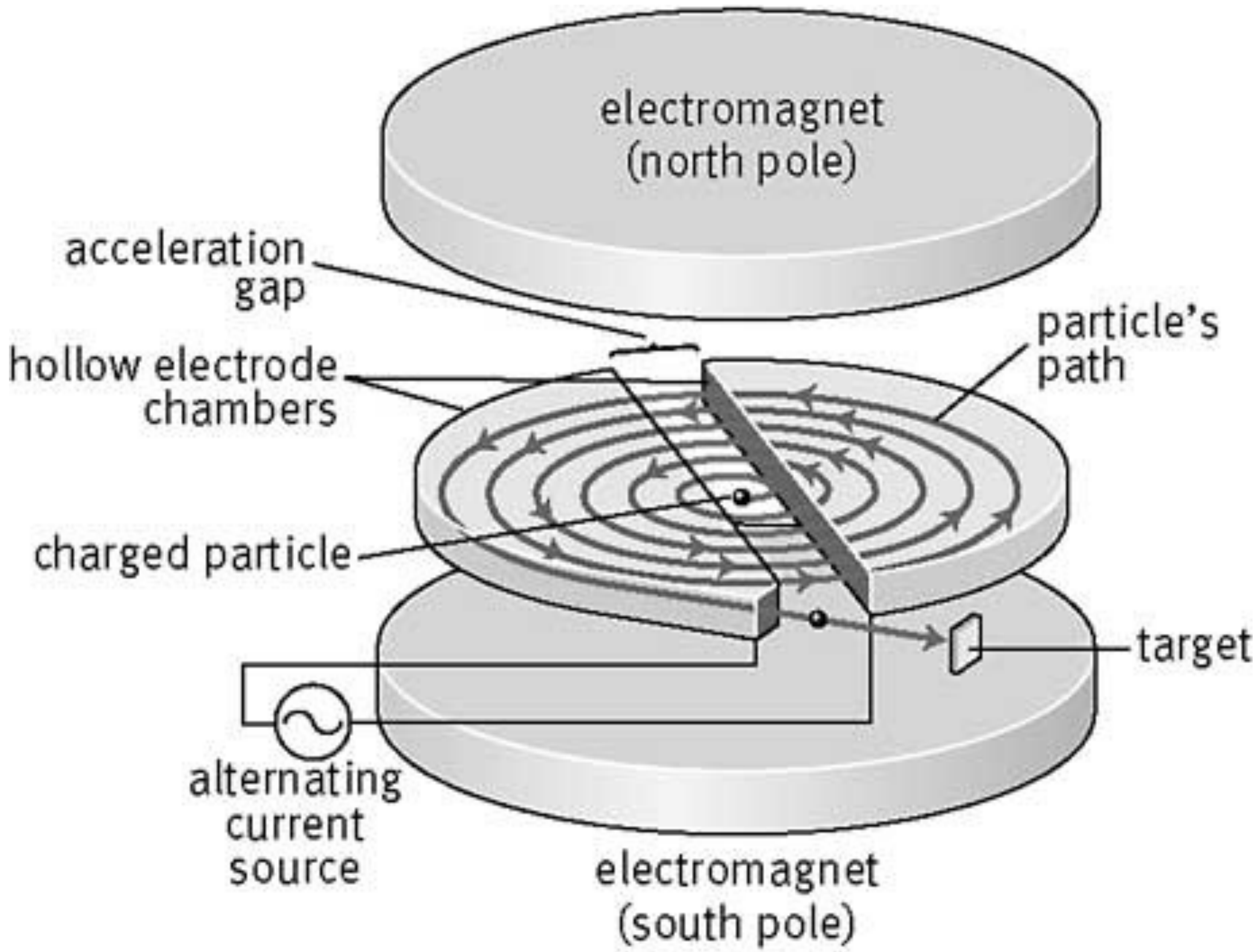
Saturne, Saclay

# Advanced Acceleration Methods

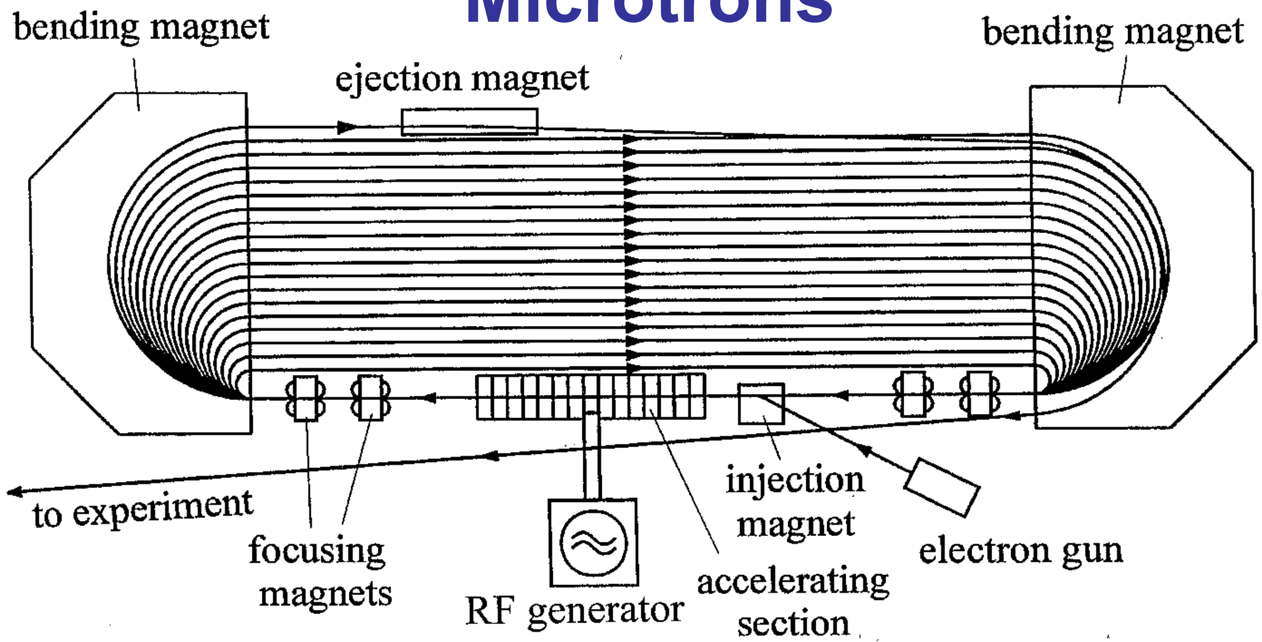
- How far do accelerating gradients go?
  - Superconducting RF acceleration: ~40 MV/m
  - CLIC: ~100 MV/m
    - Two-beam accelerator: drive beam couples to main beam
  - Dielectric wall acceleration: ~100 MV/m
    - Induction accelerator, very high gradient insulators
  - Dielectric wakefield acceleration: ~GeV/m
  - Laser plasma acceleration: ~30 GV/m
    - electrons to 1 GeV in 3.3 cm
    - particles ride in wake of plasma charge separation wave



# Cyclotrons (Again)

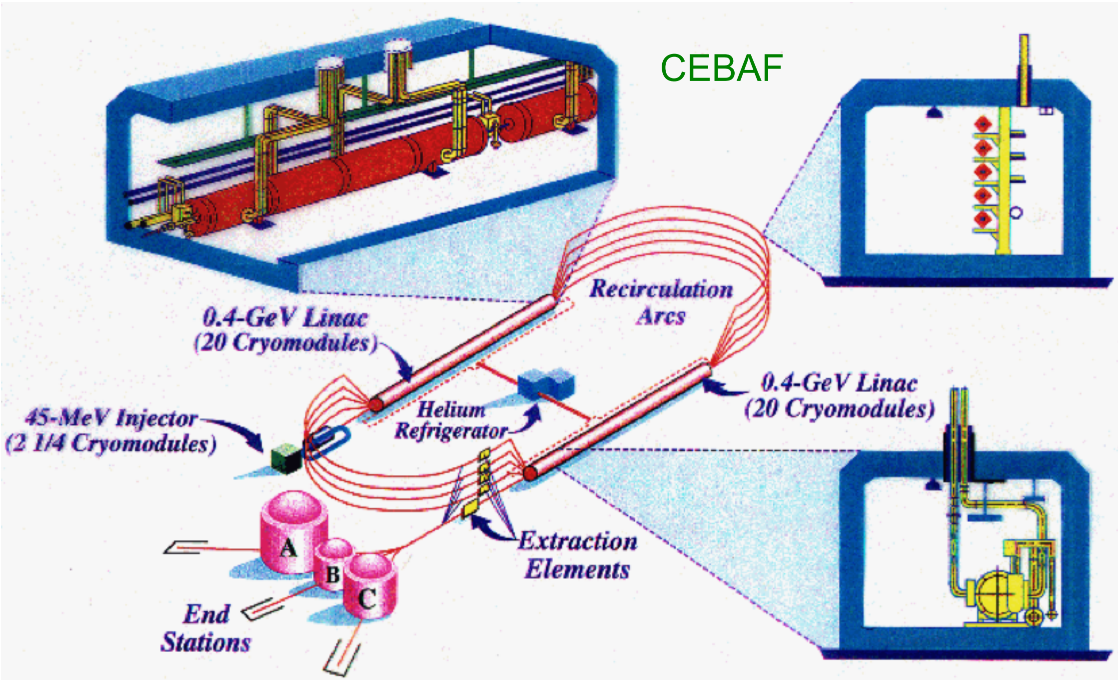


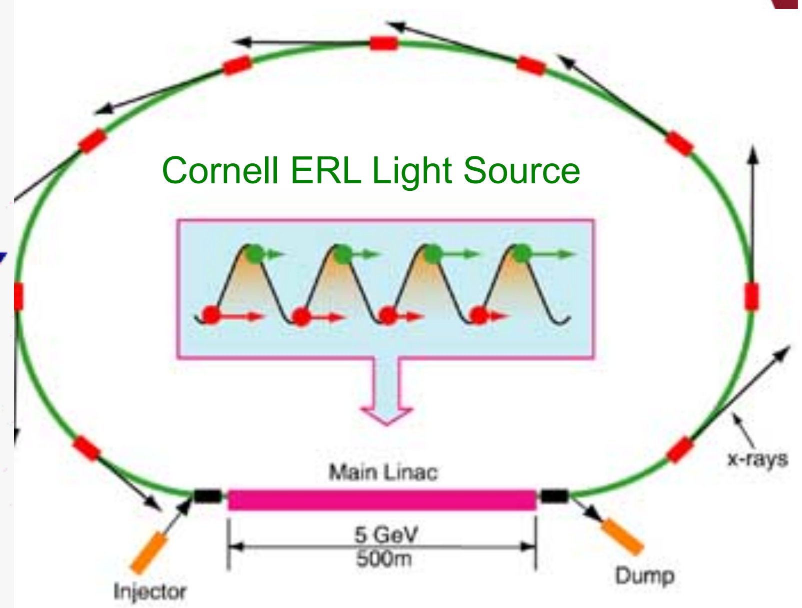
# Microtrons




- What about electrons? Microtrons are like cyclotrons
  - but each revolution electrons “slip” by integer # of RF cycles
  - Trades off large # of revs for minimal RF generation cost
  - Bends must have large momentum aperture
  - Used for medical applications today (20 MeV, 1 big magnet)
  - Mainz MAMI: 855 MeV, used for nuclear physics

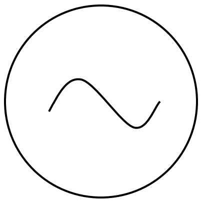
# Recirculating Linacs and ERLs



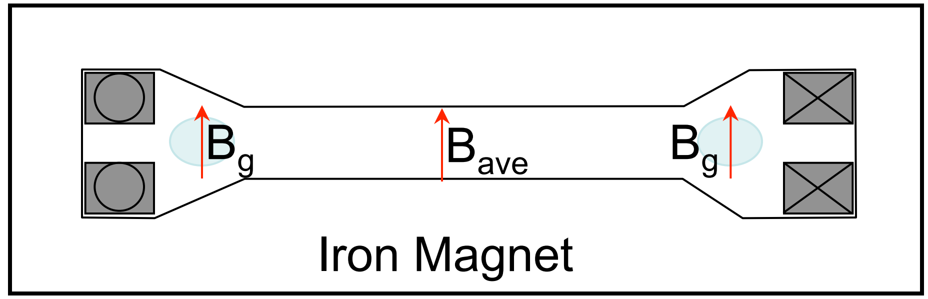




- Recirculating linacs have separate arcs, longer linacs
  - CEBAF: 4->6->12 GeV polarized electrons, 2 SRF linacs
  - Higher energy at cost of more linac, separated bends
- Energy recovery linacs recirculate exactly out of phase
  - Raise energy efficiency of linac, less beam power to dump
  - Requires high-Q SRF to recapture energy efficiently

# Betatrons



$I(t)=I_0 \cos(2\pi\omega_I t)$



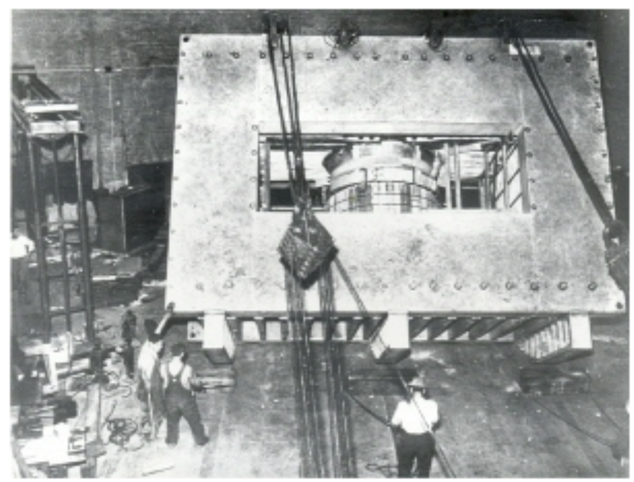

- Apply Faraday's law with time-varying current in coils
- Beam sees time-varying electric field – accelerate half the time!
- Early proofs of stability: focusing and "betatron" motion

Donald Kerst

UIUC 2.5 MeV

Betatron, 1940

Don't try this at home!!



UIUC 312 MeV

betatron, 1949

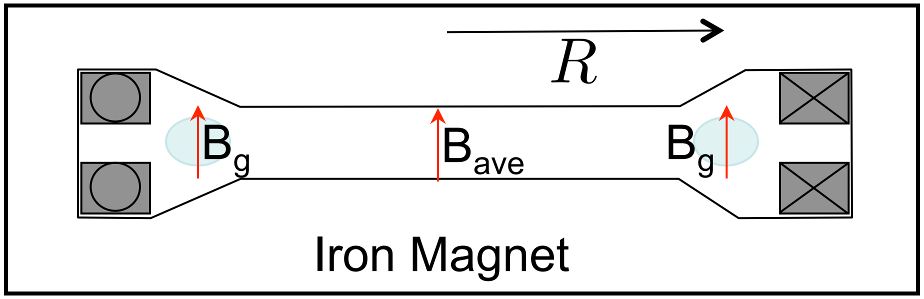
Really don't try this at home!!

# Betatron

$$I(t) = I_0 \cos(2\pi\omega t)$$

$$B_{\text{ave}}(t) = B_{\text{ave},0} \cos(2\pi\omega t)$$

$$\Phi(t) = \pi R^2 B_{\text{ave}}(t)$$




$$\text{Faraday's Law}: \quad \text{EMF } \mathcal{E} = -\frac{d\Phi(t)}{dt} = \oint \vec{E} \cdot d\vec{s}$$

$$\oint \vec{E} \cdot d\vec{s} = 2\pi R E = -\pi R^2 \frac{dB_{\text{ave}}(t)}{dt} \quad \Rightarrow \quad E = -\frac{R}{2} \frac{dB_{\text{ave}}(t)}{dt}$$

$$\text{Accelerating force}: \quad F = qE = \frac{eR}{2} \frac{dB_{\text{ave}}(t)}{dt}$$

$$\text{Centripetal force}: \quad F = \frac{dp}{dt} = eR \frac{dB_g(t)}{dt}$$

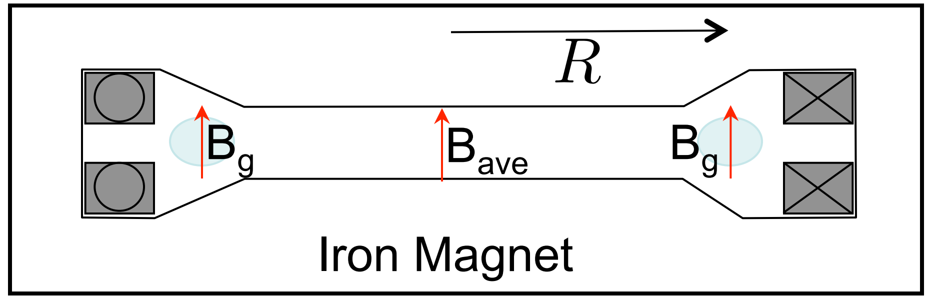
$$\Rightarrow \boxed{B_g(t) = \frac{B_{\text{ave}}(t)}{2}}$$

# Betatron

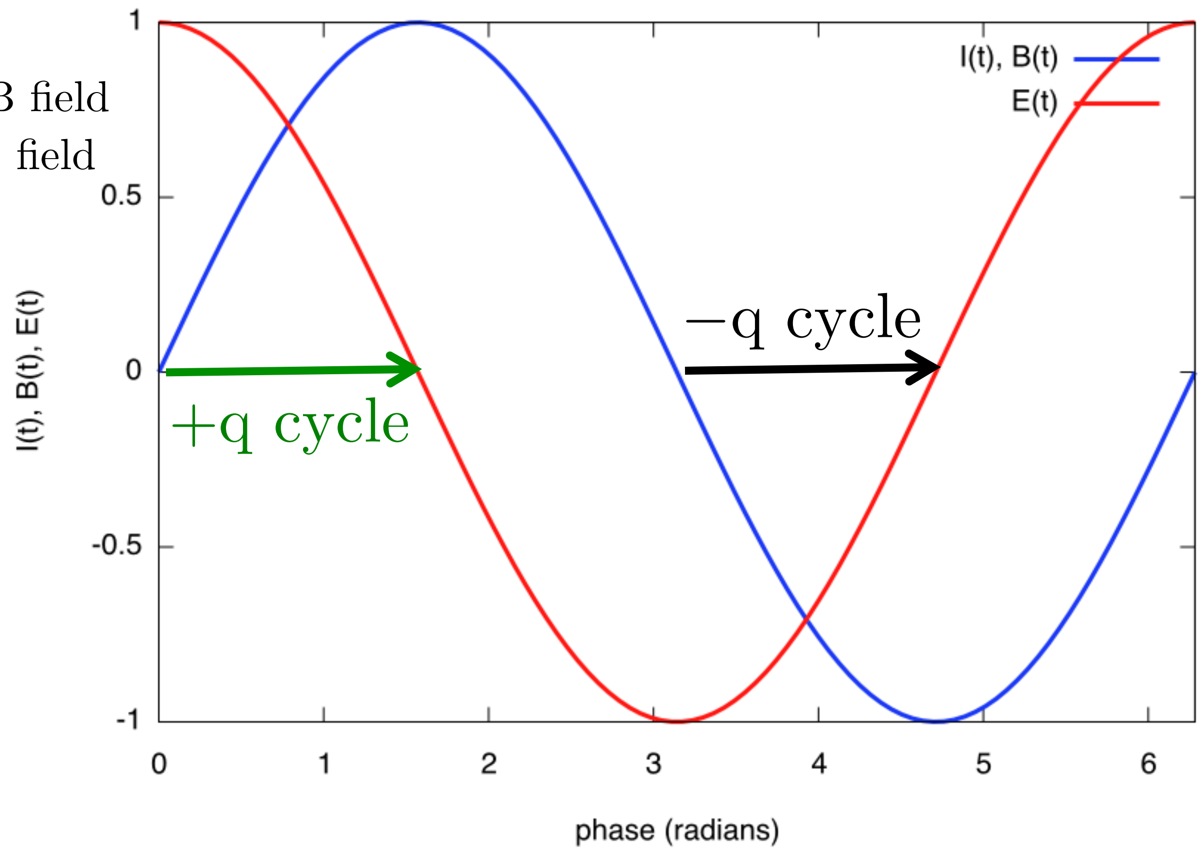
$$I(t) = I_0 \cos(2\pi\omega t)$$

$$B_{\rm ave}(t) = B_{\rm ave,0} \cos(2\pi\omega t)$$

$$\Phi(t) = \pi R^2 B_{\rm ave}(t)$$



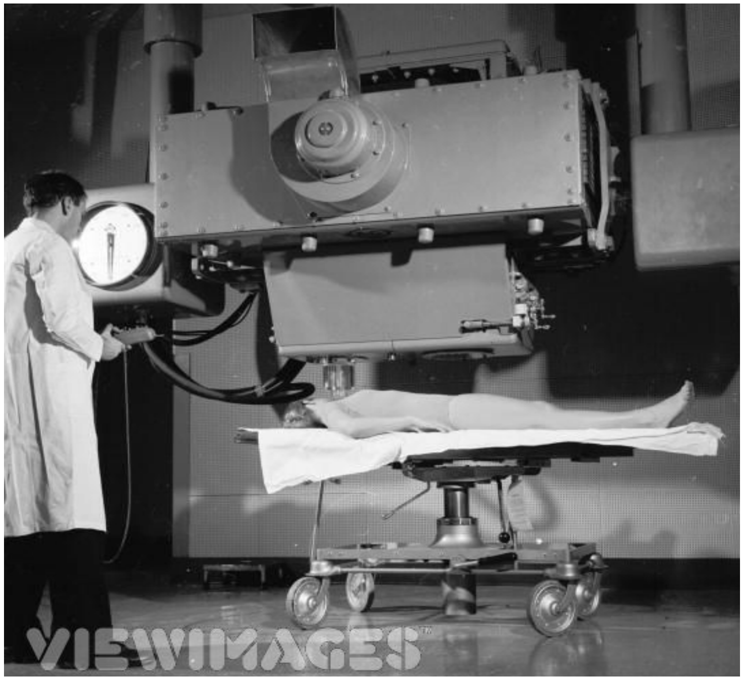

inject at high EMF, low B field
extract at 0 EMF, high B field



# Betatrons

- Betatrons produced electrons up to 300+ MeV
  - Early materials and medical research
  - Also produced medical hard X-rays and gamma rays

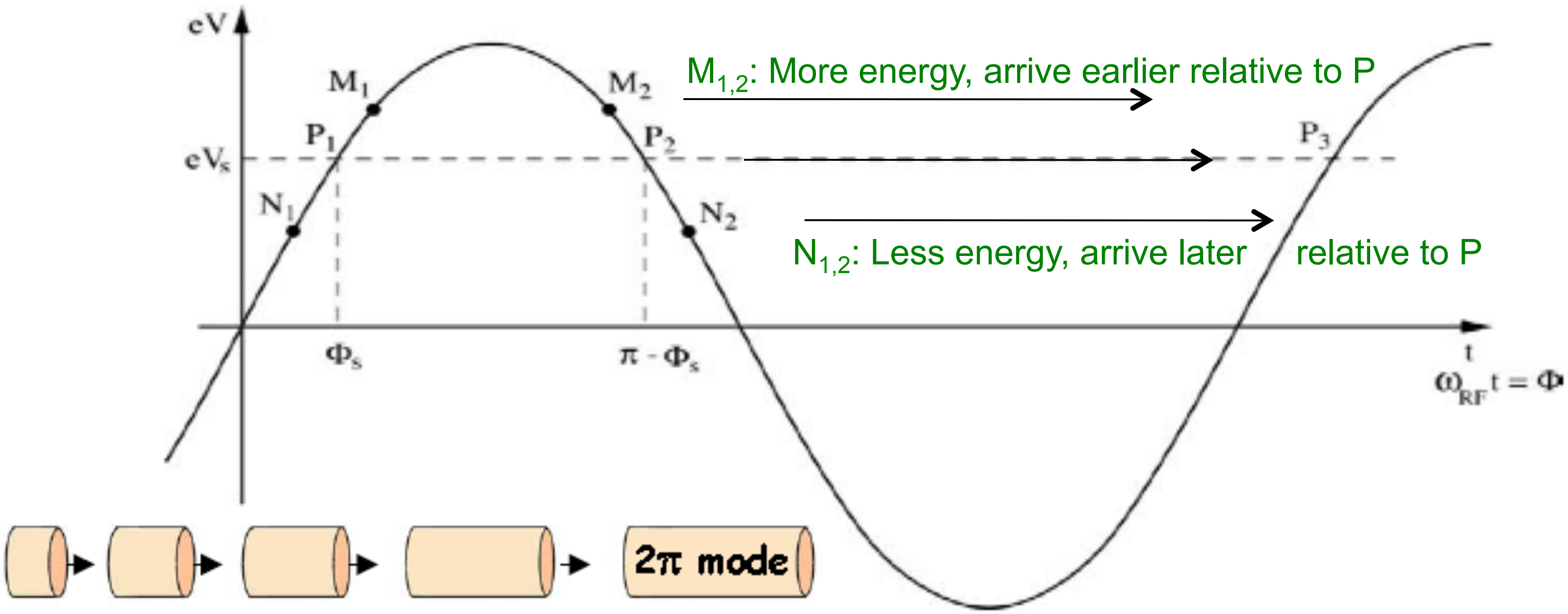
- Betatrons have their challenges
  - Linear aperture scaling
  - Large stored energy/impedance
  - Synchrotron radiation losses
  - Half-duty cycle
  - Only accelerate negatively charged particles




This will only hurt a bit…

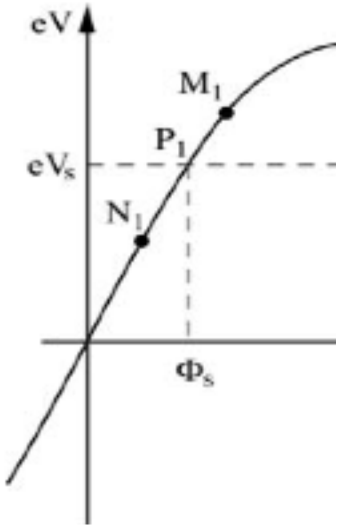
- More on betatrons/weak focusing in a bit

# Phase Stability




- Consider a series of accelerating gaps (or a ring with one gap)
  - **By design** there is a synchronous phase $\Phi_s$ that gains just enough energy to hit phase $\Phi_s$ in the next gap
  - $P_{1,2}$ are fixed points: they "ride the wave" exactly in phase
- If increased energy means increased velocity ("below transition")
  - $M_1$,$N_1$ will move towards $P_1$ (local stability) => **phase stability**
  - $M_2$,$N_2$ will move away from $P_2$ (local instability)

# Phase Stability Implies Transverse Instability




$$\frac{\partial V}{\partial t} > 0 \quad \Rightarrow \quad \frac{\partial E_z}{\partial z} < 0$$

- For phase stability, longitudinal electric field must have a negative gradient. But then Maxwell says (no plasma)

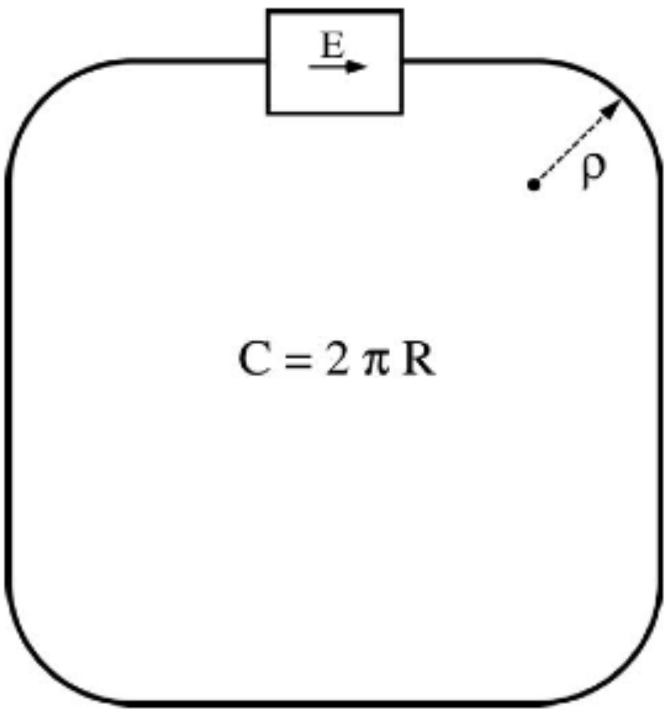
$$\vec{\nabla} \cdot \vec{E} = 0 \quad \Rightarrow \quad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \quad \Rightarrow \quad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} > 0$$

There must be some transverse defocusing/diverging force!

Any accelerator with RF phase stability (longitudinal focusing) needs transverse focusing! (solenoids, quads…)

# The Synchrotron

The synchrotron is a synchronous accelerator since there is a synchronous RF phase for which the energy gain fits the increase of the magnetic field at each turn. That implies the following operating conditions:




$e\hat{V}\sin\Phi \longrightarrow$ Energy gain per turn

$\Phi = \Phi_s = cte \longrightarrow$ Synchronous particle

$\omega_{RF} = h\omega_r \longrightarrow$ RF synchronism

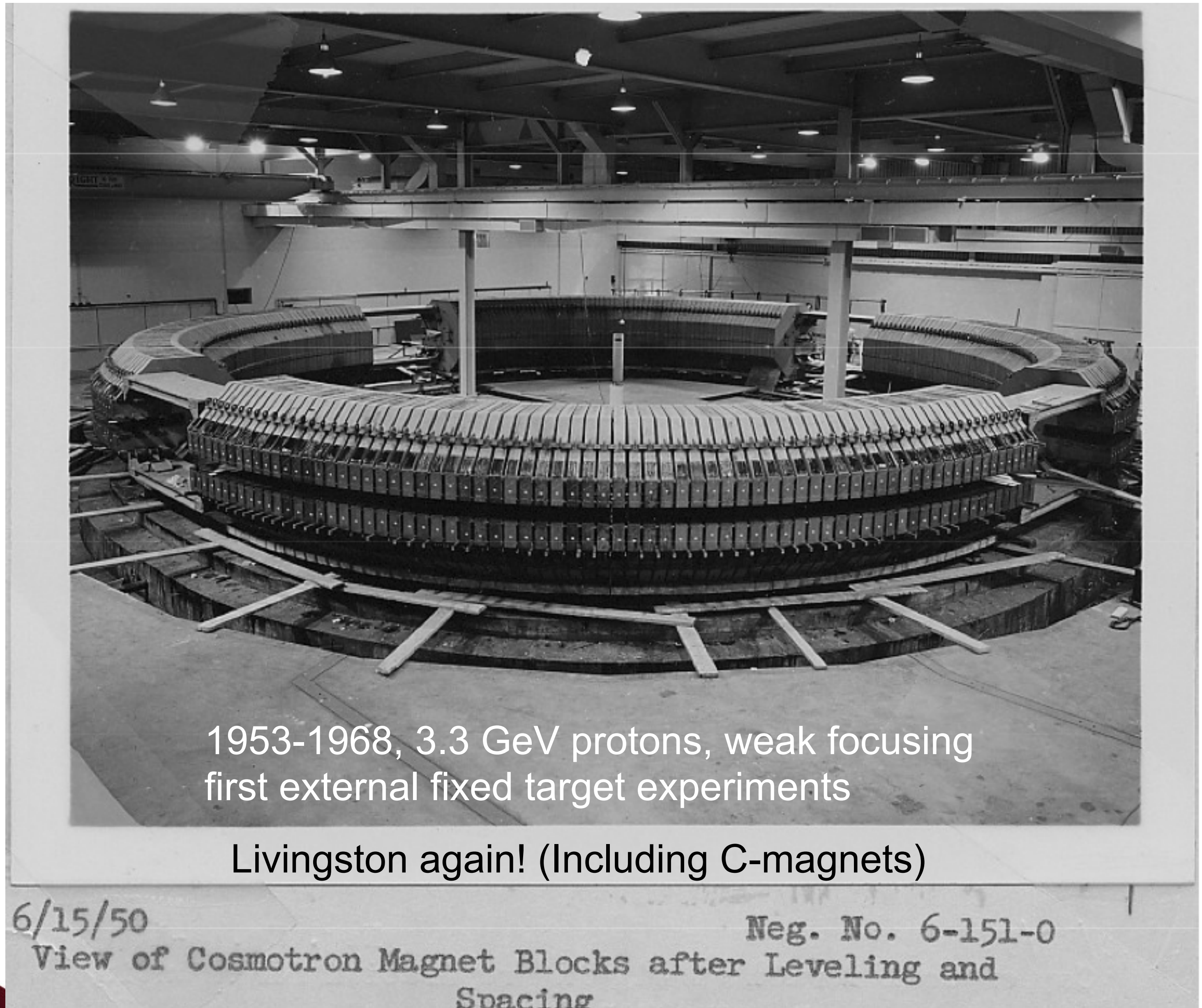
$\rho = cte \quad R = cte \longrightarrow$ Constant orbit

$B\rho = P/e \Rightarrow B \longrightarrow$ Variable magnetic field

If v = c, $\omega_r$ hence $\omega_{RF}$ remain constant (ultra-relativistic $e^-$)

Joel LeDuff, CERN Accel School July 2003

# BNL Cosmotron




1953-1968, 3.3 GeV protons, weak focusing
first external fixed target experiments

Livingston again! (Including C-magnets)

National Academy of Sciences, Biographical Memoir of M. Stanley Livingston by Ernest D. Courant

# LBL Bevatron


- Last and largest weak-focusing proton synchrotron
- 1954, Beam aperture about 4' square!, beam energy to 6.2 GeV
- Discovered antiproton 1955, 1959 Nobel for Segre/Chamberlain

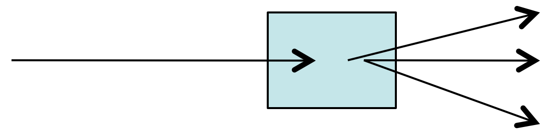
(Became Bevelac, decommissioned 1993, demolished recently)

# Fixed Target Experiments

- Why did the Bevatron need 6.2 GeV protons?
  - Antiprotons are “only” 930 MeV/$c^2$ (times 2…)
  - Bevatron used Cu target, p+n->p+n+p+pbar
  - Mandelstam variables give:

$$\frac{E_{\mathrm{cm}}^2}{c^2} = 2\left(\frac{E_1 E_2}{c^2} + p_{\mathrm{z1}} p_{\mathrm{z2}}\right) + (m_{01}c)^2 + (m_{02}c)^2$$

- Fixed Target experiment



$$(4m_{\mathrm{p0}}c)^2 < \frac{E_{\mathrm{cm}}^2}{c^2} = 2\frac{E_1 m_{\mathrm{p0}}}{c^2} + 2(m_{\mathrm{p0}}c)^2 \quad \Rightarrow \quad E_1 > 7m_{\mathrm{p0}}c^2$$

$$E_{\mathrm{cm}} = \sqrt{2E_1(m_{02}\, c^2)}$$

  - Available CM energy scales with root of beam energy
    - Main issue: forward momentum conservation steals energy

# Two Serious Problems

- These machines were getting way too big
  - Bevatron magnet was 10,000 tons
  - Apertures scale linearly with machine size, energy

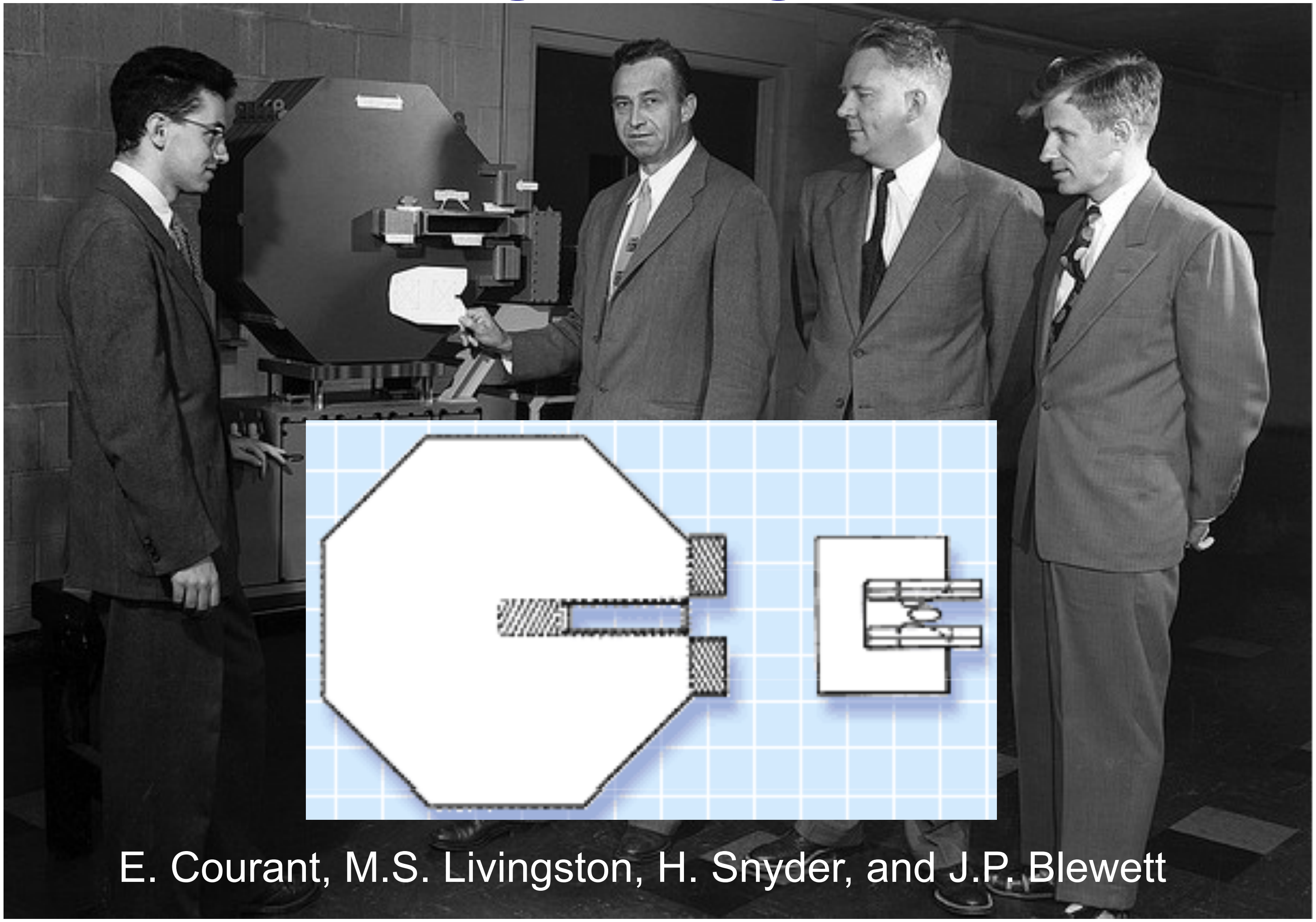
    (Length/circumference scales linearly with energy at fixed field strength too…)

- Fixed target energy scaling is painful
  - Available CM energy only scales with $\sqrt{E_{beam}}$

- Accelerator size grew with the square of desired CM energy
  - Something had to be done!!!

Strong Focusing (1952) and Colliders (1958-62ish) to the rescue!!!

# Livingston *Again*?



E. Courant, M.S. Livingston, H. Snyder, and J.P. Blewett

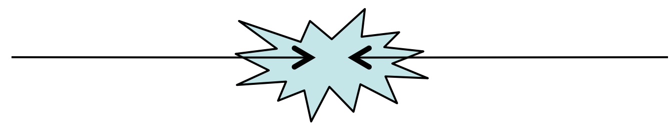
- Strong focusing (and its mathematical treatment) is really the focus (ha) of the rest of this week

# Collider Experiments

- What if the Bevatron was a collider?
  - Antiprotons are "only" 930 MeV/$c^2$ (times 2…)
  - Two-body system (Mandelstam variables) gives (again):

$$\frac{E_{\mathrm{cm}}^2}{c^2} = 2\left(\frac{E_1 E_2}{c^2}\right) + p_{z1} p_{z2} + (m_{01} c)^2 + (m_{02} c)^2$$

- Case 2: Collider



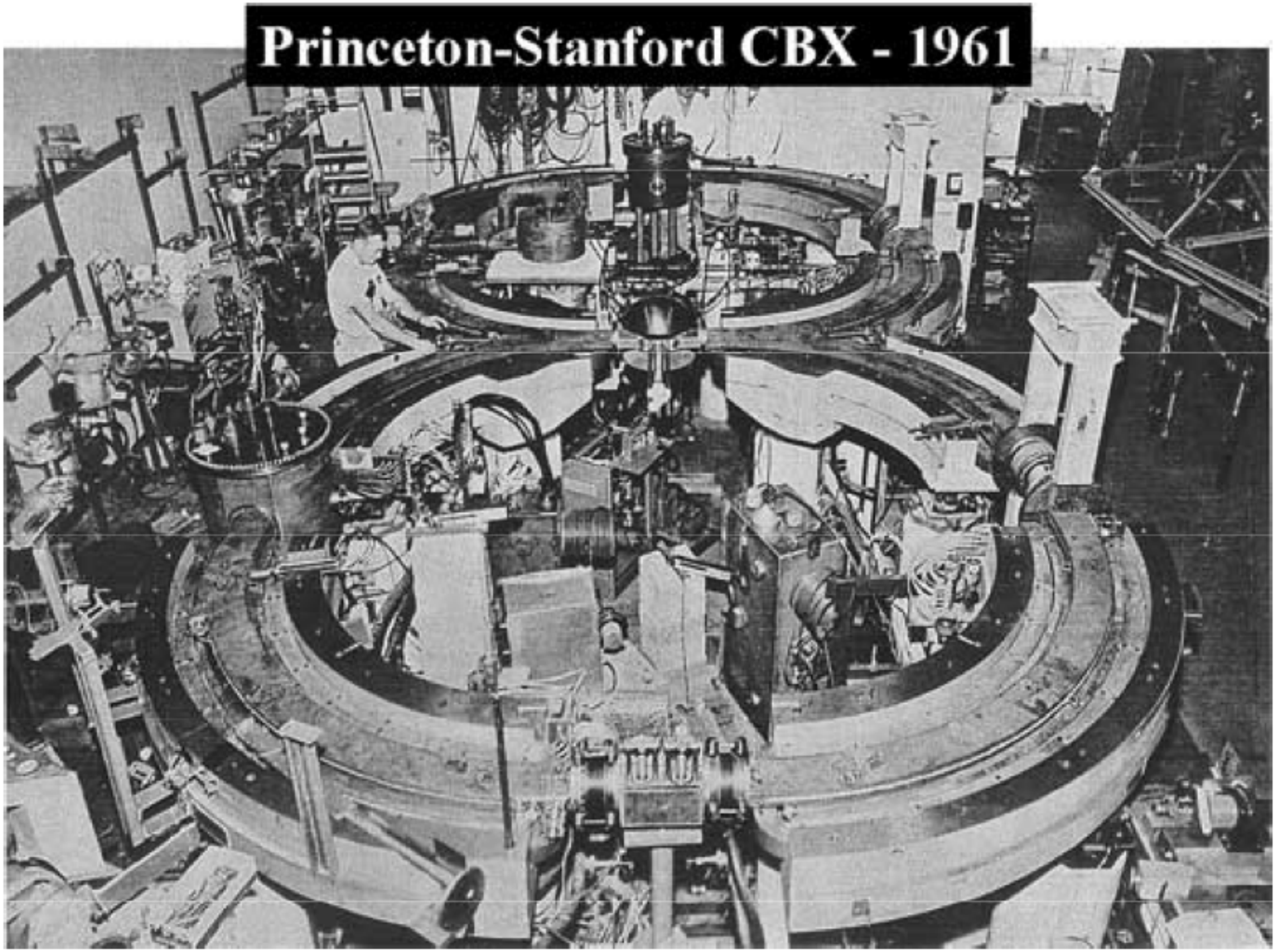
$$E_1 \gg m_{01} c^2 \qquad E_2 \gg m_{02} c^2$$

$$E_{\mathrm{cm}} = 2\sqrt{E_1 E_2} = 2E \quad \text{if } E_1 = E_2$$

  - Linear scaling with beam energy!
  - For Bevacollidatron, e- + e+ -> p+pbar is possible!

(Although the cross section is probably pretty small)

# First Electron Collider



# Cambridge Electron Accelerator

*THE CEA TEAM, 1959.* *The group that led the Cambridge Electron Accelerator (CEA) in Cambridge, Massachusetts. The machine was later converted for colliding beam experiments, testing the technique of 'low-beta' that proved so important in storage rings. Seated from left: Thomas Collins and David Jacobus. Standing from left: Fred Barrington, CEA Director Stanley Livinston, Robert Cummings, Lee Young, John Rees, William Jones, Janez Dekkra, and Kenneth Robinson (deceased).*

SLAC Beam Line, "Colliding Beam Storage Rings", John Rees, Mar 1986

# Luminosity

- **Luminosity** L is a measure of how many interactions of **cross section** σ can be created per unit time

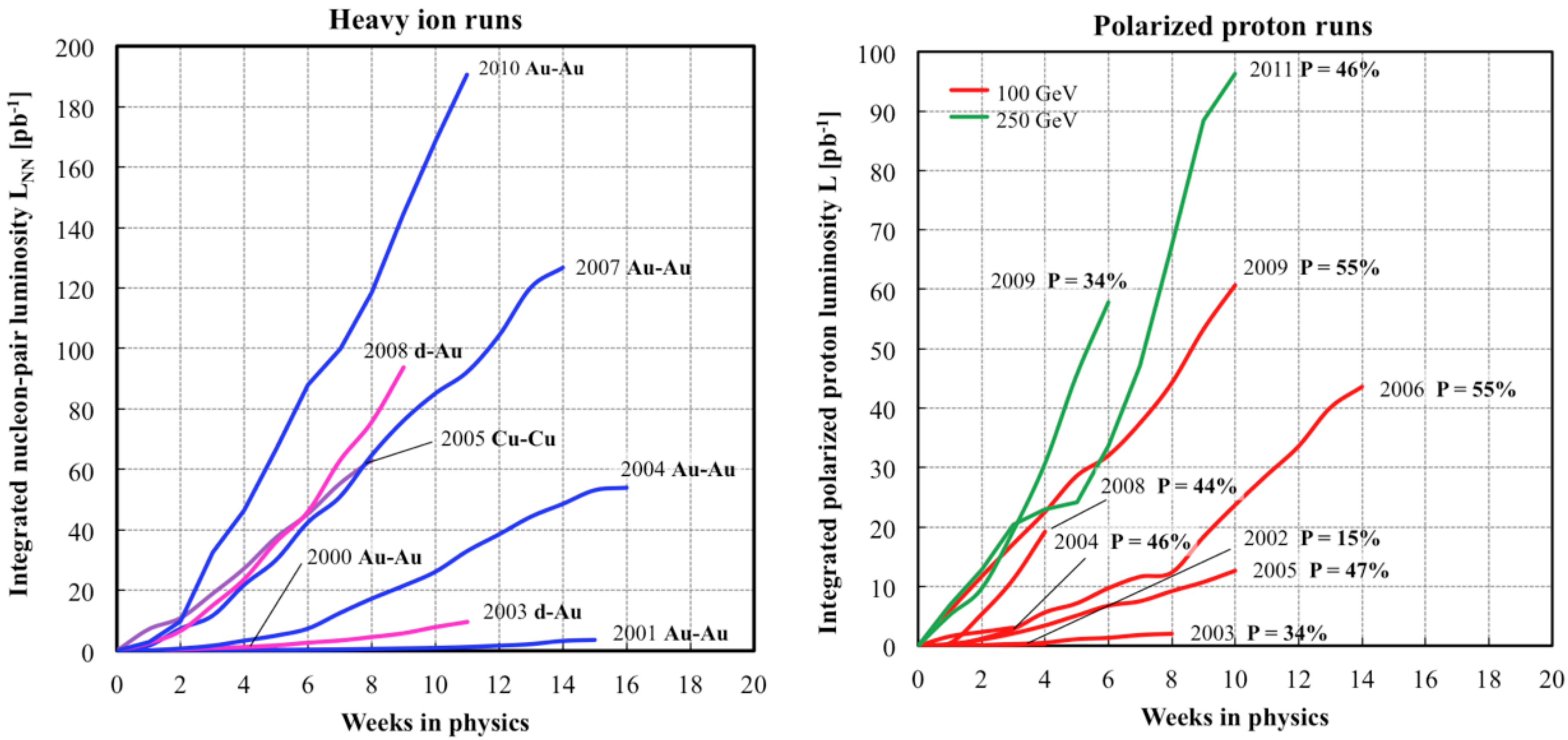
$$L\sigma = \frac{dN}{dt} \qquad N = \sigma \int L\, dt = \sigma\, L_{\rm int}$$

  - $L_{int}$ is integrated luminosity, an important factor of production for colliders
  - $[L]=cm^{-2}\ s^{-1}$, $[L_{int}]=cm^{-2}$ (1 ba=$10^{-24}$ cm; 1 $pb^{-1}=10^{36}$ $cm^{-2}$)
- For equal-sized head-on Gaussian beams in a collider

$$L = \frac{f_{\rm rev}\ h\ N_1\ N_2}{4\pi\sigma_x\sigma_y}$$

  - $\sigma_{x,y}$ are rms beam sizes, h is number of bunches
    - Colliding 100 μm 7.5e9p bunches at 100 kHz for 1 year gives about 1 $pb^{-1}$ of integrated luminosity
    - See Appendix D of the text for more details about luminosity

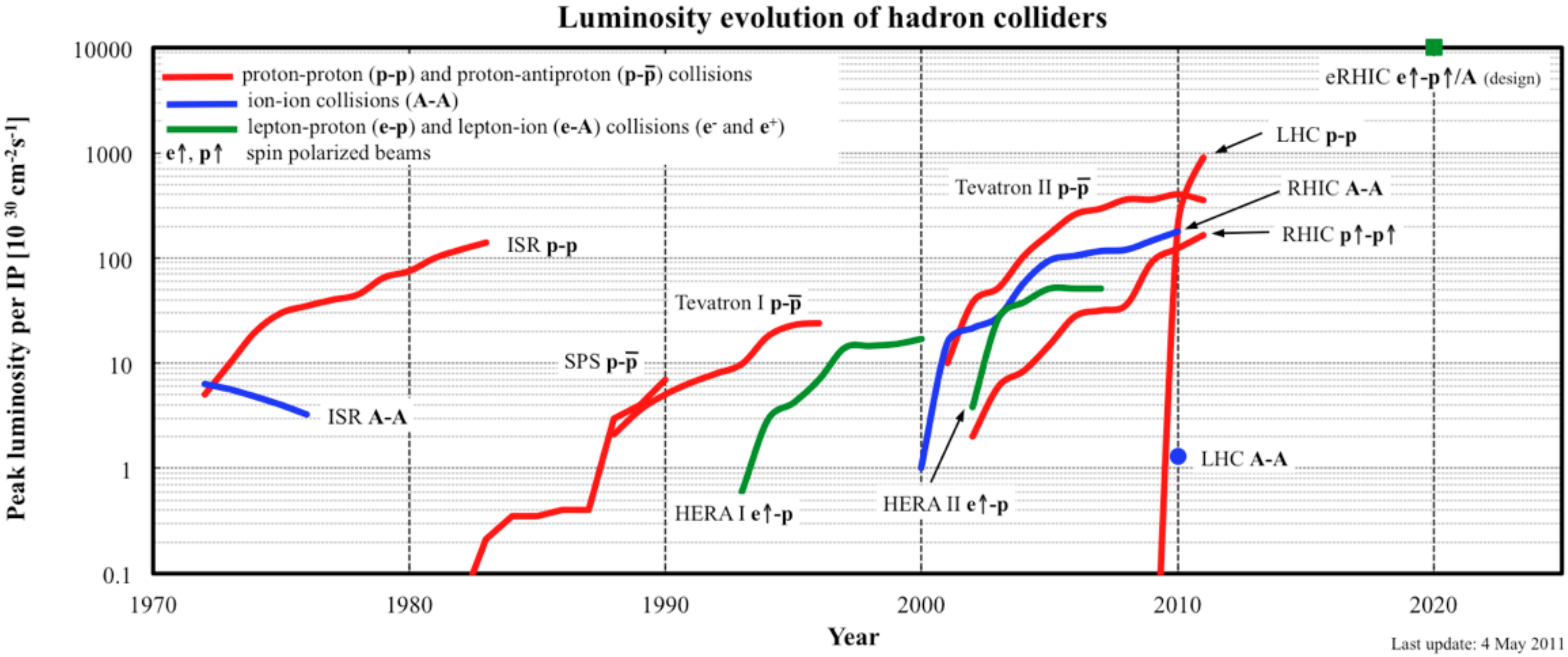
# Evolution of RHIC Collider Luminosities




Note: The nucleon-pair luminosity is defined as $L_{NN} = A_1 A_2 L$, where $L$ is the luminosity, and $A_1$ and $A_2$ are the number of nucleons of the ions in the two beam respectively.

W. Fischer, http://www.rhichome.bnl.gov/RHIC/Runs

# Evolution of Hadron Collider Luminosities

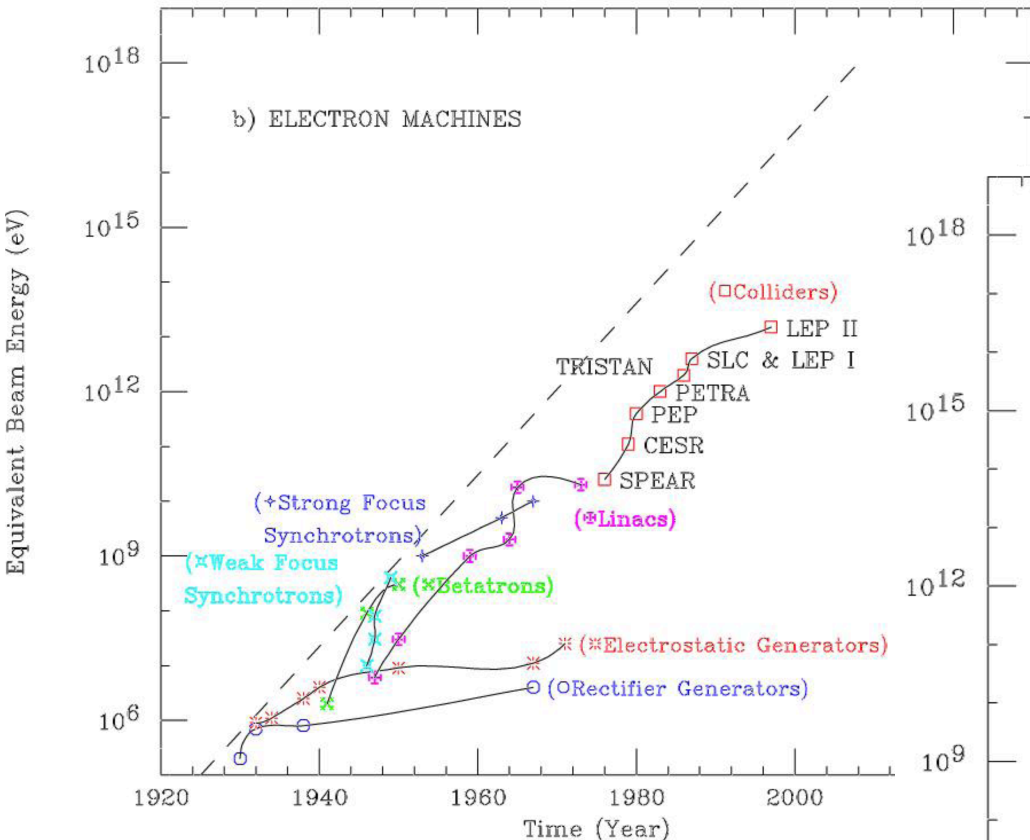


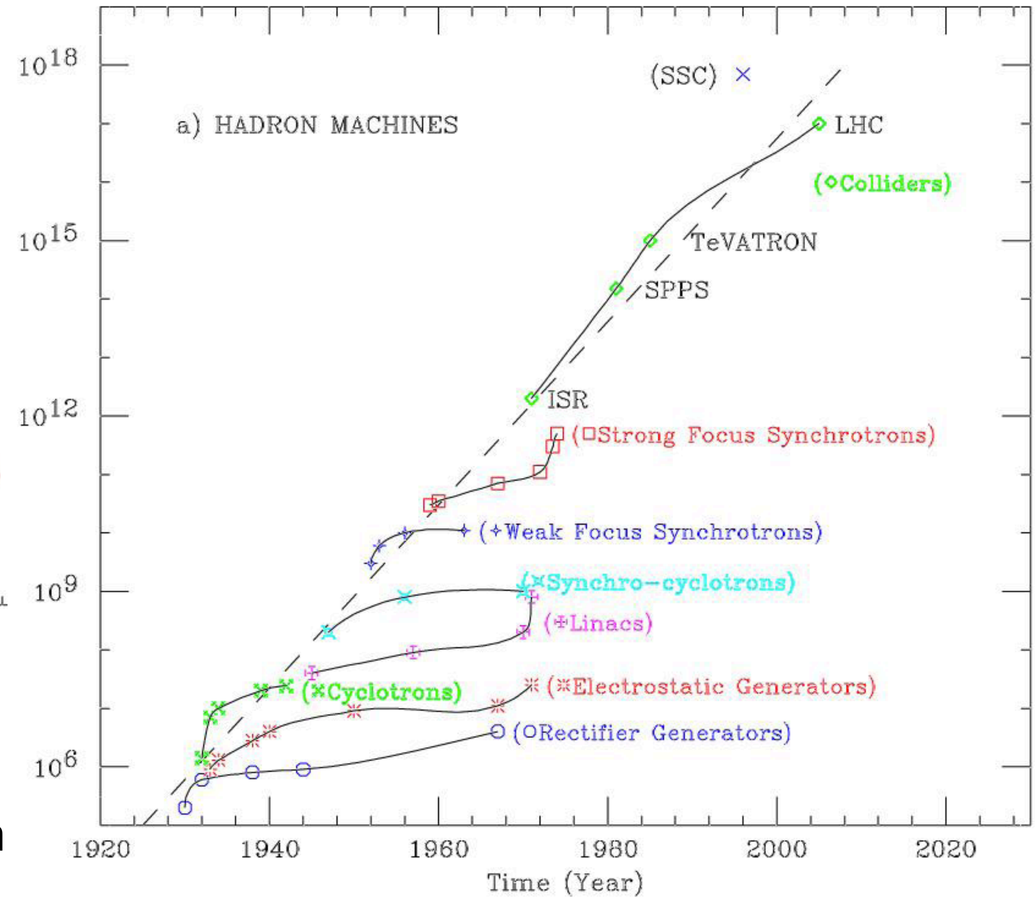

Note: For ion collisions the nucleon-pair luminosity is shown. The nucleon-pair luminosity is defined as $L_{NN} = A_1 A_2 L$, where $L$ is the luminosity, and $A_1$ and $A_2$ are the number of nucleons of the ions in the two beam respectively. The highest energies for the machines are: ISR 31 GeV, SPS 315 GeV, Tevatron 980 GeV, HERA 920 GeV (p) 27.5 GeV (e), RHIC 250 GeV, LHC 3.5 TeV.

W. Fischer, http://www.rhichome.bnl.gov/RHIC/Runs

# Livingston Plots

Livingston observed that accelerator energy was growing exponentially (in 1950!)



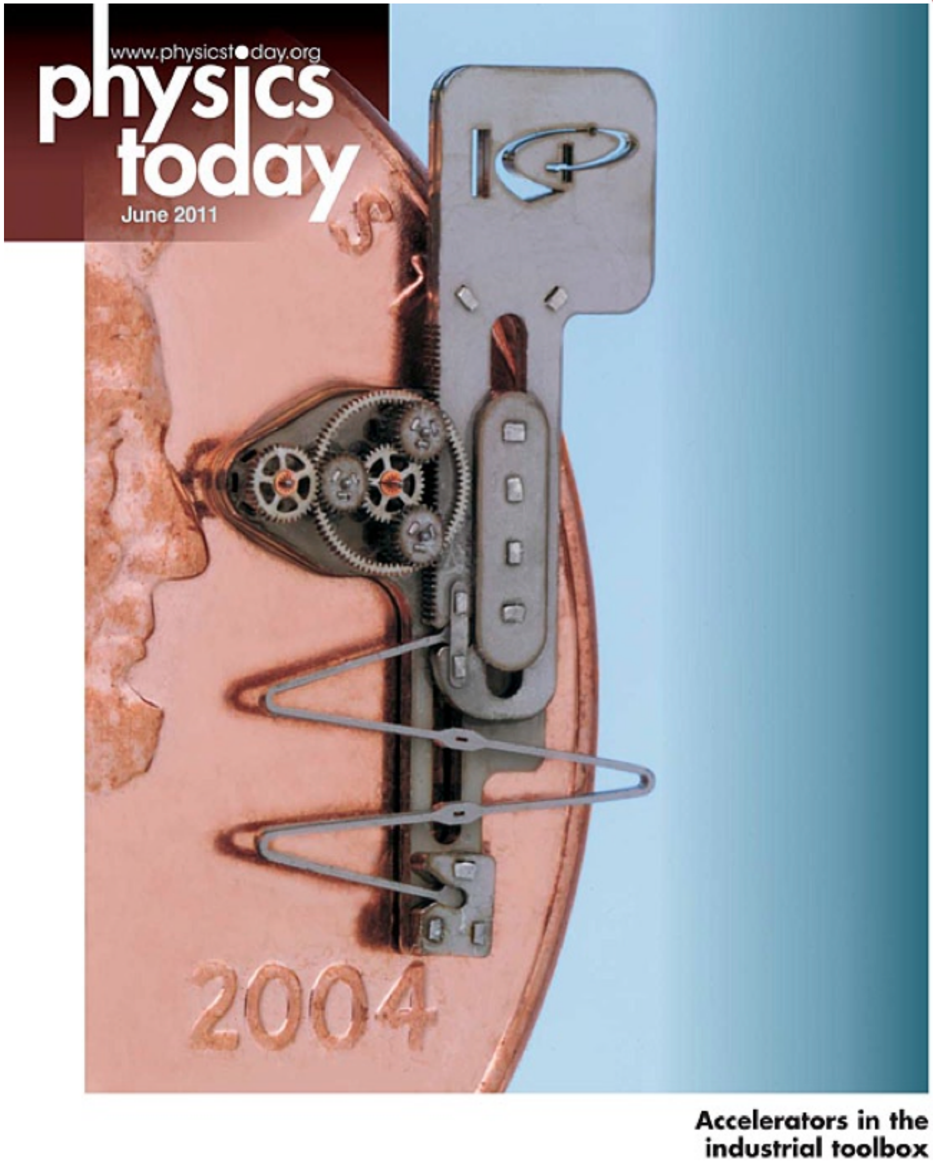

Still holds true over 60 years (!!!) later

Technologies tend to saturate then new technologies are introduced

(G. Hoffstaetter, Cornell)

# Cover Of The Rolling Stone

- Accelerators make the cover of June 2011 Physics Today
  - Micromachining example from synchrotron light
- Industrial applications



# ========= Extra Slides =========

# Lorentz Lie Group Generators I

- Lorentz transformations can be described by a Lie group where a general Lorentz transformation is

$$A = e^{L} \qquad \det A = e^{\mathrm{Tr}\, L} = +1$$

where L is 4x4, real, and traceless. With metric g, the matrix gL is also antisymmetric, so L has the general six-parameter form

$$L = \begin{pmatrix} 0 & L_{01} & L_{02} & L_{03} \\ L_{01} & 0 & L_{12} & L_{13} \\ L_{02} & -L_{12} & 0 & L_{23} \\ L_{03} & -L_{13} & -L_{23} & 0 \end{pmatrix}$$

Deep and profound connection to EM tensor $F^{\alpha\beta}$

J.D. Jackson, Classical Electrodynamics 2nd Ed, Section 11.7

# Lorentz Lie Group Generators II

- A reasonable basis is provided by six generators
  - Three generate rotations in three dimensions

$$S_{1,2,3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

  - Three generate boosts in three dimensions

$$K_{1,2,3} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

# Lorentz Lie Group Generators III

- $(S_{1,2,3})^2$ and $(K_{1,2,3})^2$ are diagonal.
- $(\epsilon \cdot S)^3 = -\epsilon \cdot S$ and $(\epsilon \cdot K)^3 = \epsilon \cdot K$ for any unit 3-vector $\epsilon$
- Nice commutation relations:

$$[S_i, S_j] = \epsilon_{ijk} S_k \quad [S_i, K_j] = \epsilon_{ijk} K_k \quad [K_i, K_j] = -\epsilon_{ijk} S_k$$

- We can then write the Lorentz transformation in terms of two three-vectors (6 parameters) $\omega, \zeta$ as

$$L = -\omega \cdot S - \zeta \cdot K \qquad A = e^{-\omega \cdot S - \zeta \cdot K}$$

  - Electric fields correspond to boosts
  - Magnetic fields correspond to rotations
  - Deep beauty in Poincare, Lorentz, Einstein connections