University Physics 226N/231N
Old Dominion University

Motion in One Dimension

Dr. Todd Satogata (ODU/Jefferson Lab)
satogata@jlab.org
http://www.toddsatogata.net/2012-ODU

Wednesday, August 29 2012

Happy Birthday to Lea Michele, Roy Oswalt, John Locke, and Stephen Wolfram
(*Mathematica* and *A New Kind of Science*)

Happy Chop Suey Day and Lemon Juice Day!

Review

- On Monday we discussed
  - What SCALE-UP is and how the class will generally work
  - The class website
    - Including links to the syllabus, schedule, and MasteringPhysics
    - **Register** in MasteringPhysics and catch up on assignments!
    - First assignments are due by next Wednesday, 9 AM
  - A bit of background about physics
  - SI and metric units and their conversions
    - Try to keep units explicitly in all your calculations
    - Google calculator is your friend (as is Wolfram Alpha)
  - Accuracy vs precision
  - Estimations
  - Significant figures and rules for their use
    - Basically how many digits you would write for a number in scientific notation (not including trailing zeros)

Quick Question

- Stretch to wake up while you think about this problem!
- Choose the sequence that correctly ranks the numbers according to the number of significant figures. (Rank from fewest to most.)

A. $0.041\times10^{9},\ 3.14\times10^{7},\ 2.998\times10^{-9},\ 0.0008.$

B. $3.14\times10^{7},\ 0.041\times10^{9},\ 0.0008,\ 2.998\times10^{-9}.$

C. $2.998\times10^{-9},\ 0.041\times10^{9},\ 0.0008,\ 3.14\times10^{7}.$

D. $0.0008,\ 0.041\times10^{9},\ 3.14\times10^{7},\ 2.998\times10^{-9}.$

E. $0.0008,\ 0.041\times10^{9},\ 2.998\times10^{-9},\ 3.14\times10^{7}.$

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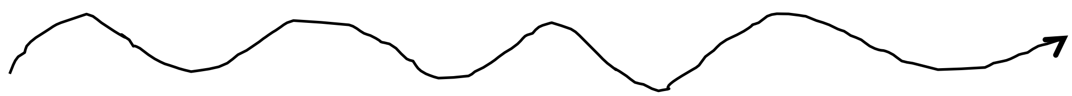
A. 0.041×10^{9}, 3.14×10^{7}, 2.998×10^{-9}, 0.0008.	2,3,4,1 significant figures
B. 3.14×10^{7}, 0.041×10^{9}, 0.0008, 2.998×10^{-9}.	3,2,1,4 significant figures
C. 2.998×10^{-9}, 0.041×10^{9}, 0.0008, 3.14×10^{7}.	4,2,1,3 significant figures
D. 0.0008, 0.041×10^{9}, 3.14×10^{7}, 2.998×10^{-9}.	1,2,3,4 significant figures
E. 0.0008, 0.041×10^{9}, 2.998×10^{-9}, 3.14×10^{7}.	1,2,4,3 significant figures

Motion in One Dimension: Position

- **Position:** an object's location in space relative to a reference point
  - Underlying assumptions include
    - A **coordinate system** (usually a Cartesian x,y,z grid; here just x!)
    - An **origin** in that coordinate system is the reference point ("zero")
    - The object's location is described by a single point
      - We'll deal with complications of orientation in space later this semester
    - That location is a distance x from the reference point

Units!

  - With these assumptions, **ALL motion is one dimensional**
    - As that point moves in time, it traces out a line (one dimension!)

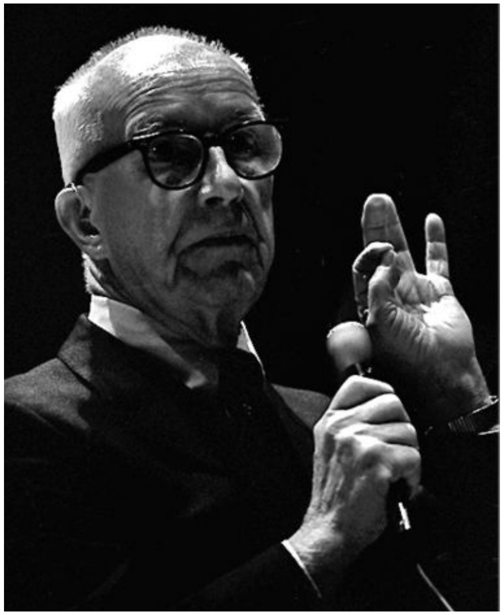


  - To simplify even more, only describe motion in the "x" direction
  - **Observe**: distances (or displacements) Δx don't depend on our choice of an origin!

(The straight line thing is tricky)

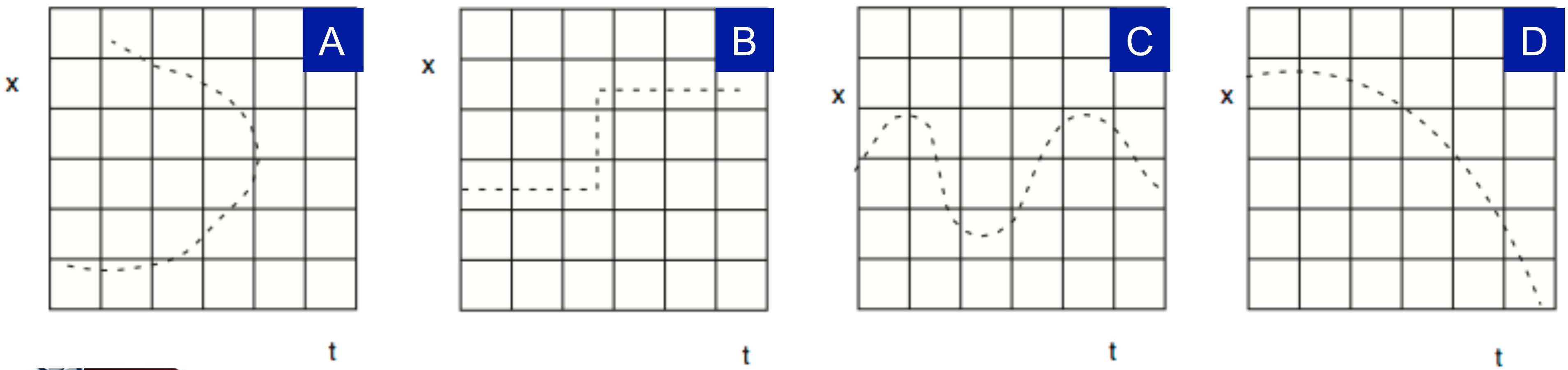
Everything you've learned in school as "obvious" becomes less and less obvious as you begin to study the universe. For example, there are no solids in the universe. There's not even a suggestion of a solid. There are no absolute continuums. There are no surfaces. **There are no straight lines.**

R. Buckminster Fuller



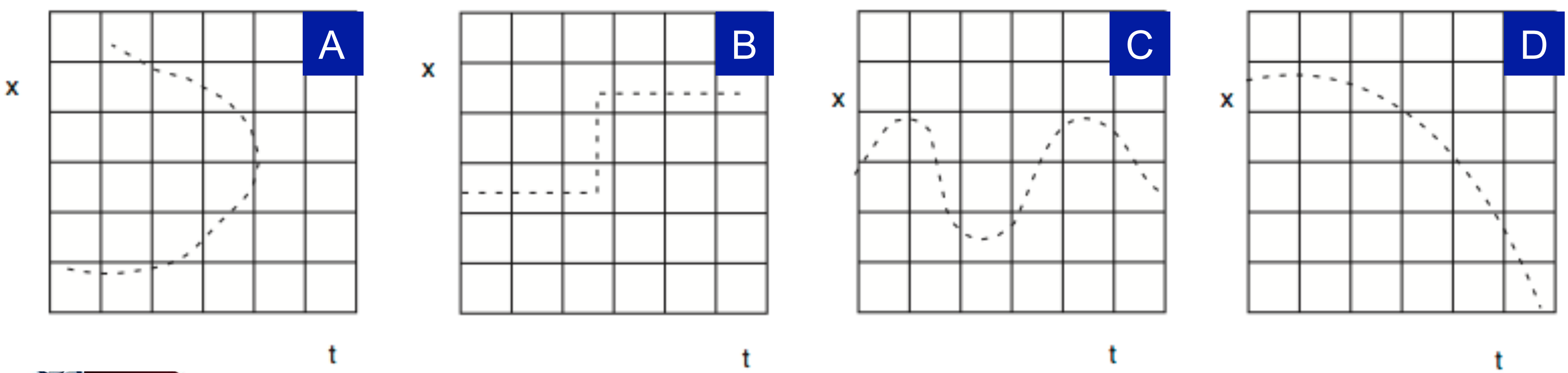
Ponderable: Graphs and Observation (5 minutes)

- Position changes over time -- it is a **function** of time: $x(t)$
- Physics involves relating physical quantities together over ranges of their possible values
  - Graphs are an excellent way of visualizing relationships between numerical quantities
- Ponderables (5 minutes)
  - These plots depict an object's position x as a function of time t.
  - For each plot, describe the motion in words or explain why it is not a type of motion we see in normal everyday objects.



Ponderable: Graphs and Observation

- A: Normal objects are usually not in two places at once, and they usually have a well-defined position for all time.
- B: Normal objects usually don't suddenly jump from one location to another.
- C: Normal objects can oscillate in time, like a mass on a spring or a pendulum (we cover those later this semester)
- D: Normal objects can move parabolically (we cover that soon!)
  - I emphasize "normal" to avoid questions of quantum mechanics.
    - Change in displacement over a given time is a slope on these plots



Motion in One Dimension: Velocity

- **Velocity:** How far an object moves Δx in a given time interval Δt : $v = \Delta x / \Delta t$

Units!

  - Velocity, like position and displacement, is a **vector** with magnitude and direction
    - We use **speed** for just the magnitude in common language
  - Velocity, like position, is a **function** of time. Over a finite time period, we call this **average velocity**:

$$\boxed{\bar{v}(t) = \Delta x(t) / \Delta t}$$

  - You've had calculus so you can see what's coming!

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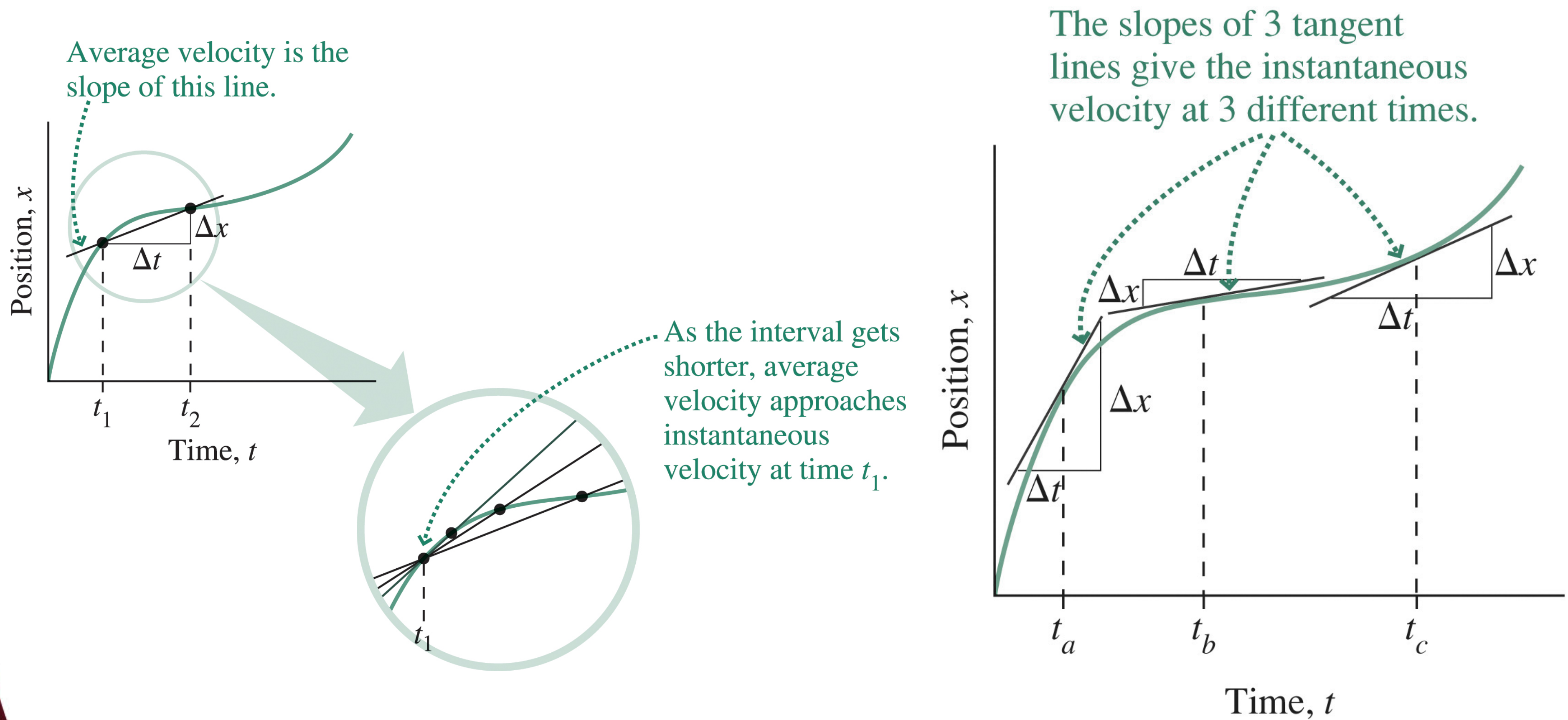
  - Indeed, **instantaneous velocity** is the **derivative** of position

$$\boxed{v(t) = dx(t)/dt}$$

    - Remember that a derivative is really just the slope of a curve as measured *very* locally around a point on that curve at time t

Velocity is a Slope

– Velocity is the derivative (slope) of the curve of x(t)



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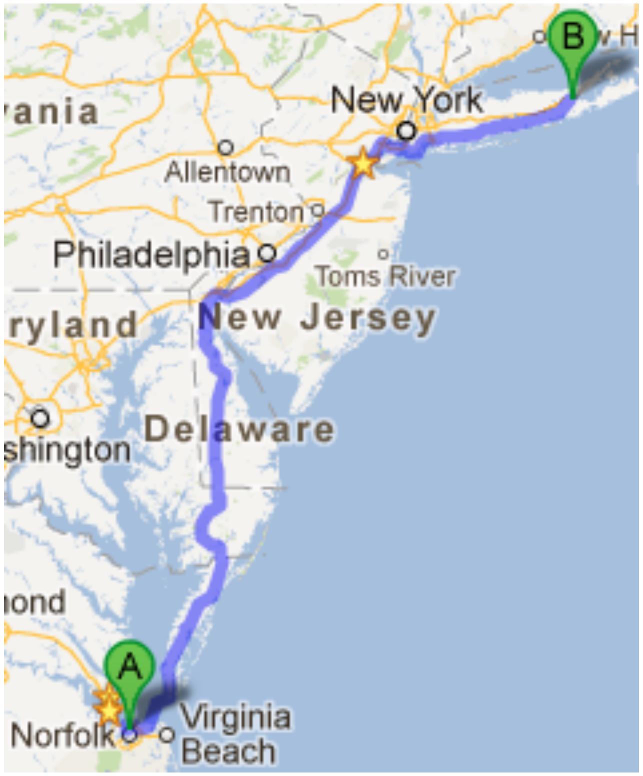
- That means that velocity is **also** a function and we can plot velocity v(t) like we plot position x(t)
  - And then we can take the derivative of the velocity too!
  - But first, a ponderable…

Ponderable (10 minutes)

- Someone you know might be driving to New York this weekend (for a fantasy football draft)
  - Distance to his destination is 440 miles.
  - Fortunately the speed limit is 65 mph.
  - What does the driver's average speed need to be if he wants to get there in 8.0 hours? (Note: two significant figures!)
  - What does his average speed need to be if he hits a traffic jam where his average speed is 20 mph for 2.0 of those hours?
  - Can you tell whether his instantaneous velocity ever exceeded the speed limit in either case?



Acceleration

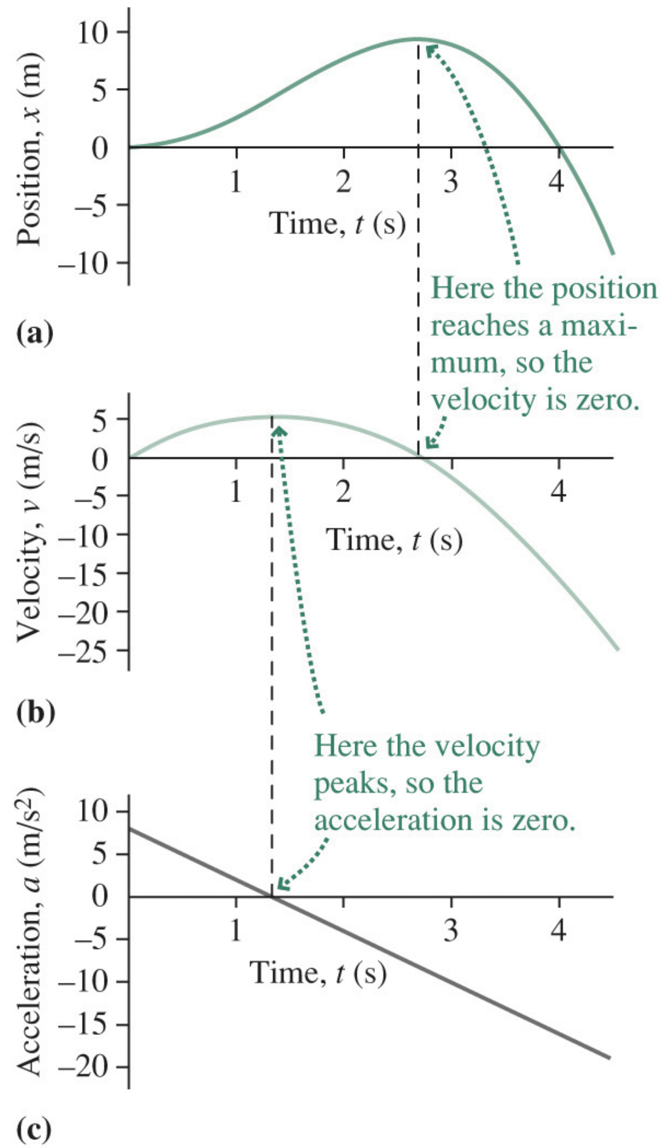
- **Acceleration** is the rate of change of velocity.
  - Exactly like velocity was the rate of change of position!
  - **Average velocity** over a time interval Δt is defined as the change in velocity divided by the time:

$$\boxed{\bar{a} = \frac{\Delta v}{\Delta t}}$$

  - **Instantaneous acceleration** is the limit of the average acceleration as the time interval becomes arbitrarily short:

$$\boxed{a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}}$$

– Acceleration is the slope of v(t)



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Position, Velocity, and Acceleration

- Individual or absolute values of position, velocity, and acceleration are not related.
    - Instead, velocity depends on the *rate of change* of position.
    - Acceleration depends on the *rate of change* of velocity.
        - These are all **relative** quantities, and **not** based on absolute position or position of the origin
        - This makes our description of this motion **universal**
    - An object can be at position $x = 0$ and still be *moving.*
    - An object can have zero velocity and still be *accelerating*.

- At the peak of its trajectory, a juggling thud has
    - Maximum vertical displacement from my hand
    - Zero vertical velocity
    - Constant negative acceleration due to the force of gravity

Ponderable (10 minutes)

- The best way to start to learn to juggle is by juggling one item with one hand

  Even 5 year olds are physicists

- Draw graphs of x(t), v(t), and a(t) for the juggling "thud" when
  - I successfully throw it up in the air, then catch it in the same place as it comes down
  - I throw it up in the air, then miss it, and it lands on the ground

  Hence the name "thud"

Label your axes!

Constant Acceleration

- When acceleration is constant, then position, velocity, acceleration, and time are related by

$$v = v_0 + at$$

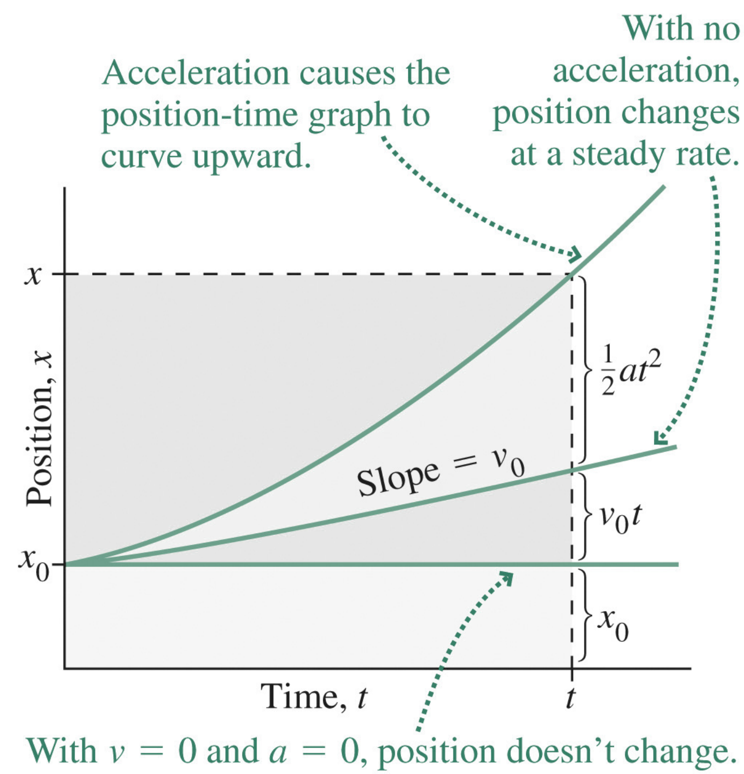
$$x = x_0 + \tfrac{1}{2}(v_0 + v)t$$

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

where x_0 and v_0 are initial values at time $t = 0$, and x and v are the values at an arbitrary time t.

- With constant acceleration
  - Velocity is a linear function of time
  - Position is a quadratic function of time



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The Acceleration of Gravity

- The acceleration of gravity at any point is exactly the same for all objects, regardless of mass.
- Near Earth's surface, the value of the acceleration is essentially constant at $g = 9.8\,\mathrm{m/s^2} = 32\,\mathrm{ft/s^2}$
- Therefore the equations for constant acceleration apply:
  - In a coordinate system with *y* axis upward, they read

$$v = v_0 - gt$$

$$y = y_0 + \tfrac{1}{2}(v_0 + v)t$$

$$y = y_0 + v_0 t - \tfrac{1}{2} g t^2$$

$$v^2 = v_0^2 - 2g(y - y_0)$$

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This strobe photo of a falling ball shows increasing spacing resulting from the acceleration of gravity.

(Honors Ponderable)

- Assume that the 13 ball in the photo is a "standard" 2.25 inch diameter billiard ball
- How fast is the strobe flashing between images of the falling ball?
- What is the ball's approximate instantaneous velocity in the first image? In the last?

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This strobe photo of a falling ball shows increasing spacing resulting from the acceleration of gravity.

Example: The Acceleration of Gravity

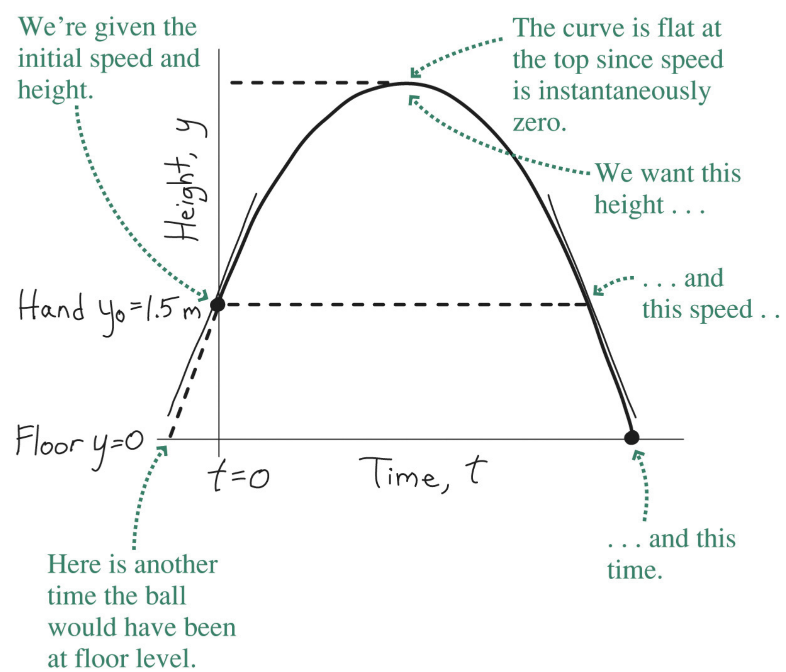
- A ball is thrown straight up at 7.3 m/s, leaving your hand 1.5 m above the ground. Find its maximum height and when it hits the floor.
  - At the maximum height the ball is instantaneously at rest (even though it's still *accelerating*). Solving the last equation with $v = 0$ gives the maximum height:

$$0 = v_0^2 - 2g\left(y - y_0\right)$$

or

$$y = y_0 + \frac{v_0^2}{2g} = 1.5\text{ m} + \frac{(7.3\text{ m/s})^2}{(2)(9.8\text{ m/s}^2)} = 4.2\text{ m}$$

- Setting $y = 0$ in the third equation gives a quadratic in time; the result is the two values for the time when the ball is on the floor:
  $t = -0.18$ s and $t = 1.7$ s
- The first answer tells when the ball *would have been* on the floor if it had always been on this trajectory; the second is the answer we want.



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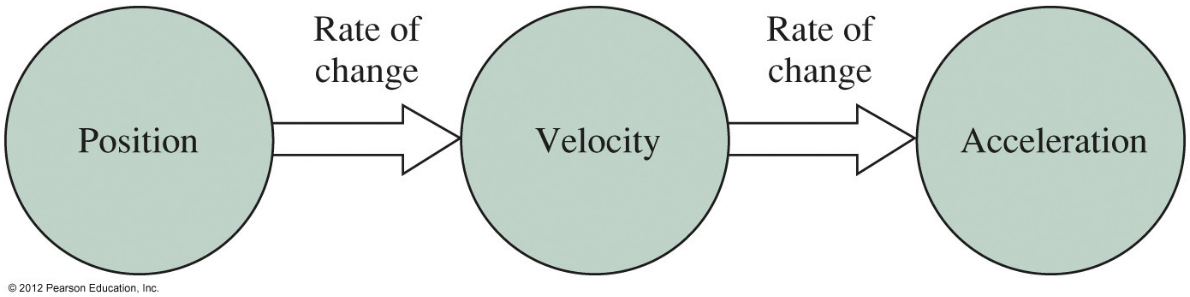
Summary

- Position, velocity, and acceleration are the fundamental quantities describing motion.
  - Velocity is the rate of change of position.
  - Acceleration is the rate of change of velocity.



- When acceleration is constant, simple equations relate position, velocity, acceleration, and time.
  - An important case is the acceleration due to gravity near Earth's surface.
  - The magnitude of the gravitational acceleration is g = 9.8 m/s^2 = 32 ft/s^2.

$$v = v_0 + at$$

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

=========== EXTRA SLIDES ===========

(Honors Estimation)

- These are extra problems if the honors students are feeling up to the challenge. I' ll include at least one in every class.

- About how far away is the Earth's horizon…

  - For a 6' tall person standing at ground level?
  - For someone looking out the window of a plane flying at 30,000 feet?

  - Assume a spherical Earth, of course… ☺

  - (Hint: To within a few percent, each time zone of the Earth is about 1000 miles at the equator. We' ll use this later in the semester.)