University Physics 226N/231N
Old Dominion University

Motion in One Dimension

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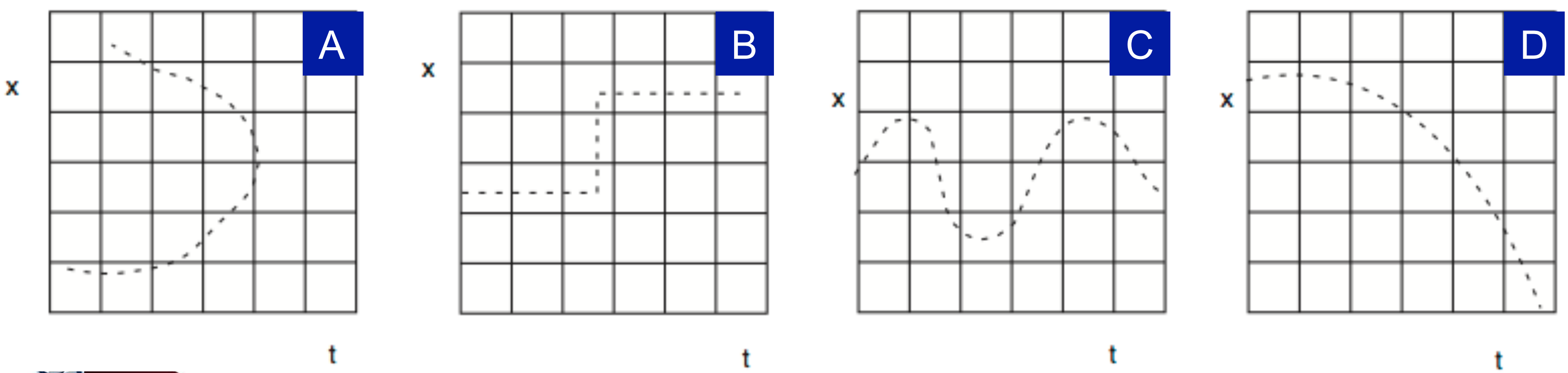
Happy Birthday to Hermann von Helmholtz, H. David Politzer (Physics Nobel 2004), and Larry Fitzgerald (Arizona Cardinals)

Happy International Blog Day, Eat Outside Day, and last blue moon 'til 2015!

Ponderable: Graphs and Observation

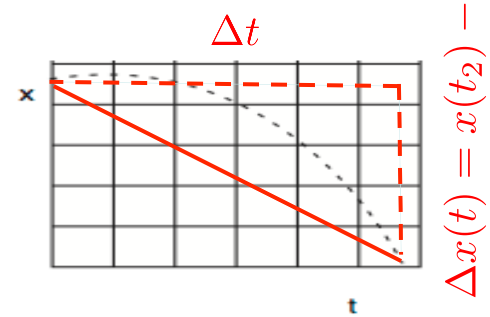
- A: Normal objects are usually not in two places at once, and they usually have a well-defined position for all time.
- B: Normal objects usually don't suddenly jump from one location to another.
- C: Normal objects can oscillate in time, like a mass on a spring or a pendulum (we cover those later this semester)
- D: Normal objects can move parabolically (we cover that soon!)
  - I emphasize "normal" to avoid questions of quantum mechanics.
    - Change in displacement over a given time is a slope on these plots



Motion in One Dimension: Velocity

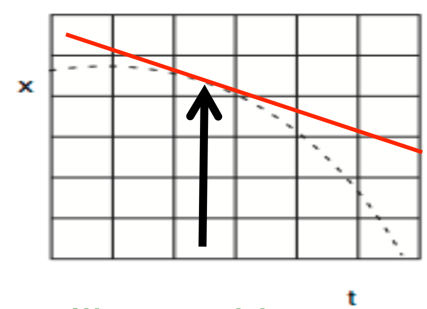
- **Velocity:** How far an object moves Δx in a given time interval Δt : $v = \Delta x / \Delta t$
  - Velocity, like position and displacement, is a **vector** with **magnitude** and **direction**
    - We use **speed** for just the magnitude in common language
  - Velocity, like position, is a **function** of time. Over a finite time period, we call this **average velocity**:

$$\bar{v}(t) = \Delta x(t) / \Delta t$$



  - (Let's skip the calculus for now…)
  - **Instantaneous velocity** is the **slope** of position over a very small time, as

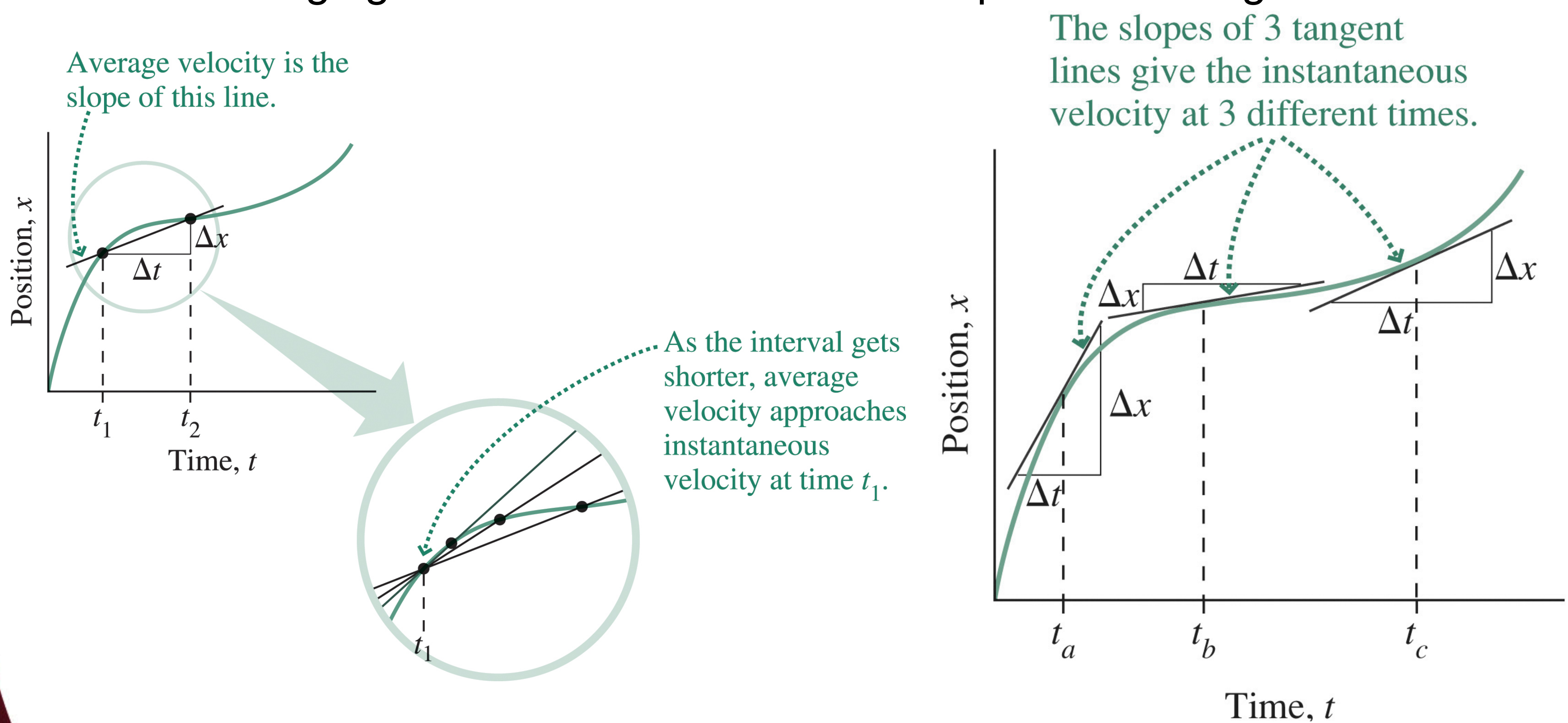
$$v(t) = \Delta x(t) / \Delta t \quad (\text{for very small } \Delta \text{t})$$



We can figure this out for any time t, and it's continuous like position

Velocity is a Slope

– Velocity is the slope of the curve of x(t): how fast position is changing with time. Note that it can be positive or negative!



- That means that velocity is **also** a function and we can plot velocity v(t) like we plot position x(t)
  - And then we can figure out how the velocity is changing (take slopes) too!

Acceleration

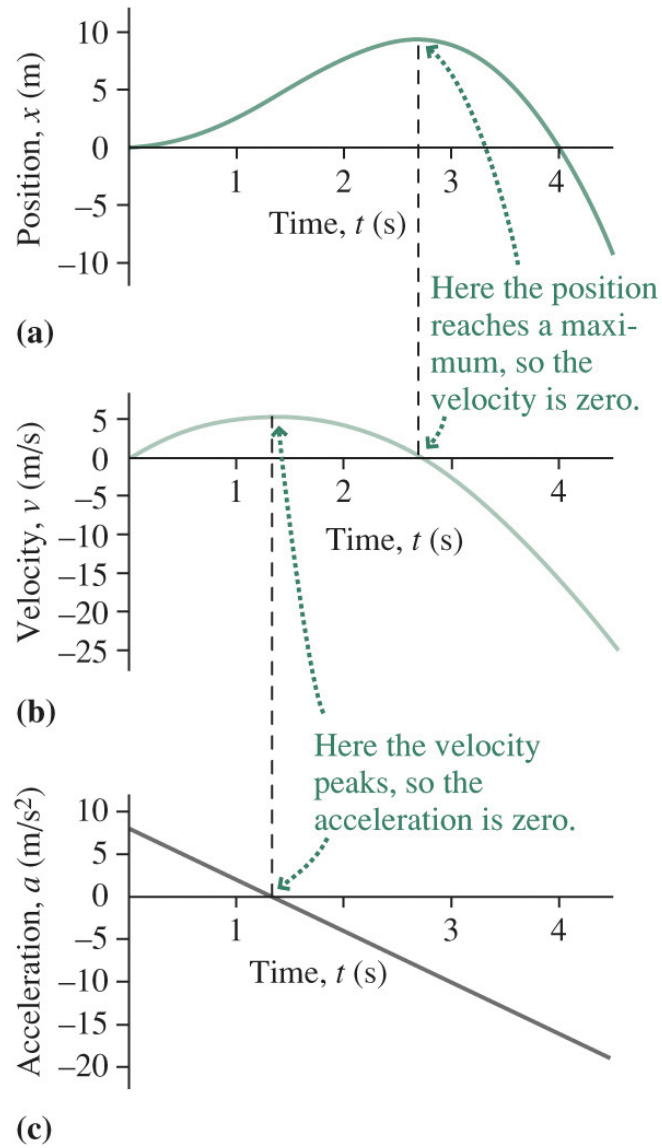
- **Acceleration** is the rate of change of velocity.
  - Exactly like velocity was the rate of change of position!
  - **Average velocity** over a time interval Δt is defined as the change in velocity divided by the time:

$$\boxed{\bar{a} = \frac{\Delta v}{\Delta t}}$$

  - **Instantaneous acceleration** is the limit of the average acceleration as the time interval becomes arbitrarily short:

$$\boxed{a = \frac{\Delta v}{\Delta t} \quad \text{(for very small } \Delta \text{t)}}$$

– Acceleration is the slope of v(t)



Position, Velocity, and Acceleration

- Individual or absolute values of position, velocity, and acceleration are not related.
  - Instead, velocity depends on the *rate of change* of position.
  - Acceleration depends on the *rate of change* of velocity.
    - These are all **relative** quantities, and **not** based on absolute position or position of the origin
    - This makes our description of this motion **universal**
  - An object can be at position x = 0 and still be *moving.*
  - An object can have zero velocity and still be *accelerating*.
- At the peak of its trajectory, a juggling thud has
  - Maximum vertical displacement from my hand
  - Zero vertical velocity
  - Constant negative acceleration due to the force of gravity

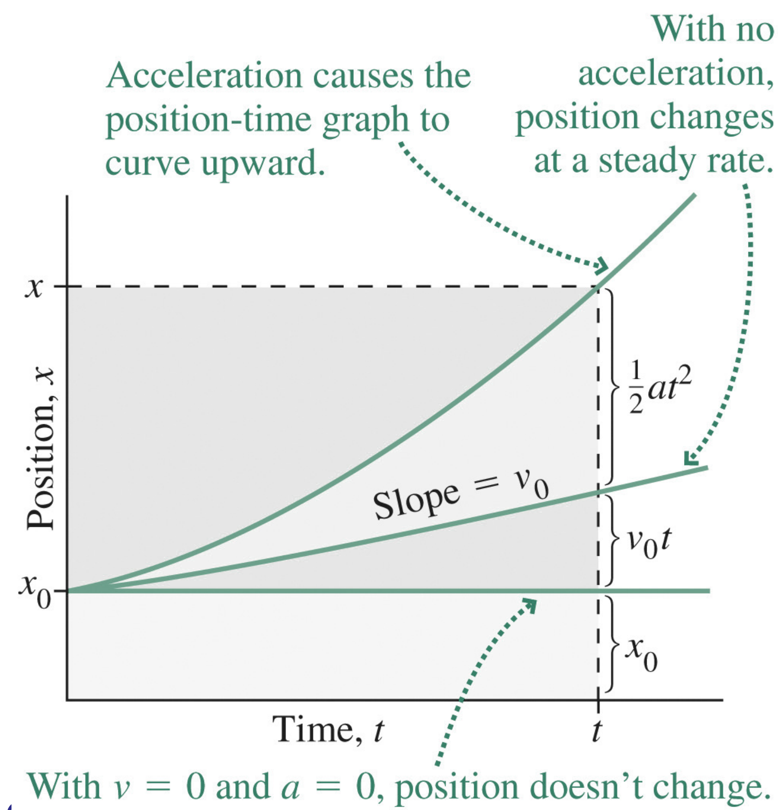
Constant Acceleration

- When **acceleration is constant**: position x, velocity v, acceleration a, and time t are related by

$$v(t) = v_0 + at$$
$$x(t) = x_0 + \frac{1}{2}\left[v_0 + v(t)\right]$$
$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v^2(t) = v_0^2 + 2a[x(t) - x_0]$$

where x_0 and v_0 are **initial values** at time $t = 0$ and $x(t)$ and $v(t)$ are the values at an arbitrary time t.

- With **constant acceleration**
  - Velocity is a linear function of time
  - Position is a quadratic function of time



The Acceleration of Gravity

- The acceleration of gravity at any point is (basically) the same for all objects, regardless of mass.
- Near Earth's surface, the value of the acceleration is essentially constant at $g = 9.8\,\mathrm{m/s^2} = 32\,\mathrm{ft/s^2}$
- Therefore the equations for constant acceleration apply:
  - In a coordinate system with *y* axis upward, they read

$$v(t) = v_0 + at$$
$$y(t) = y_0 + \frac{1}{2}\left[v_0 + v(t)\right]$$
$$y(t) = y_0 + v_0 t - \frac{1}{2} g t^2$$
$$v^2(t) = v_0^2 - 2a[y(t) - y_0]$$

This strobe photo of a falling ball shows increasing spacing resulting from the acceleration of gravity.

(Honors Ponderable)

- Assume that the 13 ball in the photo is a "standard" 2.25 inch diameter billiard ball
- How fast is the strobe flashing between images of the falling ball?
- What is the ball's approximate instantaneous velocity in the first image? In the last?

This strobe photo of a falling ball shows increasing spacing resulting from the acceleration of gravity.

Example: The Acceleration of Gravity

- A ball is thrown straight up at 7.3 m/s, leaving your hand 1.5 m above the ground. Find its maximum height and when it hits the floor.
  - At the maximum height the ball is instantaneously at rest (even though it's still *accelerating*). Solving the last equation with $v = 0$ gives the maximum height:

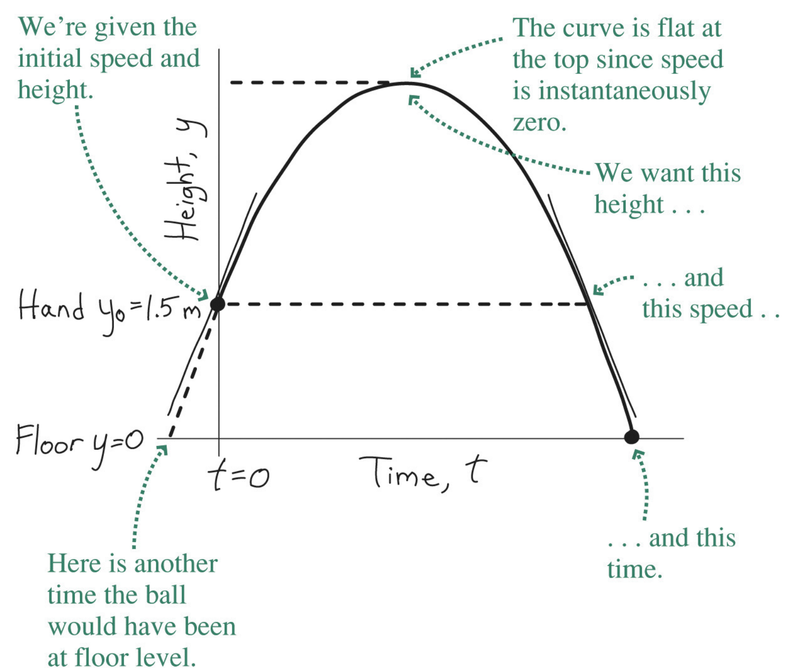
$$0 = v_0^2 - 2g\left(y - y_0\right)$$

2 Significant Figures!

or

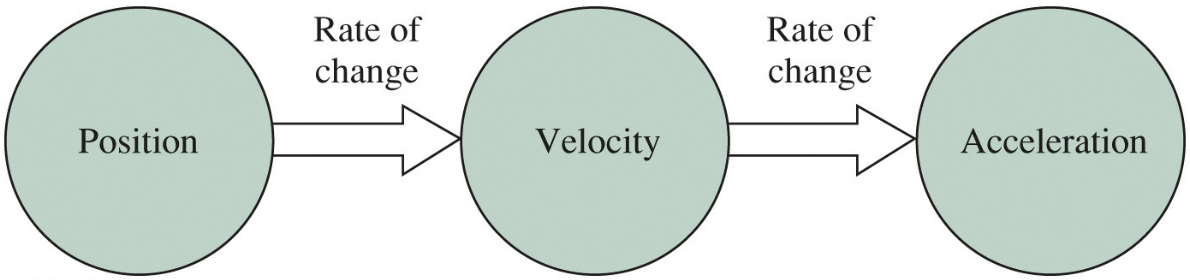
$$y = y_0 + \frac{v_0^2}{2g} = 1.5\text{ m} + \frac{(7.3\text{ m/s})^2}{(2)(9.8\text{ m/s}^2)} = 4.2\text{ m}$$

- Setting $y = 0$ in the third equation gives a quadratic in time; the result is the two values for the time when the ball is on the floor:
  $t = -0.18$ s and $t = 1.7$ s
- The first answer tells when the ball *would have been* on the floor if it had always been on this trajectory; the second is the answer we want.



Summary

- Position, velocity, and acceleration are the fundamental quantities describing motion.
  - Velocity is the rate of change of position.
  - Acceleration is the rate of change of velocity.



- When acceleration is constant, simple equations relate position, velocity, acceleration, and time.
  - An important case is the acceleration due to gravity near Earth's surface.
  - The **magnitude** of Earth's gravitational acceleration is g = 9.8 m/s^2 = 32 ft/s^2.

$$v = v_0 + at$$

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

=========== EXTRA SLIDES ===========

(Honors Estimation)

- These are extra problems if the honors students are feeling up to the challenge. I'll include at least one in every class.

- About how far away is the Earth's horizon…
  - For a 6' tall person standing at ground level?
  - For someone looking out the window of a plane flying at 30,000 feet?

  - Assume a spherical Earth, of course… ☺

  - (Hint: To within a few percent, each time zone of the Earth is about 1000 miles at the equator. We'll use this later in the semester.)