University Physics 226N/231N
Old Dominion University

Motion in One Dimension Examples
(and starting Motion in Two Dimensions)

Homework due date now Friday morning!

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Happy Birthday to Dweezil Zappa, Freddie Mercury, and Jack Daniels (yes, he was real)!

Happy Teacher's Day (in India), Cheese Pizza Day, and Be Late For Something Day!

(Amusingly, Thursday Sep 6 is Fight Procrastination Day!)

Review: Constant Acceleration

- Acceleration a (constant!)

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = a\Delta t$$
$$(area!!)$$

- Velocity v

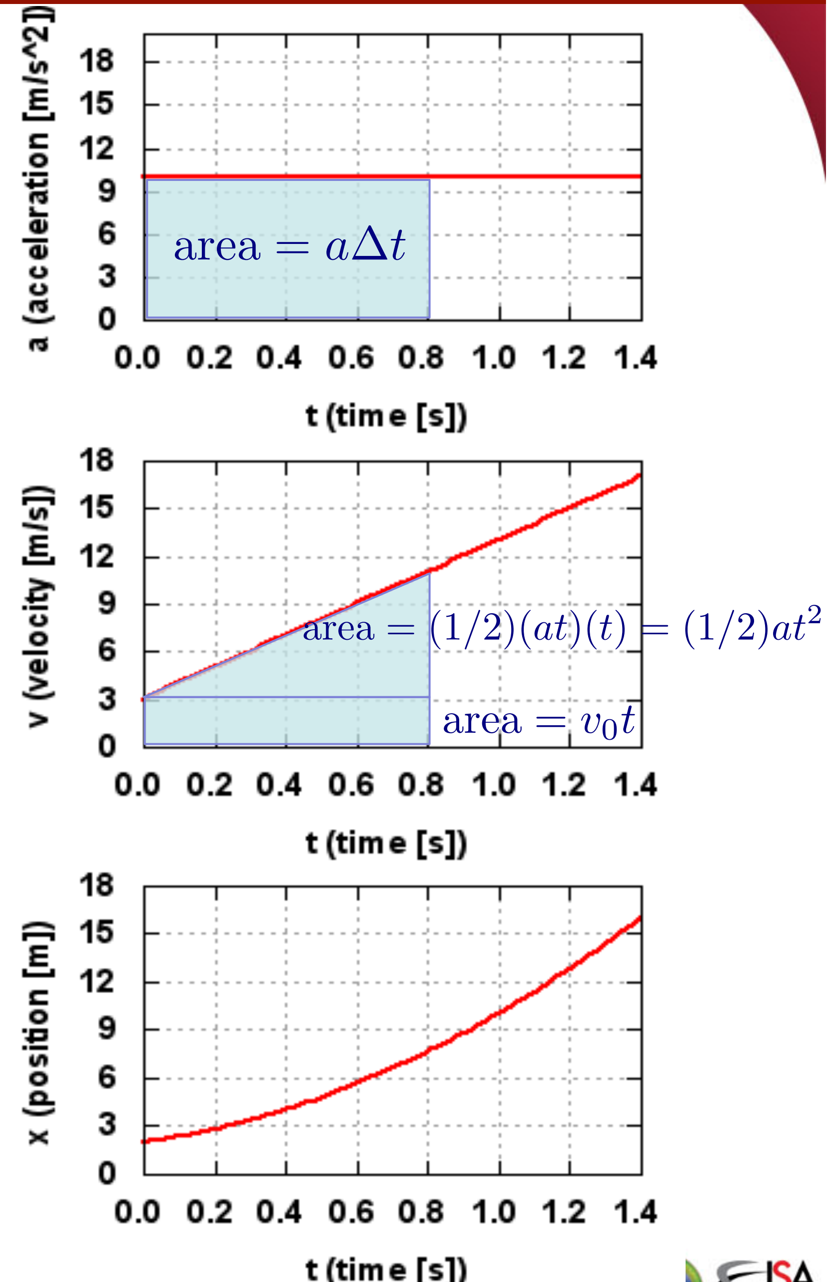
$$v = v_0 + at$$
$$v = \frac{\Delta x}{\Delta t}$$

$$t = \frac{v - v_0}{a}$$

$$\Delta x = \text{area too!}$$

- Position x

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$



Review: Constant Acceleration

gravity $a = g = -9.8 \text{ m/s}^2 = -32 \text{ ft/s}^2$

- Acceleration a (constant!)

$$a = \frac{\Delta v}{\Delta t} \qquad \Delta v = a \Delta t \quad (area!!)$$

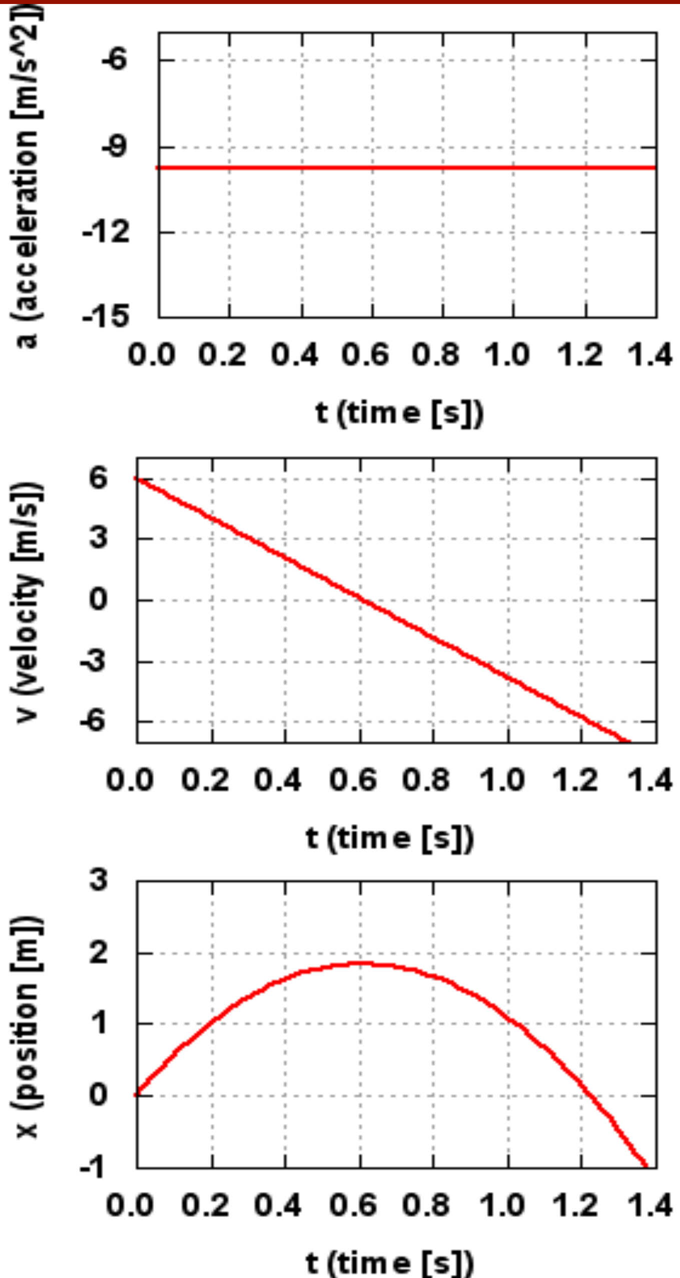
- Velocity v

$$v = v_0 + at \qquad t = \frac{v - v_0}{a}$$

$$v = \frac{\Delta x}{\Delta t} \qquad \Delta x = \text{area too!}$$

- Position x

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$



Using the Equations: Example 1 (5 minutes)

- At the moment a traffic light turns green, a car that has been waiting at the intersection accelerates with constant acceleration of a_c=3.2 m/s^2. At that same moment, a truck whizzes by at a constant velocity of v_t=20.0 m/s.
  - How far beyond the traffic light does the car catch the truck?
  - How fast is the car moving when it catches the truck?
- Say the traffic light is x=0, so initial positions of both car and truck are x_{c0}=x_{t0}=0 m.

car

$$v_{c0} = 0 \text{ m/s}$$

$$a_c = 3.2 \text{ m/s}^2$$

$$x_c = v_{c0}t + \frac{1}{2}a_c t^2 = \frac{1}{2}a_c t^2$$

truck

$$v_{t0} = 20.0 \text{ m/s}$$

$$a_t = 0.0 \text{ m/s}^2$$

$$x_t = v_{t0}t + \frac{1}{2}a_t t^2 = v_{t0}t$$

$$x_c = x_t \quad \text{when} \quad \frac{1}{2}a_c t^2 = v_{t0}t$$

Using the Equations: Example 1 (cont)

- At the moment a traffic light turns green, a car that has been waiting at the intersection accelerates with constant acceleration of a_c=3.2 m/s^2. At that same moment, a truck whizzes by at a constant velocity of v_t=20.0 m/s.
  - How far beyond the traffic light does the car catch the truck?
  - How fast is the car moving when it catches the truck?

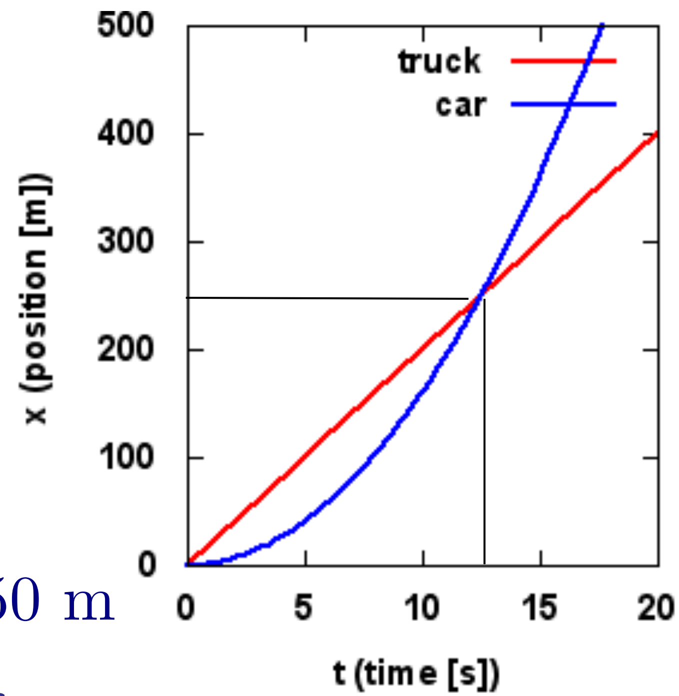
$$x_c = x_t \quad \text{when} \quad \frac{1}{2} a_c t^2 = v_{t0} t$$

$$t = \frac{2 v_{t0}}{a_c} = \frac{2(20.0 \text{ m/s})}{3.2 \text{ m/s}^2} = \boxed{12.5 \text{ s}}$$

$$v_c = v_{c0} + a_c t = a_c t$$

$$v_c = (3.2 \text{ m/s}^2)(12.5 \text{ s}) = \boxed{40 \text{ m/s}}$$

$$x = v_{t0} t = (20.0 \text{ m/s})(12.5 \text{ s}) = 250 \text{ m}$$



Using the Equations: Example 2

- I toss a bean bag up in the air at v_0=6.0 m/s.
  - How far up from my hand does it go?
  - How long does it take to come back to my hand?
    - (Assume my hand stays at the same height, x_0=0.0 m).

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \qquad x_0 = 0 \text{ m} \qquad v_0 = 6.0 \text{ m/s}$$

  - We want to know x… but we don't know t to "plug and chug"!
    - (This is typical of a frustration with the homework)

Using the Equations: Example 2 (cont)

- I toss a bean bag up in the air at v_0=6.0 m/s.
  - How far up from my hand does it go?
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$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \qquad x_0 = 0 \text{ m} \qquad v_0 = 6.0 \text{ m/s}$$

  - We want to know x… but we don't know t to "plug and chug"!
    - (This is typical of a frustration with the homework)
- You have another equation, and you know v=0 m/s at top.

$$v = v_0 + at \qquad \Rightarrow \qquad t = \frac{v - v_0}{a} = \frac{-v_0}{a} \text{ (here)}$$

  - If you have trouble going from the equation on the left to the equation on the right, please see me after class
    - Calc may not be a prerequisite for this class, but algebra is!

Using the Equations: Example 2 (cont)

- I toss a bean bag up in the air at v_0=6.0 m/s.
  - How far up from my hand does it go?
  - How long does it take to come back to my hand?
    - (Assume my hand stays at the same height, x_0=0.0 m).

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \qquad x_0 = 0 \text{ m} \qquad v_0 = 6.0 \text{ m/s}$$

  - We want to know x… but we don't know t to "plug and chug"!
    - (This is typical of a frustration with the homework)
- You have another equation, and you know v=0 m/s at top.

$$v = v_0 + at \qquad \Rightarrow \qquad t = \frac{v - v_0}{a} = \frac{-v_0}{a} \text{ (here)}$$

  - Now substitute into the first equation…

$$x - x_0 = v_0\left(\frac{-v_0}{a}\right) + \frac{1}{2}a\left(\frac{-v_0}{a}\right)^2 = \boxed{-\frac{1}{2}\frac{v_0^2}{a} = x - x_0}$$

Using the Equations: Example 2 (cont)

- I toss a bean bag up in the air at v_0=6.0 m/s.
  - How far up from my hand does it go?
  - How long does it take to come back to my hand?
    - (Assume my hand stays at the same height, x_0=0.0 m).

$$x - x_0 = -\frac{1}{2}\frac{v_0^2}{a}$$

$$x - x_0 = -\frac{1}{2}\frac{v_0^2}{a} = -\frac{1}{2}\left[\frac{(6.0 \text{ m/s})^2}{(-9.8 \text{ m/s}^2)}\right] = \frac{18.0 \text{ m}^2/\text{s}^2}{9.8 \text{ m/s}^2} = \boxed{1.8 \text{ m}}$$

- On the previous page we'd also figured out t in terms of v_0

$$t = \frac{-v_0}{a} = \frac{-6.0 \text{ m/s}}{(-9.8 \text{ m/s}^2)} = \boxed{0.61 \text{ s}}$$

Using the Equations: Example 2 (cont)

- I toss a bean bag up in the air at v_0=6.0 m/s.
  - How far up from my hand does it go?
  - How long does it take to come back to my hand?
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- On the previous page we'd also figured out t in terms of v_0

$$t = \frac{-v_0}{a} = \frac{-6.0 \text{ m/s}}{(-9.8 \text{ m/s}^2)} = \boxed{0.61 \text{ s}}$$

- Be careful! The time back to my hand is TWICE this! (up/down)

$$t = \text{time back to hand} = \boxed{1.21 \text{ s}}$$

Huh, that looks a bit familiar…

- Acceleration a (constant!)

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = a\Delta t$$
$$(area!!)$$

- Velocity v

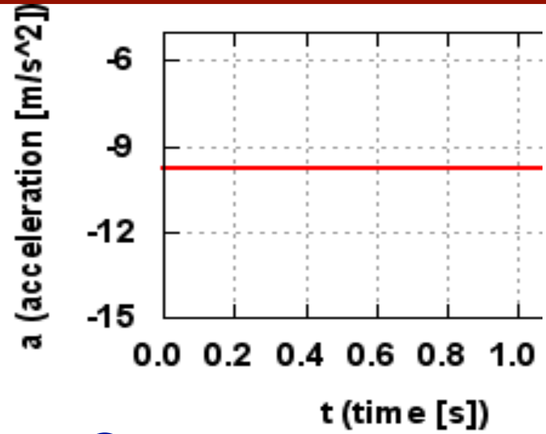
$$v = v_0 + at$$
$$v = \frac{\Delta x}{\Delta t}$$

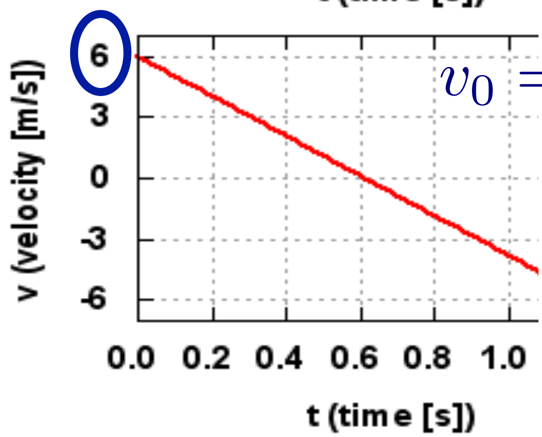
$$t = \frac{v - v_0}{a}$$

$$\Delta x = \text{area too!}$$

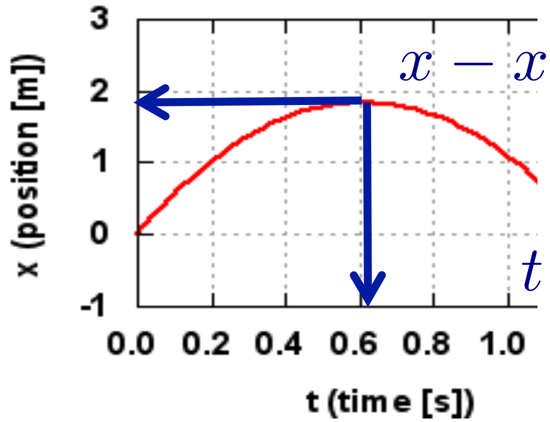
- Position x

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$





$$v_0 = 6.0\ \mathrm{m/s}$$



$$x - x_0 = 1.8\ \mathrm{m}$$

$$t = 0.61\ \mathrm{s}$$

Using the Equations: Example 3

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand.
  - How fast v_0 do I have to throw the beanbag?
  - If I toss the beanbag at t=0s and miss the catch on the way back down, at what time does the beanbag hit the floor 1.5m below my hand?
    - (Assume my hand stays at the same height, x_0=0.0 m).

$$v = v_0 + at \qquad \Rightarrow \qquad v_0 = v - at = -at \text{ (here)}$$

- We know v=0 at the ceiling, and a=g, and want to know v_0… but we don't know t to "plug and chug"!
  - (This is typical of a frustration with the homework again)

Using the Equations: Example 3 (cont)

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand.
  - How fast v_0 do I have to throw the beanbag?
  - If I toss the beanbag at t=0s and miss the catch on the way back down, at what time does the beanbag hit the floor 1.5m below my hand?
    - (Assume my hand stays at the same height, x_0=0.0 m).

$$v = v_0 + at \quad \Rightarrow \quad v_0 = v - at = -at \text{ (here)}$$

- In the previous problem we figured out an equation that eliminated time (**only** when v_f=0 m/s):

$$x - x_0 = v_0 \left(\frac{-v_0}{a}\right) + \frac{1}{2} a \left(\frac{-v_0}{a}\right)^2 = -\frac{1}{2}\frac{v_0^2}{a} = x - x_0$$

$$\Rightarrow \quad v_0 = \sqrt{-2a(x - x_0)}$$

$$v_0 = \sqrt{-2(-9.8 \text{ m/s}^2)(2.0 \text{ m})} = \boxed{6.3 \text{ m/s} = v_0}$$

Using the Equations: Example 3 (cont)

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand.
  - How fast v_0 do I have to throw the beanbag?
  - If I toss the beanbag at t=0s and miss the catch on the way back down, at what time does the beanbag hit the floor 1.5m below my hand?
    - (Assume my hand stays at the same height, x_0=0.0 m).

- For the second part, $x - x_0 = -1.5 \text{ m}$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

$$(-1.5 \text{ m}) = (6.3 \text{ m/s}) t - (4.9 \text{ m/s}^2) t^2$$

- Uh oh… This is a quadratic equation. You can't solve this using simple algebra – you have to remember the equation for the solutions of a quadratic.

Quadratic Equation Refresher

- For any quadratic equation of the form

$$ax^2 + bx + c = 0$$

This is general math:
a is NOT acceleration
x is NOT position

where a,b,c are known values, the TWO values of x that "solve" this equation can be found with the equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note that you should usually find that $b^2 - 4ac > 0$ for physics problems otherwise the square root produces imaginary numbers (which don't have good physical meanings!)

Using the Equations: Example 3 (cont)

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand.
  - How fast v_0 do I have to throw the beanbag?
  - If I toss the beanbag at t=0s and miss the catch on the way back down, at what time does the beanbag hit the floor 1.5m below my hand?
    - (Assume my hand stays at the same height, x_0=0.0 m).

$$(-1.5\ \mathrm{m}) = (6.3\ \mathrm{m/s})t - (4.9\ \mathrm{m/s^2})t^2$$

$$(-4.9\ \mathrm{m/s^2})t^2 + (6.3\ \mathrm{m/s})t + 1.5\ \mathrm{m} = 0$$

$$a = (-4.9\ \mathrm{m/s^2}) \quad b = (6.3\ \mathrm{m/s}) \quad c = 1.5\ \mathrm{m}$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6.3\ \mathrm{m/s} \pm 8.3\ \mathrm{m/s}}{-9.8\ \mathrm{m/s^2}} = \boxed{1.5\ \mathrm{s}, -0.20\ \mathrm{s}}$$

Two solutions!

Ponderable (10 minutes)

- I drop a bean bag from my hand to the floor 1.5 m below my hand.
  - How long does it take to reach the floor?
  - What is the beanbag's velocity at floor height?
    - (Assume floor height is x_0=0.0 m).

$$v = v_0 + at \qquad x = x_0 + v_0 t + \frac{1}{2} a t^2$$

Ponderable (10 minutes)

- I drop a bean bag from my hand to the floor 1.5 m below my hand.
  - How long does it take to reach the floor?
  - What is the beanbag's velocity at floor height?
    - (Assume floor height is x_0=0.0 m).

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad -1.5\ \text{m} = \frac{1}{2}at^2$$

$x-x_0$=-1.5m $\quad v_0$=0 m/s

$$t = \sqrt{\frac{2(-1.5\ \text{m})}{(-9.8\ \text{m/s}^2)}} = \boxed{\pm 0.55\ \text{s}}$$

$$v = v_0 + at \qquad v = (0\ \text{m/s}) + (-9.8\ \text{m/s}^2)(\pm 0.55\ \text{s}) = \boxed{\pm 5.4\ \text{m/s}}$$

- Note that the equations don't care about past or future!

Example 3 With a Twist (~5 minutes)

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand, AND I'm throwing it to some unfortunate student who is 5.0m away from me.
  - Here, catch! (*thud*)
  - How is this different from just throwing the beanbag straight up to graze the ceiling?
    - Do I have to throw the one to the student with a higher or lower velocity than the one straight up that grazes the ceiling?
    - Which one stays in the air a longer time?
    - At what angle from horizontal do I have to throw the one to the student? More or less than 45 degrees?

Example 3 With a Twist (Explanation)

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand, AND I'm throwing it to some unfortunate student who is 5.0m away from me.
  - Do I have to throw the one to the student with a higher or lower velocity than the one straight up that grazes the ceiling?

    Higher!
  - Which one stays in the air a longer time?
  - At what angle from horizontal do I have to throw the one to the student? More or less than 45 degrees?

    Less, but how much less?

- The time thing is **tricky** and **nonintuitive.**
- The vertical portion of the motion is the same for both cases
  - Therefore the time the bag is in the air is the **SAME** for both too!
- Two objects that have the same vertical motion take the same time for that motion **regardless** of their horizontal motion!
  - Example: a cannonball fired horizontally out of a cannon and another cannonball dropped from that cannon take the same time to hit the ground.

Onwards to Two Dimensional Motion

- We'll get further into examples of these equations next class
  - Horizontal motion is easy: acceleration is zero!
  - You've already done the hard part, vertical motion with gravity

$$v_x = v_{x0}$$

$$x = x_0 + v_{x0} t$$

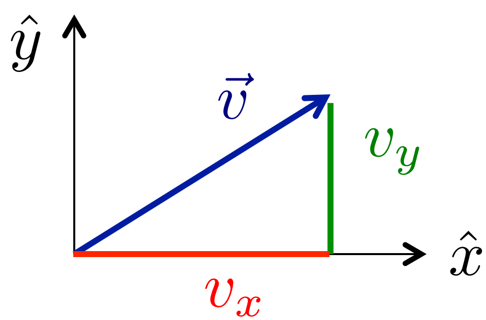
Horizontal motion (no acceleration)

$$v_y = v_{y0} + at$$

$$y = y_0 + v_{y0} t + \frac{1}{2} a t^2$$

Vertical motion (gravitational acceleration)

Components of velocity, and vectors:



$$\vec{v} = v_x \hat{x} + v_y \hat{y}$$

$$v = \sqrt{v_x^2 + v_y^2}$$

Comment about Homework

- Please email me if you want me to reset your homework set
  - (Just this once)
- Please read the yellow boxes titled "A Message from Your Instructor" – they contain hints and useful information!

A Message from Your Instructor:
Motion in one dimension (often with gravity; remember acceleration of gravity is g=-9.8 m/s^2 when the up direction is positive.

Exercise 2.38 is for 1 point(s)
Incomplete

Exercise 2.37 is for 1 point(s)
Incomplete

Exercise 2.25 is for 1 point(s)
Incomplete

Problem 2.82 is for 1 point(s)
Incomplete

A Message from Your Instructor:
The next problem is difficult if you had not had calculus yet. You can solve it without calculus when I tell you that the velocity is v=3bt^2 and the acceleration is a=6bt.

Problem 2.51 is for 1 point(s)
Incomplete

A Message from Your Instructor:
The tutorials are not required, but are available if you want to view them or play with them. I'd appreciate feedback on whether you find them useful or not. Thanks!

=========== EXTRA SLIDES ===========

(Honors Estimation)

- These are extra problems if the honors students are feeling up to the challenge. I' ll include at least one in every class.

- About how far away is the Earth' s horizon…

  - For a 6' tall person standing at ground level?
  - For someone looking out the window of a plane flying at 30,000 feet?

  - Assume a spherical Earth, of course… ☺

  - (Hint: To within a few percent, each time zone of the Earth is about 1000 miles at the equator. We' ll use this later in the semester.)