University Physics 226N/231N
Old Dominion University

Vectors and Motion in Two Dimensions

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Happy Birthday to James Van Allen, Michael Emerson, Chrissie Hynde, and Google!
Happy Granddad's Day, Acorn Squash Day, and Feel The Love Day!

Review of Vertical Motion

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand.
  - How fast v_0 do I have to throw the beanbag?
  - If I toss the beanbag at t=0s and miss the catch on the way back down, at what time does the beanbag hit the floor 1.5m below my hand?
    - (Assume my hand stays at the same height, x_0=0.0 m).

$$v = v_0 + at \qquad \Rightarrow \qquad v_0 = v - at = -at \text{ (here)}$$

- In the previous problem we figured out an equation that eliminated time (**only** when v_f=0 m/s):

$$x - x_0 = v_0 \left(\frac{-v_0}{a} \right) + \frac{1}{2} a \left(\frac{-v_0}{a} \right)^2 = -\frac{1}{2} \frac{v_0^2}{a} = x - x_0$$

$$\Rightarrow \quad v_0 = \sqrt{-2a(x - x_0)}$$

$$v_0 = \sqrt{-2(-9.8 \text{ m/s}^2)(2.0 \text{ m})} = \boxed{6.3 \text{ m/s} = v_0}$$

Review of Vertical Motion (cont)

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    - (Assume my hand stays at the same height, x_0=0.0 m).

- For the second part, $x - x_0 = -1.5 \text{ m}$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

$$(-1.5 \text{ m}) = (6.3 \text{ m/s})t - (4.9 \text{ m/s}^2)t^2$$

- Uh oh… This is a quadratic equation. You can't solve this using simple algebra – you have to remember the equation for the solutions of a quadratic.

Review of Vertical Motion (cont)

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    - (Assume my hand stays at the same height, x_0=0.0 m).

$$x - x_0 = v_0 t + \frac{1}{2} a t^2 \qquad (-1.5\ \mathrm{m}) = (6.3\ \mathrm{m/s})t - (4.9\ \mathrm{m/s^2})t^2$$

$$(-4.9\ \mathrm{m/s^2})t^2 + (6.3\ \mathrm{m/s})t + 1.5\ \mathrm{m} = 0$$

$$a = (-4.9\ \mathrm{m/s^2}) \quad b = (6.3\ \mathrm{m/s}) \quad c = 1.5\ \mathrm{m}$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6.3\ \mathrm{m/s} \pm 8.3\ \mathrm{m/s}}{-9.8\ \mathrm{m/s^2}} = \boxed{1.5\ \mathrm{s}, -0.20\ \mathrm{s}}$$

Review with a Twist (~5 minutes)

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand, AND I'm throwing it to some unfortunate student who is 5.0m away from me.
  - Here, catch! (*thud*)
  - How is this different from just throwing the beanbag straight up to graze the ceiling?
    - Do I have to throw the one to the student with a higher or lower velocity than the one straight up that grazes the ceiling?
    - Which one stays in the air a longer time?
    - At what angle from horizontal do I have to throw the one to the student? More or less than 45 degrees?

Review with a Twist (Explanation)

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand, AND I'm throwing it to some unfortunate student who is 5.0m away from me.
  - Do I have to throw the one to the student with a higher or lower velocity than the one straight up that grazes the ceiling?

Higher!

  - Which one stays in the air a longer time?
  - At what angle from horizontal do I have to throw the one to the student? More or less than 45 degrees?

- The time thing is **tricky** and **nonintuitive.**
- The vertical portion of the motion is the same for both cases
  - Therefore the time the bag is in the air is the **SAME** for both too!
- Two objects that have the same vertical motion take the same time for that motion **regardless** of their horizontal motion!
  - Example: a cannonball fired horizontally out of a cannon and another cannonball dropped from that cannon take the same time to hit the ground.

Onwards to Two Dimensional Motion

- We'll get further into examples of these equations next class
  - Horizontal motion is easy: acceleration is zero!
  - You've already done the **hard part**, vertical motion with gravity

$$v_x = v_{x0}$$
$$x = x_0 + v_{x0} t$$

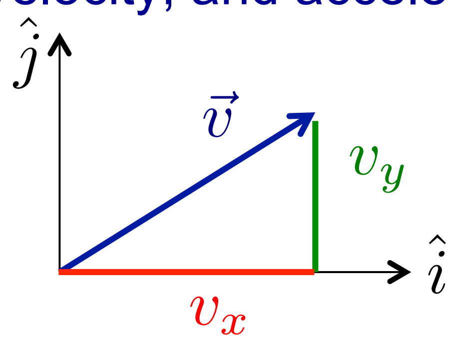
Horizontal motion (no acceleration)

$$v_y = v_{y0} + at$$
$$y = y_0 + v_{y0} t + \frac{1}{2} a t^2$$

Vertical motion (gravitational acceleration)

Position, velocity, and acceleration are all **vectors**:

Components

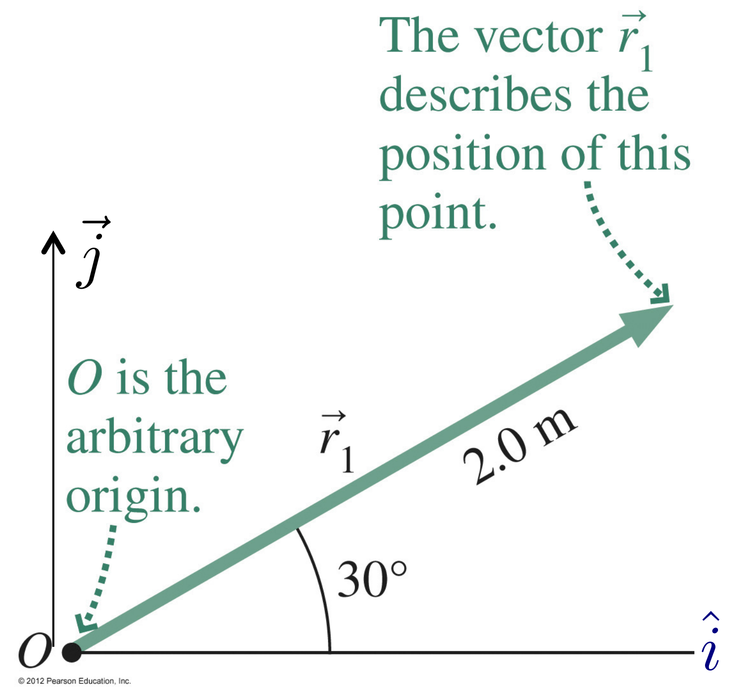


$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$
$$v = \sqrt{v_x^2 + v_y^2}$$

Vectors

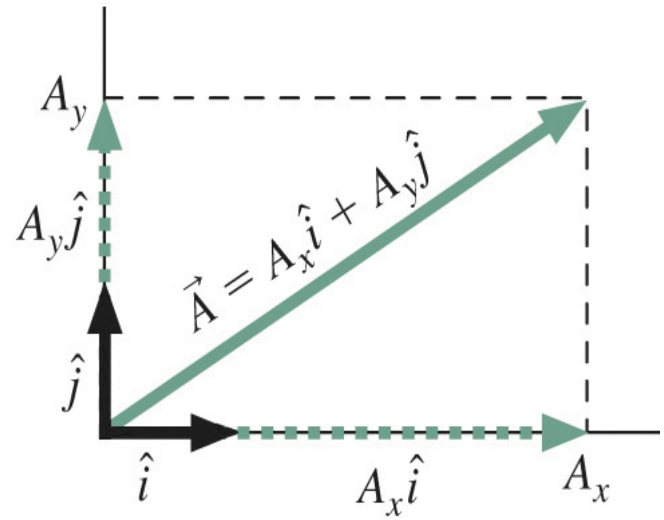
- A **vector** is a quantity that has both magnitude and direction.
  - In two dimensions it takes two numbers to specify a vector.
  - In three dimensions it takes three numbers.
  - A vector can be represented by an arrow whose length corresponds to the vector's magnitude.

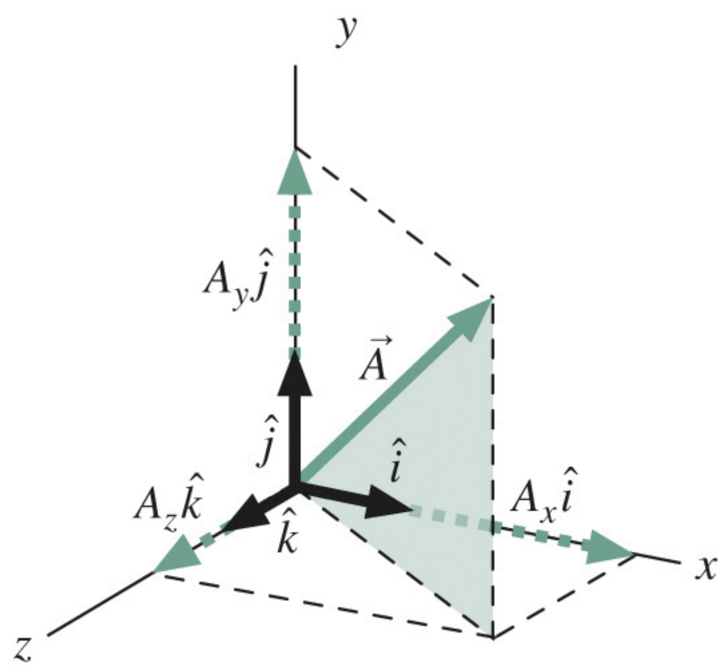
- Position is a vector quantity
  - An object's position is specified by giving its distance from an origin and its direction relative to an axis.
  - Here $\vec{r}_1$ describes a point 2.0 m from the origin at a 30° angle to the x-axis (in the $\hat{i}$ direction).



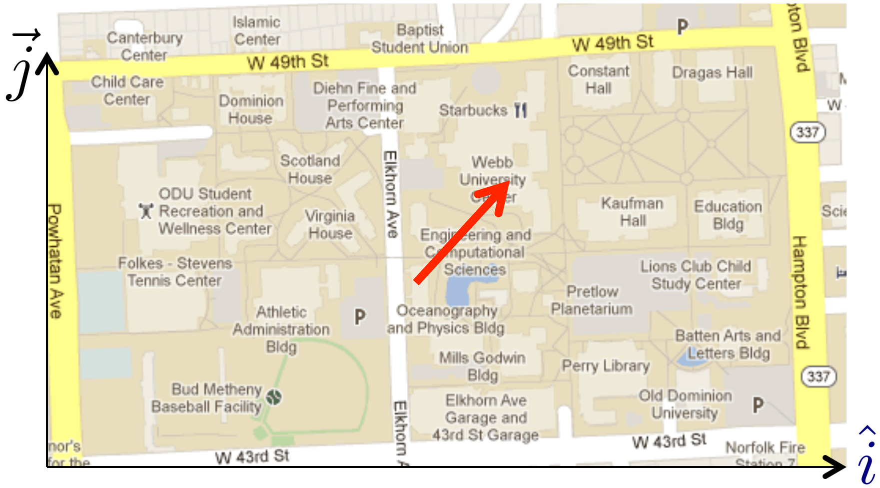
Unit Vectors

- Unit vectors have magnitude 1, **no units**, and point along the coordinate axes.
  - They're used to specify direction in compact mathematical representations of vectors.
  - Unit vectors in the *x*, *y*, and *z* directions are designated $\hat{i},\ \hat{j},\ \text{and}\ \hat{k}$
  - Any vector in two dimensions can be broken down into $\hat{i}$ and $\hat{j}$ components.
  - Any vector in three dimensions can be broken down into $\hat{i},\ \hat{j},\ \text{and}\ \hat{k}$ components.
  - Dimensions are carried in the values of the components – example, (3m)$\hat{i}$

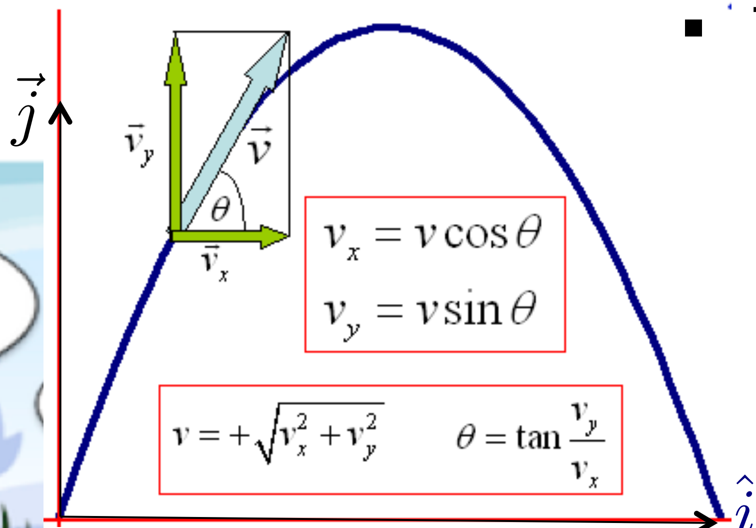




Unit Vectors Examples



- The vector from class to the Webb for coffee is about 300 feet east and 500 feet north
  - (300 feet) $\hat{i}$ + (500 feet) $\vec{j}$
  - ~580 feet ~60° north of east



Pythagorean theorem, sohcahtoa

- This projectile picture shows both position and velocity
  - Position of the projectile
  - Velocity drawn from a position at a given time
  - Each has different units
  - But they have the same axes and unit vectors, $\hat{i}$ and $\vec{j}$

Vector Arithmetic with Components

- To add vectors, add the individual components:
  - If $\vec{A} = A_x \hat{i} + A_y \hat{j}$ and $\vec{B} = B_x \hat{i} + B_y \hat{j}$ then

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$
$$\vec{A} - \vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}$$

- To multiply a vector $\vec{A}$ by a scaler (constant) c:

$$c\vec{A} = (cA_x)\vec{i} + (cA_y)\vec{j}$$

  - We can also multiply vectors in a variety of ways, but that's beyond the scope of this course…
    - And thankfully we don't need that for the physics we're learning!

Adding Vectors

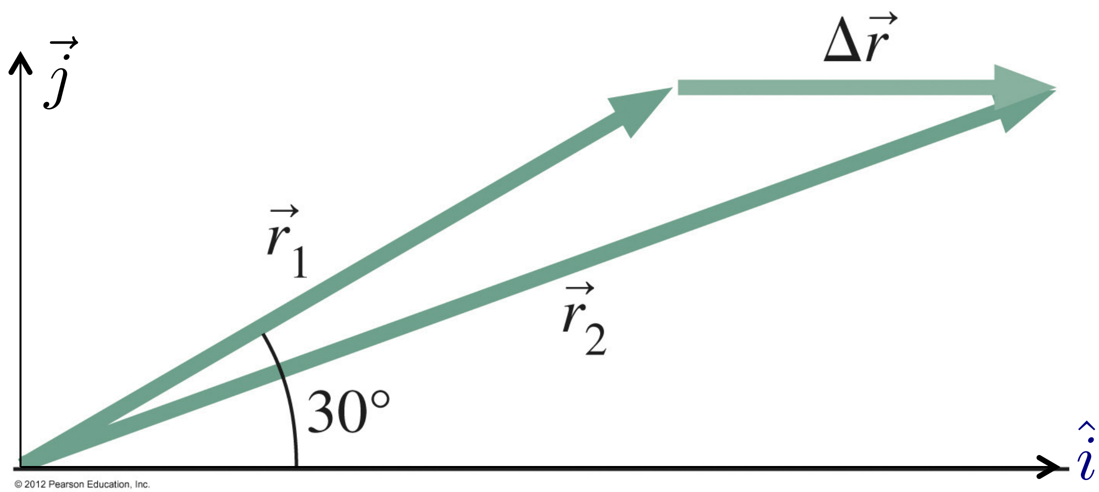
- To add vectors graphically, place the tail of the first vector at the head of the second.
  - Their sum is then the vector from the tail of the first vector to the head of the second.

– Here $\vec{r}_2$ is the sum of $\vec{r}_1$ and $\Delta\vec{r}$.

$$\vec{r}_2 = \vec{r}_1 + \Delta\vec{r}$$

$$\vec{r}_2 = r_{2x}\hat{i} + r_{2y}\hat{j}$$

$$r_{2x} = r_{1x} + \Delta r_x$$
$$r_{2y} = r_{1y} + \Delta r_y$$



Vector Arithmetic

- To multiply a vector by a scalar (a number), multiply the vector's magnitude by the scalar.
  - For a positive scalar the direction is unchanged.
  - For a negative scalar the direction reverses.

- To subtract vectors, add the negative of the second vector to the first: $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$
- Vector arithmetic is commutative and associative:

Vector addition is commutative: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.

Vector addition is also associative: $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$.