University Physics 226N/231N
Old Dominion University

Vectors and Motion in Two Dimensions

First "Midterm" is Wednesday, September 19!

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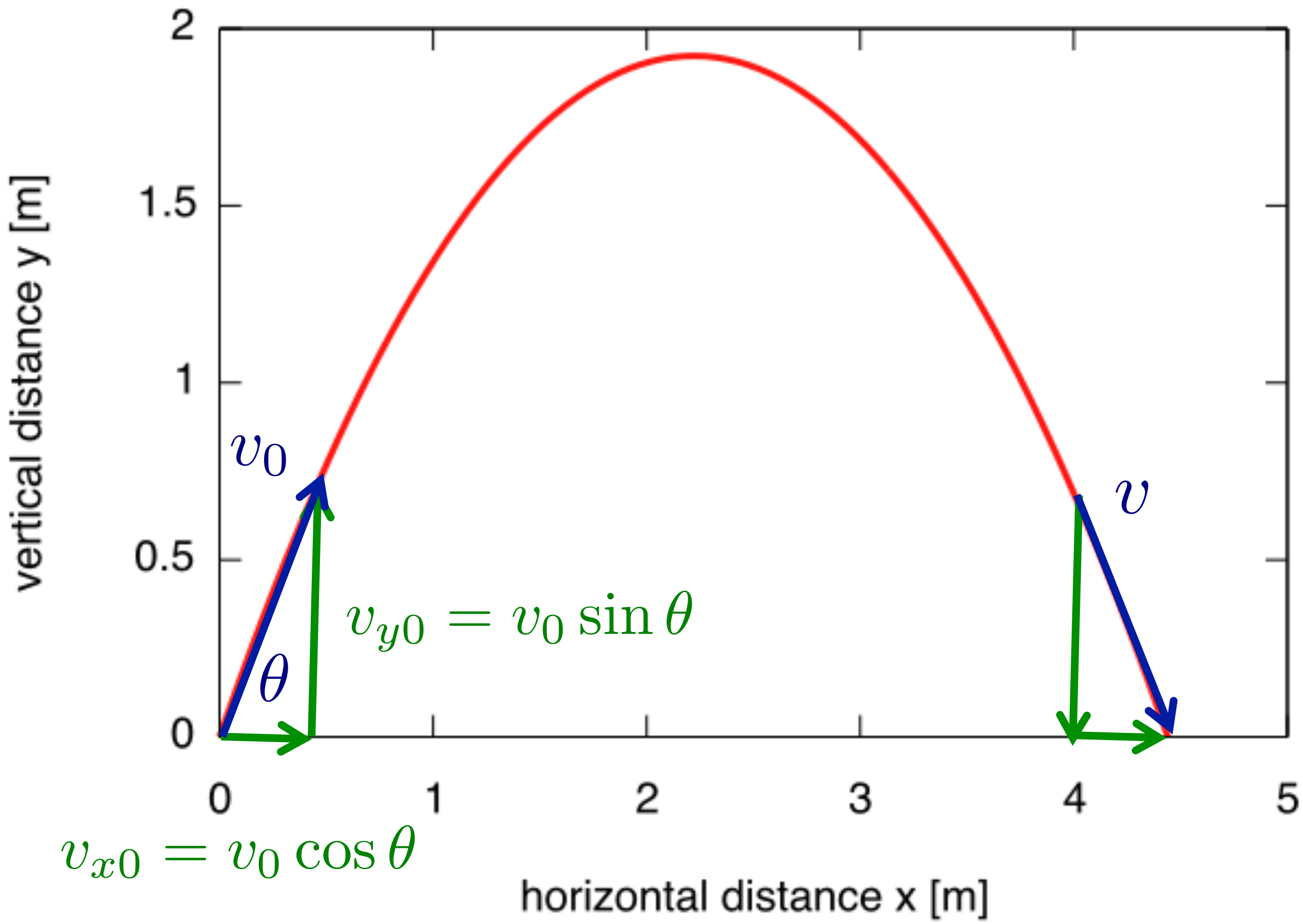
Wednesday, September 12 2012

Happy Birthday to Irene Joliot-Curie (Chemistry Nobel 1935), Andrew Luck, and Jennifer Hudson!

Happy Chocolate Milkshake Day, Video Games Day, and National Day of Encouragement!

Review: Two Dimensional Motion With Gravity

- We add constant acceleration in one direction (here vertical)
  - Gravity influencing projectile motion is one classic example



$$a_x = 0$$

$$a_y = a = g$$

Review: Two Dimensional Motion With Gravity

- We add acceleration in one direction
  - Gravity influencing projectile motion is one classic example
  - We can decompose the motion into individual directions

$$\left.\begin{aligned} v_x &= v_{x0} \\ x &= x_0 + v_{x0}t \end{aligned}\right\} \text{Horizontal motion (no acceleration)}$$

$$\left.\begin{aligned} v_y &= v_{y0} + at \\ y &= y_0 + v_{y0}t + \frac{1}{2}at^2 \end{aligned}\right\} \text{Vertical motion}$$

Horizontal motion

(no acceleration)

Vertical motion

(gravitational acceleration,

a = g = -9.8 m/s^2=-32 feet/s^2)

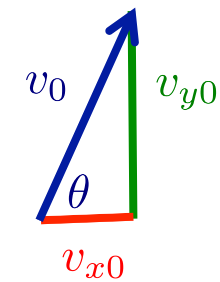
  - We've already described motion in both of these cases
  - So you already know the "hard parts" of projectile motion
    - And with that, all motion with constant acceleration in each direction
  - We have a lot more unknowns now:

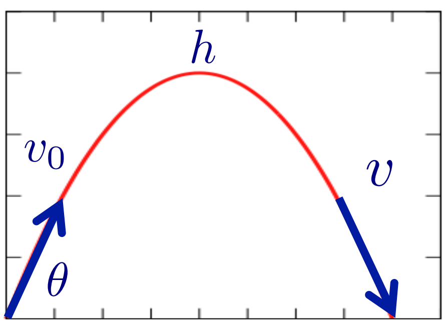
$$v_x, v_{x0}, (x - x_0) \qquad v_y, v_{y0}, (y - y_0) \qquad t$$

Example: Projectile Motion (Football Season)

- A quarterback throws a football at 45 mph. What angle does he need to throw it to hit a receiver 30 yards (90 feet) downfield? (Neglect air resistance and assume a level field. 45 mph is 66 feet/sec.)

- Call the angle θ - then we have $v_{x0} = v_0 \cos\theta$ and $v_{y0} = v_0 \sin\theta$
- We can employ a trick: from **symmetry**, $v_y = -v_{y0}$





$x - x_0 = 90$ feet

$$v_y = v_{y0} + at \quad \Rightarrow \quad t = -\frac{2v_{y0}}{a}$$

$$x - x_0 = v_{0x} t = \left(-\frac{2v_{y0}}{a}\right) v_{x0} = -\frac{2v_0^2}{a}\cos\theta\sin\theta$$

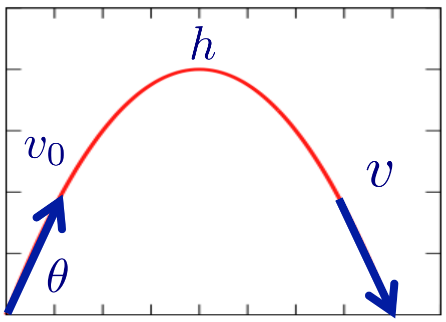
$$\text{Trigonometric identity}: \quad \sin\theta\cos\theta = \frac{1}{2}\sin(2\theta)$$

$$\sin(2\theta) = -\frac{a(x-x_0)}{v_0^2} = -\frac{(-32\ \text{feet/s}^2)(90\ \text{feet})}{(66\ \text{feet/s})^2} = 0.66$$

$$\boxed{\theta = 20.7^\circ}$$

Projectile Motion: Observations and Animation

- For **a projectile launched from ground level to ground level**, and using $v_y = -v_{y0}$, we can **derive** some interesting results



$$x - x_0 \text{ (range)} = -\frac{v_0^2}{a}\sin(2\theta)$$

Max at 45 degrees

$$h \text{ (max height)} = -\frac{v_0^2 \sin^2\theta}{2a}$$

Max at 90 degrees

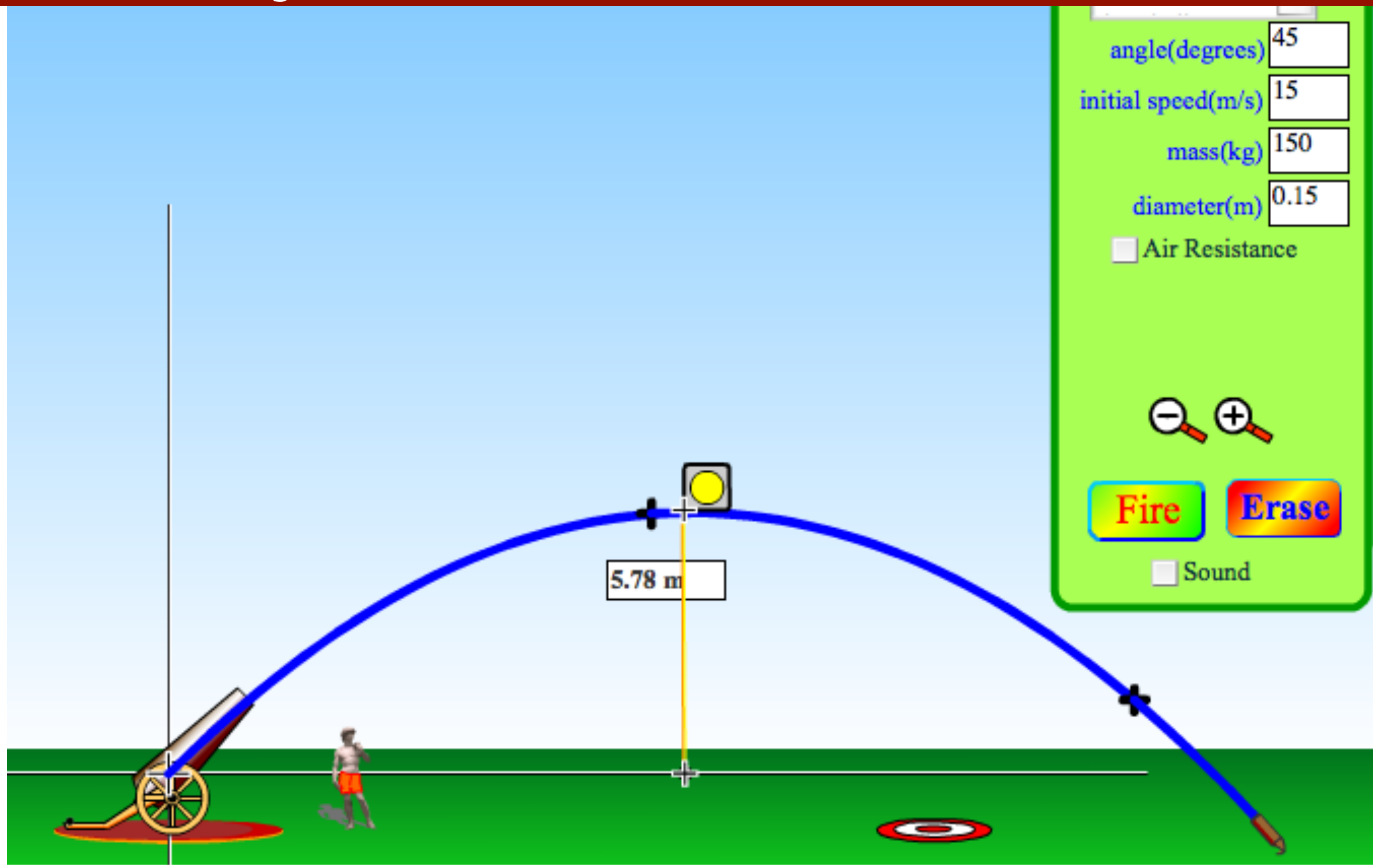
$$\frac{h}{x - x_0} = \frac{\sin^2\theta}{\sin(2\theta)} = \frac{1}{4}\tan\theta \quad \text{depends only on } \theta!$$

At a launch angle of 45 degrees to maximize the range of our projectile, the range $x - x_0 = -v_0^2/a$ is four times the achieved height.

We can experiment with this (and other projectile questions) with an applet located at

http://phet.colorado.edu/sims/projectile-motion/projectile-motion_en.html

Projectile Motion: Confirmation

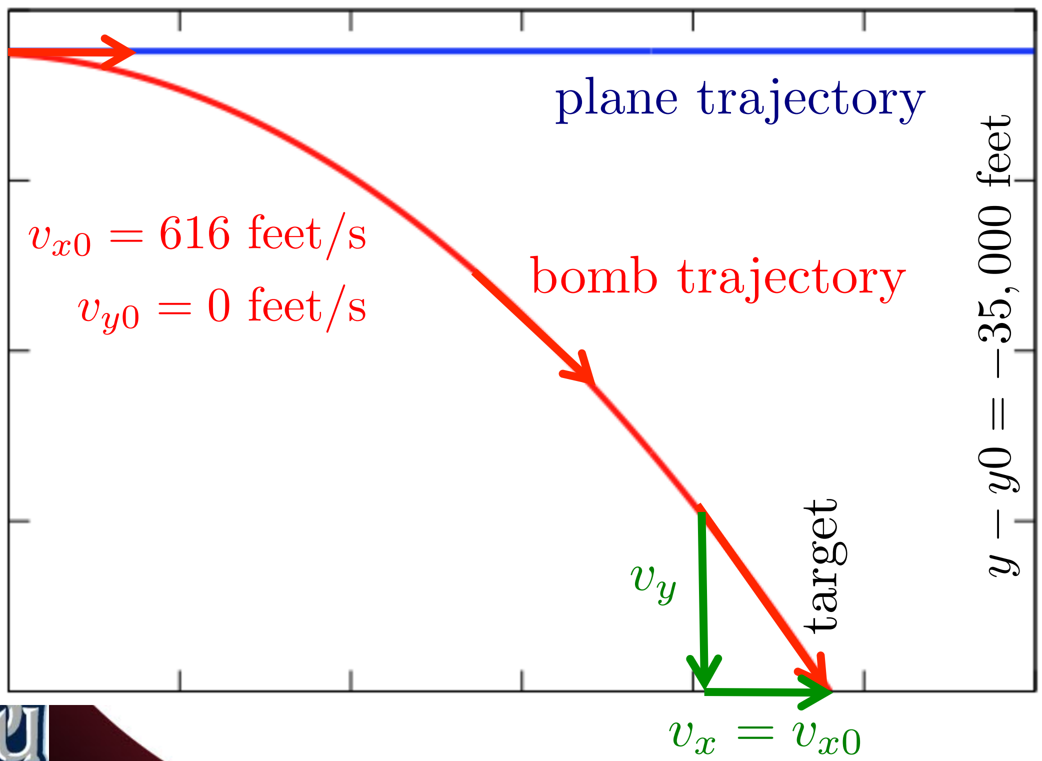


- Shoot projectile at 15 m/s at 45 degree angle
  - Height achieved is h=5.78 m, range achieved is $x-x_0$=23 m
  - Very close to $x - x_0 = 4h$ and

$$x - x_0 = -v_0^2/a = -(15\ \mathrm{m/s})^2/(9.8\ \mathrm{m/s^2}) = 22.96\ \mathrm{m}$$

Ponderable: Bomb Drop (10 minutes)

- A bomber is flying 420 mph (616 feet/s) at 35,000 feet altitude towards a ground-level target. $a=g=-32$ ft/s^2
  - How far before the target should the bomber drop the bomb?
  - Where is the bomber relative to the explosion when the bomb hits?



$$v_x = v_{x0}$$

$$x = x_0 + v_{x0}t$$

$$v_y = v_{y0} + at$$

$$\boxed{y = y_0 + v_{y0}t + \frac{1}{2}at^2}$$

We know $y-y_0$ and a, and $v_{y0}=0$, so we can find t…
Then we know v_{x0} and want to find $x-x_0$…

Ponderable: Bomb Drop (Solution)

- A bomber is flying 420 mph (616 feet/s) at 35,000 feet altitude towards a ground-level target. a=g=-32 ft/s^2
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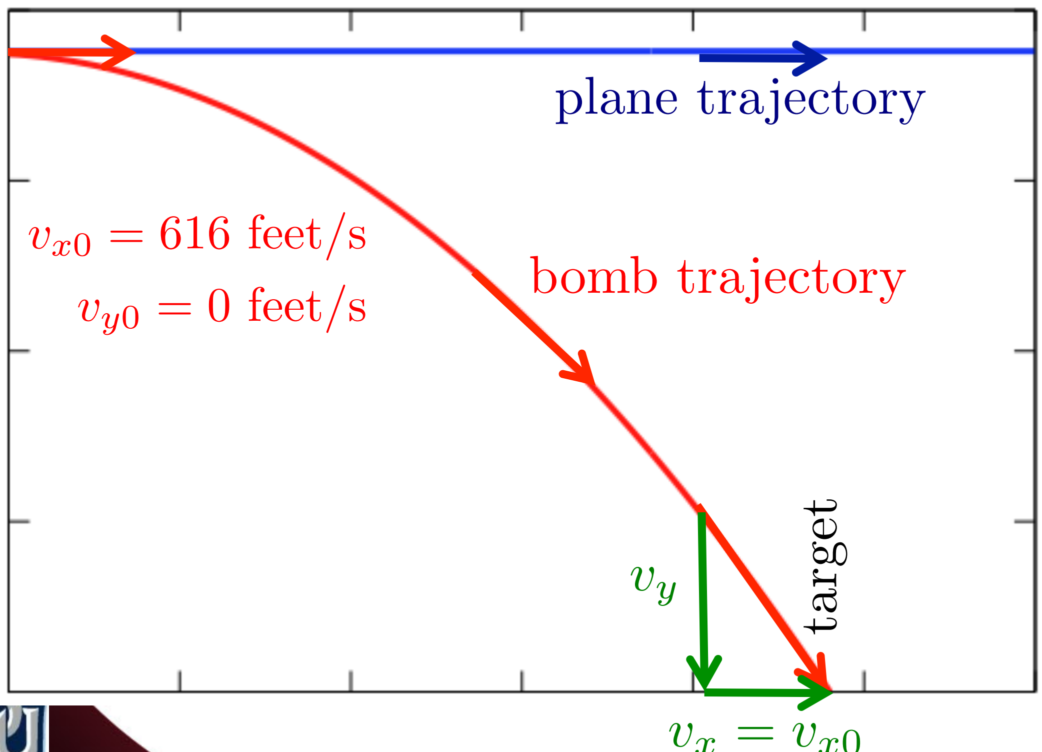


$$y - y_0 = \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2(y-y_0)}{a}} = 46.8 \text{ s}$$

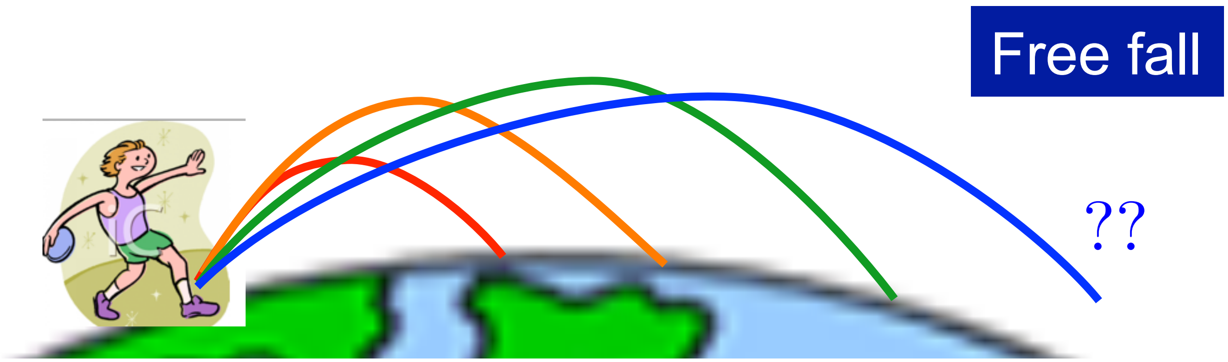
$$x - x_0 = v_{x0}t = (616 \text{ feet/s})(46.8 \text{ s})$$
$$= \boxed{5.45 \text{ miles}}$$

The bomber is directly above the explosion unless it turns or takes evasive maneuvers!

Ponderable: Falling Around The Earth

- We've been talking about projectile motion over level ground
  - The Earth is not flat: it is not "level ground" forever
  - Experience tells us that gravity always points towards the center of the Earth, wherever I am on the Earth

  - So it might be possible to shoot something really small and aerodynamic fast enough to "miss" the Earth even while the acceleration of gravity continuously makes it "fall"



Ponderable: Falling Around The Earth (10 minutes)

- Estimate (or guesstimate) how fast I have to shoot a projectile **horizontally** for it to circle around a perfectly spherical Earth and come back to me.

  How do you even start to figure this out?

All you need are:

$$g = -9.8 \text{ m/s}^2$$
$$r_e \approx 6.37 \times 10^6 \text{ m}$$

The Earth's vertical drop in a horizontal distance d is

$$\Delta y \approx -\frac{d^2}{r_e}$$

Ponderable: Falling Around The Earth (Solution)

- Estimate (or guesstimate) how fast I have to shoot a projectile **horizontally** for it to circle around a perfectly spherical Earth and come back to me.

$$\Delta y \approx -\frac{d^2}{r_e}$$

Over a small segment of time Δt,vertical velocity is

$$v_y = \frac{\Delta y}{\Delta t} = -\frac{d^2}{r_e \Delta t}$$

and vertical acceleration is

$$a_y = g = \frac{\Delta v}{\Delta t} = -\frac{d^2}{r_e \Delta t^2}$$

Ponderable: Falling Around The Earth (Solution)

- Estimate (or guesstimate) how fast I have to shoot a projectile **horizontally** for it to circle around a perfectly spherical Earth and come back to me.

"Horizontally" we're not accelerating, so horizontal velocity is distance/time:

$$v_x = \frac{d}{\Delta t} \quad \Rightarrow \quad d = v_x \Delta t$$

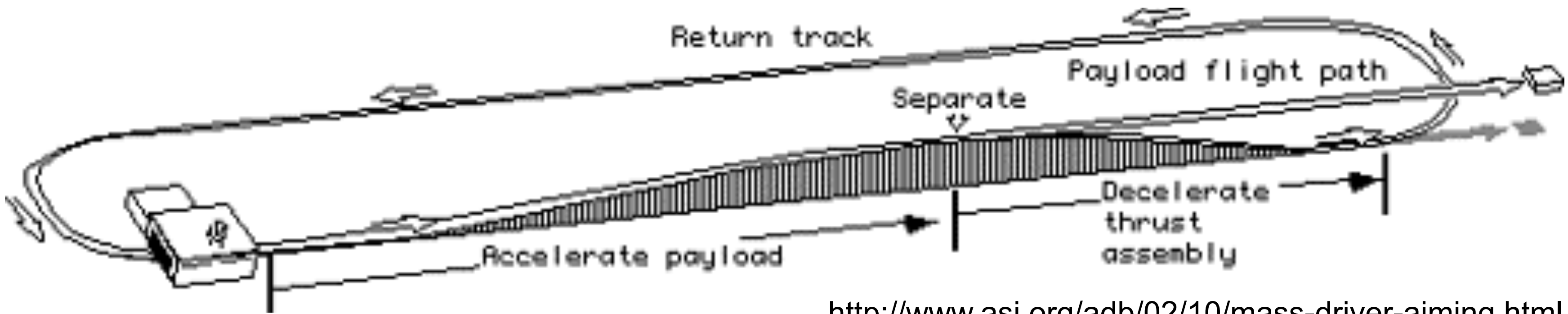
$$g = -\frac{v_x^2}{r_e}$$

$$v_x = \sqrt{-g r_e} = \boxed{7.9\ \mathrm{km/s} = v_x}$$

Over 17000 miles/hour!

Orbit time of about 1.4 hours!

Aside: Space Launches and Mass Drivers



http://www.asi.org/adb/02/10/mass-driver-aiming.html

- An Earthbound launch vehicle like this has real challenges
  - High gravity ⇨ Very high launch velocity required
    - Requires extremely long track or absurdly high acceleration
  - Dense atmosphere ⇨ Very large air resistance

- But it's been considered for the moon for decades
  - g_{moon}=-1.6 m/s^2, r_{moon}=1737 km ⇨ v_{launch}=1.67 km/s=3730 mph
  - Perhaps okay for launching materials but not people
    - At 3g acceleration, the acceleration track is 50 km (30 miles) long!
    - (At 10g acceleration, it's still 16 km or 10 miles long…)

Circular Motion and Centripetal Acceleration

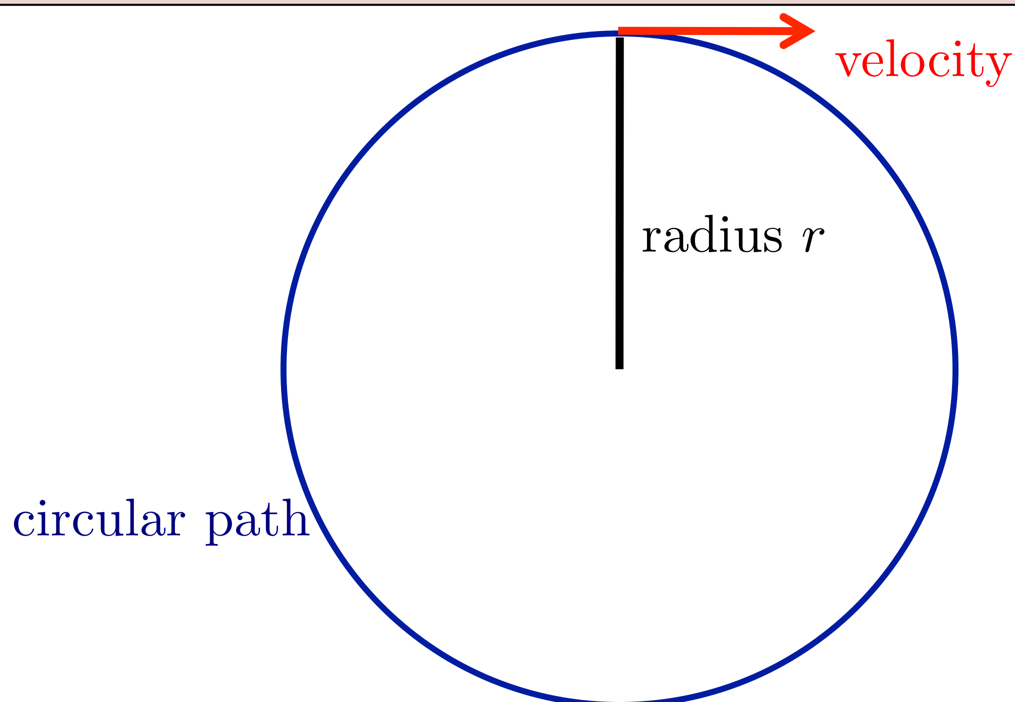
- Objects in circular gravitational orbits like this are examples of **circular motion**
  - Notice that nothing here depended on the height!
  - In fact, nothing depended on "gravity" except the constant acceleration towards the center of the earth
  - It turns out that is a pretty general result for circular motion
    - It's not easy to derive, so it's kind of a fundamental equation
    - To keep an object moving with speed v in a circle of radius r, we need a **constant acceleration**

$$\boxed{a_{\text{centrip}} = \frac{v^2}{r}}$$

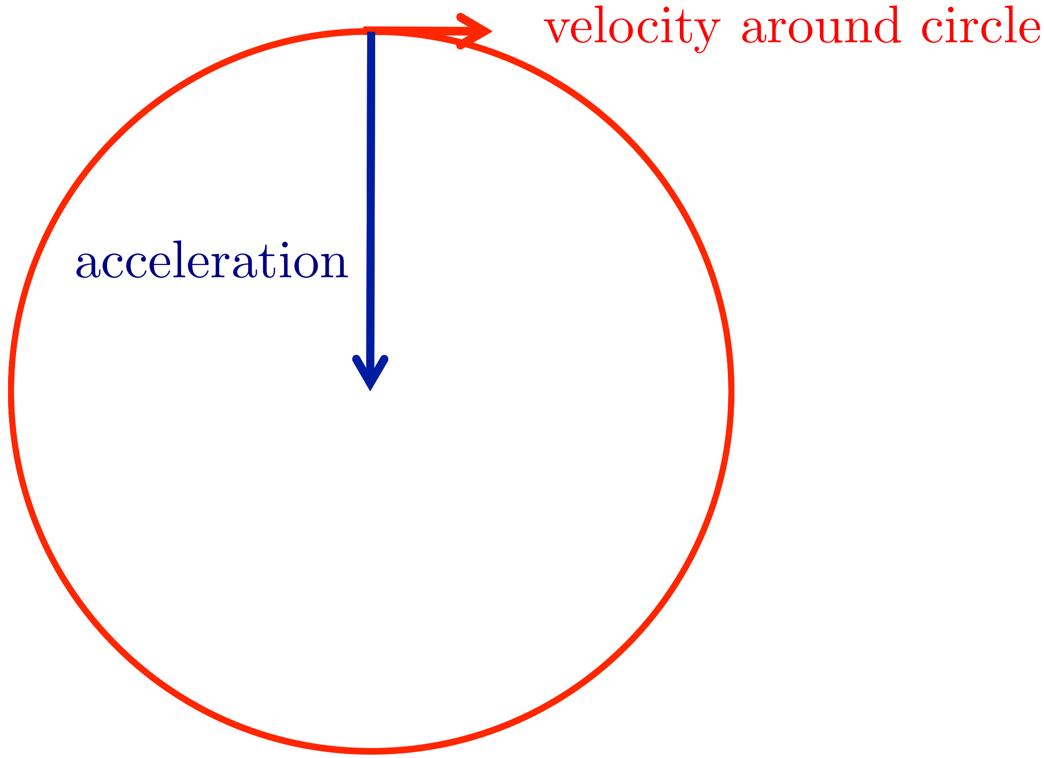
  - This is usually called **centripetal acceleration**

Ponderable (10 minutes)

- You're swinging something around on the end of a string. It moves in a circle of constant radius r with constant speed v.
  - In which direction does acceleration point when the object is at various points around the circle?
    - Remember, acceleration is how velocity changes over time…
  - Is the string tighter with a smaller radius, or a bigger radius?
  - If the string suddenly snaps, what direction does the object fly?

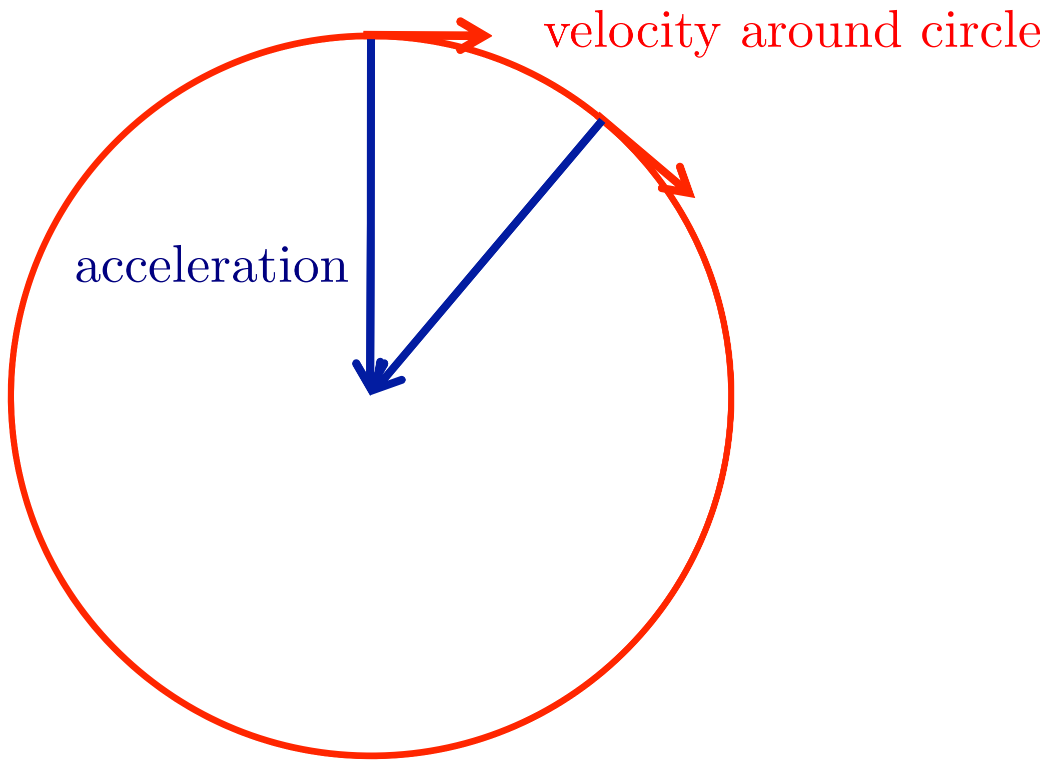


Centripetal Acceleration



- For an object moving with constant speed around a circle
  - The acceleration magnitude is constant but its direction is changing with time
  - Acceleration is always pointed towards the center of the circle
  - Remember, **acceleration is how velocity changes in time**

Centripetal Acceleration



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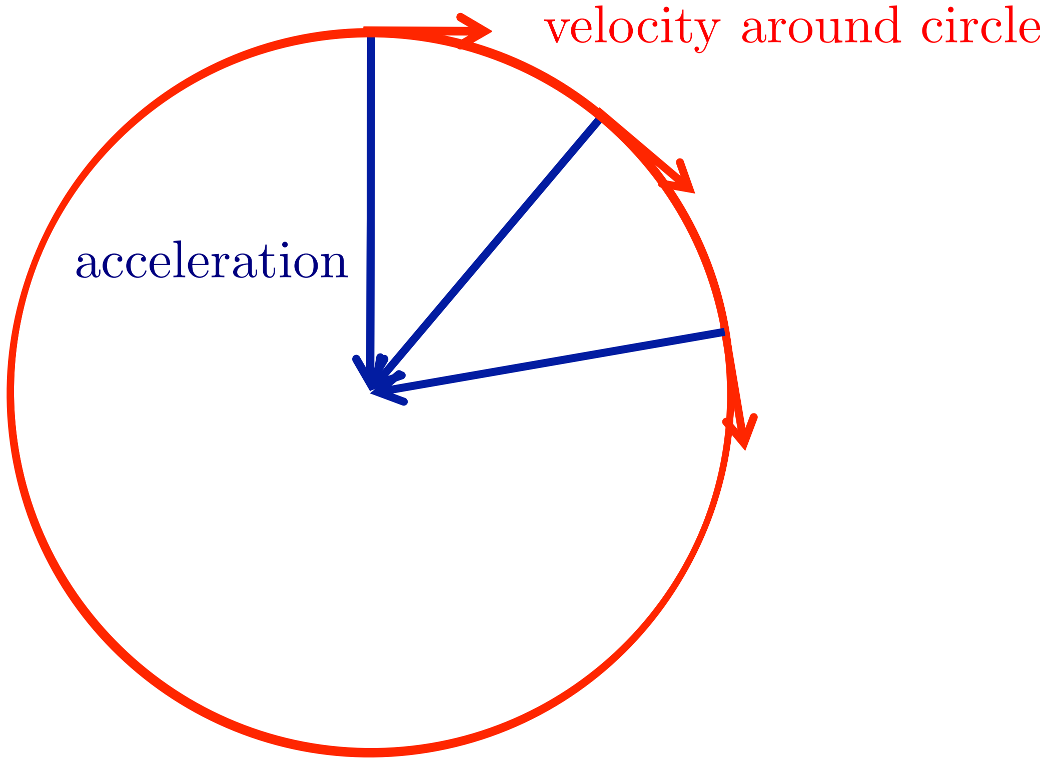

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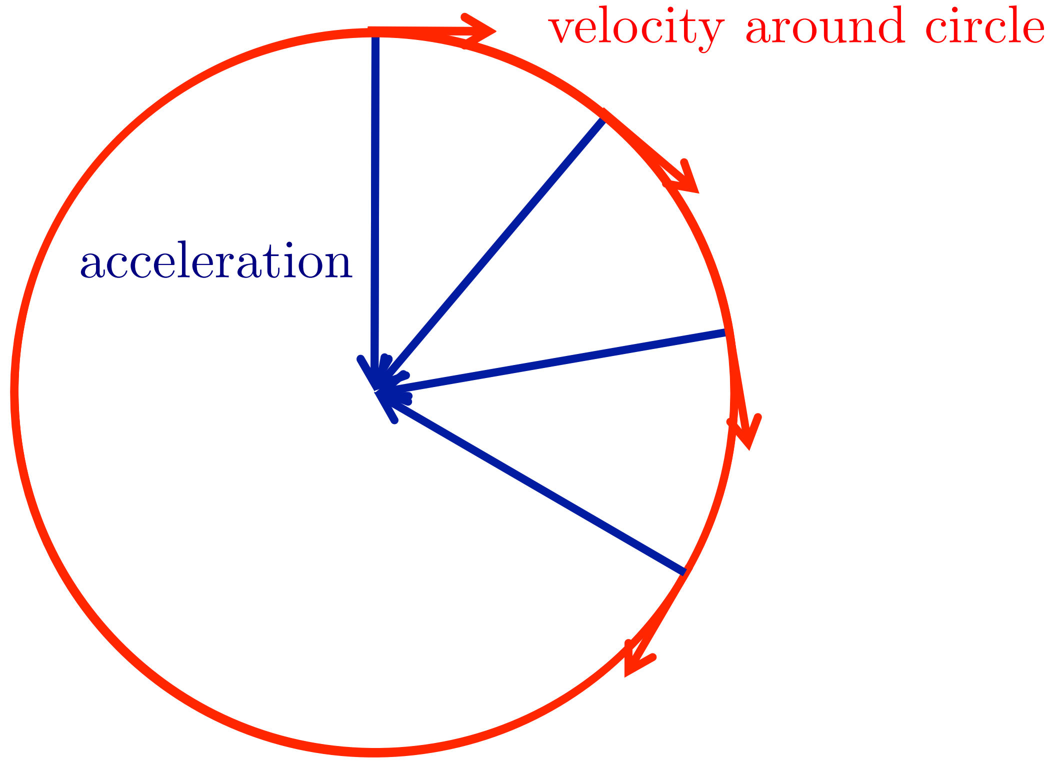

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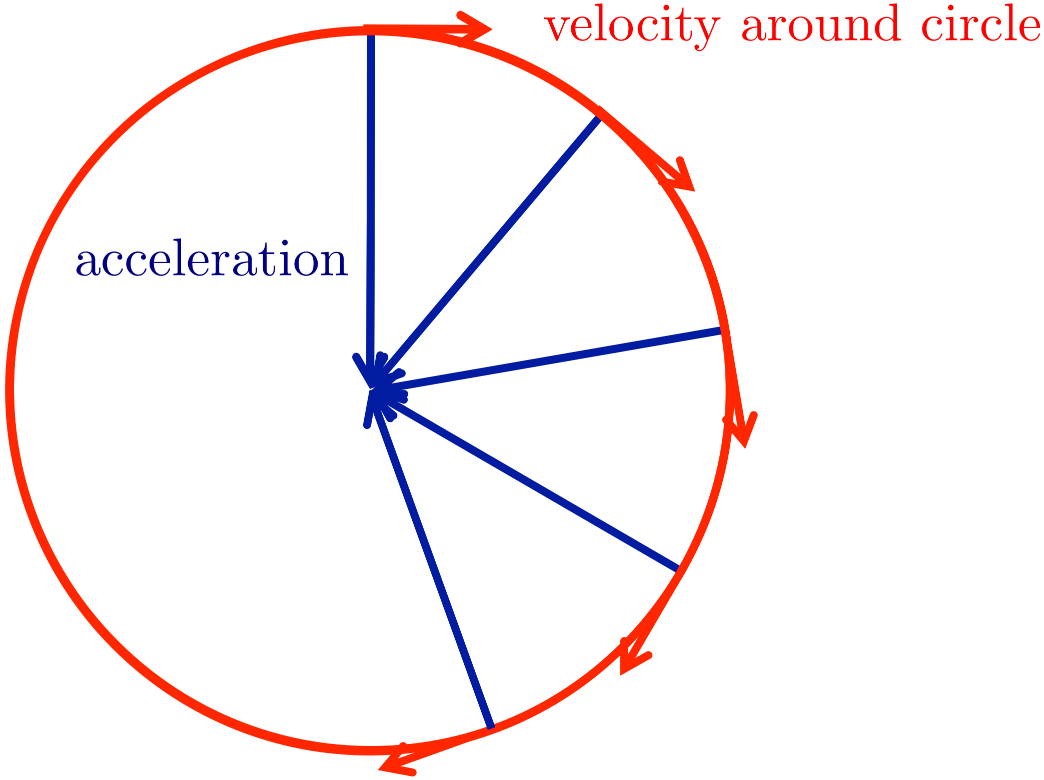
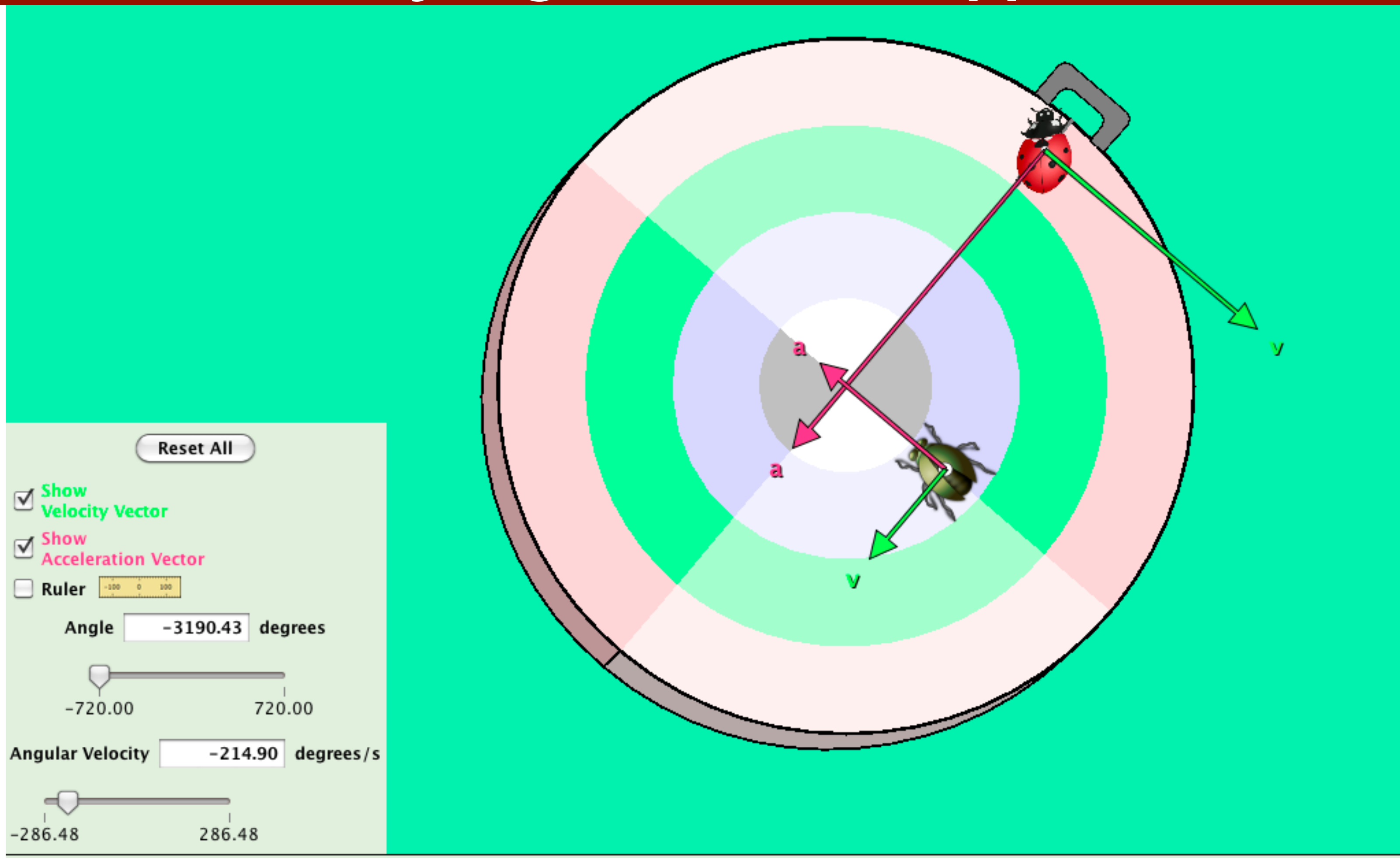
Centripetal Acceleration



- For an object moving with constant speed around a circle
  - The acceleration magnitude is constant but its direction is changing with time
  - Acceleration is always pointed towards the center of the circle
  - Applet time: http://phet.colorado.edu/en/simulation/rotation

Ladybug Revolution Applet



- Place the bugs on the disk about here
  - Adjust the angular velocity to -215 degrees/sec to see this pic
  - Differences in magnitudes **and** directions of velocity **and** acceleration
  - Does this seem to follow $a=v^2/r$?

Circular Motion: Circle Math

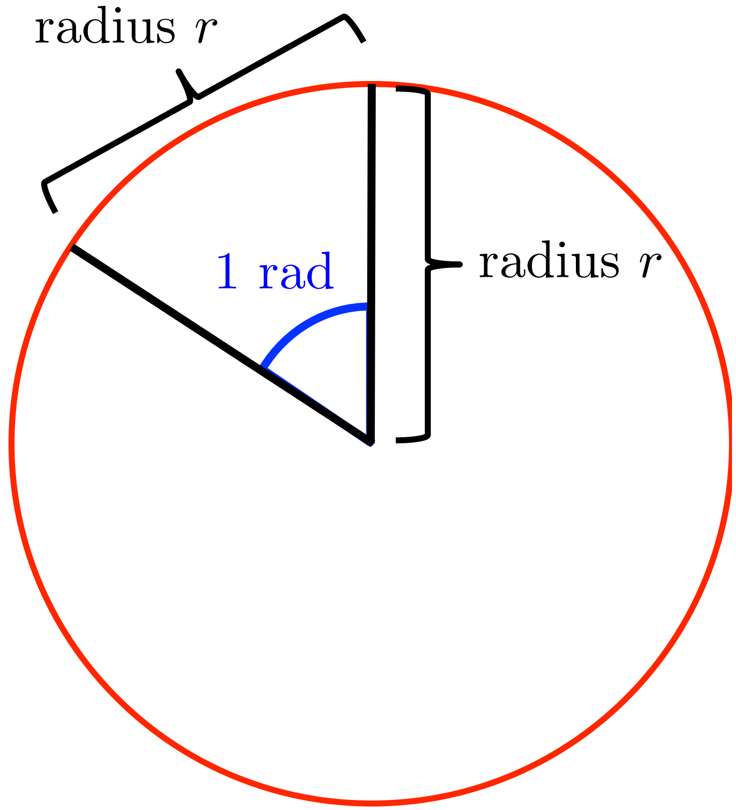
- We need language and math to describe circular motion
  - The radius of the circle determines a "length scale"
  - The other direction is "around" the circle on the circle's arc
- We define a **new unit of angle**
  - An angle of one radian intercepts a semicircular arc of length r
  - So there are 2π radians in 360°

$$\boxed{1 \text{ revolution} = 2\pi \text{ radians} = 360^\circ}$$

- More generally

$$\boxed{\text{arc length} = r\,\theta(\text{in radians})}$$

- So, e.g. one rpm angular velocity is a velocity of 2πr/minute.



Ponderable (10 minutes)

- Todd gets nostalgic and spins up his old **45 RPM** (revolutions per minute) record collection. Each record has a **7 inch total diameter**.
  - How fast is the outermost edge of the album moving in inches/sec?
  - How many "gees" of acceleration does a bug on the edge feel? ($g=32$ feet/s^2=384 inches/s^2)