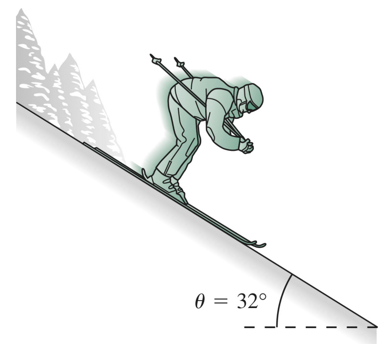
University Physics 226N/231N
Old Dominion University

Newton's Laws and Forces Examples



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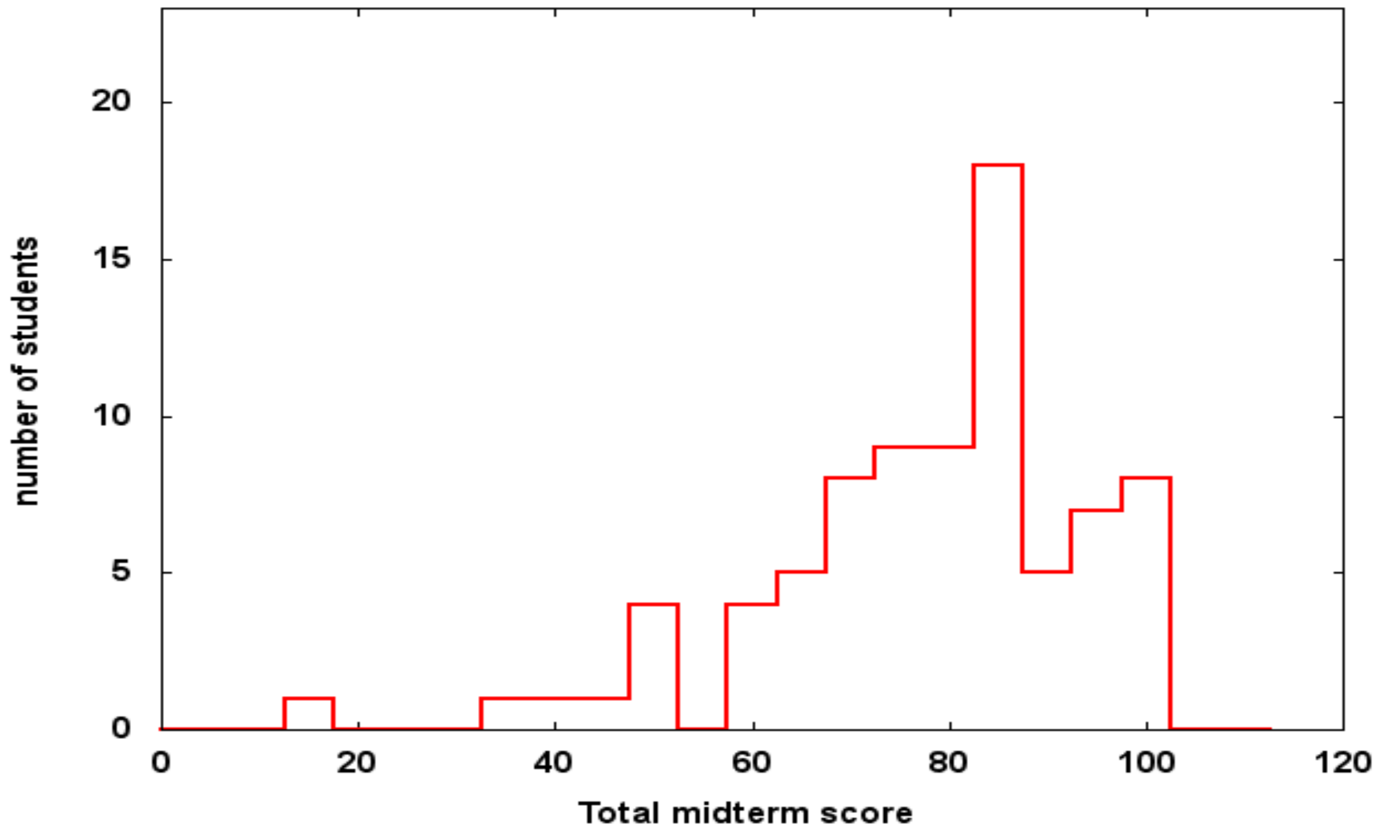
Wednesday, September 26 2012

Happy Birthday to Ivan Pavlov, Serena Williams, Olivia Newton-John, and Johnny Appleseed!

Happy Banned Books Day, Love Note Day, and Street Dancing Day

Midterm Solutions Posted



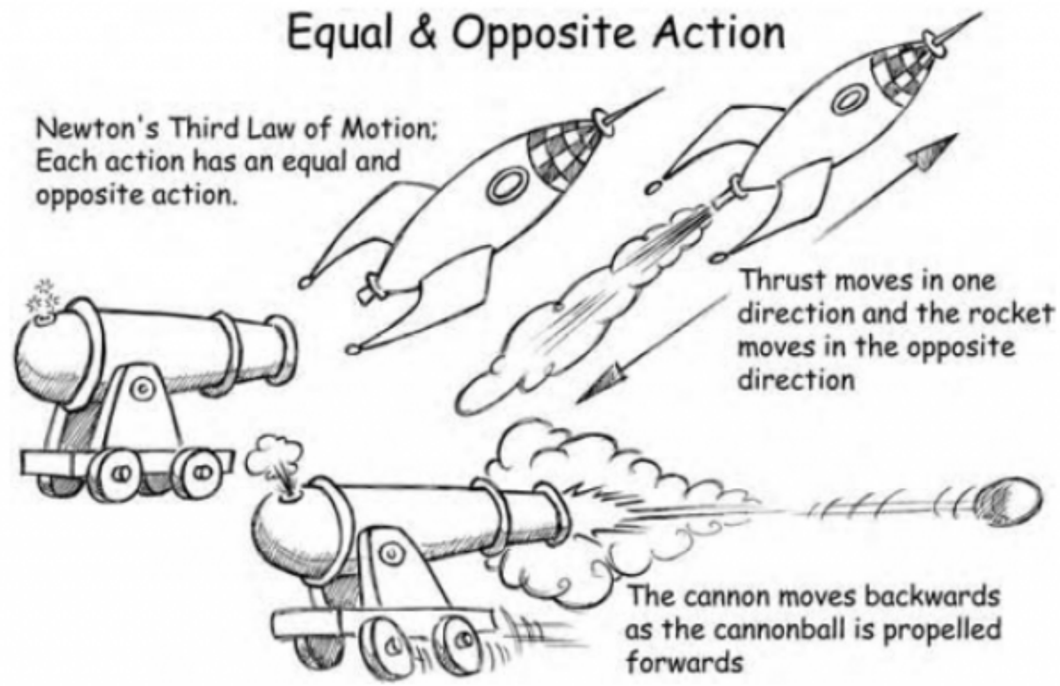
- I've graded midterms, distributing before and after class
  - Class average was about 80 with a few outliers
  - Please see me if you scored less than about 60-65
  - **Solutions and full statistics are posted on class website**
  - If you need a make-up, email me separately for arrangements

Reminders

- Newton's three laws really boil down to two
  - The net sum of all forces on an object produces acceleration

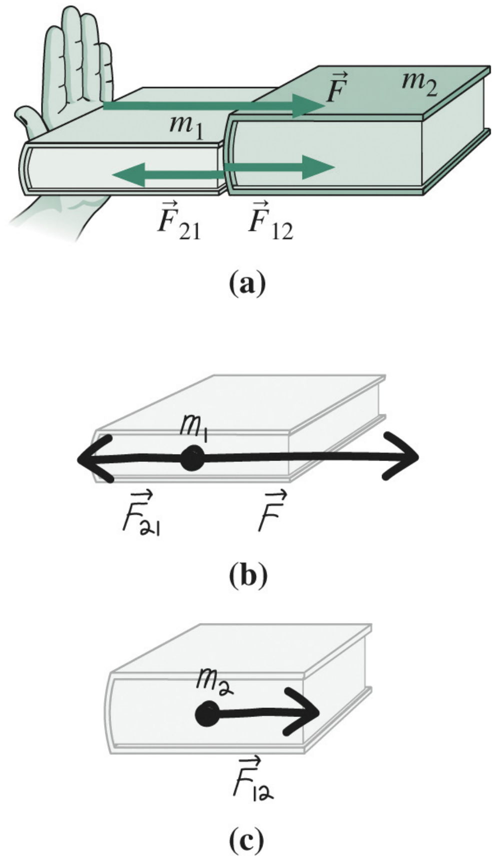
$$\boxed{\vec{F}_{\text{net}} = m\vec{a}}$$

  - Any force on an object is always matched by an equal and opposite force on the object applying the force

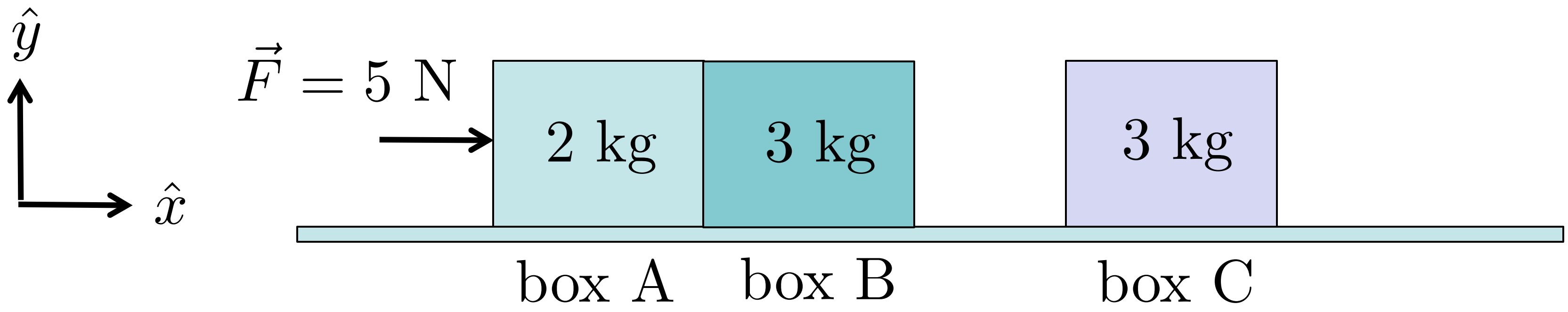


Newton's Third Law

- Forces **always** come in pairs.
  - If object A exerts a force on object B, then object B exerts an oppositely directed force of equal magnitude on A.
  - Obsolete language: "For every action there is an equal but opposite reaction."
  - The two forces always act on *different* objects; they can't cancel each other.
- Example:
  - Push on book of mass m_1 with force $\vec{F}$
  - Note third-law pair $\vec{F}_{12}$ and $\vec{F}_{21}$
  - Third law is necessary for a consistent description of motion in Newtonian physics.

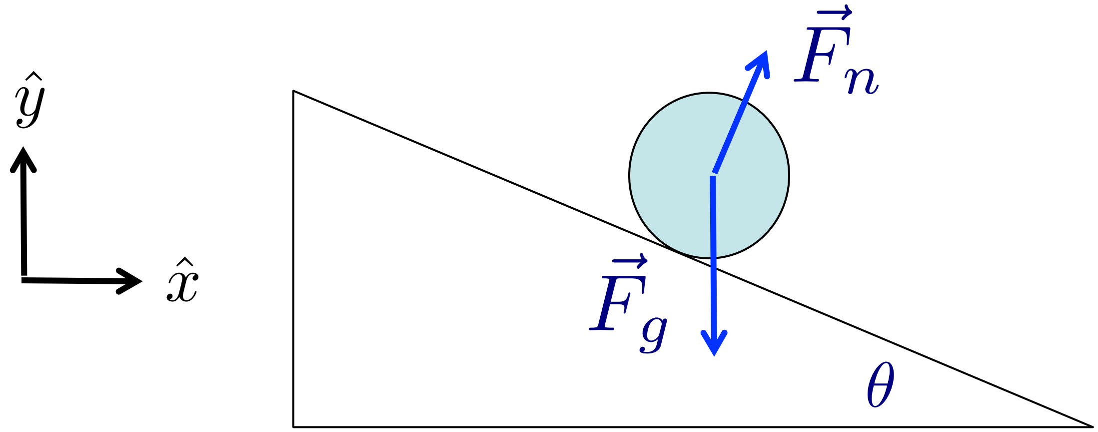


Ponderable (10 minutes)



- Draw and label **all** the forces on all three boxes above
  - Assume boxes A and B are touching
  - Assume there is no friction with the table top
  - Assume the force is constant
- What are the initial accelerations of all three boxes?
- What is the acceleration of all three boxes after boxes A and B have hit and move with box C?
- Do boxes A and B slow down (reduce velocity or speed) when they hit box C?

Newton's Second/Third Laws: Inclined Plane



$$m = 5\ \text{kg}$$

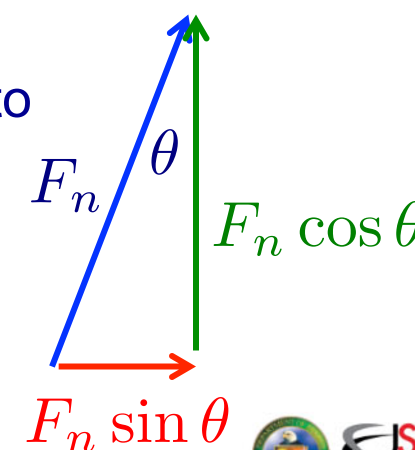
$$\theta = 20^\circ$$

$$F_g = mg = 49\ \text{N}$$

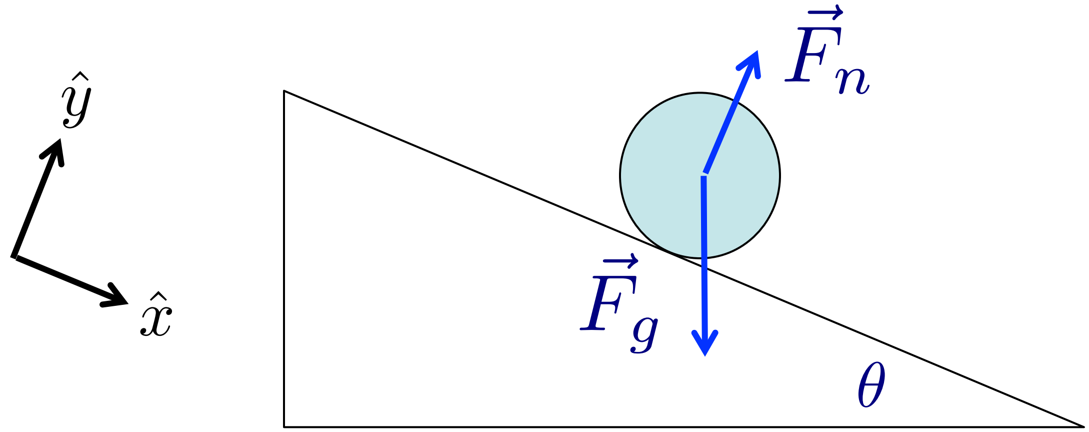
- A small ball of m=5 kg is on an inclined plane of 20 degrees
  - What are all the forces acting on the ball? (no friction for now!)
    - Note that forces are vectors: they have direction and magnitude!
    - Two forces: gravity pointing down and push of plane pointing perpendicular to the surface of the plane.
  - **What are the components of the forces**? One way to look at it is with the x,y axes shown above

$$F_{g,x} = 0 \qquad F_{n,x} = F_n \sin\theta$$

$$F_{g,y} = -F_g \qquad F_{n,y} = F_n \cos\theta$$



Newton's Second/Third Laws: Inclined Plane



$$m = 5 \text{ kg}$$

$$\theta = 20^\circ$$

$$F_g = mg = 49 \text{ N}$$

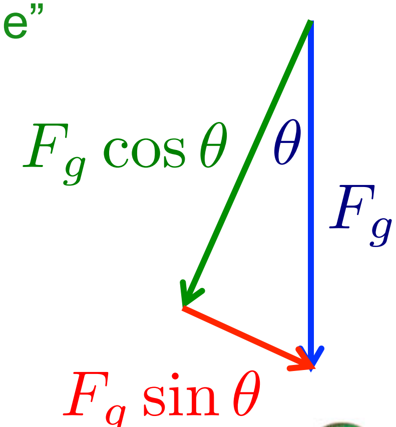
- A small ball of m=5 kg is on an inclined plane of 20 degrees
  - **What are the components of the forces**? Another way to look at it is with axes parallel to and perpendicular to the plane
    - This makes the final acceleration easier to calculate – we know that the net force and acceleration are "down the plane"

$$F_{n,x} = 0$$

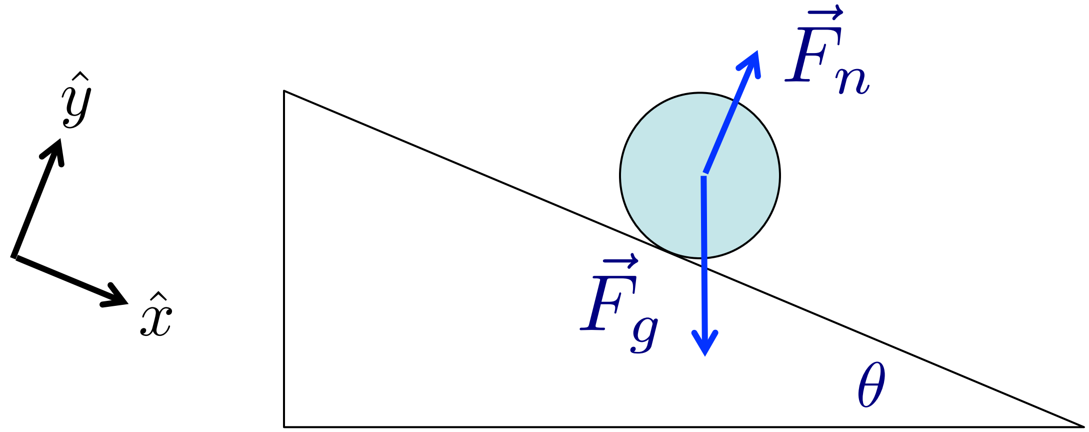
$$F_{n,y} = F_n$$

$$F_{g,x} = F_g \sin\theta$$

$$F_g, y = -F_g \cos\theta$$



Newton's Second/Third Laws: Inclined Plane

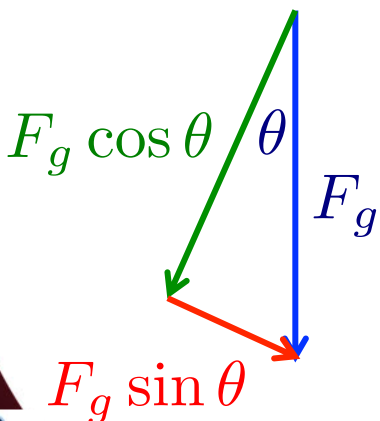


$$m = 5 \text{ kg}$$

$$\theta = 20^\circ$$

$$F_g = mg = 49 \text{ N}$$

- A small ball of m=5 kg is on an inclined plane of 20 degrees
  - **What is the normal force from the plane**, F_n?



$$F_{g,x} = F_g \sin\theta \qquad F_{n,x} = 0$$

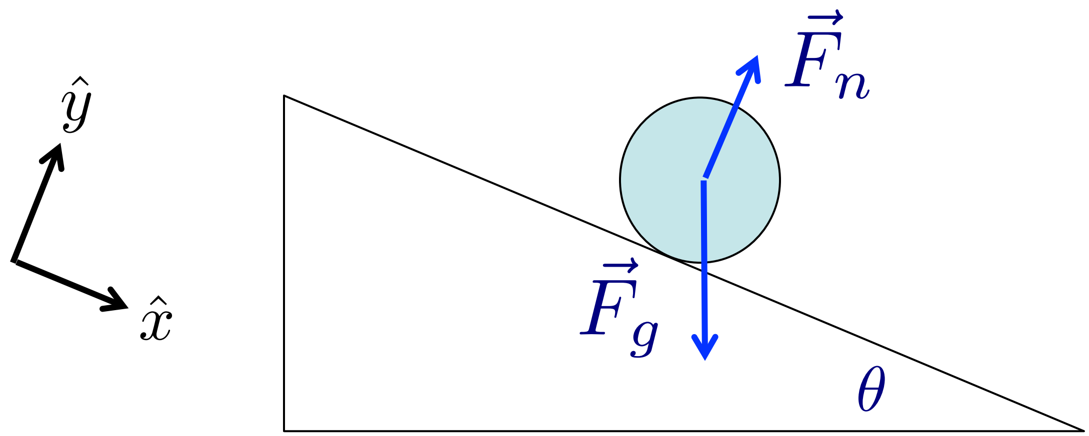
$$F_{g,y} = -F_g \cos\theta \qquad F_{n,y} = F_n$$

$$F_{\text{net,y}} = F_n - F_g \cos\theta = 0$$

No acceleration perpendicular to the plane

$$F_n = F_g \cos\theta = (49 \text{ N}) \cos(20^\circ) = \boxed{46 \text{ N} = F_n}$$

Newton's Second/Third Laws: Inclined Plane

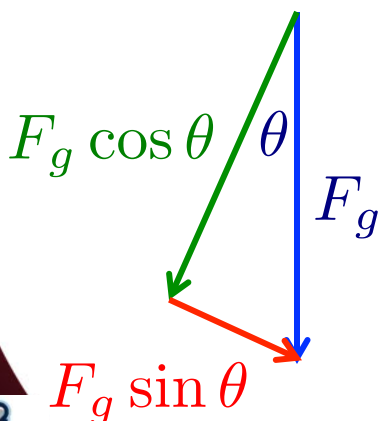


$$m = 5\ \text{kg}$$

$$\theta = 20^\circ$$

$$F_g = mg = 49\ \text{N}$$

- A small ball of m=5 kg is on an inclined plane of 20 degrees
  - **What is the acceleration of the ball down the plane?** a_x



$$F_{g,x} = F_g \sin\theta \qquad F_{n,x} = 0$$

$$F_{g,y} = -F_g \cos\theta \qquad F_{n,y} = F_n$$

$$F_{\text{net,x}} = ma_x = F_{g,x} + F_{n,x} = F_g \sin\theta + 0 = mg\sin(20^\circ)$$

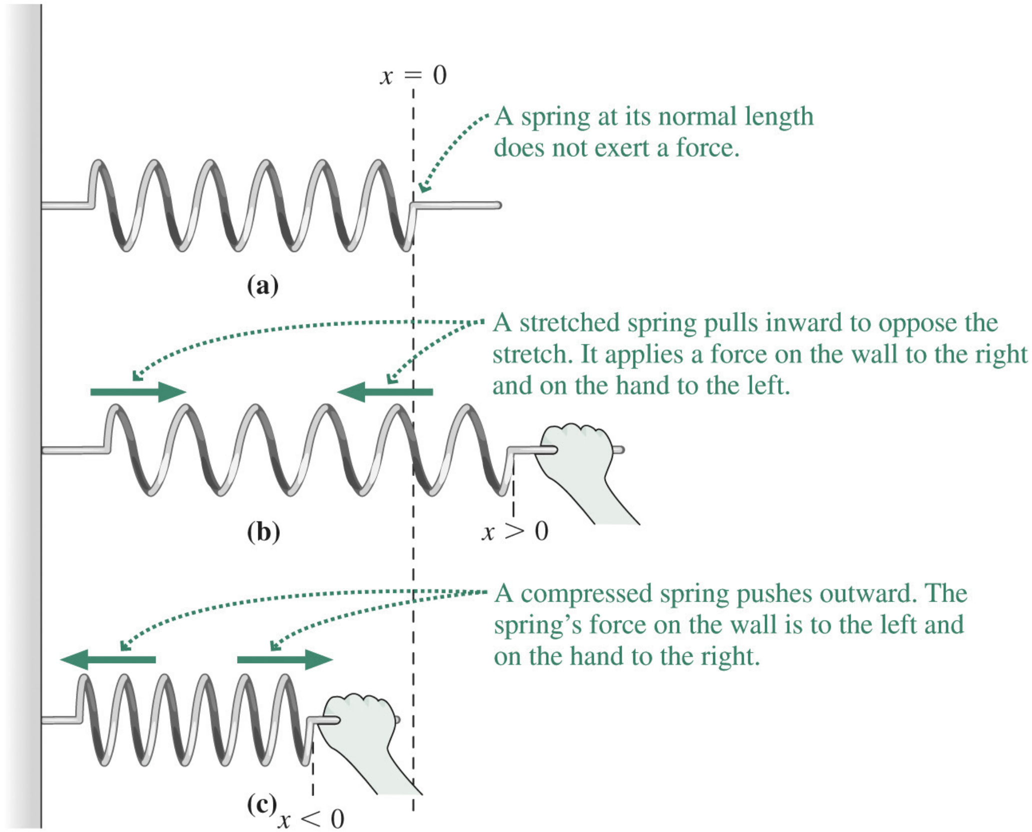
$$a_x = g\sin(20^\circ) = (9.8\ \text{m/s}^2)(0.342) = \boxed{3.35\ \text{m/s}^2 = a_x}$$

Spring Forces

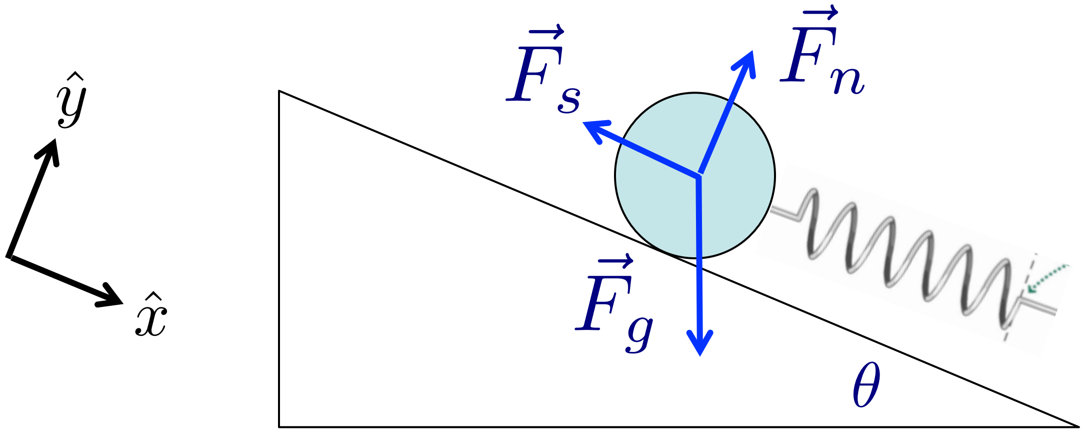
- A stretched or compressed spring produces a force proportional to the stretch or compression from its equilibrium configuration: $F_{sp} = -kx$.

k is in units of N/m

- The spring force is a **restoring force** because its direction is opposite that of the stretch or compression.
- Springs provide convenient devices for measuring force.



Ponderable



$m = 5\ \mathrm{kg}$

$\theta = 20^\circ$

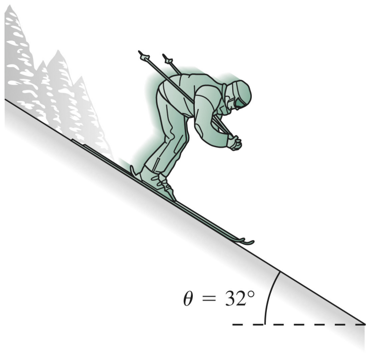
$F_g = mg = 49\ \mathrm{N}$

- A small ball of m=5 kg is on an inclined plane of 20 degrees
  - **Now we put a spring (k=2 N/m) beneath it to support it**
  - The spring compresses until the force it exerts counteracts gravity's force on the ball

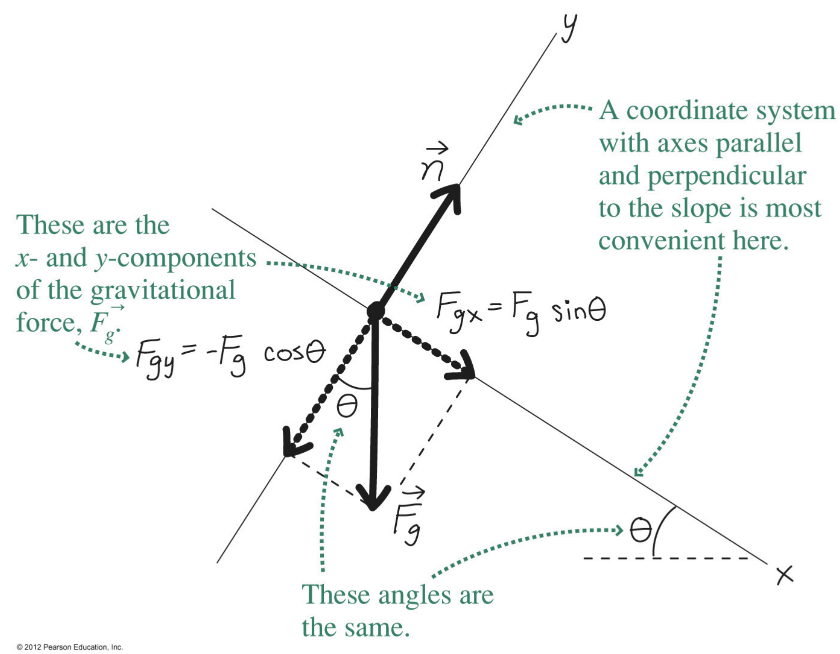
    - **How far does the spring compress?**
    - **If we double the mass of the ball, how does the spring compression distance change?**

A Typical Problem: What's the skier's acceleration? What's the force the snow exerts on the skier?

- Physical diagram:



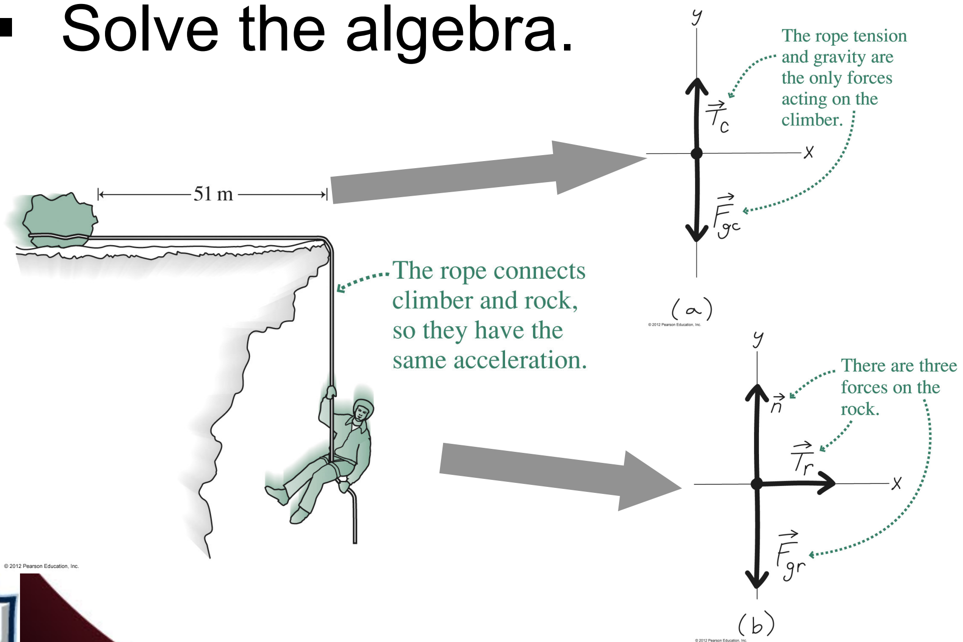
- "Free-body" diagram:



- Newton's law: $\vec{F}_{\text{net}} = \vec{n} + \vec{F}_g = m\vec{a}$
- In components:
  - x-component: $mg \sin\theta = ma$
  - y-component: $n - mg\cos\theta = 0$
- Solve (with $m = 65$ kg and $\theta = 32°$) to get the answers:
  - $a = g \sin\theta = (9.8 \text{ m/s}^2)\sin 32° = 5.2 \text{ m/s}^2$
  - $n = mg\cos\theta = (65 \text{ kg})(9.8 \text{ m/s}^2)\cos 32° = 540 \text{ N}$

Multiple Objects

- Solve problems involving multiple objects by first identifying each object and all the forces on it.
- Draw a free-body diagram for each.
- Write Newton's law for each.
- Identify connections between the objects, which give common terms in the Newton's law equations.
- Solve the algebra.



- Newton's law:

$$\text{climber: } \vec{T}_c + \vec{F}_{gc} = m_c \vec{a}_c$$

$$\text{rock: } \vec{T}_r + \vec{F}_{gr} + \vec{n} = m_r \vec{a}_r$$

- In components:

$$\text{climber, } y\text{: } T - m_c g = -m_c a$$

$$\text{rock, } x\text{: } T = m_r a$$

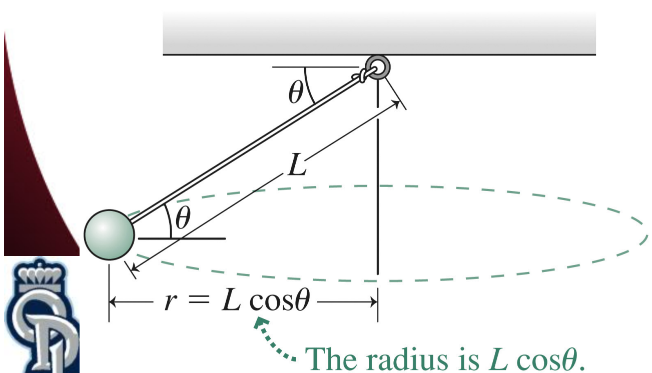
$$\text{rock, } y\text{: } n - m_r g = 0$$

- Solution: $\boxed{a = \dfrac{m_c g}{m_c + m_r}}$

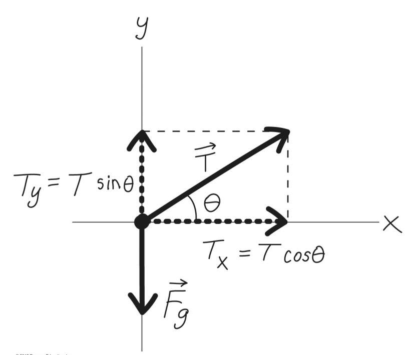
Circular Motion

- Problems involving circular motion are no different from other Newton's law problems – the geometry is just more complicated.
- Identify the forces, draw a free-body diagram, write Newton's law.
- The magnitude of the **centripetal force** on an object of mass *m* in circular motion with radius *r* is $F = ma = \frac{mv^2}{r}$
  - the acceleration has magnitude v^2/r and points toward the center of the circle

A ball whirling on a string:



Free-body diagram:



- Newton's law: $\vec{T} + \vec{F}_g = m\vec{a}$
- In components:

$$x: \quad T\cos\theta = \frac{mv^2}{L\cos\theta}$$

$$y: \quad T\sin\theta - mg = 0$$

- Solve for the ball's speed:

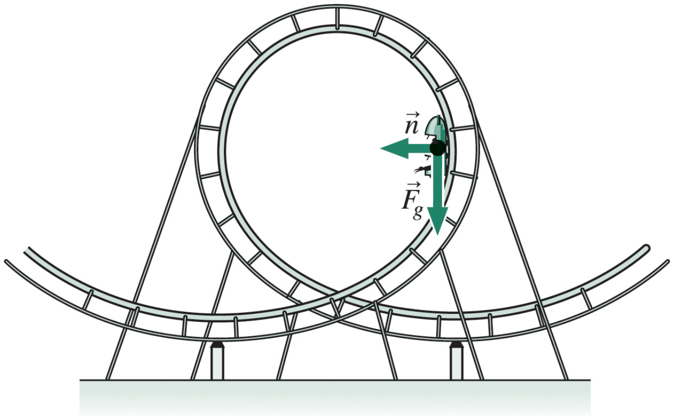
$$\boxed{v = \sqrt{\frac{TL\cos^2\theta}{m}} = \sqrt{\frac{(mg/\sin\theta)L\cos^2\theta}{m}} = \sqrt{\frac{gL\cos^2\theta}{\sin\theta}}}$$

Loop-the-Loop!

- The two forces acting on the roller-coaster car are:
  - gravity
  - normal force
- Gravity is always downward, and the normal force is perpendicular to the track.
- At the position shown, the two forces are at right angles:
  - The normal force acts perpendicular to the car's path, keeping its direction of motion changing.
  - Gravity acts opposite the car's velocity, slowing the car.
  - The net force is *not* toward the center

Newton's law :

$$\vec{n} + \vec{F}_g = m\vec{a}$$

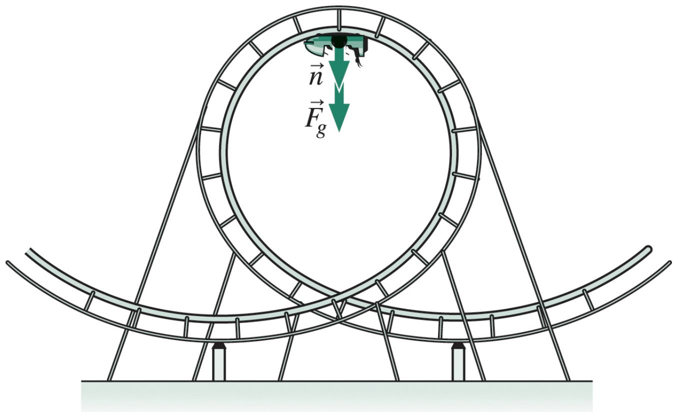


Loop-the-Loop!

- At the top of the loop, both forces are downward:

$$n_y = n,\ F_{gy} = mg \quad \Rightarrow \quad n + mg = \frac{mv^2}{r}$$

  - Solving for *v*, we obtain $v = \sqrt{nr/m + gr}$
  - For the car to stay in contact with the track, the normal force must be greater than zero.
  - So the minimum speed is the speed that let the normal force get arbitrarily close to zero at the top of the loop.
  - Then gravity alone provides the force that keeps the car in circular motion.

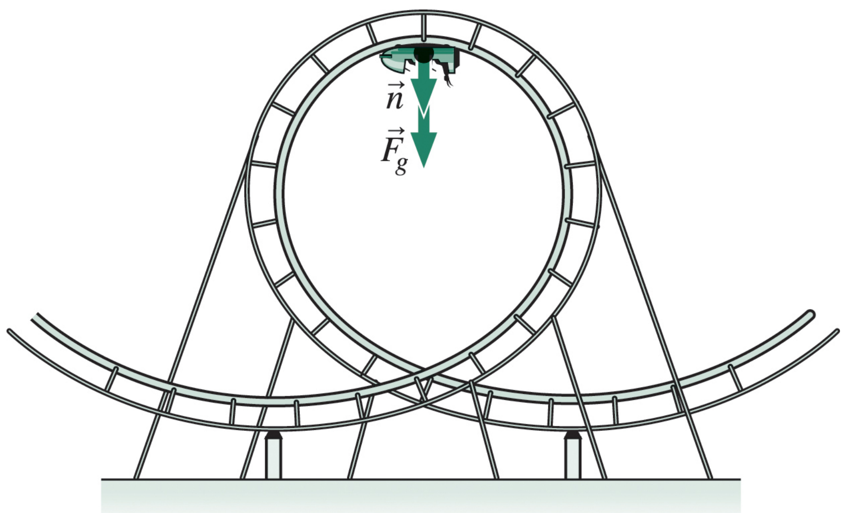


Loop-the-Loop!

- Therefore Newton's law has a single component, with the gravitational force *mg* providing the acceleration v^2/r that holds the car in its circular path:

$$\vec{F} = m\vec{a} \quad \rightarrow \quad mg = \frac{mv^2}{r}$$

  - Solving for the minimum speed at the loop top gives $v = \sqrt{gr}$.
  - Slower than this at the top, and the car will leave the track!
  - Since this result is independent of mass, car and passengers will all remain on the track as long as $v \geq \sqrt{gr}$.

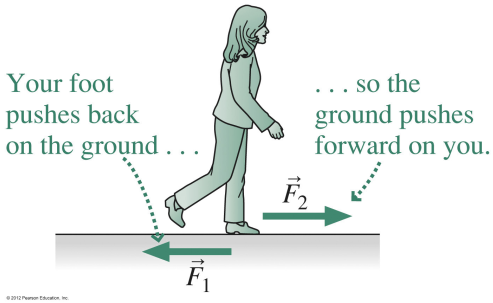


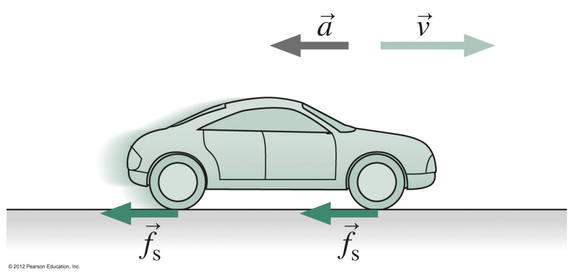
Following slides for Friday

Friction

- **Friction** is a force that opposes the relative motion of two contacting surfaces.
- **Static friction** occurs when the surfaces aren't in motion; its magnitude is $f_s \geq \mu_s n$, where n is the normal force between the surfaces and μ_s is the **coefficient of static friction.**
- **Kinetic friction** occurs between surfaces in motion; its magnitude is $f_k = \mu_k n$.

Friction is important in walking, driving and a host of other applications:

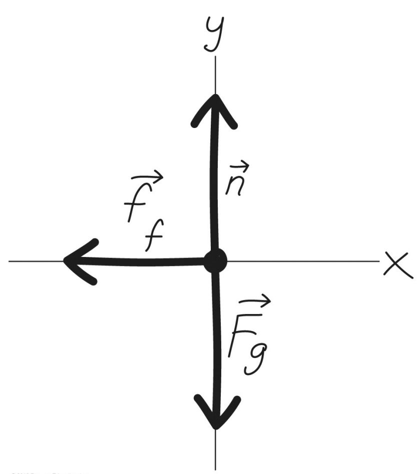




Solving Problems with Friction

- Problems with friction are like all other Newton's law problems.
- Identify the forces, draw a free-body diagram, write Newton's law.
- You'll need to relate the force components in two perpendicular directions, corresponding to the normal force and the frictional force.

- Example: A braking car: What's the acceleration?



- Newton's law: $\vec{F}_g + \vec{n} + \vec{f}_\mathrm{f} = m\vec{a}$
- In components: $x{:}\quad -\mu n = m a_x$

  $y{:}\quad -mg + n = 0$
- Solve for *a*:

  y equation gives $n = mg$,

  so x equation gives $a_x = -\dfrac{\mu n}{m} = -\mu g$