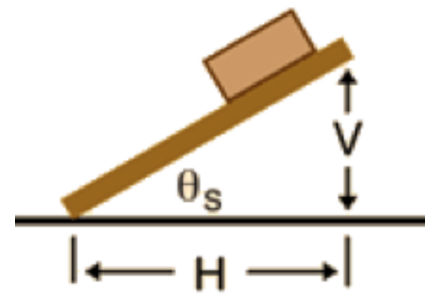
University Physics 226N/231N
Old Dominion University

Getting Loopy and Friction



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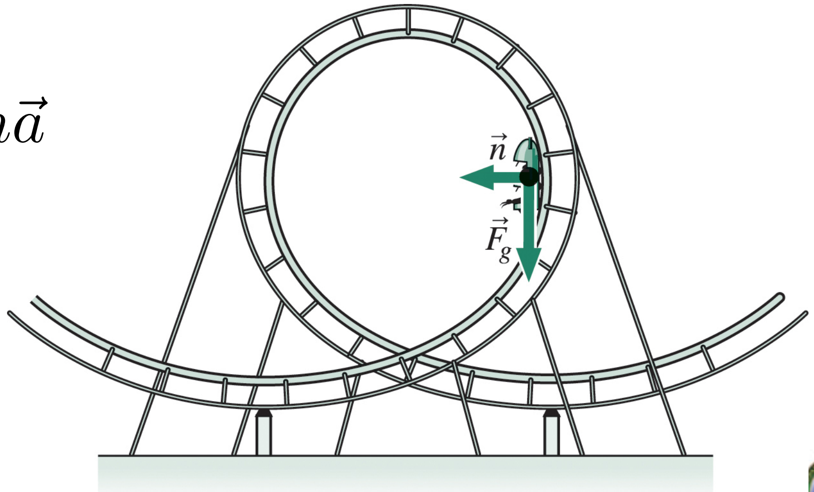
Happy Birthday to Hilary Duff, Moon Unit Zappa, Arthur Guinness, and Confucius!
Happy Ask a Stupid Question Day and Drink as Much Beer as Possible Day!

Loop the Loop!

- The two forces acting on the roller-coaster car are:
  - Gravity $\vec{F}_g$ and the "normal" force $\vec{n}$ of the track's push
- Gravity is always downward, and the normal force is perpendicular to the track.
- At the position shown, the two forces are at right angles:
  - The normal force acts perpendicular to the car's path, keeping its direction of motion changing.
  - Gravity acts opposite the car's velocity, slowing the car.
  - The net force (acceleration) is *not* toward the center

$$\text{Newton}: \quad \vec{F}_{\text{net}} = \vec{n} + \vec{F}_g = m\vec{a}$$



Loop the Loop!

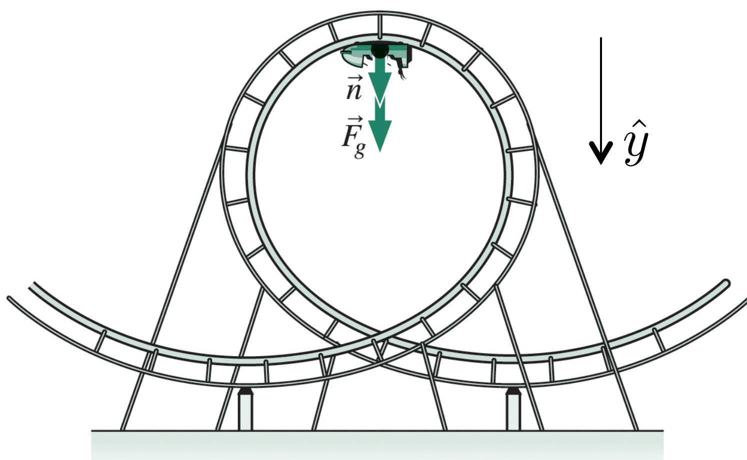
- At the top of the loop, both forces are vertically downward.
  - I selected the positive y direction to be **down** here (see diagram)

$$n_{\mathrm{y}} = n \qquad F_{\mathrm{g,y}} = mg \qquad \Rightarrow \qquad F_{\mathrm{net,y}} = n + mg = a_{\mathrm{centrip}} = \frac{mv^2}{r}$$

  - Solving for *v*, we obtain $v = \sqrt{(nr + mgr)/m} = \sqrt{(nr/m) + gr}$
  - For the car to stay in contact with the track, the normal force must be greater than zero.
  - So the minimum speed is the speed that let the normal force get arbitrarily close to zero at the top of the loop.
  - Then gravity alone (barely) provides the force that keeps the car in circular motion.
  - When n=0 (barely on track):

$$v_{\mathrm{min\ safe}} = \sqrt{gr}$$

  - Independent of mass m!!



Frictional Forces

- **Friction** is a **force** (magnitude and direction!) that opposes the relative motion (velocity) of two contacting surfaces.
  - Newton's third law! Each surface feels equal and opposite force!
  - Really a result of lots of microscopic/molecular surface forces
- We can develop a pretty good basic model of frictional forces
  - **Frictional force** between two objects moving relative to each other always **acts opposite to their relative velocity**
  - This **kinetic friction** depends on the magnitude of the normal force n between them and a **coefficient of kinetic friction** μ_k

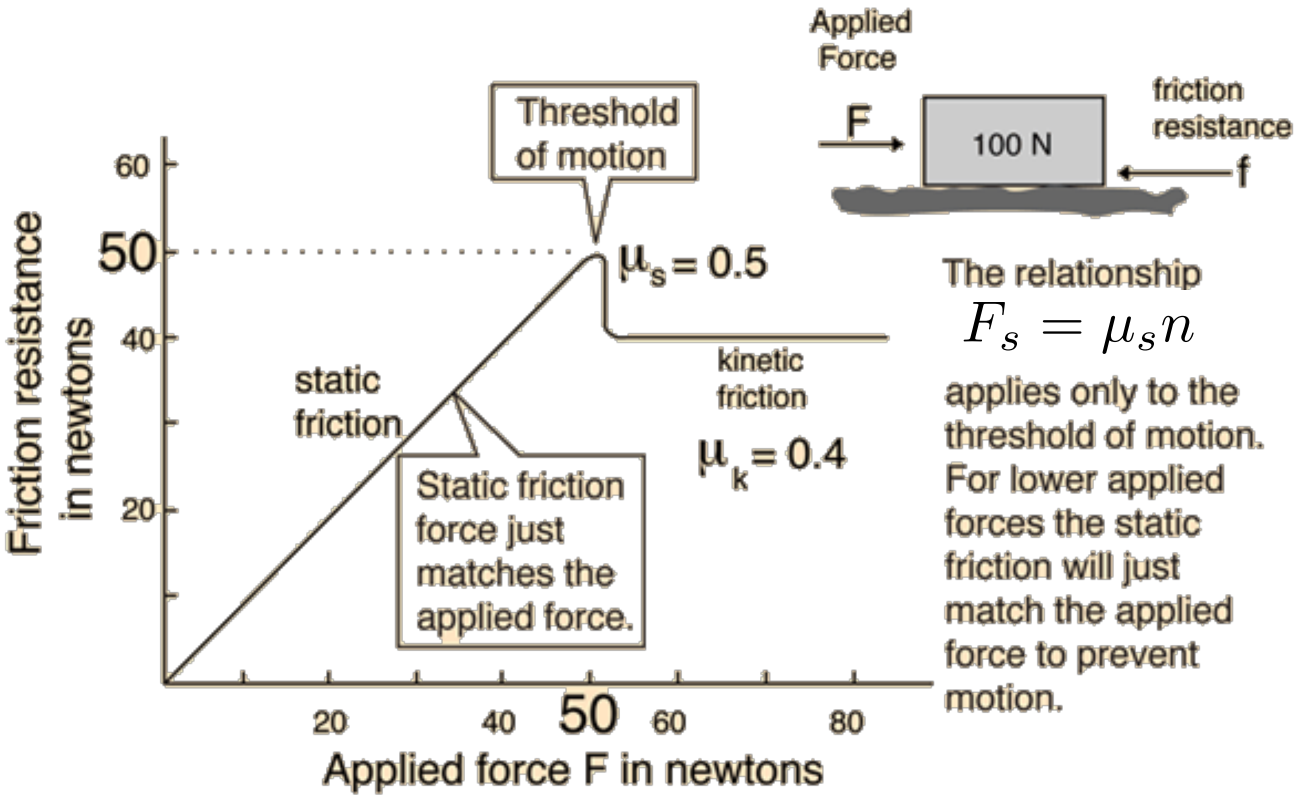
$$F_k = \mu_k n$$

  - For objects that aren't moving to each other, the **static friction** acts to cancel applied forces up to some **limit where they start moving.** This is described with a different **coefficient of static friction** μ_s

$$F_s \leq \mu_s n$$

  - Interesting note: **Frictional force does not depend on velocity!**

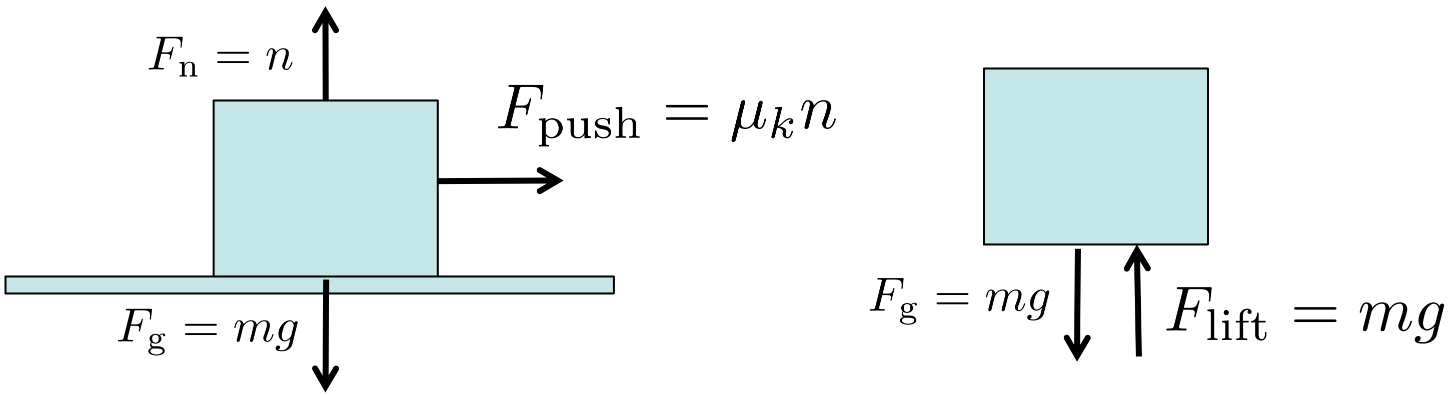
Static and Kinetic Frictional Forces



http://hyperphysics.phy-astr.gsu.edu/hbase/frict2.html

- Static friction acts to exactly cancel an applied force up to its maximum value, at which the object starts moving
  - The 100N object above does not start moving until the applied force F is greater than 50 N: $F_s = \mu_s n = (0.5)(100\ \mathrm{N}) = 50\ \mathrm{N}$
  - When the object starts moving, kinetic friction applies instead

Tangible/Ponderable (~5 minutes)

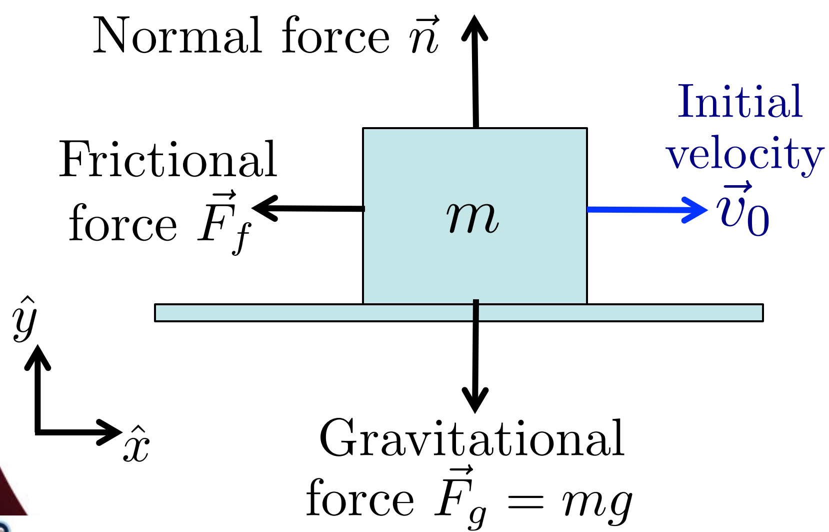


- Put a flat object on the table in front of you (e.g. cell phone, notebook..)
  - Use an object that does not roll (we haven't discussed that yet)
  - Compare the forces to push it **horizontally** at constant speed, and to hold it **vertically** still against the pull of gravity
  - Estimate the coefficient of kinetic friction for this situation
  - Try it again on a different flat surface (e.g. a white board)
- Can coefficients of kinetic or static friction be greater than 1?
  - Interesting note: a flatter, smoother surface does not necessarily mean less friction!

Example Friction Problem

- Problems with friction are like all other Newton's law problems.
  - Identify the forces, draw a diagram, identify vector components, write Newton's law and solve for unknowns.
  - You'll need to relate the force components in two perpendicular directions, corresponding to the normal force and the frictional force.

- **Example:** A box sliding to a stop due to friction on a surface



Coefficients of friction $\mu_s, \ \mu_k$

$$\text{Vertical}: F_{\text{net}} = n - mg = 0$$

$$n = mg$$

$$\text{Horizontal}: F_{\text{net}} = -F_f = -\mu_s n$$

$$F_{\text{net}} = -\mu_s mg = ma_x$$

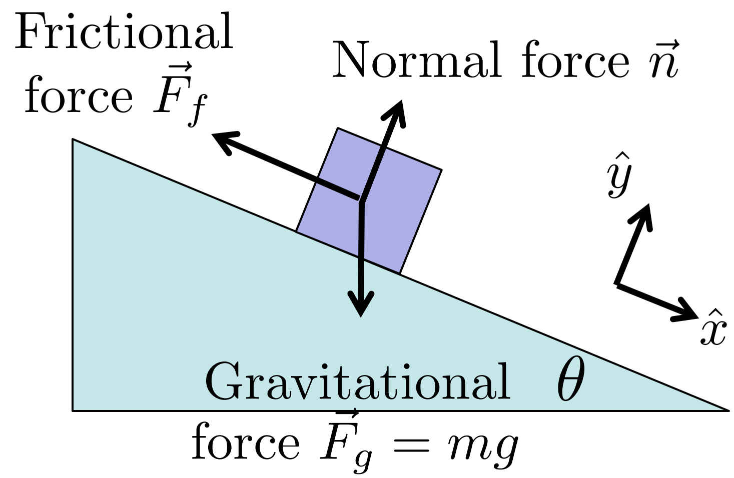
$$a_x = -\mu_s g$$

$$\text{Time to stop}: t = \frac{v - v_0}{a_x} = \boxed{\frac{v_0}{\mu_s g} = t}$$

A More Practical Friction Problem

A box of mass m sits on a surface. We incline the surface until the box just starts slipping down the surface, and measure this angle of incline θ.

What is μ_s?



$$\text{Vertical}: F_{\text{net}} = 0 = n - F_g \cos\theta$$

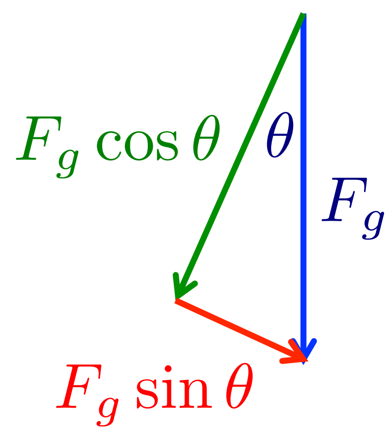
$$n = F_g \cos\theta$$

$$\text{Horizontal}: F_{\text{net}} = 0 = F_g \sin\theta - F_f$$

$$F_f = F_g \sin\theta$$

$$F_f = \mu_s n = \mu_s F_g \cos\theta$$

$$\boxed{\mu_s = \tan\theta}$$



Force being decomposed is hypoteneuse