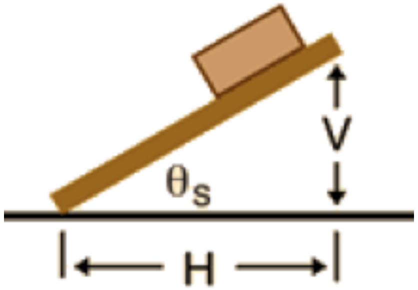
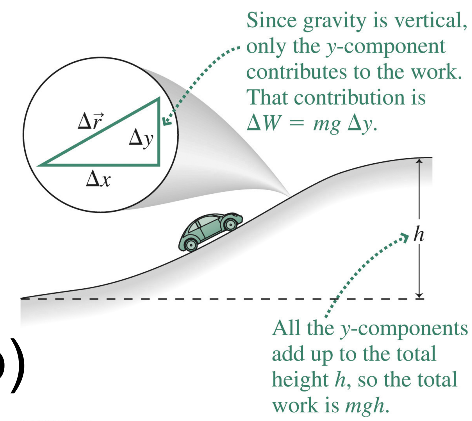


University Physics 226N/231N
Old Dominion University

Friction, Work, Energy



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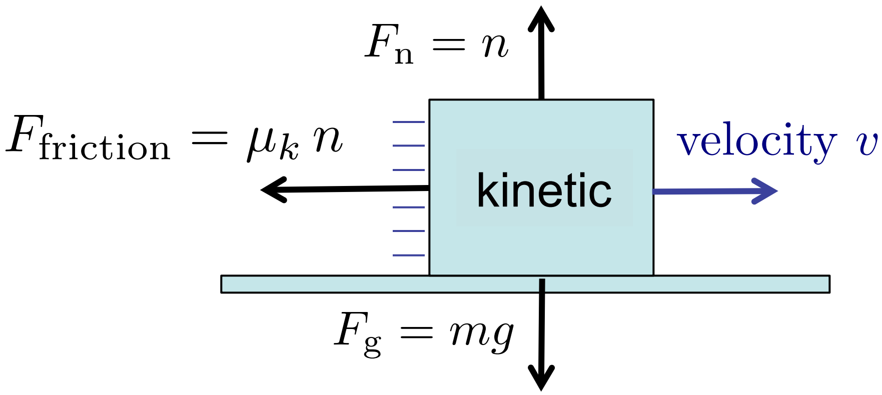
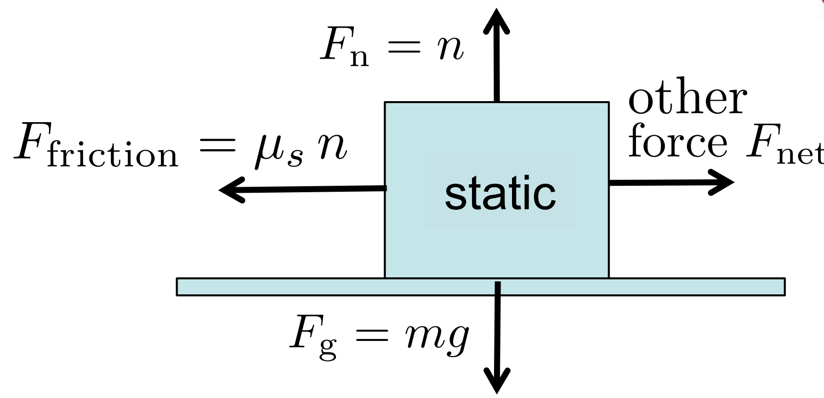
Monday, October 1 2012

Happy Birthday to Chen Ning Yang, Zach Galifianakis, William Boeing, and Walter Matthau!

Happy Postcard Day, World Vegetarian Day, and World Habitat Day!

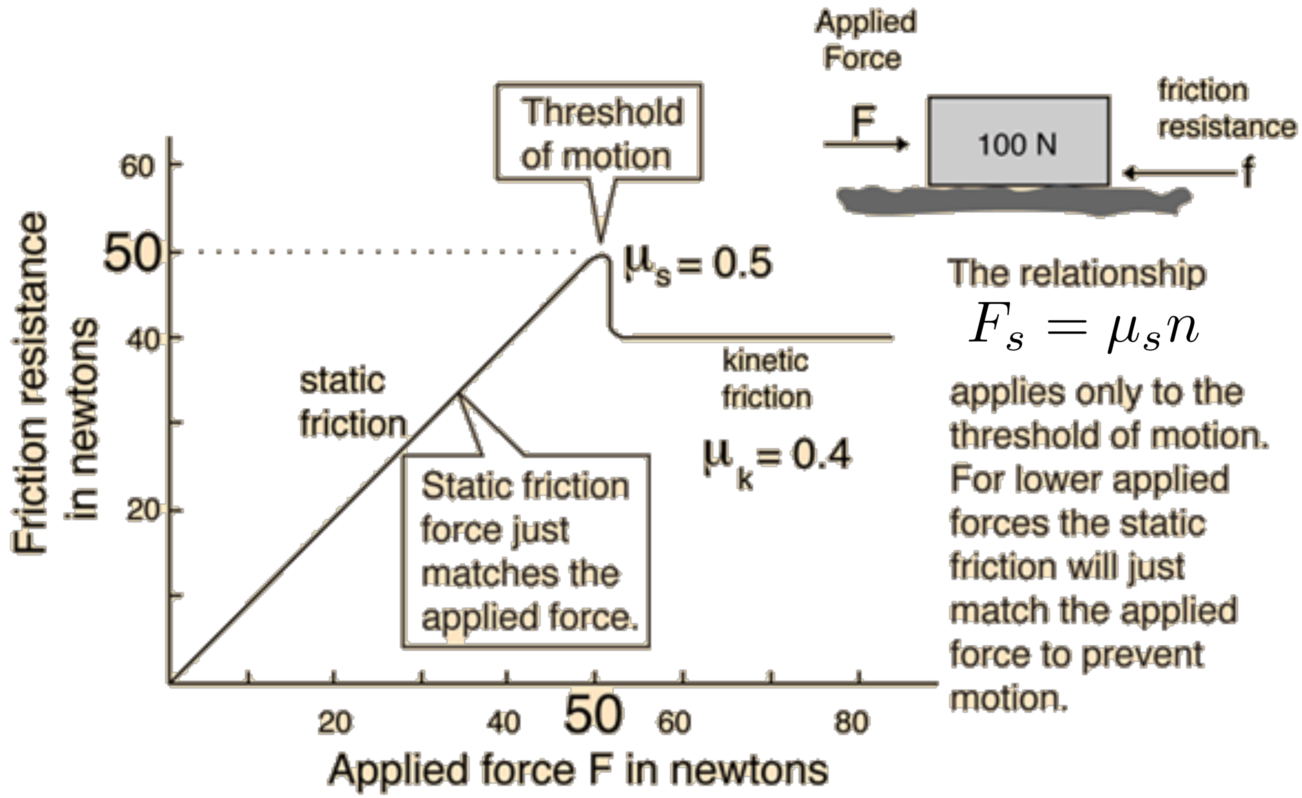
Frictional Forces





- **Friction** is a **force** (magnitude and direction!) that opposes the relative motion (velocity) of two contacting surfaces.
  - Newton's third law: Each surface feels equal and opposite force
- We have a pretty good basic model of frictional forces
  - Moving: **kinetic friction** $\boxed{F_k = \mu_k n}$ **against relative velocity**
  - Not moving: **static friction** $\boxed{F_s \leq \mu_s n}$ **against other net force**
  - This model of frictional force does **not** depend on velocity
  - Atmospheric friction (e.g. drag) is quite a lot more complicated
    - Depends on atmospheric density and viscosity, velocity, etc.

Static and Kinetic Frictional Forces



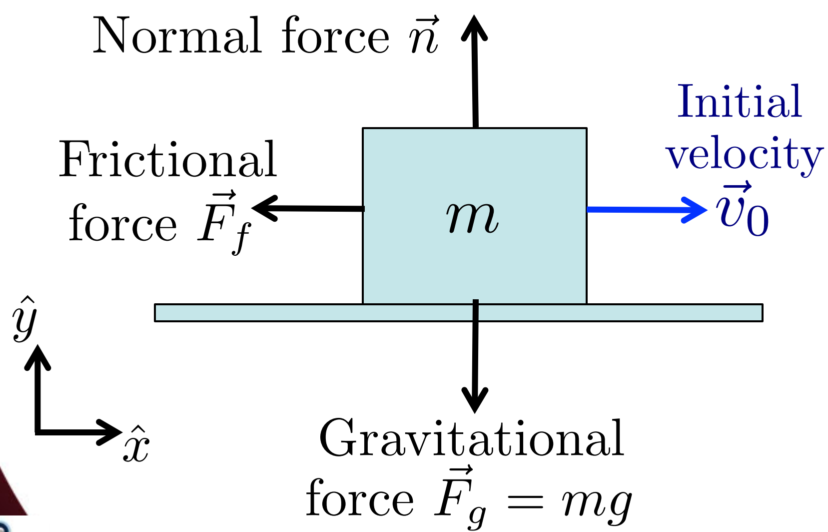
http://hyperphysics.phy-astr.gsu.edu/hbase/frict2.html

- Static friction acts to exactly cancel an applied force up to its maximum value, at which the object starts moving
  - The 100N object above does not start moving until the applied force F is greater than 50 N: $F_s = \mu_s n = (0.5)(100\text{ N}) = 50\text{ N}$
  - When the object starts moving, kinetic friction applies instead

Example Friction Problem

- Problems with friction are like all other Newton's law problems.
  - Identify the forces, draw a diagram, identify vector components, write Newton's law and solve for unknowns.
  - You'll need to relate the force components in two perpendicular directions, corresponding to the normal force and the frictional force.

- **Example:** A box sliding to a stop due to friction on a surface



Coefficients of friction $\mu_s, \ \mu_k$

$$\text{Vertical}: F_{\text{net}} = n - mg = 0$$

$$n = mg$$

$$\text{Horizontal}: F_{\text{net}} = -F_f = -\mu_s n$$

$$F_{\text{net}} = -\mu_s mg = ma_x$$

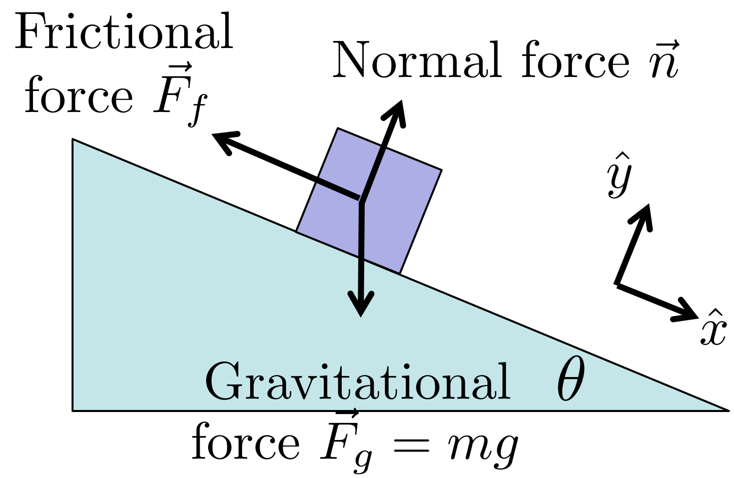
$$a_x = -\mu_s g$$

$$\text{Time to stop}: t = \frac{v - v_0}{a_x} = \boxed{\frac{v_0}{\mu_s g} = t}$$

A More Practical Friction Problem

A box of mass m sits on a surface. We incline the surface until the box just starts slipping down the surface, and measure this angle of incline θ.

What is μ_s?



$$\text{Vertical}: F_{\text{net}} = 0 = n - F_g \cos\theta$$

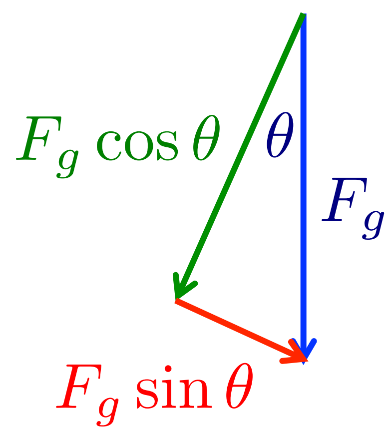
$$n = F_g \cos\theta$$

$$\text{Horizontal}: F_{\text{net}} = 0 = F_g \sin\theta - F_f$$

$$F_f = F_g \sin\theta$$

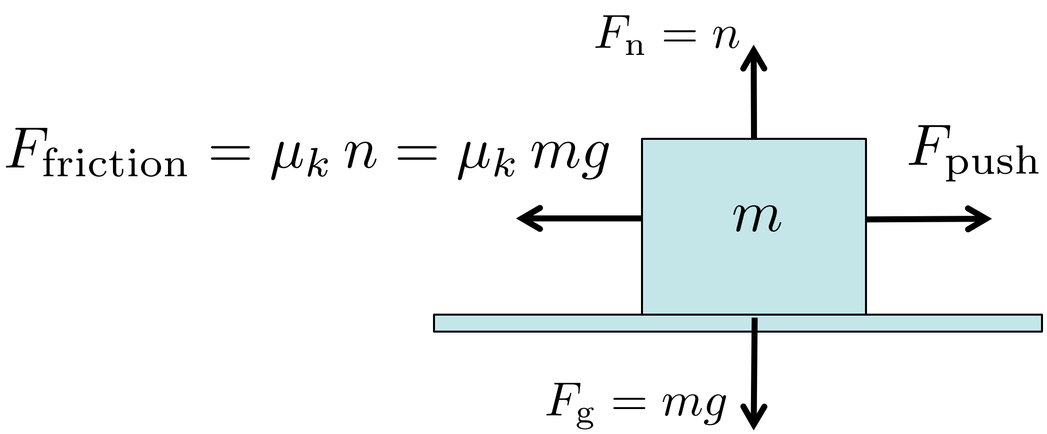
$$F_f = \mu_s n = \mu_s F_g \cos\theta$$

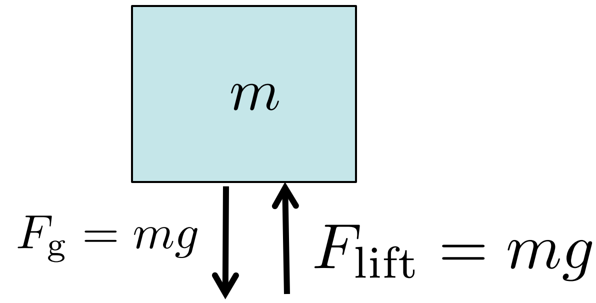
$$\boxed{\mu_s = \tan\theta}$$



Force being decomposed is hypoteneuse of triangle

Tangible/Ponderable (10 minutes)





- Put a flat object on the table in front of you (e.g. cell phone, notebook..)
  - Use an object that does not roll (we haven't discussed rolling yet)
  - Compare the forces to push it **horizontally** at constant speed, and to hold it **vertically** still against the pull of gravity
    - Estimate the coefficient of kinetic friction between your object and the surface
    - Try it again on a different level flat surface (e.g. a white board)
    - Can coefficients of kinetic or static friction be greater than 1?
    - If you tilt the surface, can you measure the angle where it starts slipping and get μ_s that way?

Some Coefficients of Friction

First Material	Second Material	Static	Kinetic
Cast Iron	Cast Iron	1.1	0.15
Aluminum	Aluminum	1.05-1.35	1.4
Rubber	Asphalt (Dry)	--	0.5-0.8
Rubber	Asphalt (Wet)	--	0.25-0.75
Rubber	Concrete (Dry)	--	0.6-0.85
Rubber	Concrete (Wet)	--	0.45-0.75
Oak	Oak (parallel grain)	0.62	0.48
Oak	Oak (cross grain)	0.54	0.32
Ice	Ice	0.05-0.5	0.02-0.09
Teflon	Steel	0.2	--
Teflon	Teflon	0.04	--

http://physics.info/friction

Quantities and their Relationships: Impulse

- We've covered several physics relationships so far in this course
  - Many are relationships between position, velocity, acceleration, time, and force
  - We've even **defined** the vectors velocity, acceleration, force, and momentum this way

$$\vec{v} \equiv \frac{d\vec{x}}{dt} \qquad \vec{a} \equiv \frac{d\vec{v}}{dt} \qquad \vec{F} \equiv m\vec{a} \qquad \vec{p} \equiv m\vec{v}$$

  - We can define change in **impulse** as a force times the time it's applied

$$\Delta\vec{I} = \vec{F}\Delta t$$

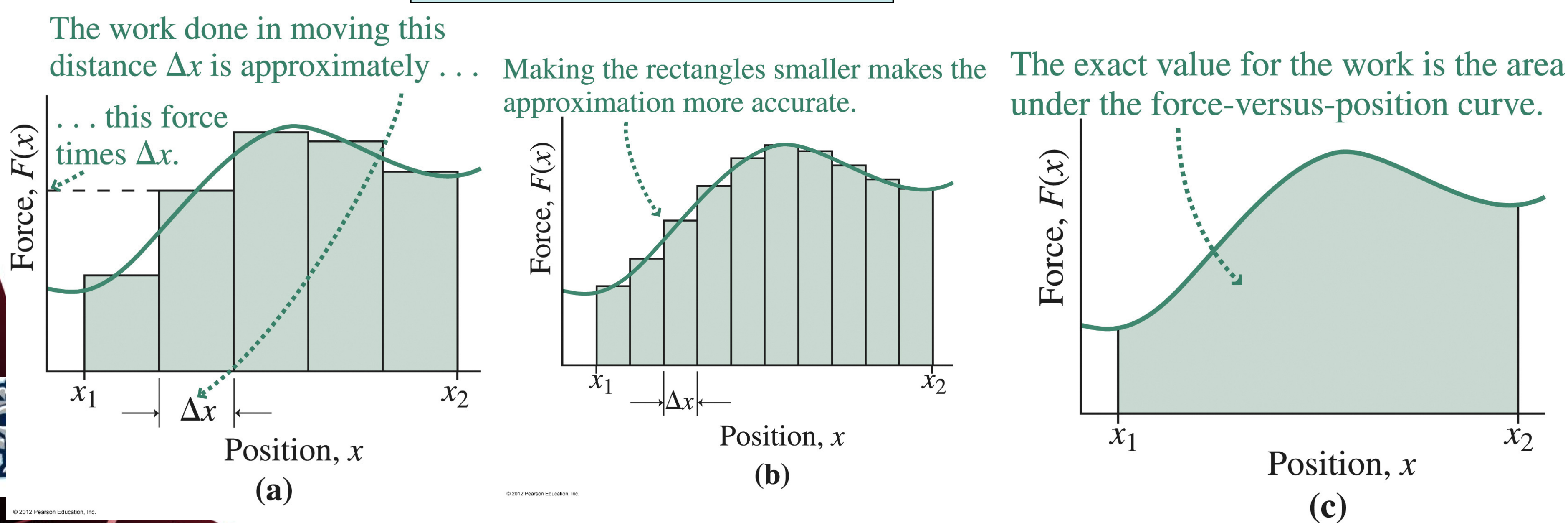
  - Adding these up (or integrating!) gives the total impulse

$$\vec{I} = \int \vec{F}\, dt$$

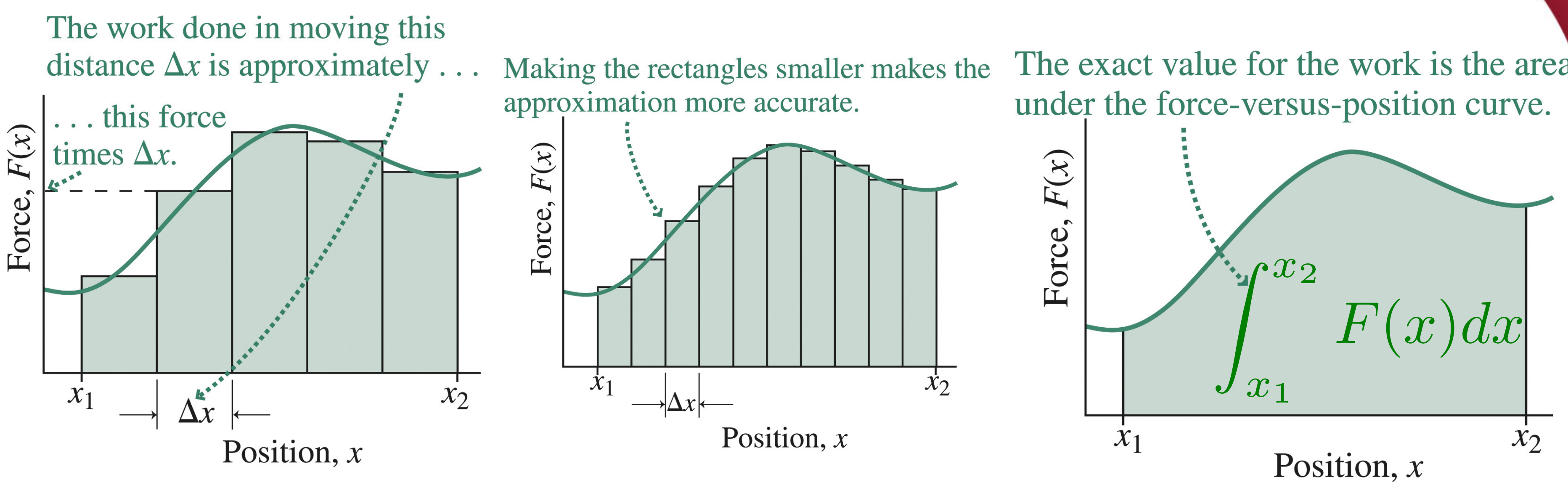
Quantities and their Relationships: Work

- Impulse is useful, but another combination is even moreso
  - We define **work** W as **net force applied times the distance its applied in**
    - The force can be different at various spots so we really need to add together (roughly constant) force over small distances
    - This is really another integral, like impulse. It's a scaler (units: 1 J=1 N-m)

$$\boxed{W \equiv \int F\,dx}$$ work in one dimension

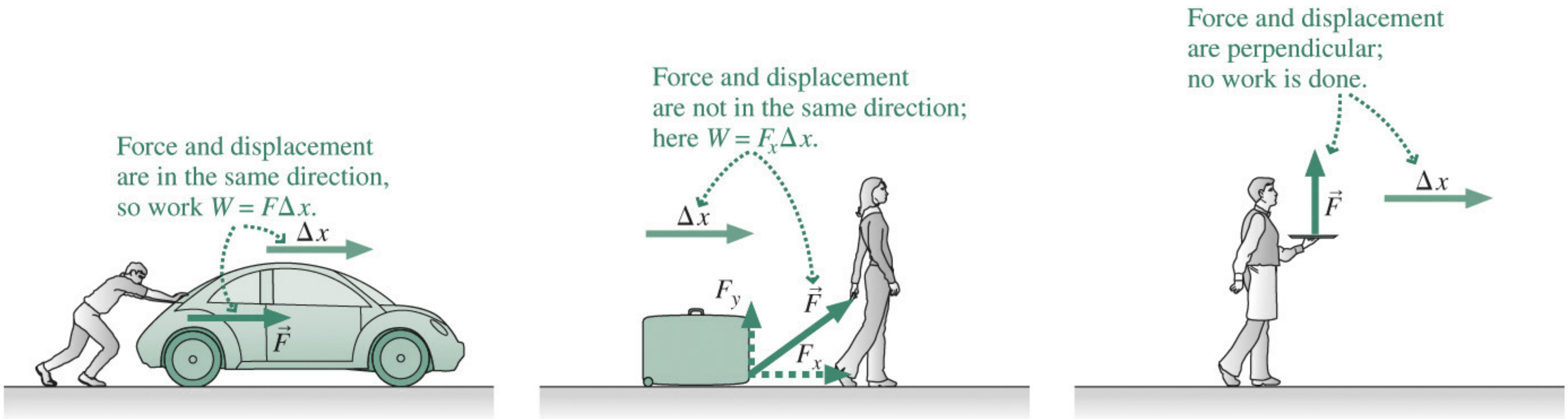


A Quick Aside: Integrals



- An integral is really a combination of two things:
  - A **sum of rectangular areas** that are sketched "under" a curve
    - Since these are areas, the integral has the same units as the units of the x-axis times the units of the y-axis – whatever those are.
    - In calculus they often ignore those limits. In physics, we can't.
  - A **limit** of that sum of areas as the width of each rectangle gets smaller and smaller (and approaches zero)
    - We add together more and more rectangles in this process
    - There are **definite** and **indefinite** integrals

Examples of Work (from a Physicist ☺)



- Work is really a bookkeeping tool
  - We'll relate it to how the **energy** of an object or system changes
  - We only count force and displacement in the same direction
  - Forces applied perpendicular to Δx or when Δx=0 do no work!
    - Holding an object still against gravity, or moving it horizontally
    - Static frictional forces (since there is no Δx)

Ponderable (10 minutes)

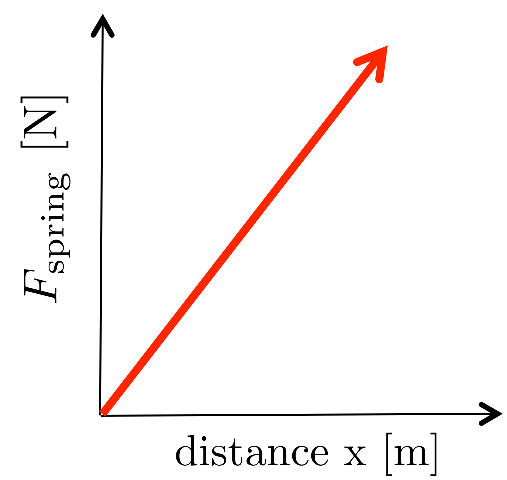
- Remember, **work W** in one dimension is **defined** as

$$W \equiv \int F dx = F\Delta x \quad \text{for constant } F$$

$$1 \text{ Joule} = 1 \text{ N} - \text{m}$$

  - What is the work done to lift an object of mass m over a distance h against the force of gravity? (Assume $v_{final}=v_0=0$ m/s)
    - This is quite straightforward: work is force times distance

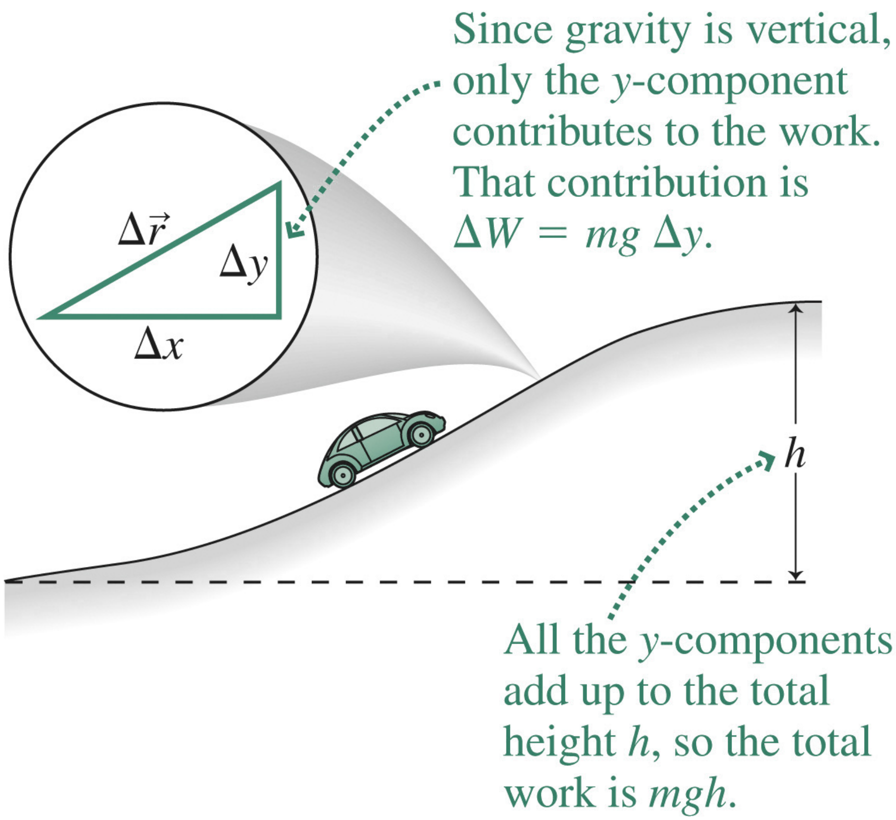
- A harder one: the force of spring relates to one end's displacement by $F_{\text{spring}} = -kx$
  - What is the work done to displace a spring by a distance x in terms of the spring constant k and the distance x?
    - This is a bit less straightforward: work is force times distance but the force is not constant; it's larger as we compress or stretch the spring



Work Done Against Gravity

- The work done by an agent lifting an object of mass *m* against gravity depends only on the vertical distance *h*:

$$W = mgh$$



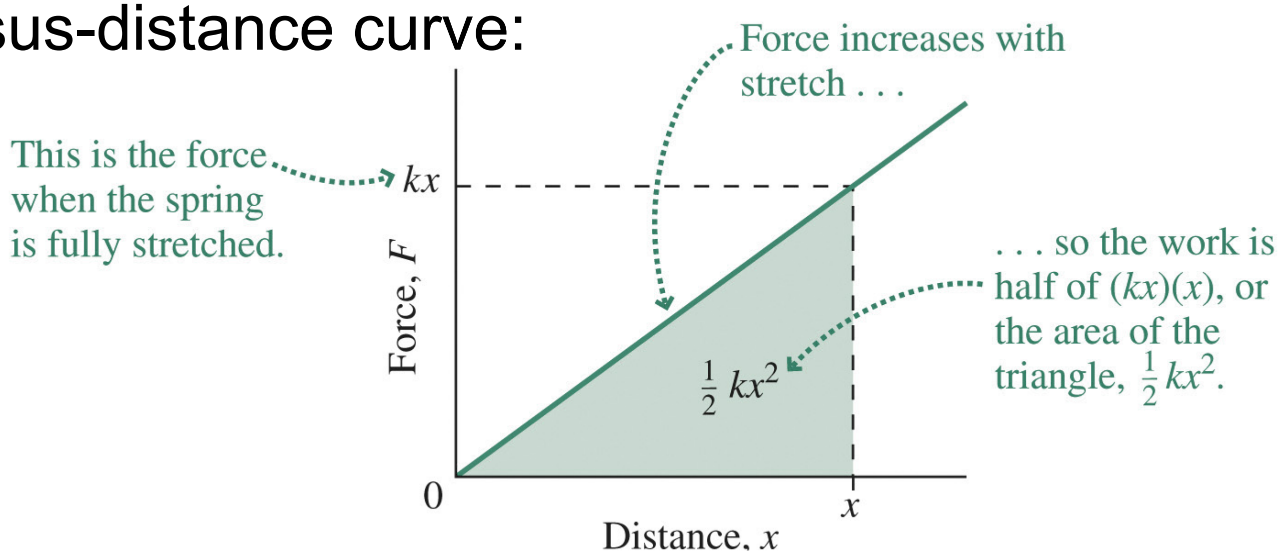
- The work is **positive** if the object is raised (moved against the force of gravity) and **negative** if it's lowered (moved with the force of gravity).
- The horizontal motion

Work Done in Stretching a Spring

- A spring exerts a force $F_{\text{spring}} = -kx$
- Someone stretching a spring exerts a force $F_{\text{stretch}} = +kx$, and the work done is

$$W = \int_0^x F(x)\,dx = k\int_0^x x\,dx = \left(\frac{1}{2}kx^2\right)\Big|_0^x = \boxed{\frac{1}{2}kx^2 = W}$$

- In this case the work is the area under the triangular force-versus-distance curve:



Energy

- **Energy**: the capacity of an object to perform work
  - **Energy** is what we add up when we do our bookkeeping
  - **Work** is how energy moves through application of forces
- How do we do the energy bookkeeping for a system?
  - Add up energy from a variety of different sources and things that we know can do work
  - **Conservation of energy:** total energy for a system is constant

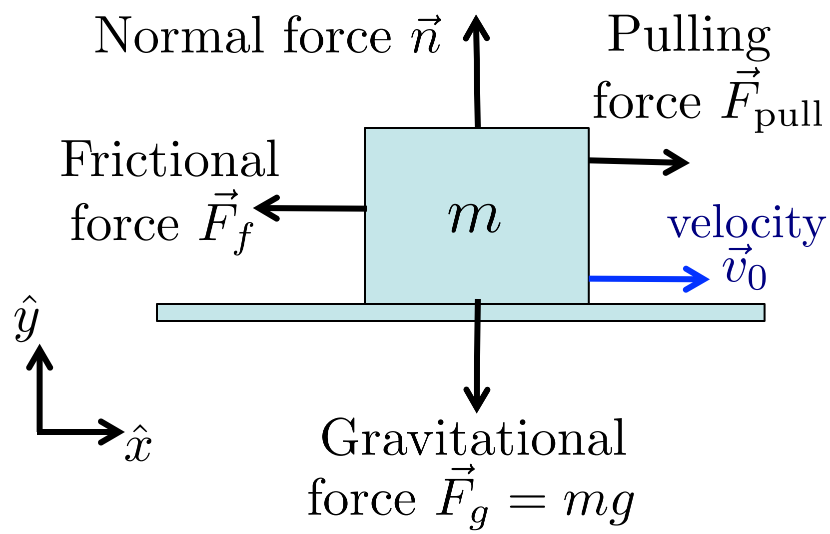
- Kinetic energy: energy of object's motion, $KE = \frac{1}{2} mv^2$
- Gravitational potential energy: energy from the potential of falling a certain distance under constant gravity: $PE_g = mg\Delta y$
- Spring potential energy: $PE_s = \frac{1}{2} kx^2$
- Energy lost to friction over distance Δx: $E_f = \mu_x n \Delta x$
- Chemical energy, nuclear energy, and others…

Work and Net Work

- **Energy**: the capacity of an object to perform work
  - **Energy** is what we add up when we do our bookkeeping
  - **Work** is how energy moves through application of forces

- Since work involves transfer of energy, and we want to account for all energy, it's important to account for all forces

- Example: Pulling a box against friction at constant velocity
  - Net sum of forces on box is zero
  - So **work done on box** is zero
  - But I still do work (I'm exerting a force over a distance)
  - The energy of my work goes into **frictional losses**

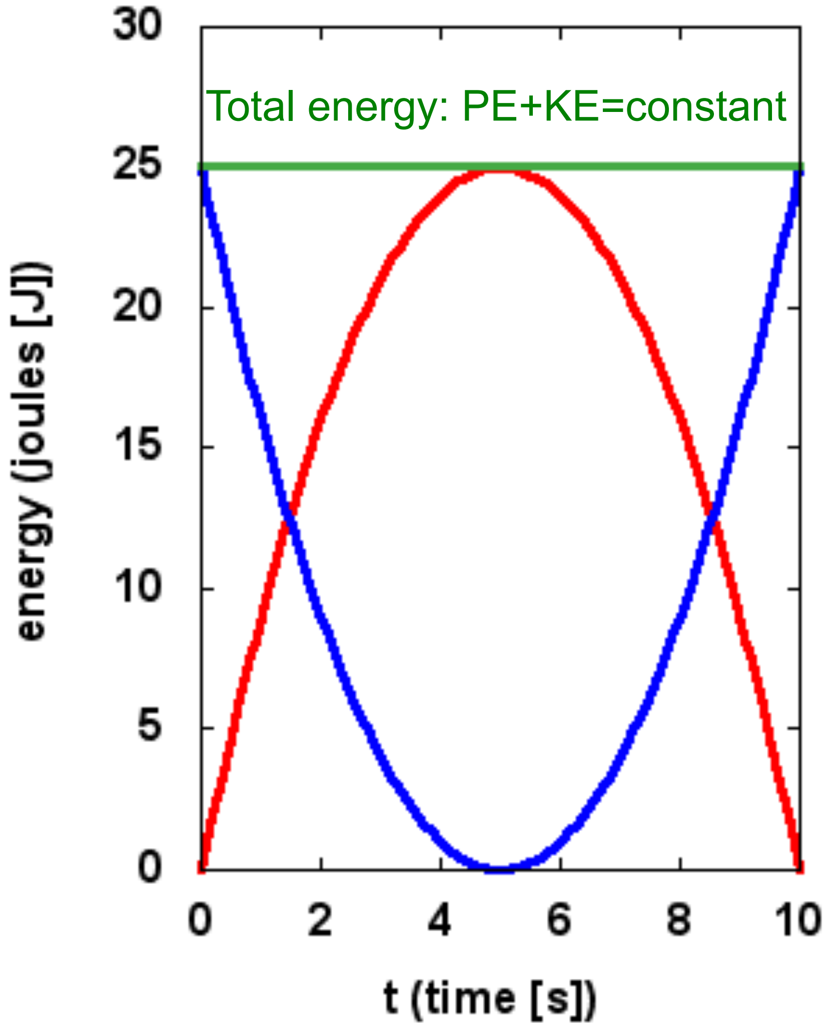


Coefficients of friction $\mu_s, \ \mu_k$

Example: Ball Toss

- Consider your professor tossing a juggling ball upwards
  - I do some work on it to add energy to the system
  - The system is now the ball!

- At start, h=0 m and all energy is kinetic energy
- As the ball moves up, potential energy grows and kinetic energy goes down
- At top, all energy is potential energy since v=0 m/s and KE=0 J
- As the ball comes back down, potential energy is released and kinetic energy grows again



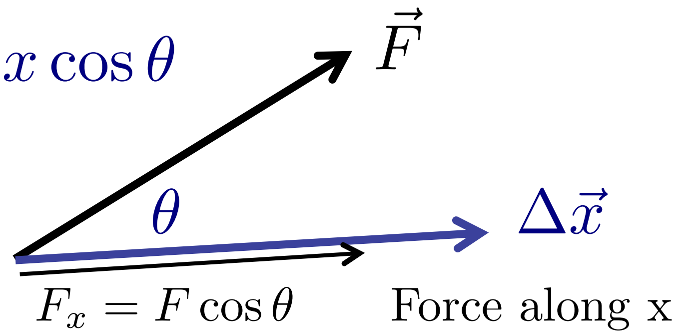
Kinetic energy: KE = ½ mv^2

Potential energy: PE_g=mgh

Work in Multiple Dimensions

- Work is adding up (integrating) force along a distance
  - But remember that force perpendicular to the distance moved does no work
  - So what we're really doing is adding up the **component** of force that is **along** the direction of motion

$$W = F_x \Delta x = F \Delta x \cos\theta$$

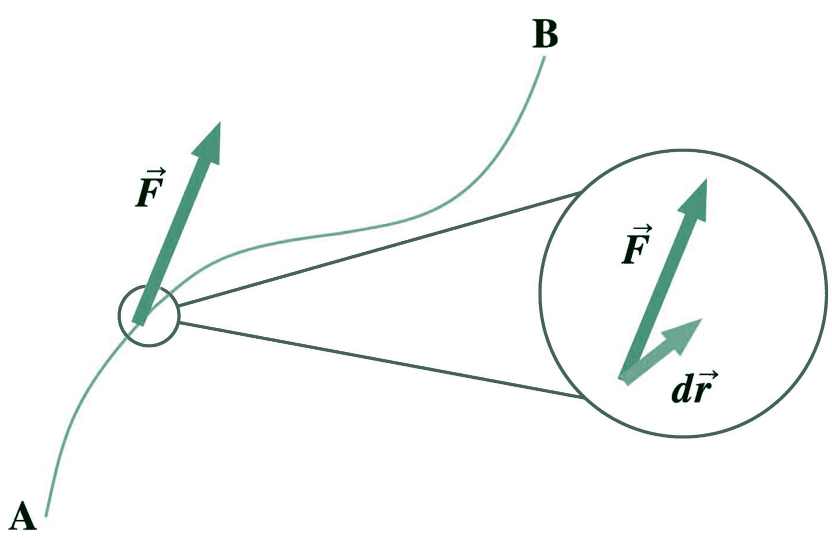


- This process of taking the "product" of two vectors and getting a scaler by looking at the component is a **dot product**

$$W = \int \vec{F} \cdot d\vec{x} = \int F dx \cos\theta$$

A Varying Force in Multiple Dimensions

- In the most general case, an object moves on an arbitrary path subject to a force whose magnitude and whose direction relative to the path may vary with position.
- In that case the integral for the work becomes a **line integral**, the limit of the sum of scalar products of infinitesimally small displacements with the force at each point.



$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot \vec{r}$$