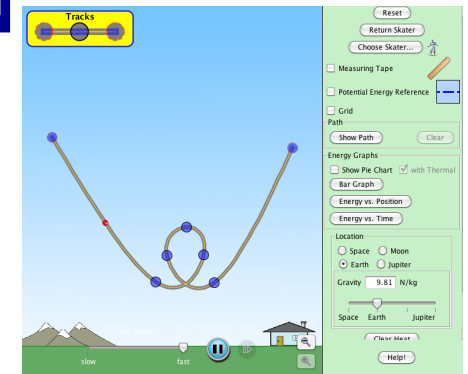


# University Physics 226N/231N Old Dominion University

## Work, Energy, Power, and Energy Conservation



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<http://www.toddsatogata.net/2012-ODU>

Wednesday, October 3 2012

Happy Birthday to Gwen Stefani, India Arie, Keb' Mo', and Stevie Ray Vaughan!

Happy Caramel Custard Day, Techies Day, and Virus Appreciation Day!



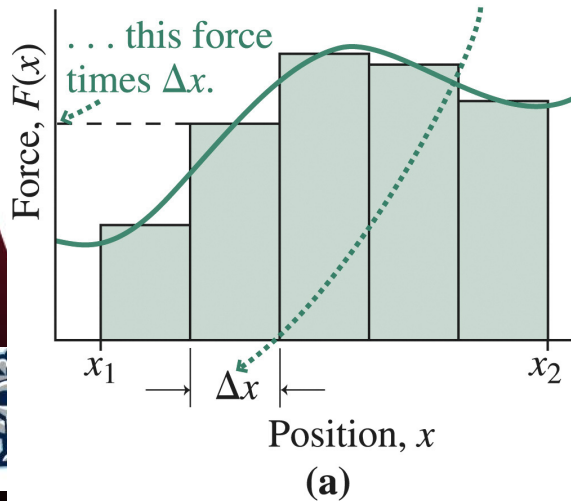
# Review: Work

- Impulse is useful, but another combination is even moreso
  - We define **work**  $W$  as **net force applied times the distance its applied in**
    - The force can be different at various spots so we really need to add together (roughly constant) force over small distances
    - This is really another integral, like impulse. It's a scaler (units: 1 J=1 N-m)

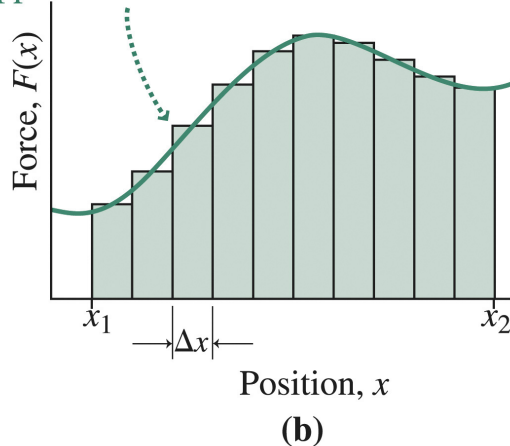
$$W \equiv \int F dx$$

work in one dimension

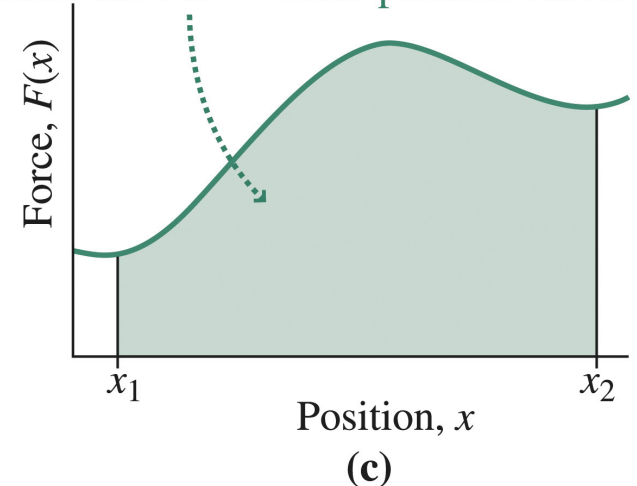
The work done in moving this distance  $\Delta x$  is approximately ...



Making the rectangles smaller makes the approximation more accurate.

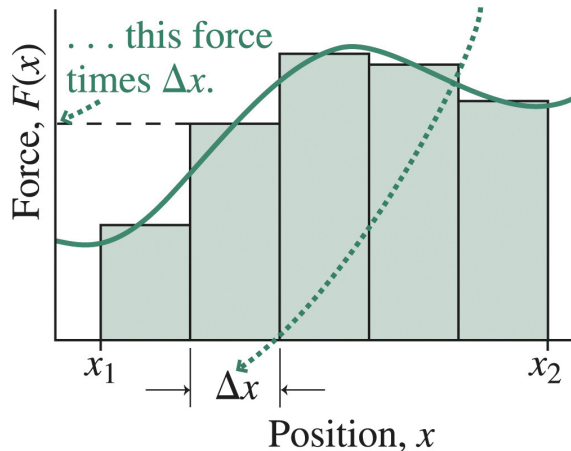


The exact value for the work is the area under the force-versus-position curve.

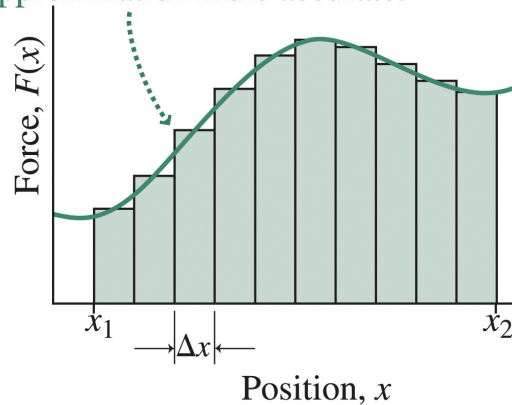


# Review: Integrals

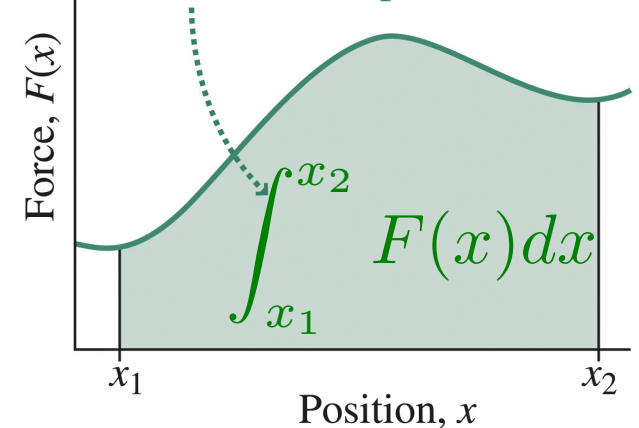
The work done in moving this distance  $\Delta x$  is approximately . . .



Making the rectangles smaller makes the approximation more accurate.



The exact value for the work is the area under the force-versus-position curve.

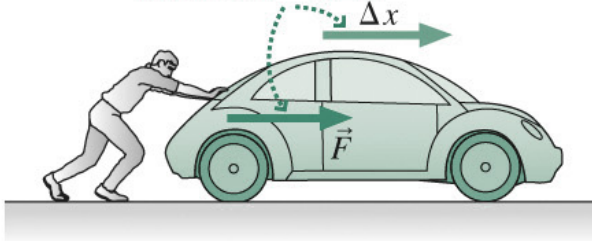


- An integral is really a combination of two things:
  - A **sum of rectangular areas** that are sketched “under” a curve
    - Since these are areas, the integral has the same units as the units of the x-axis times the units of the y-axis – whatever those are.
    - In physics, **we must pay careful attention to units of the axes.**
  - A **limit** of that sum of areas as the width of each rectangle gets smaller and smaller (and approaches zero)
    - We add together more and more rectangles in this process
    - There are **definite** and **indefinite** integrals

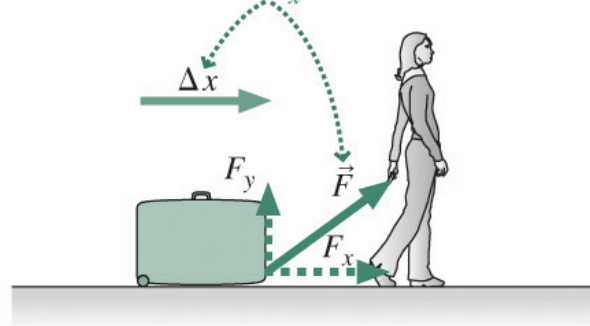


# Examples of Work (from a Physicist 😊)

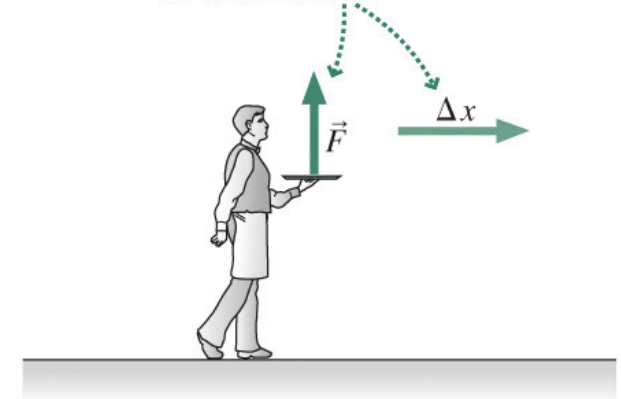
Force and displacement are in the same direction, so work  $W = F\Delta x$ .



Force and displacement are not in the same direction; here  $W = F_x\Delta x$ .



Force and displacement are perpendicular; no work is done.



- Work is really a bookkeeping tool
  - We'll relate it to how the **energy** of an object or system changes
  - **We only count force and displacement in the same direction**
  - Forces applied perpendicular to  $\Delta x$  or when  $\Delta x=0$  do no work!
    - Holding an object still against gravity, or moving it horizontally
    - Static frictional forces (since there is no  $\Delta x$ )

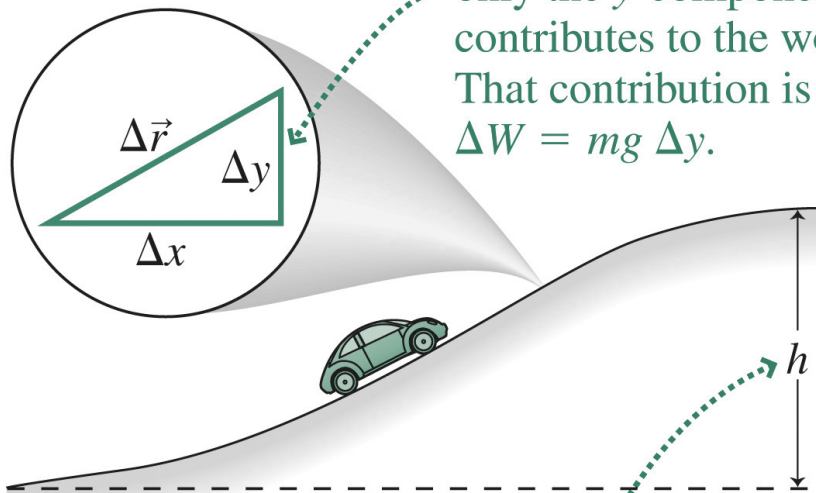


# Work Done Against Gravity

- The work done by something lifting an object of mass  $m$  against gravity depends only on the vertical distance  $h$ :

$$W = mgh$$

Since gravity is vertical, only the  $y$ -component contributes to the work. That contribution is  $\Delta W = mg \Delta y$ .



All the  $y$ -components add up to the total height  $h$ , so the total work is  $mgh$ .

- The work is **positive** if the object is raised (moved against the force of gravity) and **negative** if it's lowered (moved with the force of gravity).
- The horizontal motion only does (positive) work that is lost to friction



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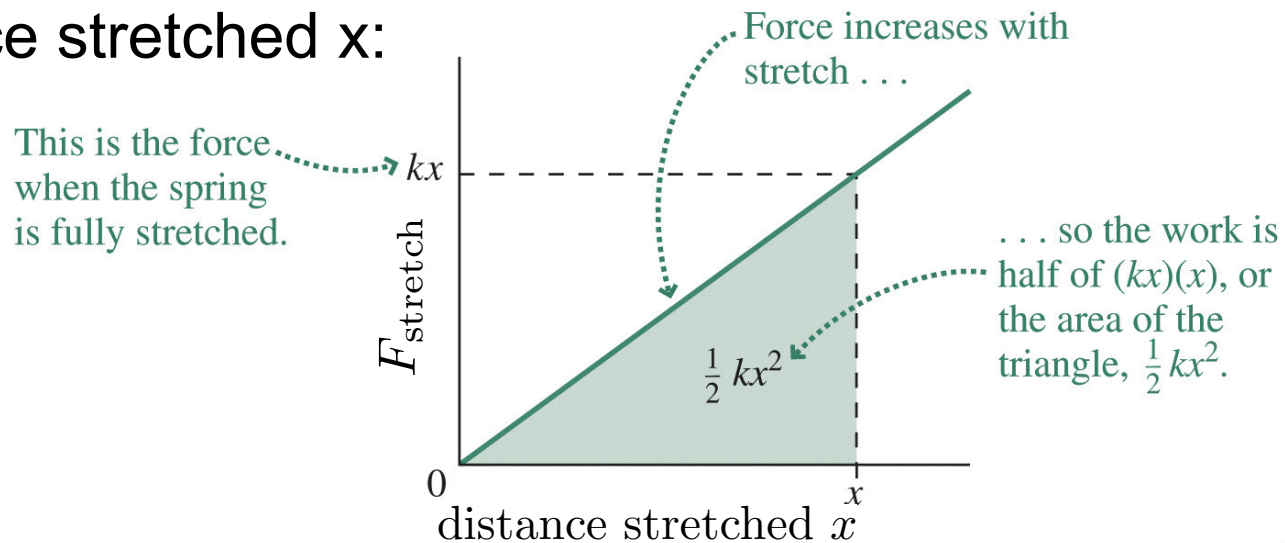


# Work Done in Stretching a Spring

- A spring exerts a force  $F_{\text{spring}} = -kx$
- Someone stretching a spring exerts a force  $F_{\text{stretch}} = +kx$ , and the work done is

$$W = \int_0^x F(x) dx = k \int_0^x x dx = \left( \frac{1}{2} kx^2 + c \right) \Big|_0^x = \boxed{\frac{1}{2} kx^2 = W}$$

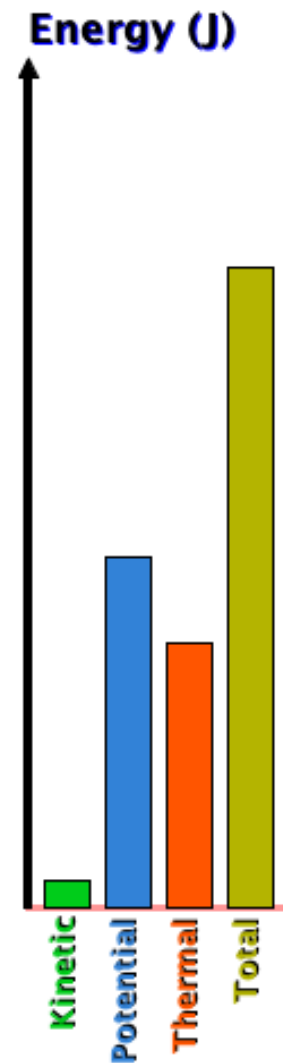
- Here the work is the triangular area under the plot of force  $F$  vs distance stretched  $x$ :





# Energy

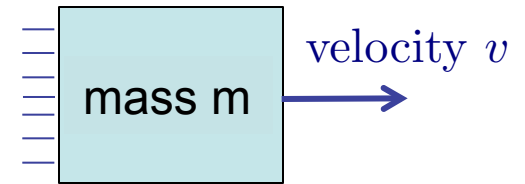
- **Energy**: the capacity of an object to perform work
  - **Energy** is what we add up when we do our bookkeeping
  - **Work** is how energy moves through application of forces
    - We'll see several examples through this class
    - Think of work as “energy exchange” in the bookkeeping
    - If I “do work”, I lose energy and it goes somewhere else
      - **But it doesn't just disappear**
- How do we do the energy bookkeeping for a system?
  - Add up energy from a variety of different sources and things that we know can do work
  - **Conservation of energy: total energy for a system is constant as the system changes over time**
    - This appears to be a **true fundamental law of nature**
    - If energy changes, the bookkeeping wasn't done right



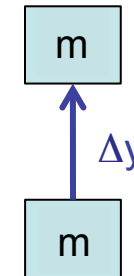
# Some Energy Bookkeeping Terms

- Kinetic energy: energy of object's motion

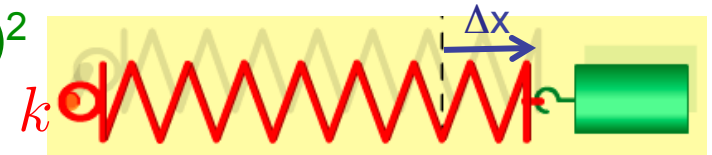
$$KE = \frac{1}{2} mv^2$$



- Gravitational potential energy: energy from the potential of falling a certain distance under constant gravity:  $PE_g = mg\Delta y$



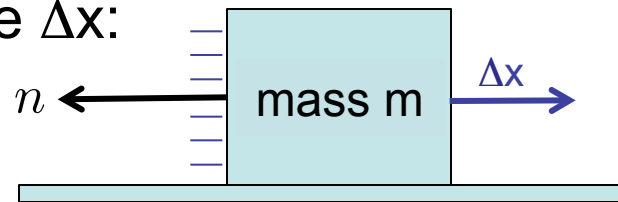
- Spring potential energy:  $PE_s = \frac{1}{2} k(\Delta x)^2$



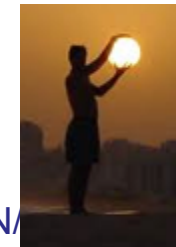
- Energy lost to friction over distance  $\Delta x$ :

$$E_f = \mu_k n \Delta x$$

$$F_{\text{friction}} = \mu_k n$$



- Chemical energy, nuclear energy, and others...

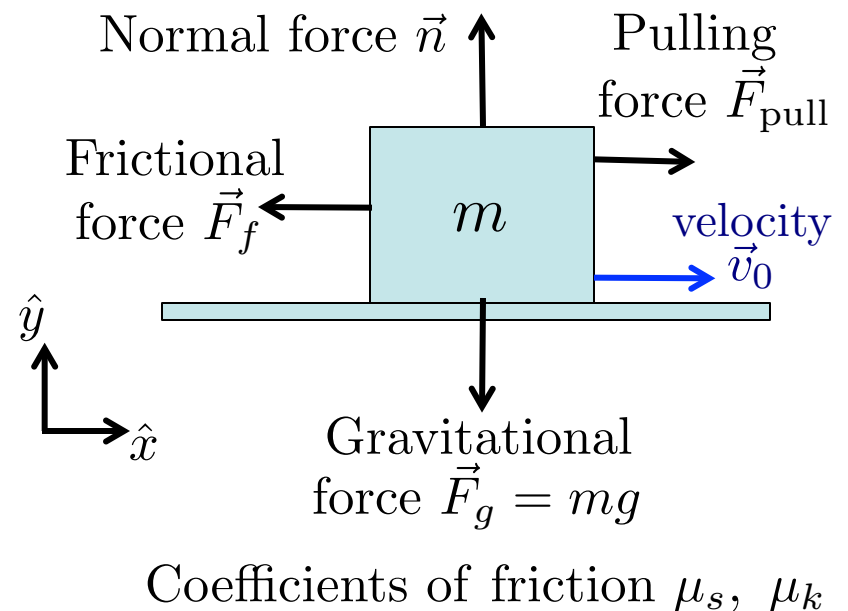




# Work and Net Work

- **Energy**: the capacity of an object to perform work
  - **Energy** is what we add up when we do our bookkeeping
  - **Work** is how energy moves through application of forces
- Since work involves transfer of energy, and we want to account for all energy, it's important to account for all forces

- Example: Pulling a box against friction at constant velocity
  - Net sum of forces on box is zero
  - So **work done on box** is zero
  - But I still do work (I'm exerting a force over a distance)
  - The energy of my work goes into **frictional losses**



# Example Friction Problem Revisited

- Some problems become **much easier** with conservation of energy
- Example:** A box sliding to a stop due to friction on a surface
  - If it's got an initial velocity  $v_0$ , how far does it slide before it stops?
  - Initial total energy is all kinetic:  $E_{\text{init}} = (1/2)mv_0^2$
  - It stops when it's lost all its energy to friction:

$$E_{\text{friction}} = \mu_k n \Delta x = \mu_k mg \Delta x$$

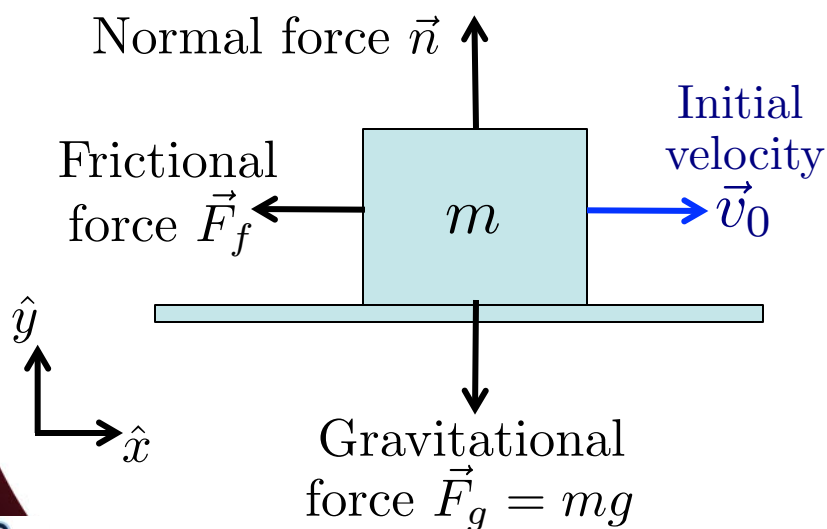
Apply Conservation of Energy

$$E_{\text{init}} = E_{\text{friction}}$$

$$\frac{1}{2}mv_0^2 = \mu_k mg \Delta x$$

$$\Delta x = \frac{v_0^2}{2\mu_k g}$$

Still need kinematics to get time to stop..

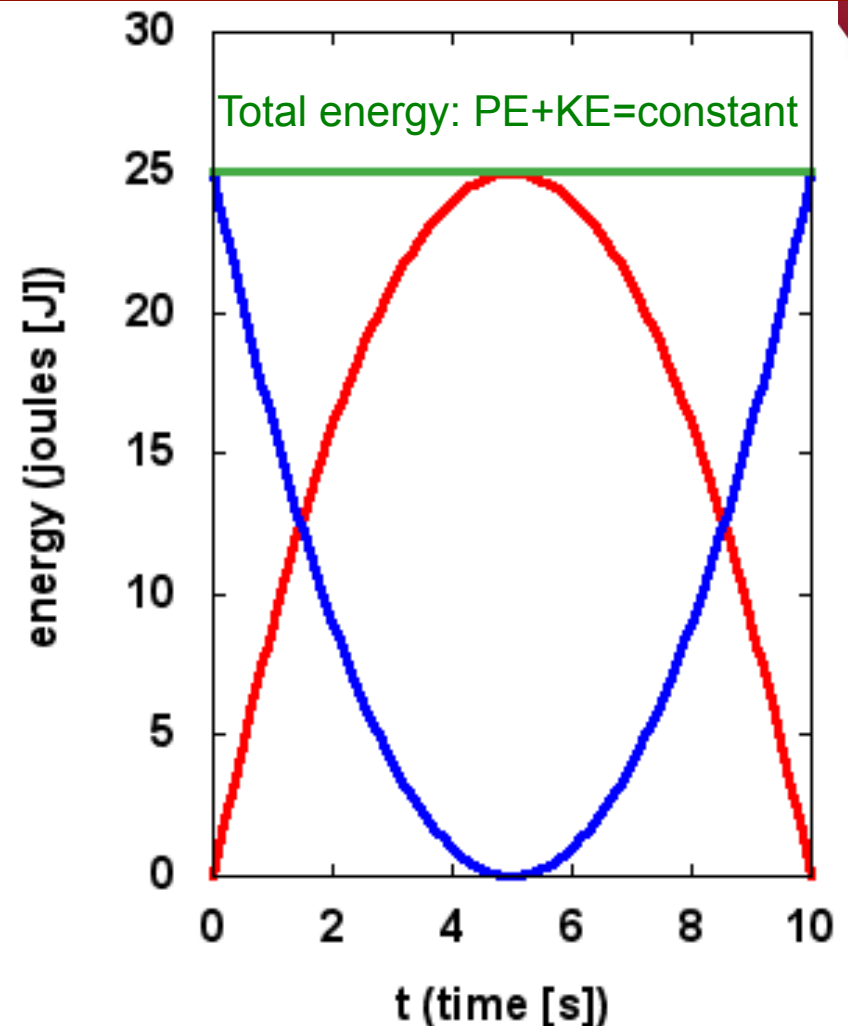


Coefficients of friction  $\mu_s, \mu_k$



# Example: Ball Toss

- Consider your professor tossing a juggling ball upwards
  - I do some work on it to add energy to the system
  - The system is now the ball!
- At start,  $h=0$  m and all energy is kinetic energy
- As the ball moves up, potential energy grows and kinetic energy goes down
- At top, all energy is potential energy since  $v=0$  m/s and  $KE=0$  J
- As the ball comes back down, potential energy is released and kinetic energy grows again

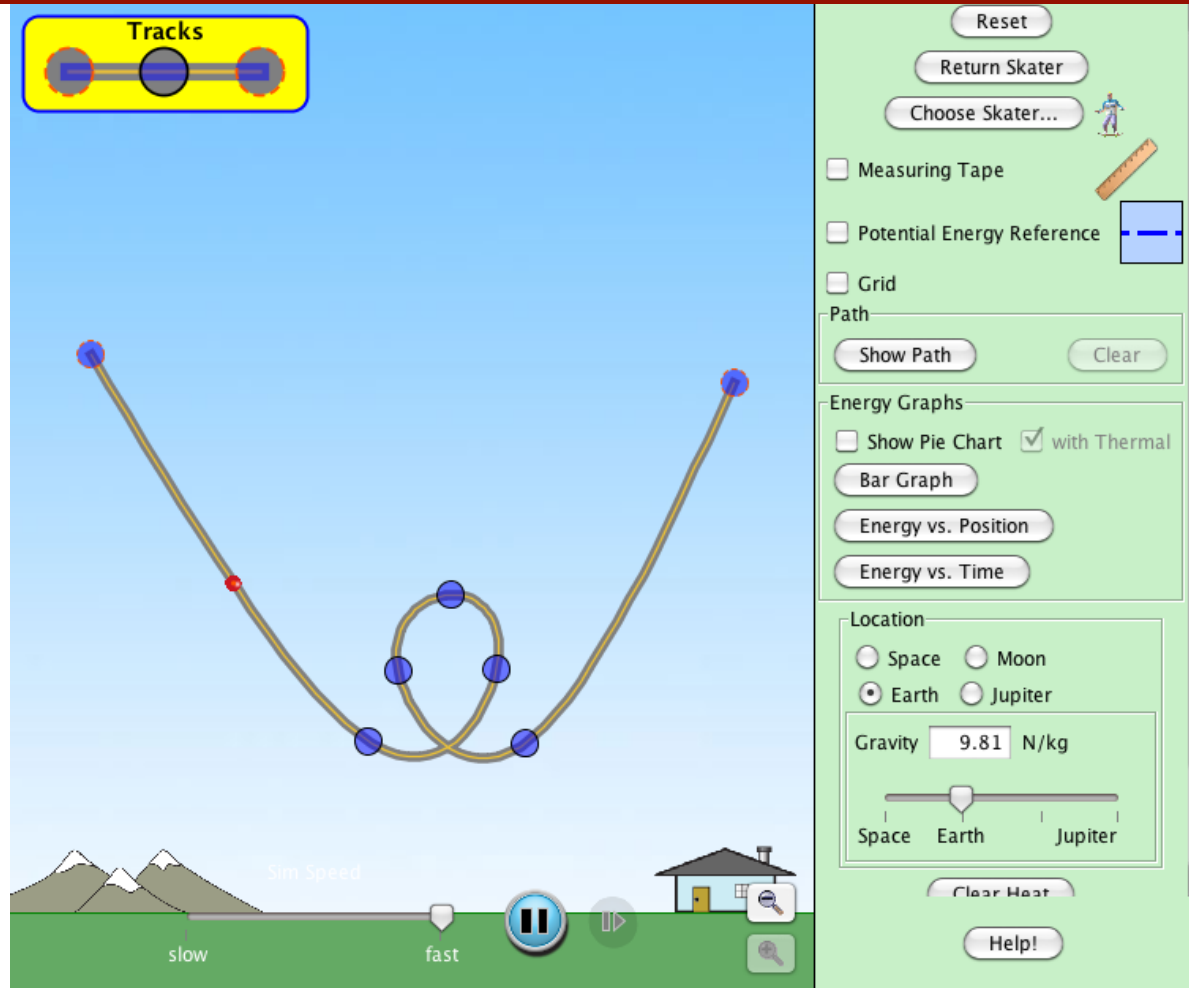


Kinetic energy:  $KE = \frac{1}{2} mv^2$

Potential energy:  $PE_g = mgh$



# Tangible: Skate Park (20 minutes)

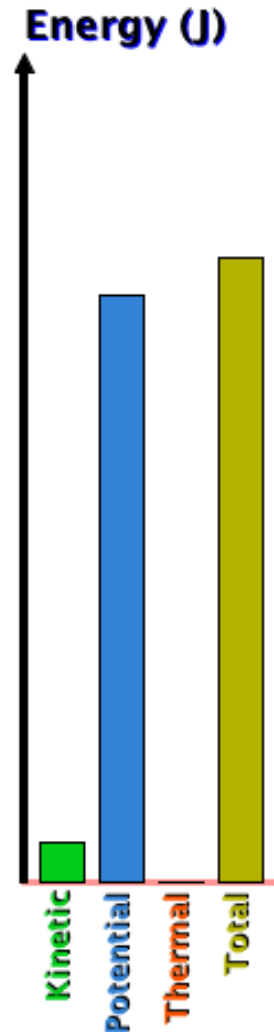


- Let's take a look at a fun simulation of a skate park
  - There's a link on the class web page
- The next slide has a few questions to answer



# Tangible: Skate Park Questions (20 minutes)

- Lift the skater and watch their motion back and forth
  - Click the “bar graph” button to see this energy graph →
    - Which energy is biggest when the skater is at peak?
    - Which energy is biggest when the skater is at bottom?
  - Click the “grid” button and note the height of the skater at peak on both sides
    - Does this change when you move the center blue dot to change the slopes of both sides of the skater ramp?
  - Click the “Track friction” button and raise the coefficient of friction a little bit
    - How does the energy chart change? Is energy still conserved? What if you didn’t count the thermal energy?
  - Play with the plots of energy vs position and vs time
  - Load the “loop.xml” file from the class website (save it to the desktop and open with applet File/Open menu)
    - This is a loop-the-loop! What height do I drop the skater from so he just barely completes the loop the loop?
    - What happens to energy if he falls and bounces?



# Skate Park Loop The Loop

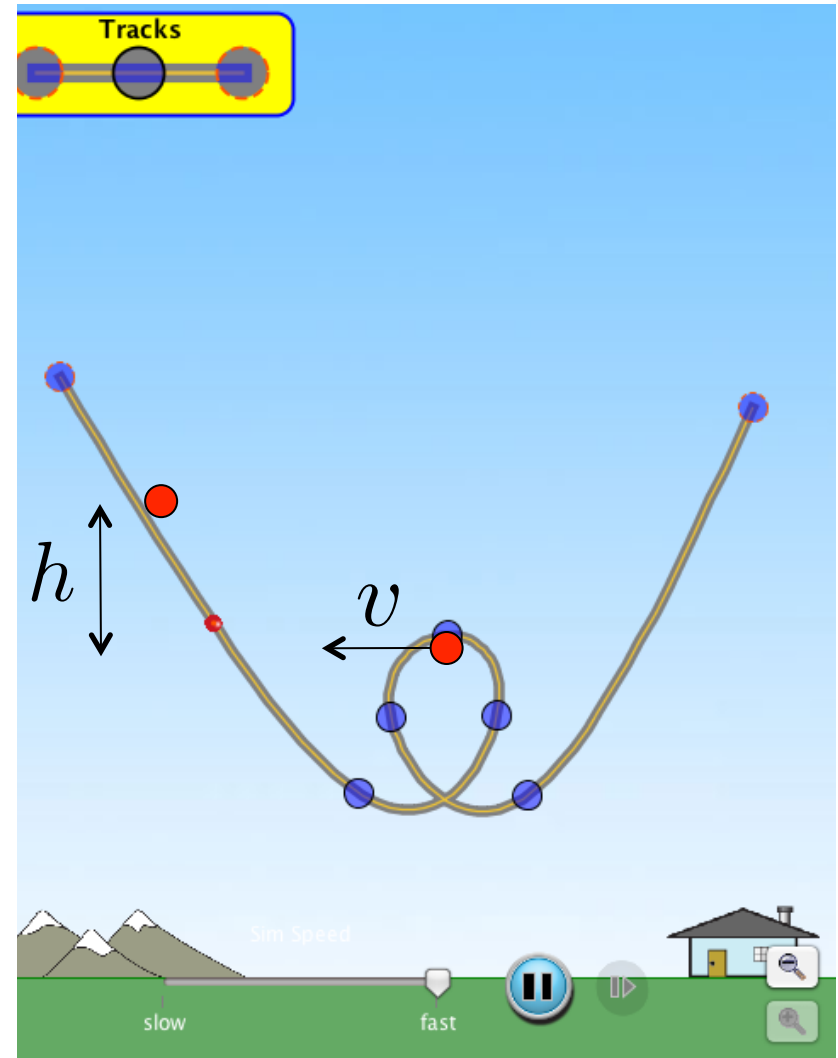
- We investigated the roller coaster loop design problem
- How does energy conservation help us calculate where to start the roller coaster so it makes it safely around the loop?

$$v_{\text{safe}} = \sqrt{gr} \quad (\text{calculated before})$$

$$E_{\text{init}} = PE_g = mgh$$

$$E_{\text{top}} = \frac{1}{2}mv_{\text{safe}}^2 = \frac{1}{2}mgr$$

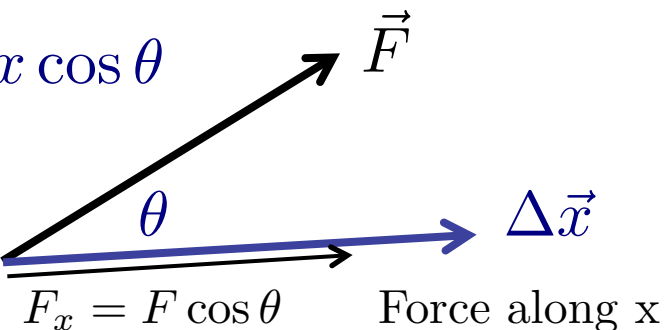
$$E_{\text{init}} = E_{\text{top}} \quad \Rightarrow \quad \boxed{h = \frac{r}{2}}$$





# Work in Multiple Dimensions

- Work is adding up (integrating) force along a distance
  - But remember that force perpendicular to the distance moved does no work
  - So what we're really doing is adding up the **component** of force that is **along** the direction of motion

$$W = F_x \Delta x = F \Delta x \cos \theta$$


The diagram illustrates the decomposition of a force vector  $\vec{F}$  into a component along the direction of displacement  $\Delta \vec{x}$ . The angle between the two vectors is  $\theta$ . The component of the force along the displacement is labeled  $F_x = F \cos \theta$  and "Force along x".

- This process of taking the “product” of two vectors and getting a scalar by looking at the component is a **dot product**

$$W = \int \vec{F} \cdot d\vec{x} = \int F dx \cos \theta$$



# Power and Energy

- **Power** is the *rate* at which work is done or at which energy is used or produced. If work  $\Delta W$  is done in time  $\Delta t$ , then the **average power** over this time is

$$\bar{P} = \frac{\Delta W}{\Delta t} \quad (\text{average power})$$

- When the rate changes continuously, the **instantaneous power** is

$$P = \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

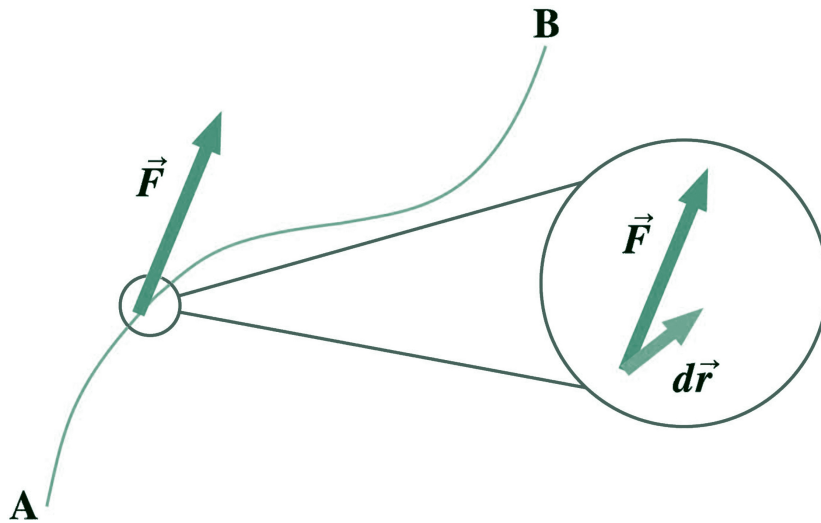
- Power is measured in **watts** (W), with  $1 \text{ W} = 1 \text{ J/s}$ .
- Total work or energy follows from power by multiplying (for constant power) or integrating (for varying power):

$$W = P\Delta t \quad \text{or} \quad W = \int_{t_1}^{t_2} P dt$$



# A Varying Force in Multiple Dimensions

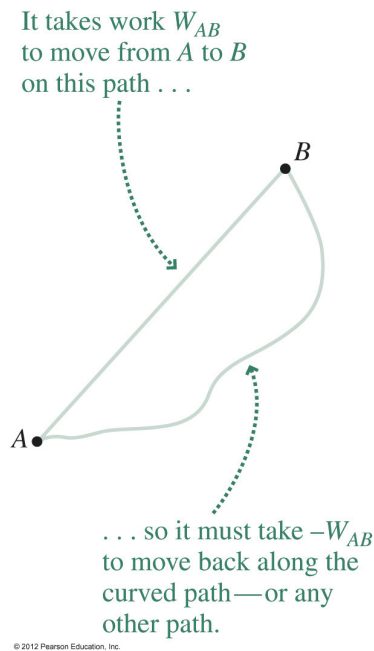
- In the most general case, an object moves on an arbitrary path subject to a force whose magnitude and whose direction relative to the path may vary with position.
- In that case the integral for the work becomes a **line integral**, the limit of the sum of scalar products of infinitesimally small displacements with the force at each point.



$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

# Conservative and Nonconservative Forces

- A **conservative force** stores any work done against it, and can “give back” the stored work as kinetic energy.
- For a conservative force, the work done in moving between two points is independent of the path:



- Because the work done by a conservative force is path independent, the work done in going around any *closed path* is zero:

$$\oint \vec{F} \cdot d\vec{r} = 0$$

• A **nonconservative force** does not store work done against it, the work done may depend on path, and the work done going around a closed path need not be zero.



# Conservative and Nonconservative Forces

- Examples of conservative forces include
  - Gravity
  - The static electric force
  - The force of an *ideal* spring (i.e. no heat generated)
- Nonconservative forces include
  - Friction
  - The electric force in the presence of changing magnetism



# Potential Energy

- The “stored work” associated with a conservative force is called **potential energy**.
  - Potential energy is stored energy that can be released as kinetic energy.
- The change in potential energy is defined as the negative of the work done by a conservative force acting over any path between two points:

$$\Delta U_{AB} = - \int_A^B \vec{F} \cdot d\vec{r}$$

- Potential energy change is independent of path.
- Only *changes* in potential energy matter.
- We’re free to set the zero of potential energy at any convenient point.



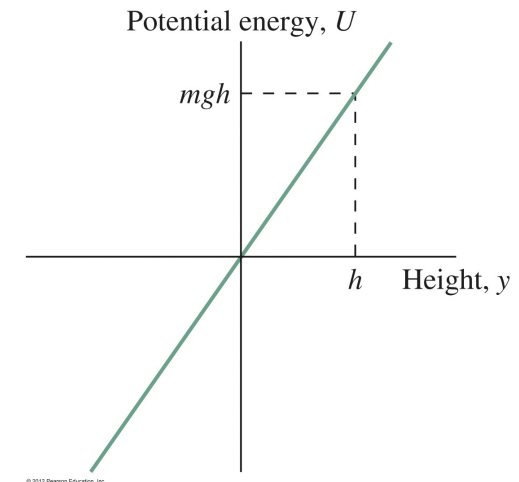


# Two Common Forms of Potential Energy

- **Gravitational potential energy** stores the work done against gravity:

$$\Delta U = mg \Delta y$$

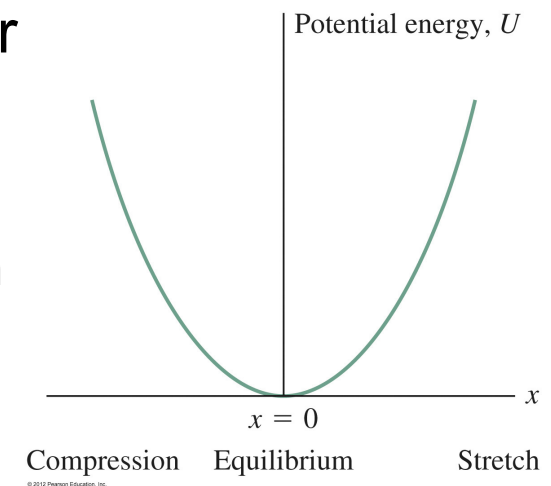
- Gravitational potential energy increases linearly with height  $y$ .
- This reflects the *constant* gravitational force near Earth's surface.



- **Elastic potential energy** stores the work done in stretching or compressing springs or spring-like systems:

$$U = \frac{1}{2} kx^2$$

- Elastic potential energy increases *quadratically* with stretch or compression  $x$ .
- This reflects the *linearly increasing* spring force.
- Here the zero of potential energy is taken in the spring's equilibrium configuration.



# Conservation of Mechanical Energy

- By the work-energy theorem, the change in an object's kinetic energy equals the net work done *on* the object:

$$\Delta K = W_{\text{net}}$$

- When only conservative forces act, the net work is the negative of the potential-energy change:  $W_{\text{net}} = -\Delta U$
- Therefore when only conservative forces act, any change in potential energy is compensated by an opposite change in kinetic energy:

$$\Delta K + \Delta U = 0$$

- Equivalently,

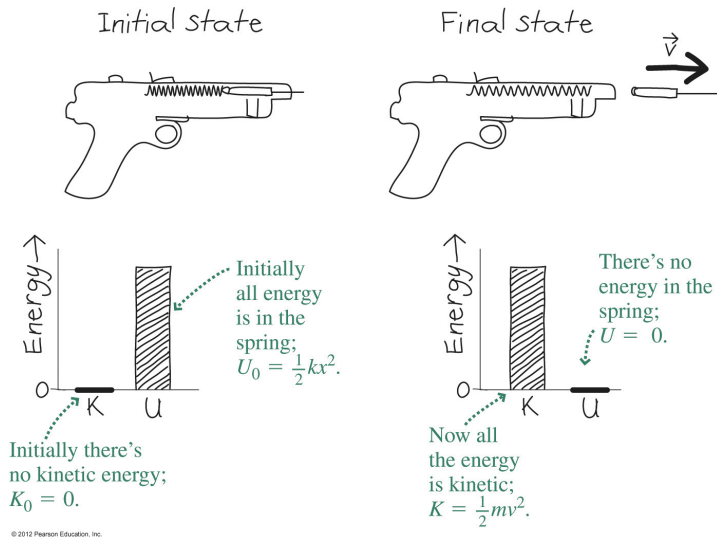
$$K + U = \text{constant} = K_0 + U_0$$

- Both these equations are statements of the law of **conservation of mechanical energy.**



# Examples

- A spring-loaded dart gun
  - What's the dart's speed?

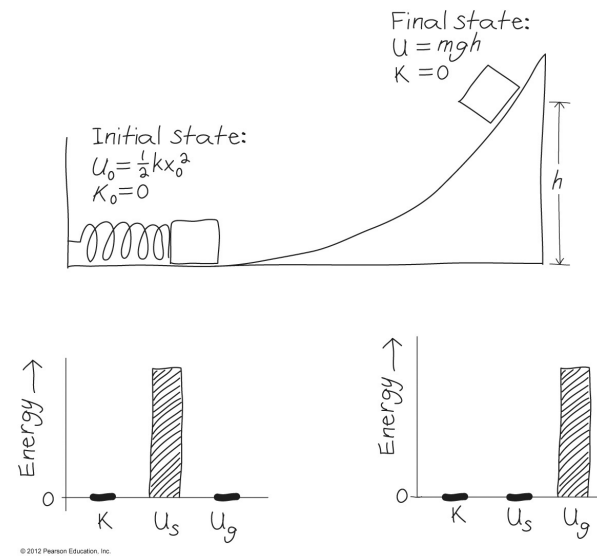


- $K + U = K_0 + U_0$  becomes

$$\frac{1}{2}mv^2 + 0 = 0 + \frac{1}{2}kx^2$$

- So  $v = \sqrt{k/m} x$  where  $x$  is the initial spring compression.

- A spring and gravity
  - How high does the block go?



- $K + U = K_0 + U_0$  becomes

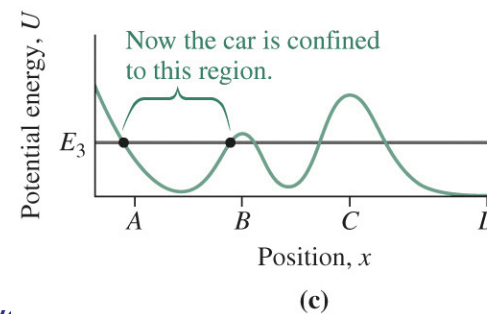
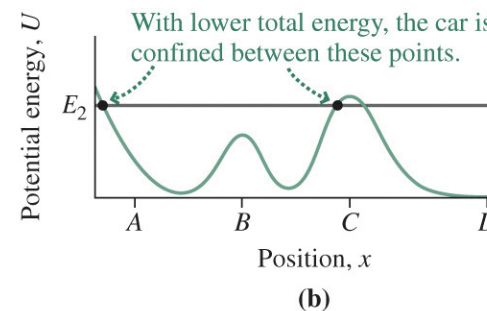
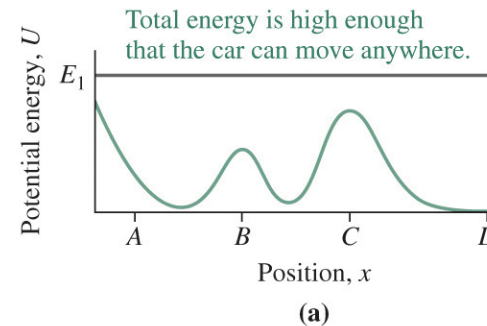
$$0 + mgh = 0 + \frac{1}{2}kx^2$$

- So  $h = \frac{kx^2}{2mg}$



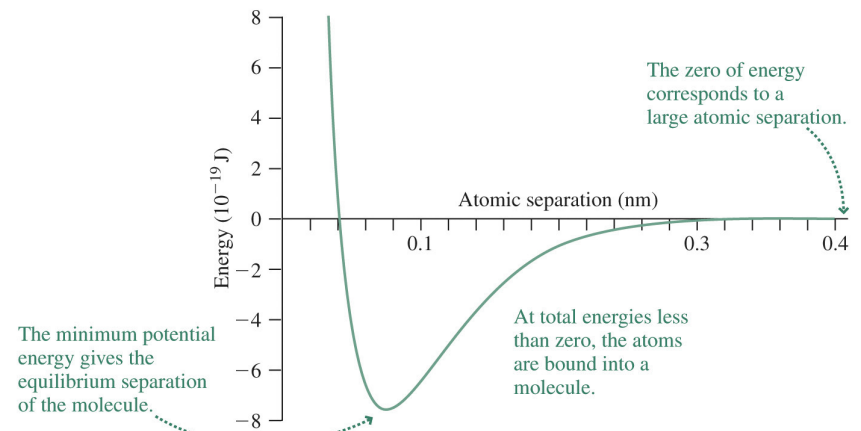
# Potential-Energy Curves

- Potential-energy curves depict a system's potential energy as a function of position or other quantities representing the system's configuration.
  - An object with a given total energy can be "trapped" in a "potential well" established by points where its total energy equals its potential energy.
  - These points are **turning points**, beyond which the object cannot move given its fixed total energy.
- Potential-energy curves for a roller-coaster car with three different total energies:



# Potential-Energy Curve for a Molecule

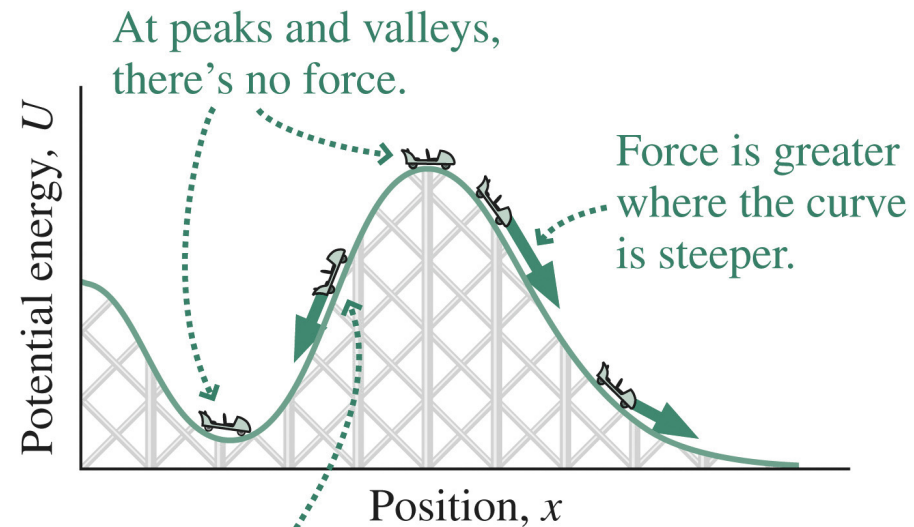
- Potential-energy curves help determine the structure of systems, from molecules to engineered systems to stars.
- The potential-energy curve for a pair of hydrogen atoms shows potential energy as a function of atomic separation.
  - The minimum in the graph shows the equilibrium separation of the H<sub>2</sub> molecule.
  - It's convenient to take the zero of potential energy at infinite separation.
  - Then negative energies represent bound states of the hydrogen molecule.
  - Positive states represent separated hydrogen atoms.



# Force and Potential Energy

- Force is greatest where potential energy increases most rapidly.
- Mathematically, the component of force in a given direction is the negative derivative of the potential energy with respect to position in that direction:

$$F_x = -\frac{dU}{dx}$$



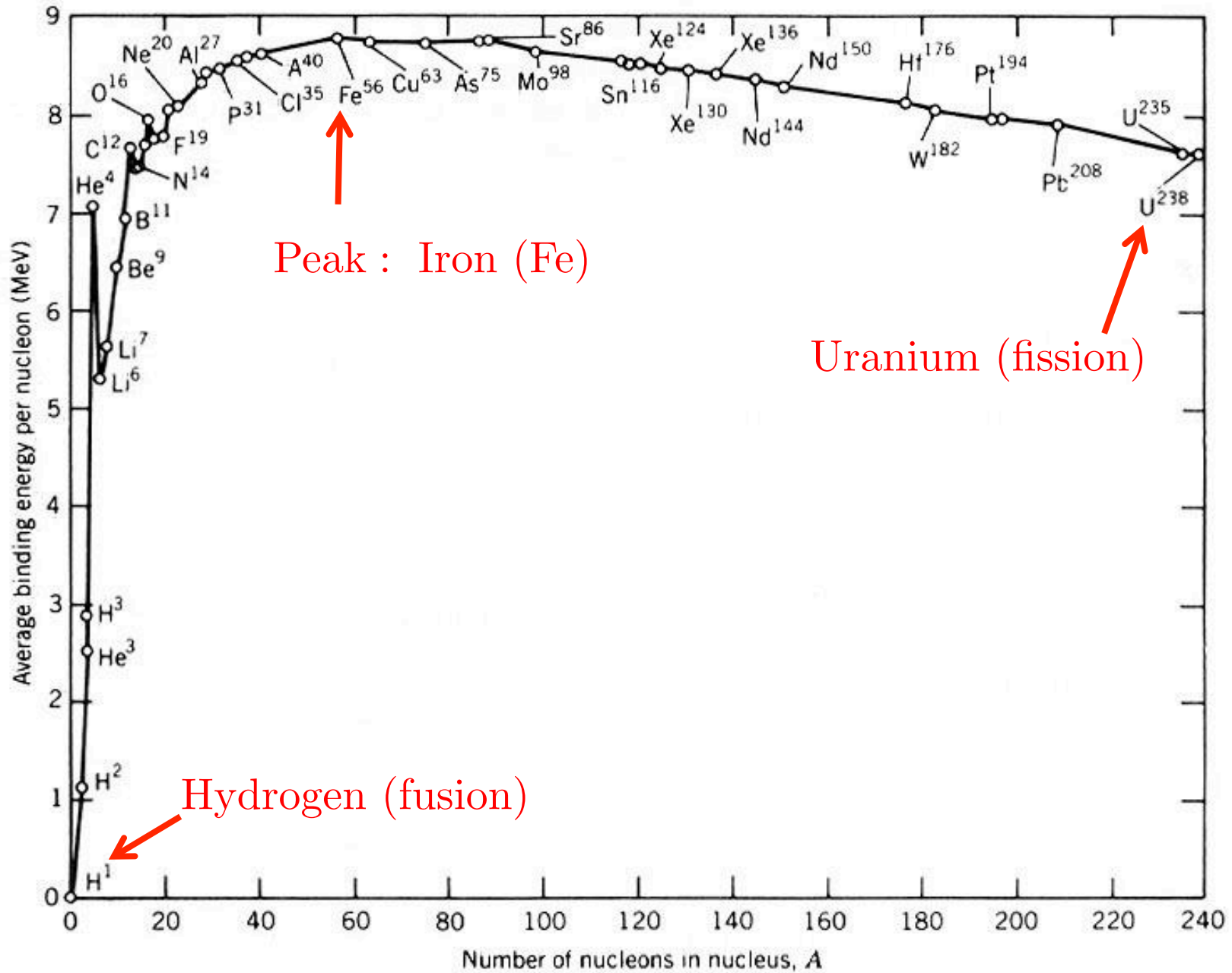
When the curve is rising to the *right*, force is to the *left*.

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# Interlude: The Curve of Binding Energy

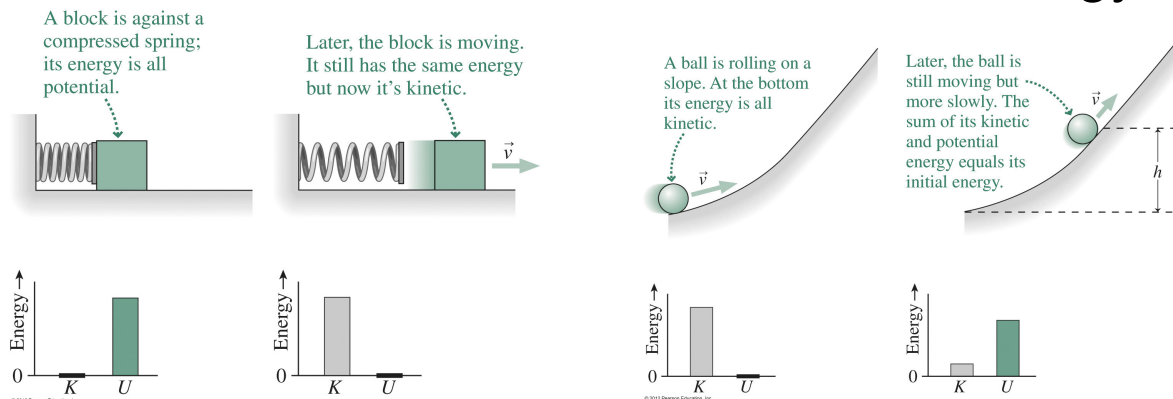


# Summary

- **Potential energy** is stored energy that can be converted to kinetic energy.
- The change in potential energy is the negative of the work done by a conservative force as an object is moved on any path between two points:

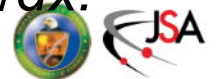
$$\Delta U = -\int_A^B \mathbf{F} \cdot d\mathbf{r}$$

- When only conservative forces act, the total mechanical energy  $K + U$  is conserved:



- **Potential-energy curves** describe potential energy as a function of position or configuration.

Force is the negative derivative of potential energy:  $F_x = -dU/dx$ .



## Clicker Question 1

- Suppose it takes the same amount of work to push a trunk across a rough floor as it does to lift a weight the same distance straight upward. How do the amounts of work compare if the trunk and the weight are moved instead on curved paths between the same starting and ending points?
  - A. The two amounts of work will remain equal to each other.
  - B. The amount of work to move the trunk will be greater.
  - C. The amount of work to move the weight will be greater.



## Clicker Question 2



- Gravitational force actually decreases with height, but that decrease is negligible near Earth's surface. To account for the decrease, how would the exact value of the potential-energy change associated with a height change  $h$  compare with  $mgh$ , where  $g$  is the gravitational acceleration at Earth's surface?
  - A. The potential energy change would be equal to  $mgh$ .
  - B. The potential energy change would be greater than  $mgh$ .
  - C. The potential energy change would be less than  $mgh$ .



## Clicker Question 3



- A bowling ball is tied to the end of a long rope and suspended from the ceiling. A student stands at one side of the room, holds the ball to her nose, and then releases it from rest. Should she duck as it swings back?
- A. She should duck!
- B. She does not need to duck.



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ODU University Physics 226N/231N 31

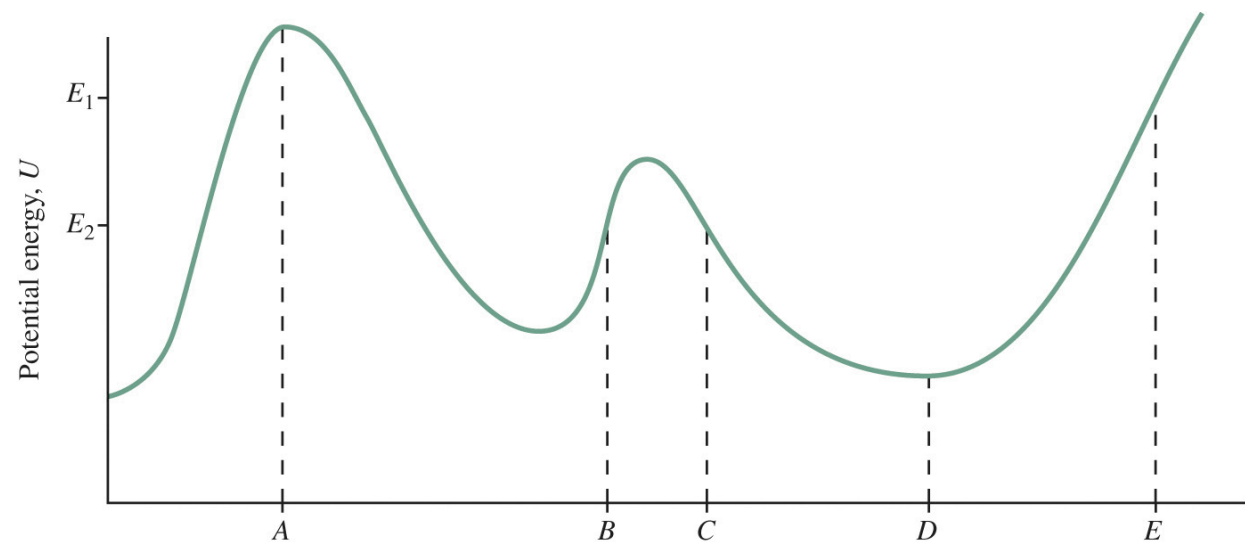


## Clicker Question 4



- This figure shows the potential energy for an electron in a microelectronic device. From among the labeled points, find the point **where the force on the electron is greatest**.

- A. Point *A*
- B. Point *B*
- C. Point *C*
- D. Point *D*
- E. Point *E*

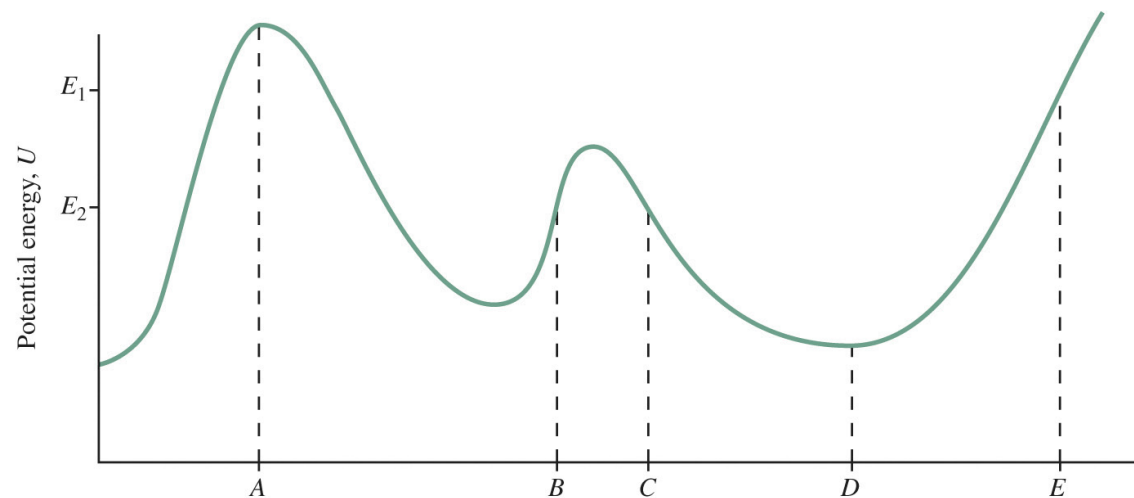


## Clicker Question 5



- This figure shows the potential energy for an electron in a microelectronic device. From among the labeled points, find the rightmost point where the force on the electron points to the left.

- A. Point *A*
- B. Point *B*
- C. Point *C*
- D. Point *D*
- E. Point *E*



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## Clicker Question 6



- Can a system have negative potential energy?
  - A. No, because a negative potential energy is unphysical.
  - B. No, because the kinetic energy of a system must be equal to its potential energy.
  - C. Yes, as long as the total energy remains positive.
  - D. Yes, as long as the total energy remains negative.
  - E. Yes, because the choice of the zero of potential energy is arbitrary.



## Clicker Question 7



- Which of the following statements is true about the total mechanical energy of a system?
  - A. It is constant if acted upon by a conservative force.
  - B. It is constant if acted upon by a nonconservative force.
  - C. It is always positive.
  - D. It is equally divided between its kinetic energy and potential energy.



## Clicker Question 8



- To get to the second floor of the physics building, student A takes the elevator straight up, while student B walks up the stairway. Assuming that both students have the same mass, which of the following statements is true about their gravitational potential energy after reaching the second floor?
  - A. Student A has greater potential energy because he gets to the second floor faster.
  - B. Student B has greater potential energy because he has traveled a greater distance.
  - C. Both students have the same potential energy.
  - D. It is impossible to tell because potential energy depends on the time elapsed and distance traveled.



## Clicker Question 9



- A spring has a spring constant of 100 N/m. How much potential energy does it store when stretched by 10 cm?

A. 50 J

B. 10 J

C. 5.0 J

D. 0.50 J

E. 0.10 J

