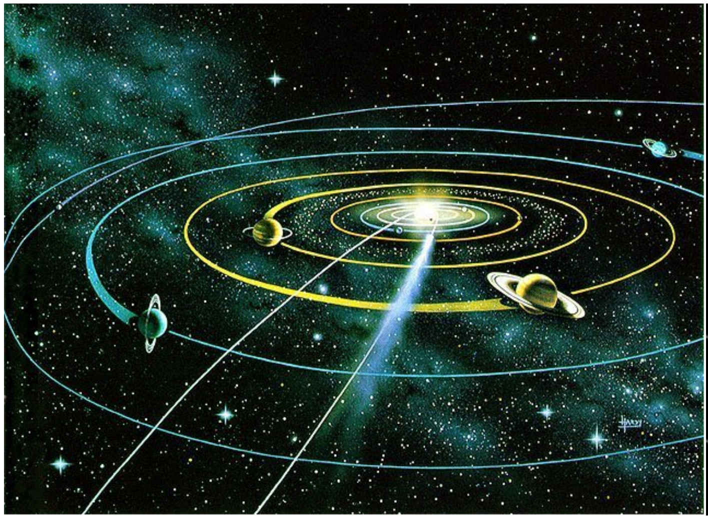
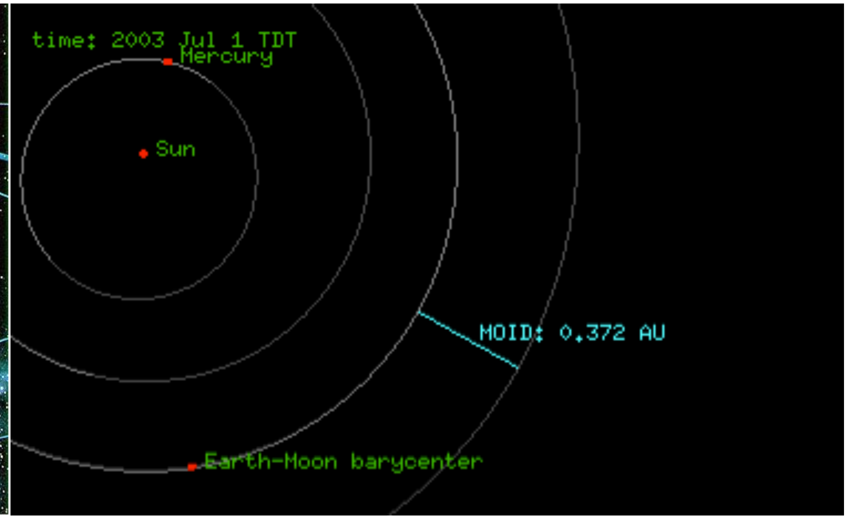
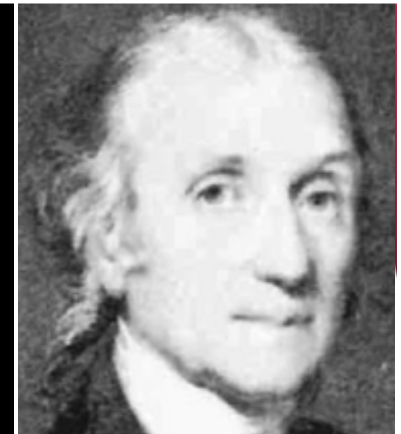
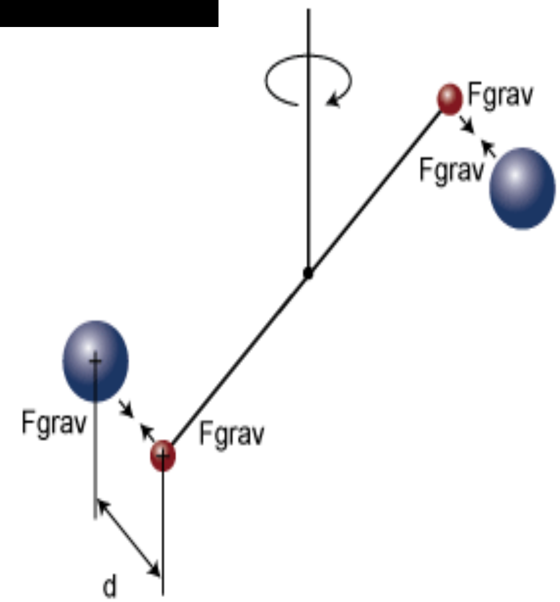




# University Physics 226N/231N
# Old Dominion University
# The Gravitational Force





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Wednesday, October 10 2012

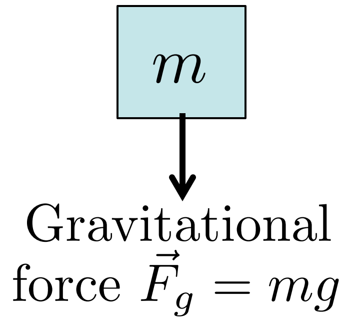
Happy Birthday to **Henry Cavendish**, Ryan Mathews, Yoenis Céspedes (Go A's!), and Thelonious Monk!

Happy Cake Decorating Day, Double Tenth Day, and Oppa Gangnam Style Day!

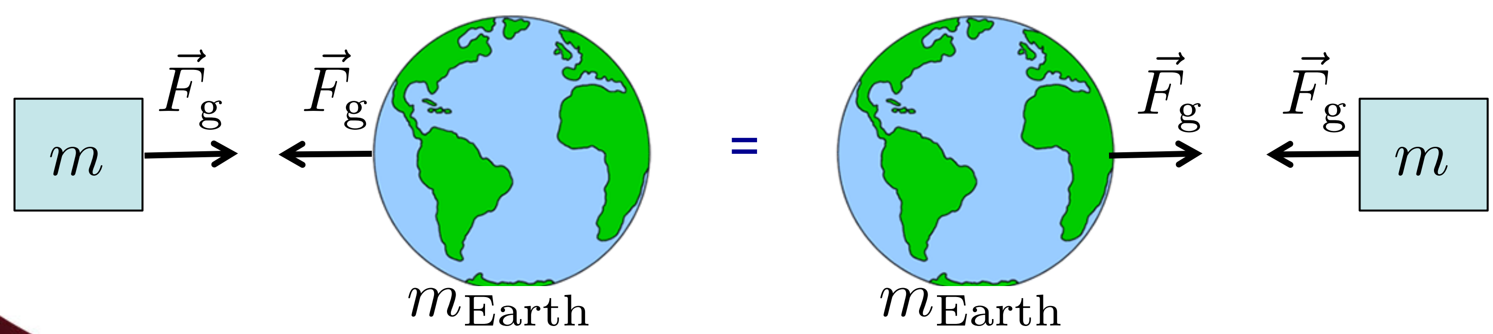
(Okay, I really made up the Oppa Gangnam Style Day thing)

# Gravity Reconsidered

- We've learned a lot about gravity so far
  - But what we've learned about it **must be an approximation**
  - We've assumed the gravitational force is independent of height and the mass of the earth, $m_{\mathrm{Earth}}$




- But Newton's third law tells us that the box also exerts an equal and opposite force on the earth
  - Gravitational forces should be the same if we interchange the masses of the earth and box: g is proportional to $m_{\mathrm{Earth}}$ !

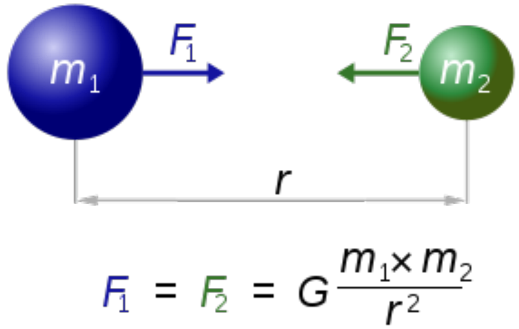


# Gravity Reconsidered

- The gravitational force must also get weaker when objects are further apart
  - Otherwise we'd live in a very crowded little universe
  - Some experiments showed that our original approximation of the gravitational force between two objects is more generally

$$\boxed{F_g = \frac{G m_1 m_2}{r^2}}$$

$$G = 6.67384 \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$$




  - This becomes our more familiar $F_g = m_1 a = m_1 g$ when

$$m_2 = m_{\mathrm{Earth}} = 5.972 \times 10^{24}\ \mathrm{kg}$$

and $$r = r_{\mathrm{Earth}} \approx 6371\ \mathrm{km}$$

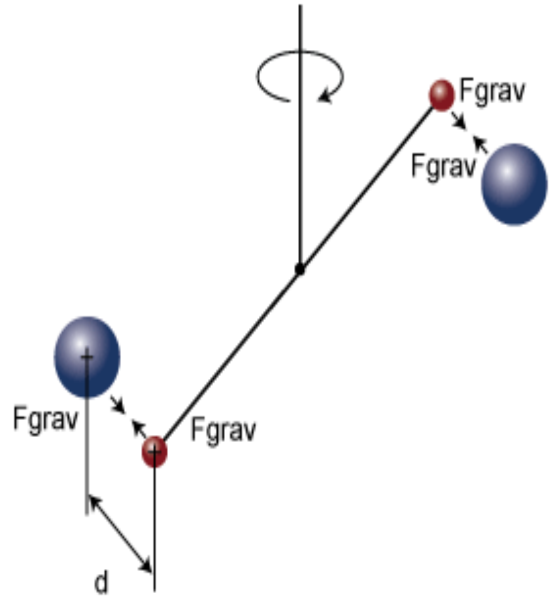
# Ponderable

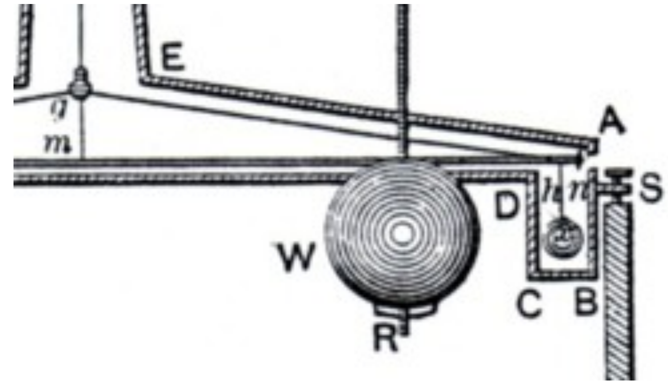
- How do you devise an **experiment** to measure a force that's this weak?
  - For two 1 kg objects 1m apart, $F_g = 6.67 \times 10^{-11}\ \mathrm{N}$
  - Earth's gravity on one of those is $W = mg = 9.8\ \mathrm{N}$
  - That's a huge difference!

- And yet Henry Cavendish devised an experiment to measure the force of gravity between a 158 kg ball and a 0.73 kg ball.

… in 1797.

# 1797: The Cavendish Experiment

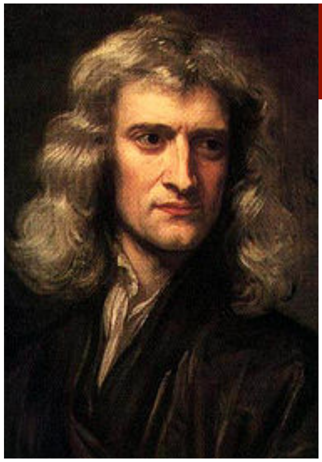



- A balance used to measure force or weight has friction

- Cavendish invented the **torsion balance** to measure gravity
  - The twisting angle on a string is proportional to the force applied times the distance the force is applied at
  - His balance was exquisite for its time, with a sensitivity of less than $10^{-8}$ N, or the weight of a very small grain of sand

# A Law of Universal Gravitation



$$\boxed{F_{\mathrm{g}} = \frac{Gm_1m_2}{r^2}}$$

$$G = 6.67384 \times 10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}$$

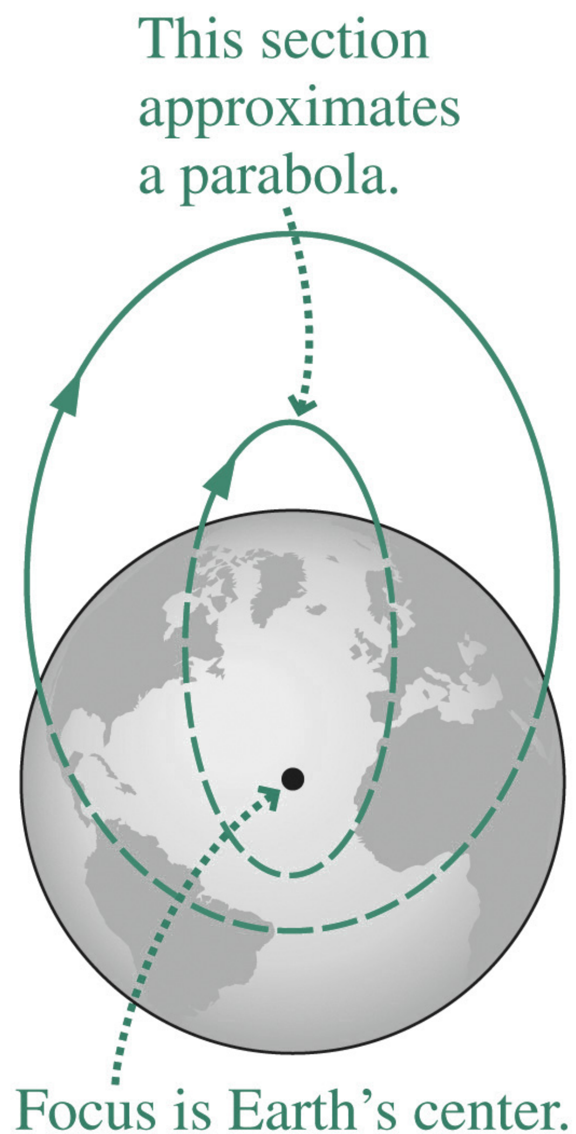

- This formula for the general gravitational force was developed by Newton (1687)
  - It's a much much better approximation, but again only an approximation!
  - Superceded by Einstein's general theory of relativity (1916)
  - But this equation works well enough to pretty much all everyday phenomena
- Technically for point particles, but works perfectly well for spheres where centers are distance r apart
  - Gravity is always attractive along the vector $\vec{r}$
  - We know of no classic examples of anti-gravity
    - Not even anti-matter, which still has positive mass: $E = mc^2$

# Orbits: Our Old Ponderable

- Newton explained orbits using universal gravitation and his laws of motion:
  - Bound orbits are generally elliptical.
  - In the special case of a circular orbit, the orbiting object “falls” around a gravitating mass, always accelerating toward its center with the magnitude of its acceleration remaining constant.
  - Unbound orbits are hyperbolic or (borderline case) parabolic.

# Projectile Motion and Orbits

- The "parabolic" trajectories of projectiles near Earth's surface are actually sections of elliptical orbits that intersect Earth.
- The trajectories are parabolic only in the approximation that we can neglect Earth's curvature and the variation in gravity with distance from Earth's center.



# Circular Orbits

- In a circular orbit, gravity provides the centripetal force of magnitude $F_{\text{centrip}} = mv^2/r$ needed to keep an object of mass *m* in its circular path about a much more massive object of mass M:

$$F_{\text{g}} = F_{\text{centrip}} \quad \Rightarrow \quad \frac{GMm}{r^2} = \frac{mv^2}{r}$$

- Orbital speed v (solve for v): $\boxed{v = \sqrt{GM/r}}$
- Orbital period (time to go 2πr with velocity v): $\boxed{T^2 = \frac{4\pi^2 r^3}{GM}}$
  - This is a derivation of Kepler's **observation** for planetary motion that the period squared goes like the orbit radius cubed:

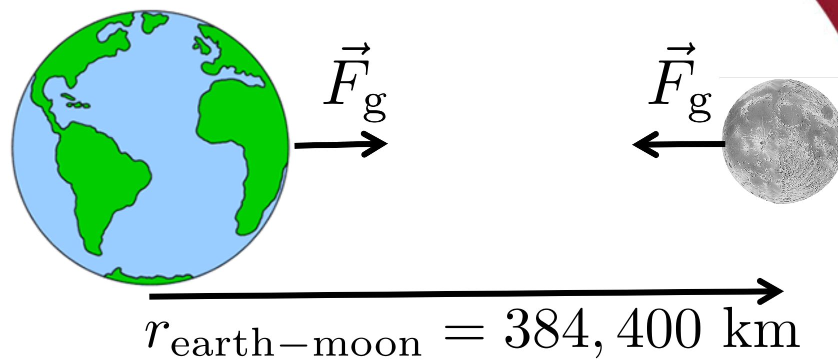
$$T^2 \propto r^3$$

  - For satellites in low-Earth orbit, the period T is about 90 minutes.

# Ponderable

$$\boxed{F_g = \frac{Gm_1 m_2}{r^2}}$$

$$G = 6.67384 \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$$




- What is the gravitational force between the Earth and its moon?

$$m_{\mathrm{Earth}} = 5.9722 \times 10^{24}\ \mathrm{kg} \qquad m_{\mathrm{moon}} = 7.3477 \times 10^{22}\ \mathrm{kg}$$

- What is the gravitational force between Todd and his laptop?

$$m_{\mathrm{Todd}} = 100\ \mathrm{kg} \qquad m_{\mathrm{laptop}} = 1.8\ \mathrm{kg} \qquad r = 0.5\ \mathrm{m}$$

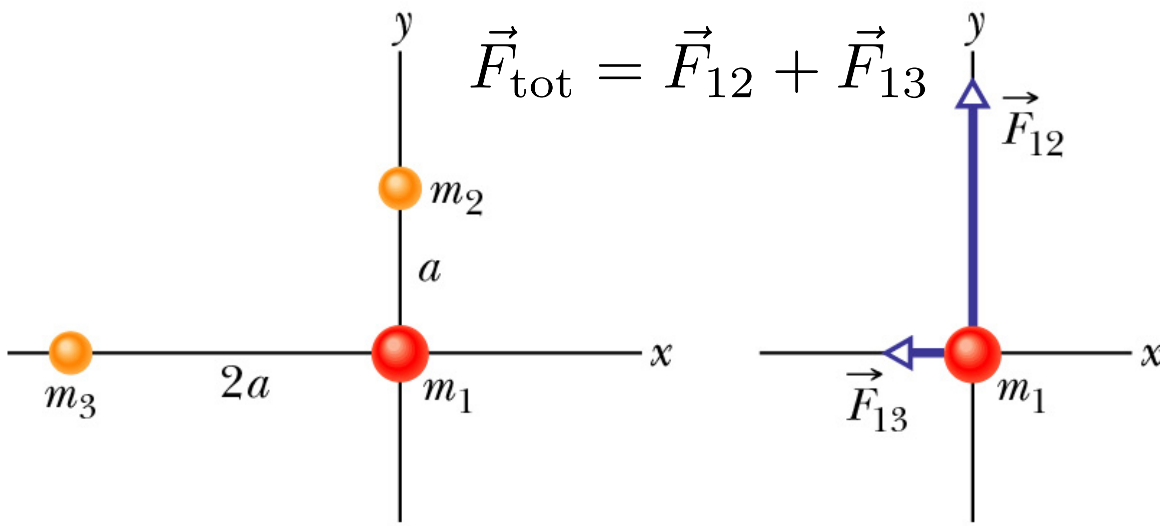
- What is the altitude of geosynchronous satellites above Earth?

$$r_{\mathrm{Earth}} = 6370\ \mathrm{km} \qquad T^2 = \frac{4\pi^2 r^3}{GM}$$

# Multiple Objects

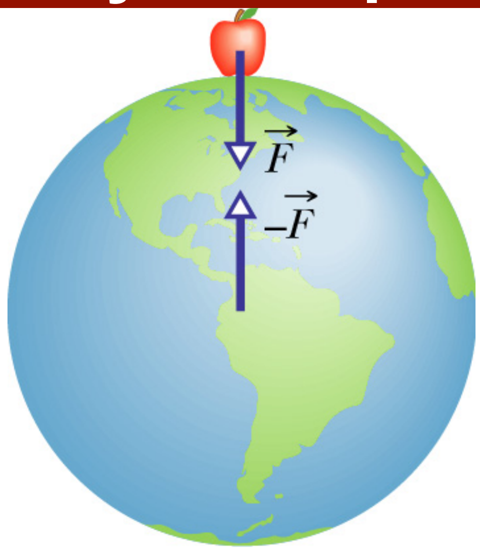
- To calculate the total gravitational force from many objects, we add together the vectors of their individual forces
  - This is known as the **principle of superposition**
  - This is (mostly) a general rule for vectors that you already know

$$\vec{F}_{\rm tot} = \vec{F}_{12} + \vec{F}_{13}$$




  - This summation extends to any number of forces: integral!
    - We can calculate the gravitational force between any two oddly shaped objects by adding up (integrating) gravitational forces between all the combinations of all their pieces

# Gravity and Spheres




- Newton had invented calculus so he could do these sums for simple geometric objects
- One cool result was **Newton's Shell Theorem**
  - A uniform spherical shell of matter attracts an object that is outside the shell as if all the shell's mass was concentrated at its center
- This is why we can dodge calculus and simplify calculations of the force of gravity for spherical objects (like planets)

# Revisiting g

- Recall that we have $F_{\rm g} = mg$ near Earth's surface
- And gravity says $$F_{\rm g} = \frac{Gmm_{\rm Earth}}{r^2}$$
- So $g = \dfrac{Gm_{\rm Earth}}{r^2_{\rm Earth}}$ on the surface of the Earth

- This applies to any nearly spherical cosmic body!
  - Again, an approximation (spherical, uniform mass, rotation…)

- On Mars:

$$m_{\rm Mars} = 6.4 \times 10^{23}\ {\rm kg}$$

$$r_{\rm Mars} = 3400\ {\rm km}$$

$$g_{\rm Mars} = 3.7\ {\rm m/s^2} = 0.38\ g_{\rm Earth}$$

On Moon:

$$m_{\rm Moon} = 7.3 \times 10^{22}\ {\rm kg}$$

$$r_{\rm Moon} = 1737\ {\rm km}$$

$$g_{\rm Moon} = 1.6\ {\rm m/s^2} = 0.16\ g_{\rm Earth}$$

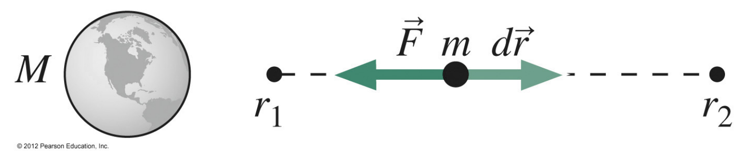
# Gravitational Potential Energy

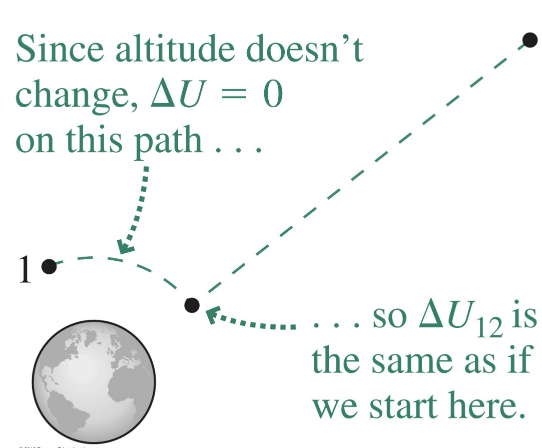
- Because the gravitational force changes with distance, it's necessary to integrate to calculate how gravitational potential energy U changes over large distances. This integration gives

$$\Delta U_{12} = G m_1 m_2 \int_{r_1}^{r_2} r^{-2} dr = G m_1 m_2 (-r^{-1}) \Big|_{r_1}^{r_2} = G m_1 m_2 \left( \frac{1}{r_1} - \frac{1}{r_2} \right)$$

- This result holds regardless of whether the two points are on the same radial line.
- It's convenient to take the zero of gravitational potential energy at infinity. Then the gravitational potential energy becomes

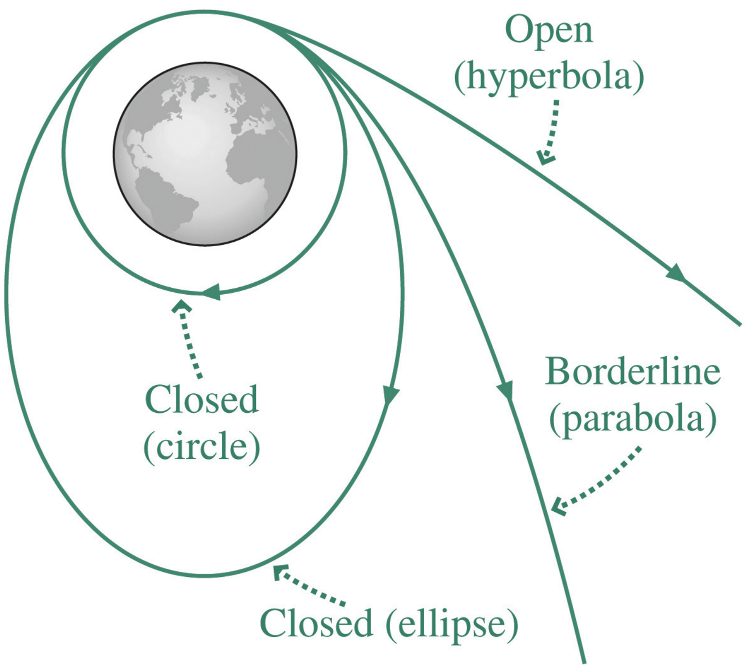
$$\boxed{U(r) = -\frac{G m_1 m_2}{r}}$$

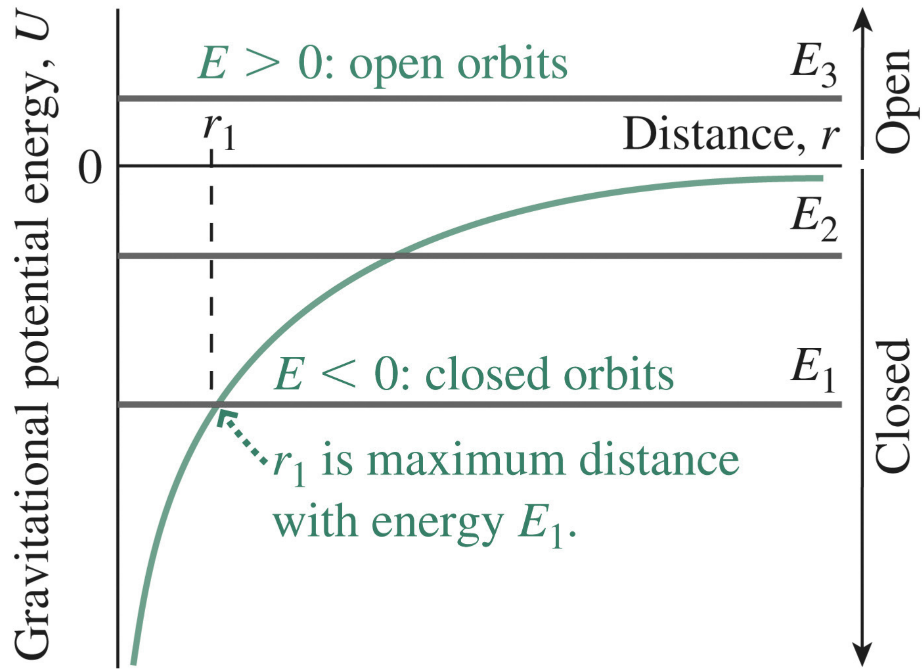


# Energy and Orbits

- The total energy $E = K + U$, the sum of kinetic energy $K$ and potential energy $U$, determines the type of orbit:
  - $E < 0$: The object is in a bound, elliptical orbit.
    - Special cases include circular orbits and the straight-line paths of falling objects.
  - $E > 0$: The orbit is unbound and hyperbolic.
  - $E = 0$: The borderline case gives a parabolic orbit.



# Escape Velocity

- An object with total energy *E* less than zero is in a bound orbit and can't escape from the gravitating center.
- With energy *E* greater than zero, the object is in an unbound orbit and can escape to infinitely far from the gravitating center.
- The minimum speed (launching directly away from the center of the earth) required to escape is given by

$$0 = K + U = \tfrac{1}{2} m v^2 - \frac{GMm}{r}$$

- Solving for *v* gives the escape velocity:

$$v_{\mathrm{esc}} = \sqrt{\frac{2GM}{r}}$$

  – Escape velocity from Earth's surface is about 11 km/s.

# Energy in Circular Orbits

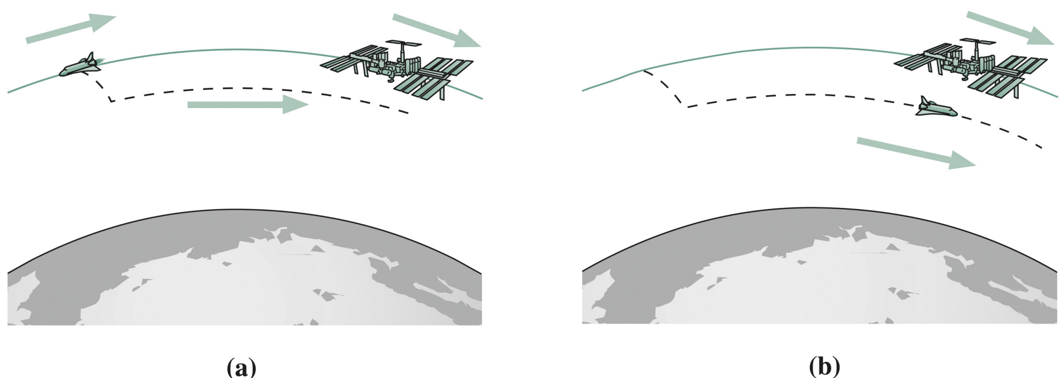
- In the special case of a circular orbit, kinetic energy and potential energy are precisely related:

$$U = -2K$$

- Thus in a circular orbit the total energy is

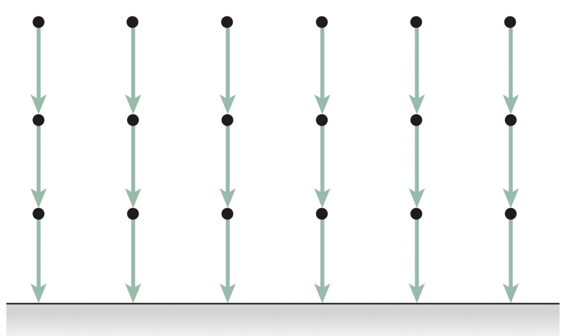
$$E = K + U = -K = \tfrac{1}{2}U = -\frac{GMm}{2r}$$

  – This negative energy shows that the orbit is bound.
  – The lower the orbit, the lower the total energy—but the faster the orbital speed.
    - This means an orbiting spacecraft needs to lose energy to gain speed.

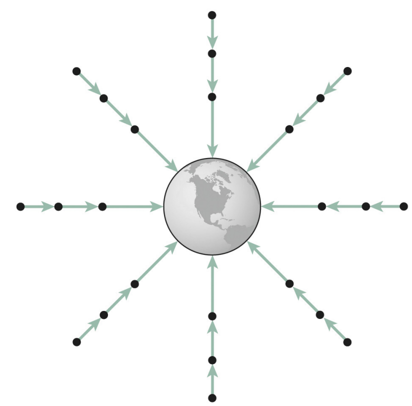


# The Gravitational Field

- It's convenient to describe gravitation not in terms of "action at a distance" but rather in terms of a **gravitational field** that results from the presence of mass and that exists at all points in space.
  - A massive object creates a gravitational field in its vicinity, and other objects respond to the field *at their immediate locations.*
  - The gravitational field can be visualized with a set of vectors giving its strength (in N/kg; equivalently, m/s$^2$) and its direction.



Near Earth's surface



On a larger scale

# Millennium Simulation

- http://www.mpa-garching.mpg.de/galform/press/

- The biggest and most detailed supercomputer simulation of the evolution of the Universe from a few hundred thousand years after the Big Bang to the present day.

- The Millennium Simulation used 10 billion particles to track the evolution of 20 million galaxies over the history of the universe.

- A 3-dimensional visualization of the Millennium Simulation. The movie shows a journey through the simulated universe. During the two minutes of the movie, we travel a distance for which light would need more than 2.4 billion years.