

Your name: _____

Please show your work, write neatly, and put a box or circle around each of your answers.

Assume Earth's gravitational acceleration ($g = 9.8 \text{ m/s}^2$) unless otherwise specified.

There are 6 problems. Some are quick and some are a bit more involved.

Remember to include units where appropriate!

(Yes, there are 105 possible points. That's intentional.)

1. (10 points) Three objects **hang** from an elevator ceiling as shown in Figure 1. The masses of the boxes are $m_1 = 10 \text{ kg}$, $m_2 = 5.0 \text{ kg}$, and $m_3 = 5.0 \text{ kg}$. The elevator and its contents are accelerating upwards with acceleration $a=1.0 \text{ m/s}^2$. What are the (different) tensions in each of the ropes, T_1 , T_2 , and T_3 ? Express your answers using two significant figures.

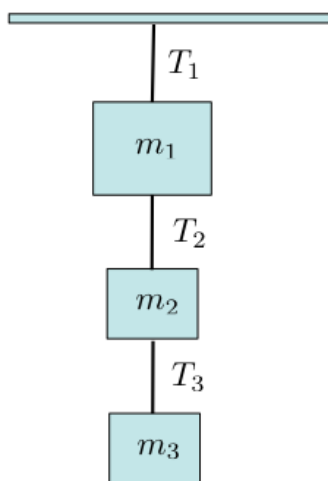


Figure 1: Figure for Problem 1. $m_1 = 10 \text{ kg}$, $m_2 = 5.0 \text{ kg}$, $m_3 = 5.0 \text{ kg}$. All are accelerating upwards with $a=1.0 \text{ m/s}^2$. Find T_1 , T_2 , and T_3 .

Solution: It's most straightforward to proceed from the lowest box to higher boxes, since each box's motion depends on the tensions above and below it. Writing Newton's 2nd law for m_3 , with the positive direction pointing **up**:

$$\sum F_{\text{on } m_3} = m_3 a \quad \Rightarrow \quad T_3 - m_3 g = m_3 a$$

$$T_3 = m_3(a + g) = (5.0 \text{ kg})(1.0 \text{ m/s}^2 + 9.8 \text{ m/s}^2) = \boxed{54 \text{ N} = T_3}$$

Then for m_2 :

$$\sum F_{\text{on } m_2} = m_2 a \quad \Rightarrow \quad T_2 - T_3 - m_2 g = m_2 a$$

$$T_2 = T_3 + m_2(a + g) = 54 \text{ N} + (5.0 \text{ kg})(1.0 \text{ m/s}^2 + 9.8 \text{ m/s}^2) = 108 \text{ N} \approx \boxed{110 \text{ N} = T_2}$$

Then for m_1 :

$$\sum F_{\text{on } m_1} = m_1 a \quad \Rightarrow \quad T_1 - T_2 - m_1 g = m_1 a$$

$$T_1 = T_2 + m_1(a + g) = 108 \text{ N} + (10.0 \text{ kg})(1.0 \text{ m/s}^2 + 9.8 \text{ m/s}^2) = 216 \text{ N} \approx \boxed{220 \text{ N} = T_1}$$

2. A box of mass $m = 3.0$ kg is being pushed up a *frictionless* ramp with force F_{push} . The ramp that is inclined at angle $\theta = 35^\circ$ to the horizontal, as shown in Figure 2.

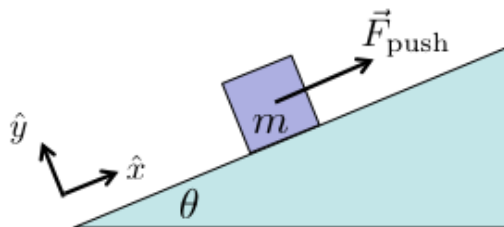
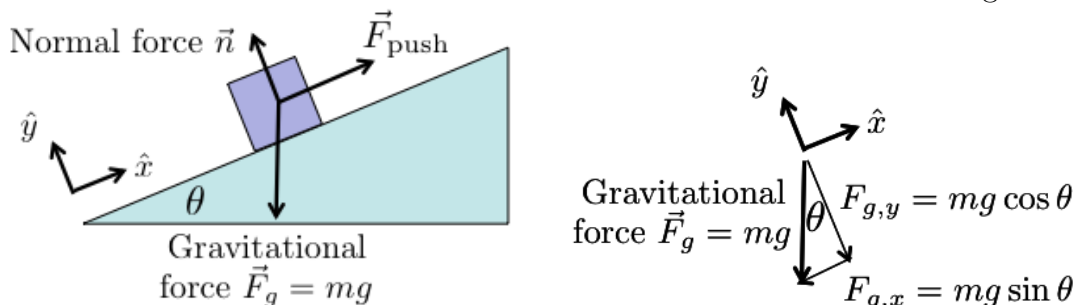


Figure 2: Figure for Problem 2.

- (a) (5 points) What is the pushing force F_{push} to keep the box is moving at constant speed?
- (b) (5 points) What is the work done by the push when it raises the box by a *vertical* distance $\Delta y = 1.5$ m?

Solution: The box is moving at constant speed, so $a = 0$ and $\sum F = 0$ in both \hat{x} and \hat{y} directions. Note that there is **no friction**. There are three forces acting on the box:



The gravitational force $F_g = mg$ is broken down into components on the right. Using Newton's 2nd law in the \hat{x} direction:

$$\sum F_x = 0 = F_{\text{push}} - F_{g,x} = F_{\text{push}} - mg \sin \theta$$

$$F_{\text{push}} = mg \sin \theta = (3.0 \text{ kg})(9.8 \text{ m/s}^2) \sin(35^\circ) = \boxed{16.9 \text{ N} = F_{\text{push}}}$$

There are several ways to compute the work for part (b). We can use the definition of work, $W = Fd \cos \phi$ where ϕ is the angle between the force and the direction the object is moved. Here the force and box's motion are in the same direction (\hat{x}) so $\phi = 0^\circ$ and $W = Fd$. The distance the object must be moved in the \hat{x} direction to rise 1.5 m vertically is $d = (1.5 \text{ m}) / \sin \theta$, so then

$$W = Fd = (16.86 \text{ N})(1.5 \text{ m} / \sin(35^\circ)) = (16.86 \text{ N})(2.615 \text{ m}) = \boxed{44.1 \text{ J} = W}$$

We can also realize that the box's kinetic energy is not changing ($\text{KE} = (1/2)mv^2$ and v is constant), so all our work must be going into raising the object's gravitational potential energy. Then we have

$$W = mg\Delta y = (3.0 \text{ kg})(9.8 \text{ m/s}^2)(1.5 \text{ m}) = \boxed{44.1 \text{ J} = W}$$

3. A ball of mass $m = 1.0$ kg is swinging in a semicircular path from a thin string of length $L = 1.0$ m. The ball's maximum height above its lowest point is $h = 0.2$ m, at which point the string makes an angle θ with the vertical as shown in Figure 3.

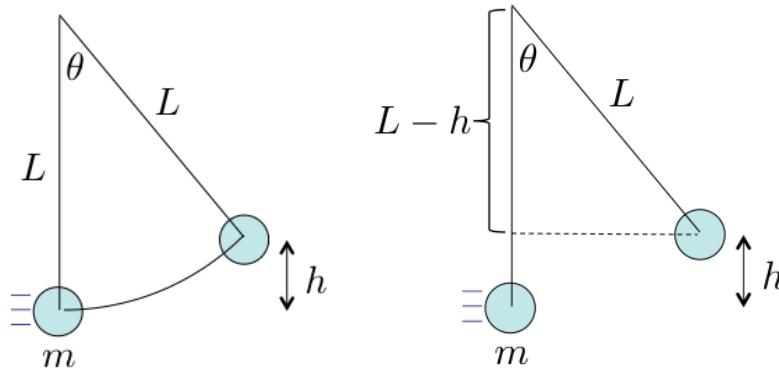


Figure 3: Figure for Problem 3, including sketch for part (a) on right. $m = 1.0$ kg, $L = 1.0$ m, and $h = 0.2$ m.

- (a) (10 points) Find the angle at the peak of the ball's swing, θ .

Solution: This problem was a lot easier if you did a similar problem in the practice test. Here the “trick” is to draw the dotted line in the right side of the above figure, and realize that the length of the vertical segment is $L - h$. This is a right triangle, with

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{L - h}{L} = \frac{0.8 \text{ m}}{1.0 \text{ m}} = 0.8 \quad \Rightarrow \quad \theta = \cos^{-1}(0.8) = \boxed{36.9^\circ = \theta}$$

- (b) (10 points) Find the maximum speed of the ball at the bottom of its trajectory.

Solution: Here you should use the conservation of energy. When the ball is at the bottom of its trajectory, all of its energy is kinetic energy. When the ball is at angle θ , it is not moving and all its energy is gravitational potential energy. So

$$\frac{1}{2}mv_{\text{bottom}}^2 = mgh \quad \Rightarrow \quad v_{\text{bottom}} = \sqrt{2gh} = \sqrt{2(9.8 \text{ m/s}^2)(0.2 \text{ m})} = \boxed{1.98 \text{ m/s} = v_{\text{bottom}}}$$

- (c) (5 points) Find the maximum tension of the string.

Solution: The ball is moving along a circular arc, so the net forces on it must be equal to the centripetal force, $F_{\text{centrip}} = mv_{\text{bottom}}^2/L$. There are only two forces acting on it: tension pulling up, and gravity pulling down. So we have

$$\begin{aligned} \sum F &= ma_{\text{centrip}} = \frac{mv_{\text{bottom}}^2}{L} = T - mg \\ T &= mg + \frac{mv_{\text{bottom}}^2}{L} = (1.0 \text{ kg})(9.8 \text{ m/s}^2) + \frac{(1.0 \text{ kg})(1.98 \text{ m/s})^2}{1.0 \text{ m}} \\ &= \boxed{13.7 \text{ N} = T} \end{aligned}$$

4. Two masses $m_1 = 2 \text{ kg}$ and $m_2 = 3 \text{ kg}$ are hung in a corner from a set of strings as in Figure 4. None of the masses are moving.

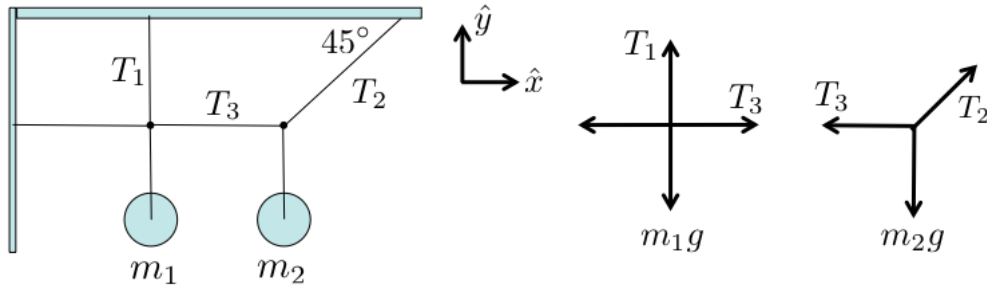


Figure 4: Figure for Problem 4, including diagrams for solution.

- (a) (5 points) What is the tension T_1 ?

Solution: I've drawn a coordinate system and force diagrams for each of the points where strings meet in Fig. 4. None of the objects are moving, so this is again a statics problem: in both \hat{x} and \hat{y} directions, there is no acceleration and $\sum F = 0$. T_1 is easy to find from $\sum F_y = 0$ for the left force diagram:

$$\sum F_y = T_1 - m_1g = 0 \Rightarrow T_1 = m_1g = (2 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{19.6 \text{ N} = T_1}$$

- (b) (5 points) What is the tension T_2 ?

Solution: T_2 is only in the right force diagram, so we have to use this diagram to find T_2 . Here $\sum F_y = 0$ so we have

$$\sum F_y = T_2 \sin(45^\circ) - m_2g = 0 \Rightarrow T_2 = \frac{m_2g}{\sin(45^\circ)} = \frac{(3 \text{ kg})(9.8 \text{ m/s}^2)}{\sin(45^\circ)} = \boxed{41.6 \text{ N} = T_2}$$

- (c) (5 points) What is the tension T_3 ?

Solution: T_3 can't be found from the left force diagram since we don't know the other unlabeled tension that points off to the left. We have to find it from the horizontal components of the right force diagram:

$$\sum F_x = T_2 \sin(45^\circ) - T_3 = 0 \Rightarrow T_3 = T_2 \sin(45^\circ) = (41.6 \text{ N}) \sin(45^\circ) = \boxed{29.4 \text{ N} = T_3}$$

- (d) (5 points) If the distance between the masses is $r = 0.3 \text{ m}$, what is the magnitude of the gravitational force between them (in Newtons)?

Solution: The force of gravitational attraction between two masses is given by

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(2 \text{ kg})(3 \text{ kg})}{(0.3 \text{ m})^2} = \boxed{4.4 \times 10^{-9} \text{ N} = F_g}$$

5. You are building a spring device that pushes boxes across a floor a distance $L_f = 0.2$ m, and then up a $\theta = 20^\circ$ ramp of length $L_r = 0.5$ m. Your spring has spring constant $k = 46$ N/m, the mass is $m = 2.5$ kg, and the coefficient of sliding friction between the floor (or ramp) and box is $\mu_k = 0.1$.

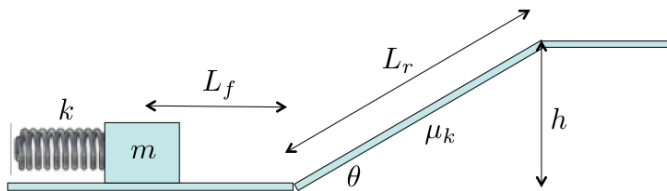


Figure 5: Figure for Problem 5.

- (a) (5 points) What is the height h at the end of the ramp?

Solution: This can be found from the length of the ramp L_r and the angle θ :

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{h}{L_r} \quad \Rightarrow \quad h = L_r \sin \theta = (0.5 \text{ m}) \sin(20^\circ) = \boxed{0.17 \text{ m} = h}$$

- (b) (5 points) If the box is moved to the top of the ramp, what is its change in gravitational potential energy (in Joules)?

Solution: The gravitational potential energy change from moving an object a vertical distance h is mgh , so

$$\Delta U_g = mgh = (2.5 \text{ kg})(9.8 \text{ m/s}^2)(0.17 \text{ m}) = \boxed{4.19 \text{ J} = \Delta U_g}$$

- (c) (5 points) As the box slides across the floor and up the ramp, what energy is lost to friction (in Joules)?

Solution: Energy lost to friction is the work done against friction, and work is force times distance. The force of friction while sliding is $F_{\text{friction}} = \mu_k n$ where n is the normal force, or the force perpendicular to the floor or ramp. This normal force n is different for the floor and the ramp! On the floor, it is the box's full weight mg , while on the ramp, it's only a portion of the box's full weight, $mg \cos \theta$. We can then combine all this to write:

$$\begin{aligned} W_{\text{friction}} &= F_{\text{friction},f} L_f + F_{\text{friction},r} L_r \\ &= \mu_k mg L_f + \mu_k mg \cos \theta L_r \\ &= (0.1)(2.5 \text{ kg})(9.8 \text{ m/s}^2)(0.2 \text{ m}) + (0.1)(2.5 \text{ kg})(9.8 \text{ m/s}^2)(\cos(20^\circ))(0.5 \text{ m}) \\ &= \boxed{1.64 \text{ J} = W_{\text{friction}}} \end{aligned}$$

- (d) (10 points) What distance x must you compress the spring so it gives the box just enough energy to make it to the top of the ramp?

Solution: The term "energy" here is a suggestion that you use conservation of energy to solve this portion of the problem. Initially there is energy stored in the spring. When the box is released and travels up the ramp, some energy is lost to friction and the remainder goes to the gravitational potential energy of the box at its new height. Energy stored in the spring is $(1/2)kx^2$, and the other energies were calculated in parts (b) and (c):

$$\begin{aligned} \frac{1}{2}kx^2 &= \Delta U_g + W_{\text{friction}} = (4.19 \text{ J}) + (1.64 \text{ J}) = 5.83 \text{ J} \\ x &= \sqrt{\frac{2(5.83 \text{ J})}{k}} = \sqrt{\frac{11.66 \text{ J}}{(46 \text{ N/m})}} = \boxed{0.50 \text{ m} = x} \end{aligned}$$

- (e) (10 point super-bonus) If you compress the spring by twice the distance found in part (c), the box will move so fast that it will “launch” off the end of the ramp. What is the distance that the box is in the air?

Solution: When the spring is compressed by twice the distance found in (c), it delivers four times as much energy to the box. But that original energy is also the energy lost to friction and gravitational potential energy to get the box to the top of the ramp, so the box’s final kinetic energy at the top of the ramp is

$$KE_{\text{top of ramp}} = \frac{1}{2}mv_{\text{top of ramp}}^2 = 4(5.83 \text{ J}) - (5.83 \text{ J}) = 3(5.83 \text{ J}) = 17.49 \text{ J}$$
$$v_{\text{top of ramp}} = \sqrt{\frac{2(17.49 \text{ J})}{2.5 \text{ kg}}} = 3.74 \text{ m/s}$$

At this point the box leaves the ramp with this velocity and at the ramp’s angle of 20° . We can then use the range equation from projectile motion to find out the distance that the box travels before it lands back on the ledge:

$$\begin{aligned} \text{range} &= \frac{v_{\text{top of ramp}}^2 \sin(2\theta)}{g} = \frac{(3.74 \text{ m/s})^2 \sin(40^\circ)}{(9.8 \text{ m/s}^2)} \\ &= \boxed{0.96 \text{ m} = \text{range}} \end{aligned}$$

If you instead interpreted “distance the box is in the air” as the maximum height, there’s a projectile motion formula for that too:

$$\begin{aligned} \text{height} &= \frac{v_{\text{top of ramp}}^2 \sin^2 \theta}{2g} = \frac{(3.74 \text{ m/s})^2 \sin^2(20^\circ)}{2(9.8 \text{ m/s}^2)} \\ &= \boxed{0.29 \text{ m} = \text{height}} \end{aligned}$$

6. A plane flies through a loop-the-loop of radius $r = 50$ m as pictured in Figure 6.

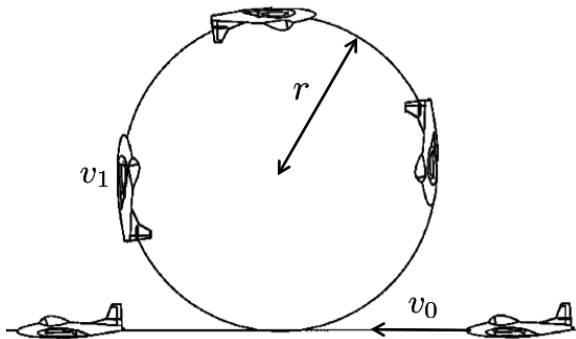


Figure 6: Figure for Problem 6.

- (a) (5 points) What **initial speed** v_0 does the plane need to have to safely make the circular loop?

Solution: This was the toughest problem of this exam, and most folks just used the formula derived in class for the minimum velocity of a looping object at the **top** of a loop-the-loop. This is

$$v_{\text{top}} = \sqrt{gr} = \sqrt{(9.8 \text{ m/s}^2)(50 \text{ m})} = 22.14 \text{ m/s}$$

The kinetic energy of the plane at the top of the loop is $KE_{\text{top}} = (1/2)mv_{\text{top}}^2$, and its gravitational potential energy relative to the bottom of the loop is $\Delta U_g = mgh = 2mgr$ since $h = 2r$ (the diameter of the loop). At the bottom of the loop, all of this energy is kinetic energy, so we have

$$\begin{aligned} KE_{\text{bottom}} &= KE_{\text{top}} + \Delta U_g = \frac{1}{2}mv_{\text{top}}^2 + 2mgr \\ \frac{1}{2}mv_0^2 &= \frac{1}{2}mv_{\text{top}}^2 + 2mgr \\ v_0^2 &= v_{\text{top}}^2 + 4gr = gr + 4gr = 5gr \\ v_0 &= \sqrt{5gr} = \sqrt{5(9.8 \text{ m/s}^2)(50 \text{ m})} = \boxed{49.5 \text{ m/s} = v_0} \end{aligned}$$

- (b) (5 points) What is the plane's speed v_1 when it is one-fourth of the way through the loop?

Solution: Using the same approach, the energy of the plane is split between the kinetic energy and the gravitational potential energy $mgh = mgr$ one-fourth of the way through the loop. So we have

$$\begin{aligned} KE_{\text{bottom}} &= KE_{\text{middle}} + \Delta U_g = \frac{1}{2}mv_1^2 + mgr \\ \frac{1}{2}mv_0^2 &= \frac{1}{2}mv_1^2 + mgr \\ v_0^2 &= v_1^2 + 2gr \\ v_1 &= \sqrt{v_0^2 - 2gr} = \sqrt{3gr} = \sqrt{3(9.8 \text{ m/s}^2)(50 \text{ m})} = \boxed{38.3 \text{ m/s} = v_1} \end{aligned}$$

- (c) (5 points) What **initial speed** v_0 does the plane need to have to safely make the circular loop if the pilot is flying a jet show on Mars, where $g_{\text{Mars}} = 3.7 \text{ m/s}^2$?

Solution: Here we use the equation we found in part (a) and substitute a different value for g :

$$v_0 = \sqrt{5g_{\text{Mars}}r} = \sqrt{5(3.7 \text{ m/s}^2)(50 \text{ m})} = \boxed{30.4 \text{ m/s} = v_0}$$

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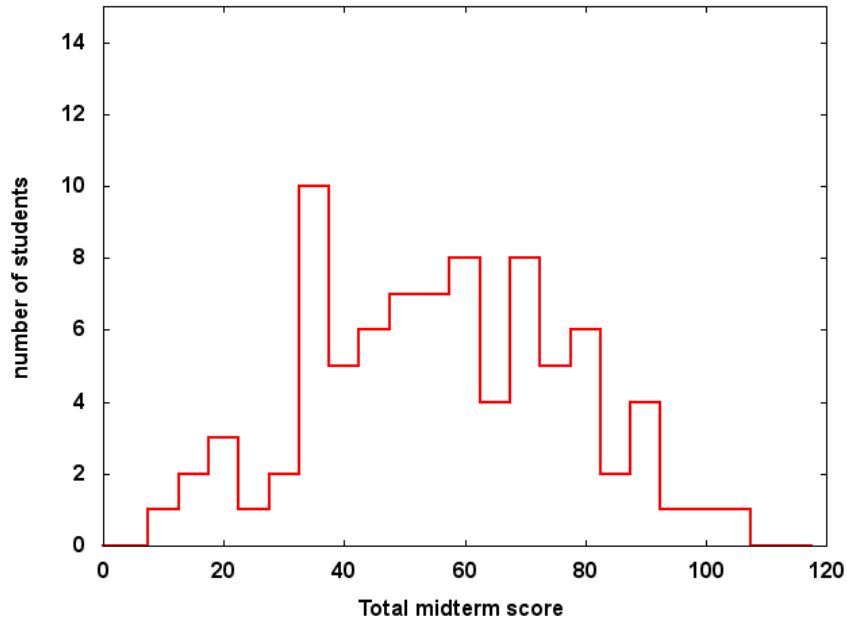


Figure 7: Distribution of scores for total test (out of 105 possible points).

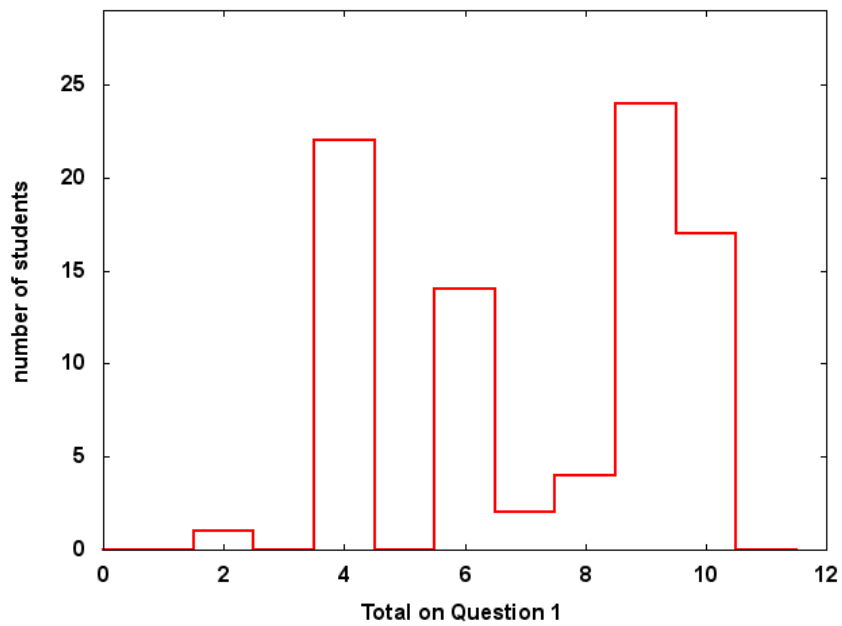


Figure 8: Distribution of scores for Question 1 (out of 10 possible points).

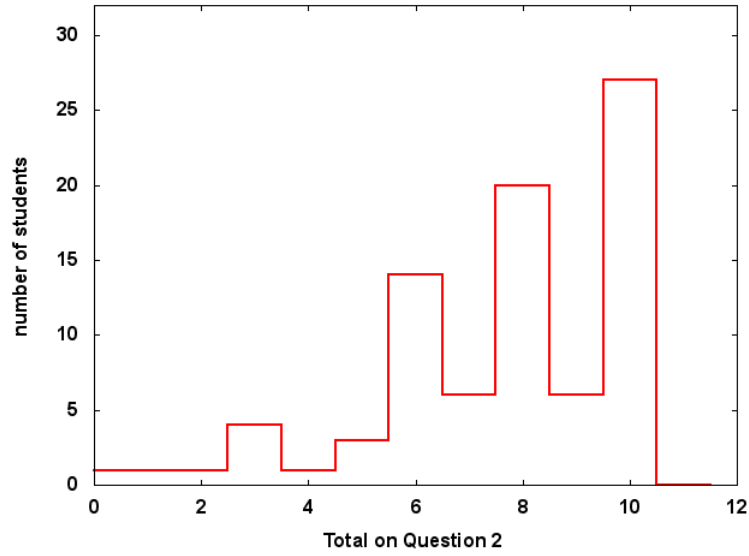


Figure 9: Distribution of scores for Question 2 (out of 10 possible points).

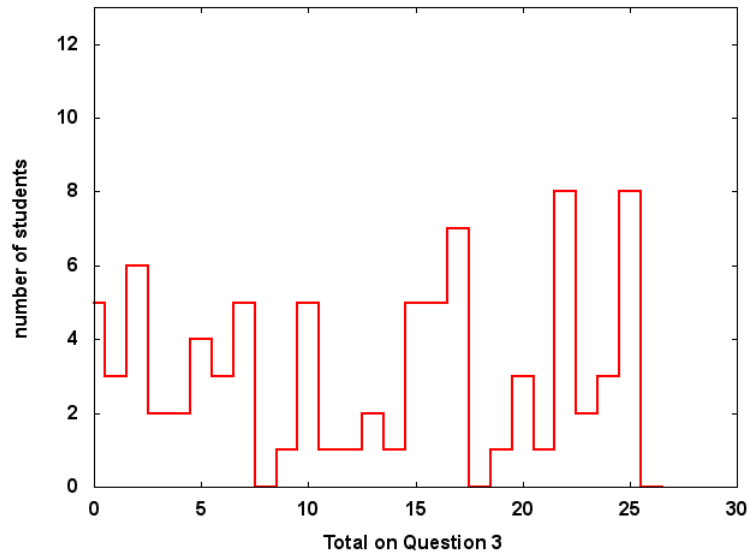


Figure 10: Distribution of scores for Question 3 (out of 25 possible points plus 10 point superbonus).

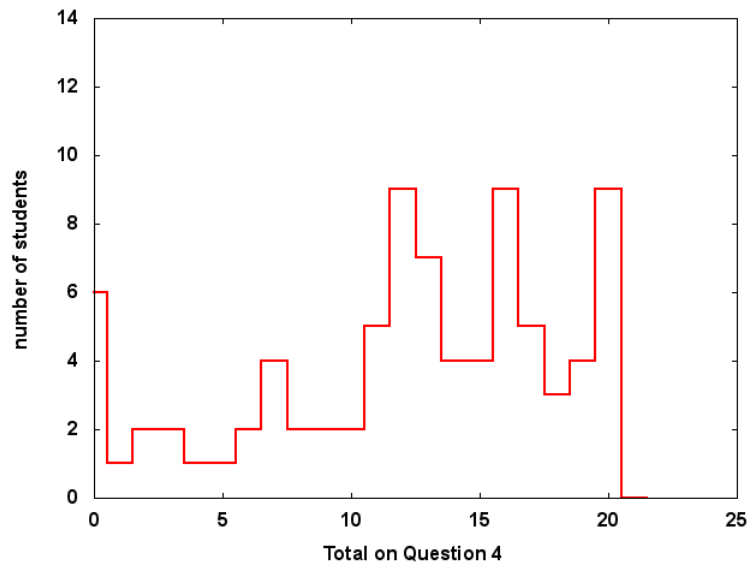


Figure 11: Distribution of scores for Question 4 (out of 20 possible points).

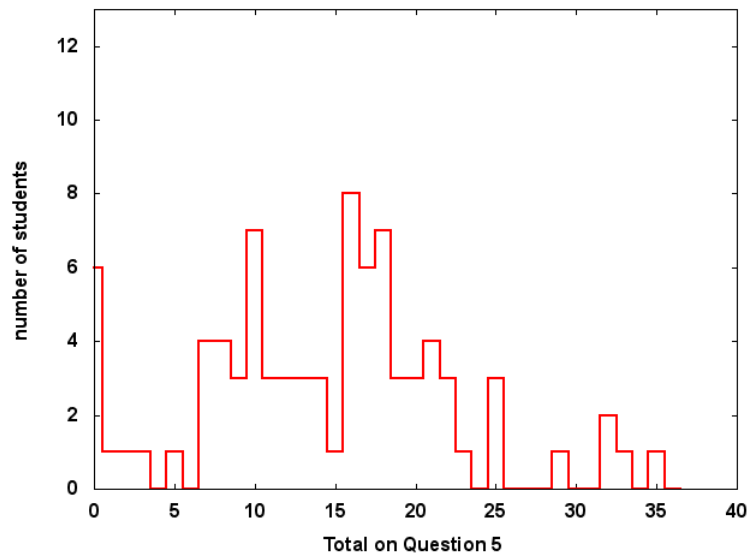


Figure 12: Distribution of scores for Question 5 (out of 25 possible points).

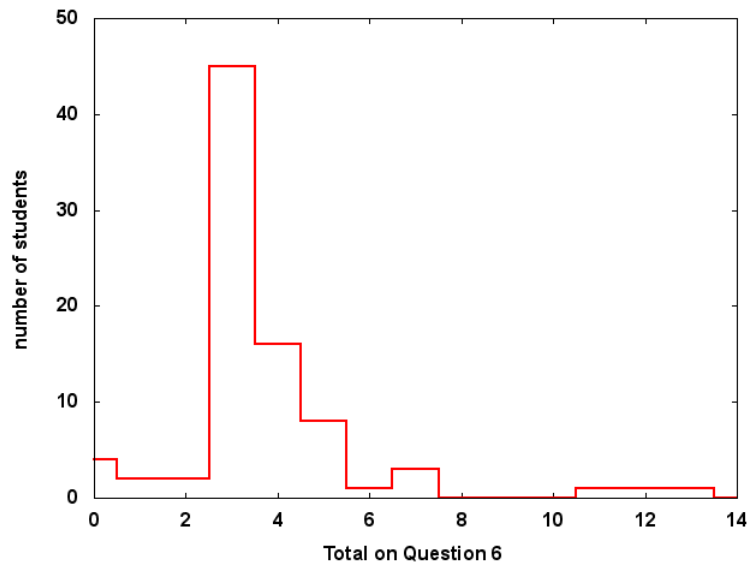


Figure 13: Distribution of scores for Question 6 (out of 15 possible points).