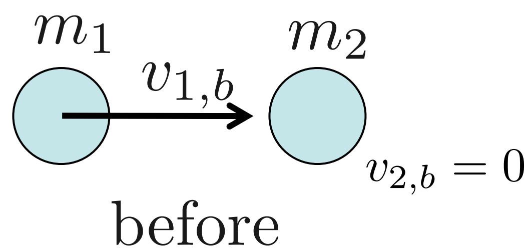


# University Physics 226N/231N
# Old Dominion University
# Momentum and Collisions


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Happy Birthday to Subrahmanyan Chandresekhar (1983 Nobel), John Lithgow, Trey Parker, and Jose Bautista!
Happy New Friends Day, Evaluate Your Life Day, and Seafood Bisque Day!

# Momentum and the Center of Mass

- Remember that the center of mass obey's Newton's 2$^{nd}$ law
  - Like the skier's c.m. traveling on a parabolic trajectory

$$\vec{F}_{\rm net,external} = m\vec{a}_{\rm cm}$$

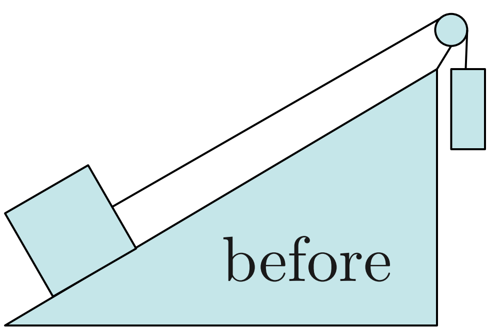
  - What about internal forces? If all internal forces are equal and opposite (Newton's 3$^{rd}$ law) then the net *internal* forces are zero

$$\vec{F}_{\rm net,internal} = 0$$

If we could force as something that changes with time, then here this "something" would be constant (conserved) if we count it for all objects within our system…

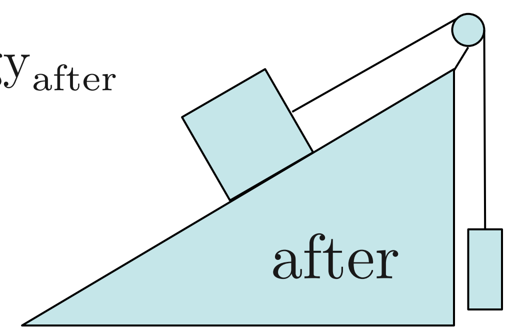
# Conservation Laws, Before and After

- We learned about **conservation of energy** earlier
- We're also learning about **conservation of momentum**…
- In any conservation problem, we usually have two pictures
  - A "before" picture and an "after" picture
  - For each picture, we add together all places that have a given property (e.g. add up all energies, horizontal momenta, …)
  - **Conservation laws** state that this accounting must add up to the same value **at all times** for the system, including our before and after pictures




$$\sum \mathrm{Energy}_{\mathrm{before}} = \sum \mathrm{Energy}_{\mathrm{after}}$$

$$\sum \vec{p}_{\mathrm{before}} = \sum \vec{p}_{\mathrm{after}}$$




($\vec{p}$ components)

$$\sum p_{\mathrm{x,before}} = \sum p_{\mathrm{x,after}}$$

$$\sum p_{\mathrm{y,before}} = \sum p_{\mathrm{y,after}}$$

$$\sum p_{\mathrm{z,before}} = \sum p_{\mathrm{z,after}}$$

# Conservation of Linear Momentum

When the net external force is zero, $\vec{F}_{\text{net}} = m\vec{a} = \frac{d\vec{p}}{dt} = 0$ $\boxed{\vec{p} \equiv m\vec{v}}$

Linear Momentum

The sum of all momenta of all objects in the system is constant:

$$\boxed{\sum \vec{p} = \text{constant}}$$

$$\sum p_x = \text{constant}$$

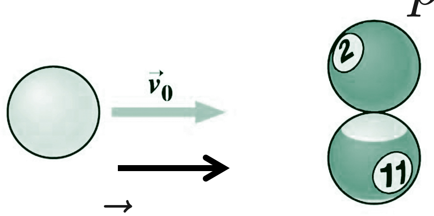
$$\sum p_y = \text{constant}$$

$$\sum p_z = \text{constant}$$

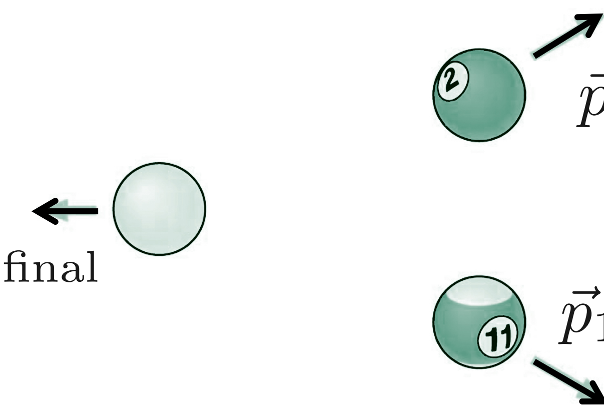
This is the **conservation of linear momentum.**

Example: A "system" of three billiard balls:

– Initially two are at rest; all the momentum is in the left-hand ball:




– Later they're all moving, but the **total** momentum remains the same:

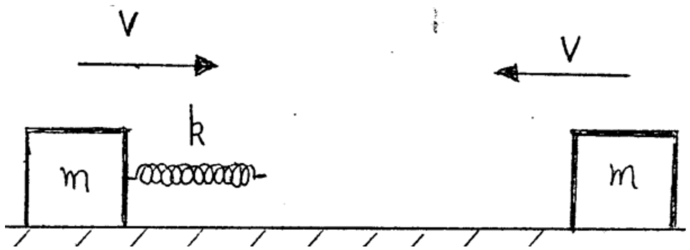


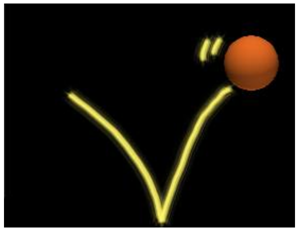
# Collisions

- A **collision** is a brief, intense interaction between objects.
  - Examples: balls on a pool table, a tennis ball and racket, baseball and bat, football and foot, an asteroid colliding with a planet.
  - The collision time is short compared with the time of the overall motion that we're interested in
  - Internal forces of the collision are so large that we can **neglect any external forces** acting on the system during the brief collision time.
  - **Linear momentum is essentially conserved during collisions**
  - The objects technically do not need to touch!

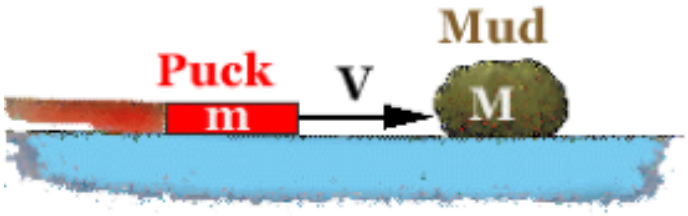
# Elastic and Inelastic Collisions

- In an **elastic collision**, the internal forces of the collision are conservative (a bounce, like a spring)
  - An elastic collision conserves kinetic energy as well as linear momentum.



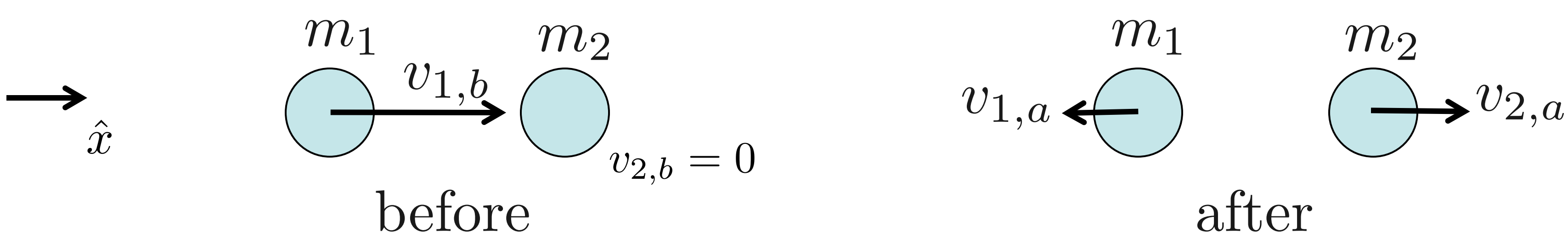



- In an **inelastic collision**, the forces are not conservative and mechanical energy is lost (a splat, like a crash)
  - In a **totally inelastic collision**, the colliding objects stick together to form a single composite object.
  - Some energy has gone to deformation and internal friction, but some **may** be left over in motion of the composite object.




# Elastic Collisions: Example

- Very bouncy things bouncing off each other are a pretty good example of **elastic** collisions
  - Example: Two pucks on an air hockey table bouncing off each other




  - Conservation of momentum (momentum before/after are equal):

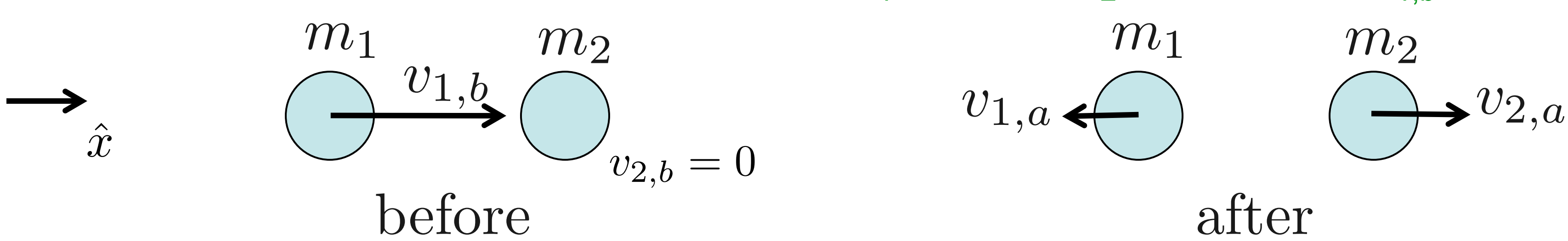
$$m_1 v_{1,b} = m_1(-v_{1,a}) + m_2(v_{2,a}) = -m_1 v_{1,a} + m_2 v_{2,a}$$

  - Conservation of energy (energy before/after are equal):

$$\frac{1}{2} m_1 v_{1,b}^2 = \frac{1}{2} m_1 v_{1,a}^2 + \frac{1}{2} m_2 v_{2,a}^2 \qquad \Rightarrow m_1 v_{1,b}^2 = m_1 v_{1,a}^2 + m_2 v_{2,b}^2$$

# Elastic Collisions: Example Continued

- Example: Two pucks on an air hockey table bouncing off each other
  - Find the final velocities if $m_1$=0.2 kg, $m_2$=0.5 kg, and $v_{1,b}$=1 m/s




- Conservation of momentum (momentum before/after are equal):

$$m_1 v_{1,b} = m_1(-v_{1,a}) + m_2(v_{2,a}) = -m_1 v_{1,a} + m_2 v_{2,a}$$

$$(0.2 \text{ kg m/s}) = -(0.2 \text{ kg}) v_{1,a} + (0.5 \text{ kg}) v_{2,a} \qquad v_{1,a} = 2.5 v_{2,a} - 1 \text{ m/s}$$

- Conservation of energy (energy before/after are equal):

$$\frac{1}{2} m_1 v_{1,b}^2 = \frac{1}{2} m_1 v_{1,a}^2 + \frac{1}{2} m_2 v_{2,a}^2 \qquad \Rightarrow m_1 v_{1,b}^2 = m_1 v_{1,a}^2 + m_2 v_{2,b}^2$$

$$(0.2 \text{ kg})(1 \text{ m/s})^2 = (0.2 \text{ kg}) v_{1,a}^2 + (0.5 \text{ kg}) v_{2,a}^2 \qquad 1.0 \ (\text{m/s})^2 = v_{1,a}^2 + 2.5 v_{2,a}^2$$

$$1.0 = 8.75 v_{2,a}^2 - 5 v_{2,a} + 1.0$$

$$\boxed{v_{2,a} = (5/8.75) \text{ m/s} = 0.57 \text{ m/s}} \qquad \boxed{v_{1,a} = 0.43 \text{ m/s}}$$

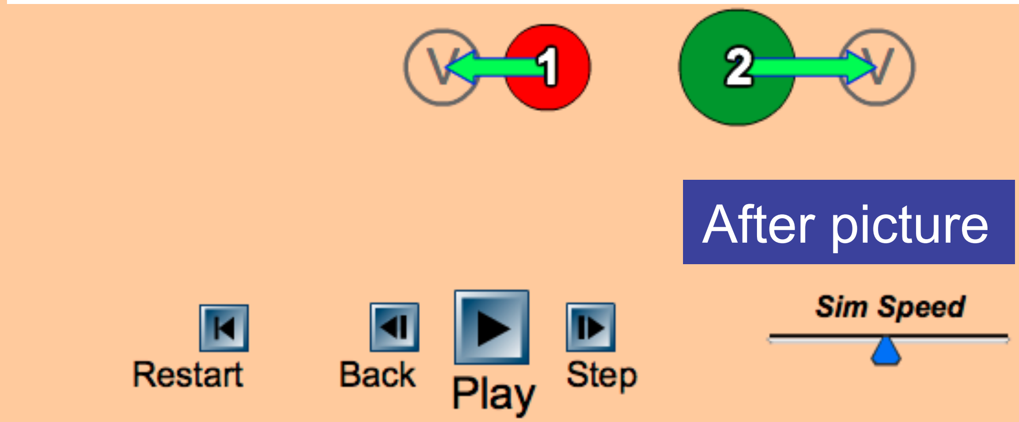
# Elastic Collisions: Example Continued

http://phet.colorado.edu/sims/collision-lab/collision-lab_en.html


Before picture

| Ball | Mass kg | Position m | Velocity m/s | Momentum kg m/s |
|---|---|---|---|---|
| 1 | 0.20 | 1.00 | 1.00 | 0.20 |
| 2 | 0.50 | 2.00 | 0.00 | 0.00 |

- Click “More Data”
- Enter values
- Press “Play”




After picture

| Ball | Mass kg | Position m | Velocity m/s | Momentum kg m/s |
|---|---|---|---|---|
| 1 | 0.20 | 1.71 | -0.43 | -0.09 |
| 2 | 0.50 | 2.10 | 0.57 | 0.29 |

$$\boxed{v_{1,a} = 0.43 \text{ m/s}}$$

$$\boxed{v_{2,a} = 0.57 \text{ m/s}}$$

# Tangible (10 minutes)

- http://phet.colorado.edu/sims/collision-lab/collision-lab_en.html
  - This GUI is linked to the class website for today
- Reproduce the situation on the previous slide

| Ball | Mass kg | Position m | Velocity m/s | Momentum kg m/s |
|---|---|---|---|---|
| 1 | 0.20 | 1.00 | 1.00 | 0.20 |
| 2 | 0.50 | 2.00 | 0.00 | 0.00 |

Click “More Data”
Enter values
Press “Play”
Press “Restart”

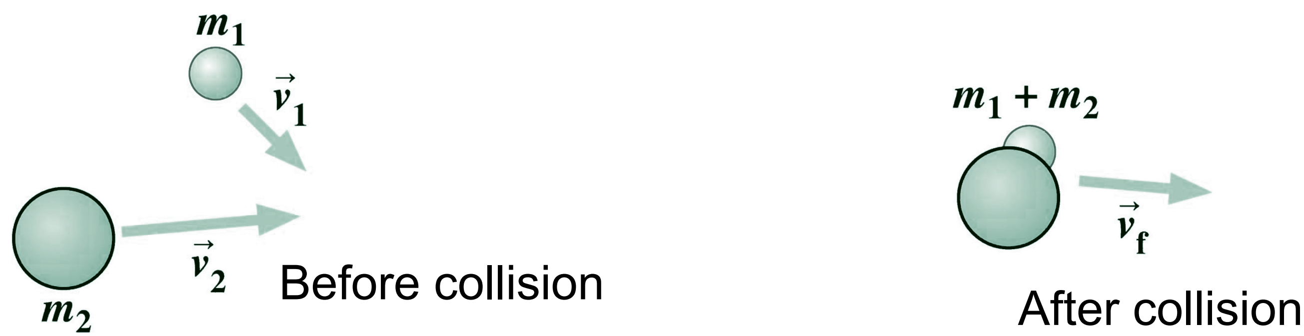
- Change masses to be equal: how does the motion look?
  - Does this make sense? What about $m_1$=0.01 kg and $m_2$=5 kg
- Use the following settings
  - Turn on Kinetic Energy
  - How does motion change when you change the elasticity?

| Ball | Mass kg | Position m | Velocity m/s | Momentum kg m/s |
|---|---|---|---|---|
| 1 | 0.20 | 1.00 | 1.00 | 0.20 |
| 2 | 0.20 | 2.00 | 0.00 | 0.00 |

  - Elastic collisions: objects bounce off each other
    - Total energy is conserved, we can use conservation of energy
  - Inelastic collisions: objects stick together or internally deform
    - Total energy is NOT conserved, we can’t use energy conservation

# Totally Inelastic Collisions and Solution Strategy

- Totally inelastic collisions are governed entirely by conservation of momentum.
  - Since the colliding objects join to form a single composite object, there is only one final velocity




- Solve even 2D collision problems like statics problems
  - Draw the picture and axes; label all masses, velocities, angles
  - Decompose all vectors (velocities) into components
  - Write equations for conservation of momentum in each direction
  - Write conservation of energy IF the collision is elastic
  - Solve the equations for any unknowns

# Tangible (5 minutes)

- http://phet.colorado.edu/sims/collision-lab/collision-lab_en.html
- Click the "Advanced" tab at the top of the screen

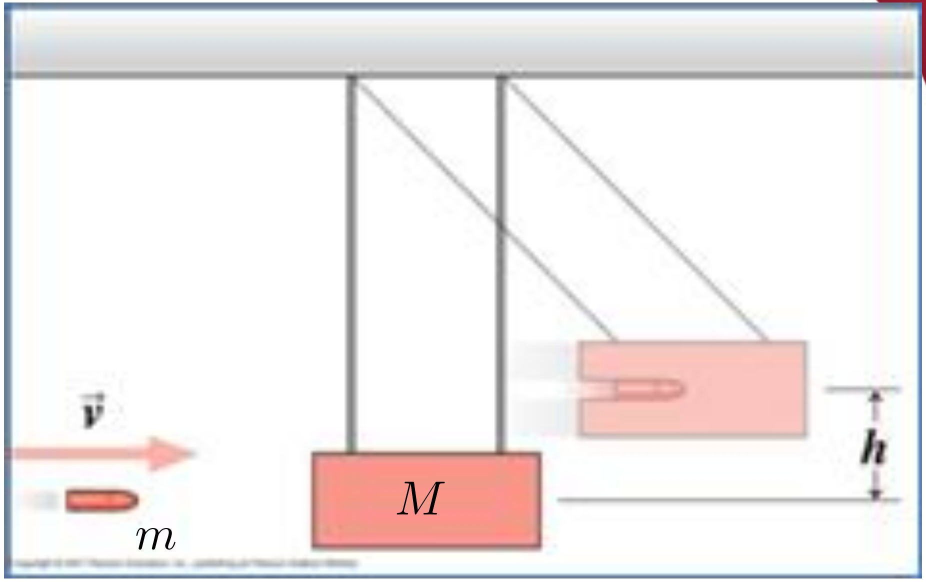
| Ball | Mass (kg) | Position (m) x | Position (m) y | Velocity (m/s) Vx | Velocity (m/s) Vy |
|---|---|---|---|---|---|
| 1 | 0.200 | 0.500 | 0.120 | 1.000 | 0.000 |
| 2 | 0.200 | 1.500 | 0.230 | 0.000 | 0.000 |

- Click "More Data", "Show Paths"
- Enter values
- Press "Play"
- Press "Restart"

  - What is the angle between the two trajectories after collision?
- Change the initial y values and/or velocity and rerun the simulation several times
  - How does the angle between the two trajectories after collision change as you change these parameters?
    - We can prove this angle is constant using conservation of momentum
  - You are learning something that professional pool players know from years of experience
    - They also know that spinning the balls changes this behavior

# Example: Ballistics Velocity Measurement

A bullet with mass m is shot with velocity v into a hanging block of mass M. If the block swings up to height h, what is the bullet's initial velocity v?




This is an **inelastic** collision. Call the velocity of the block plus bullet just after the collision $v_{\mathrm{B}}$

$$\sum p_{\mathrm{x,before}} = \sum p_{\mathrm{x,after}}$$

$$mv = (M+m)v_B$$

$$v_B = \left(\frac{m}{M+m}\right) v$$

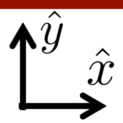
Conservation of energy from just after the bullet hits the block to the top of the swing:

$$\mathrm{KE}_{\mathrm{init}} = \mathrm{PE}_{\mathrm{grav,final}}$$

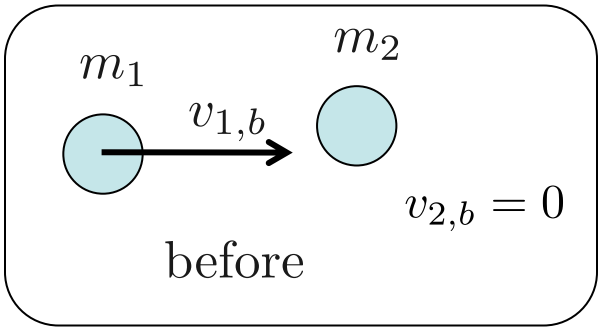
$$\frac{1}{2}(M+m)v_B^2 = (M+m)gh \qquad v_B = \sqrt{2gh}$$

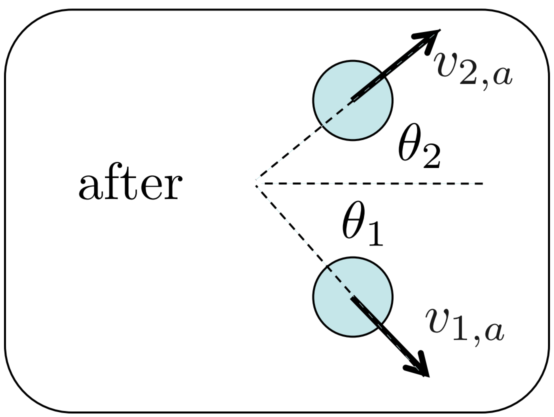
$$\text{So} \quad \boxed{v = \left(\frac{M+m}{m}\right)\sqrt{2gh}}$$

# Example: Two Dimensional Elastic Collision




$$\text{If } m_1 = m_2 = m, \text{ show that } \theta_1 + \theta_2 = 90^\circ$$




$$\sum p_{y,b} = 0 \text{ (vertical velocities are zero)}$$

$$\sum p_{y,a} = m v_{2,a} \sin\theta_2 - m v_{1,a} \sin\theta_1$$

$$\text{Cons. of vert. momentum}: \sum p_{y,b} = \sum p_{y,a}$$

$$\Rightarrow v_{2,a} \sin\theta_2 = v_{1,a} \sin\theta_1$$

$$\sum p_{x,b} = m v_{1,b}$$

$$\sum p_{x,a} = m v_{2,a} \cos\theta_2 + m v_{1,a} \cos\theta_1$$

$$\text{Cons. of horz. momentum}: \sum p_{x,b} = \sum p_{x,a}$$

$$\Rightarrow v_{1,b} = v_{2,a} \cos\theta_2 + v_{1,a} \cos\theta_1$$

$$\text{Cons. of energy}: m v_{1,b}^2 = m v_{2,a}^2 + m v_{1,a}^2$$

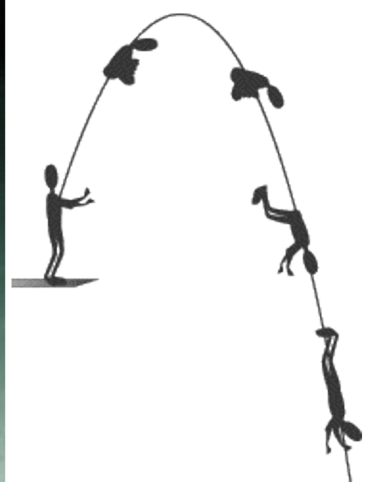
$$\Rightarrow v_{1,b}^2 = v_{2,a}^2 + v_{1,a}^2$$

$$\text{algebra gives } \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 = 0$$

$$\text{trig identity}: \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 = \cos(\theta_1 + \theta_2)$$

$$\cos(\theta_1 + \theta_2) = 0 \quad \Rightarrow \quad \theta_1 + \theta_2 = 90^\circ$$

# Next Week: Rotational Motion



- Describe the rotational motion of rigid bodies
  - We'll develop an analogy between new quantities describing rotational motion and familiar quantities from one-dimensional linear motion
- Calculate the rotational inertias of objects made of discrete and continuous distributions of matter
  - Rotational inertia is the rotational analog of mass
- We'll start to handle more interesting problems involving both linear and rotational motion
- We'll describe rolling motion