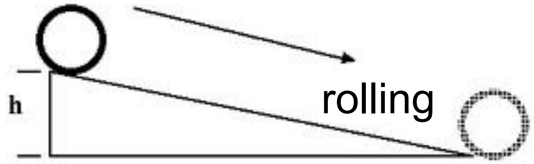
University Physics 226N/231N
Old Dominion University
Rotational Motion



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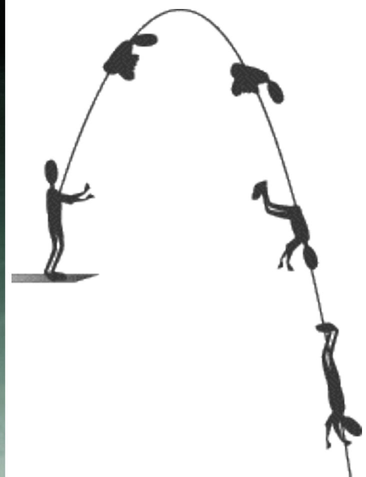
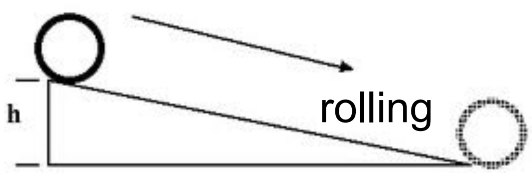
Happy Birthday to Clinton Davisson (1937 Nobel), Franz Liszt, Ichiro Suzuki, Spike Jonze, and Plan B!

Happy Fechner Day, Used Car Day, and National Nut Day!

This Week: Rotational Motion





- Describe the rotational motion of rigid bodies
  - We'll develop an analogy between new quantities describing rotational motion and familiar quantities from one-dimensional linear kinematics
- Calculate the rotational inertias of objects made of discrete and continuous distributions of matter
  - Rotational inertia is the rotational analog of mass
  - Gyroscopes!!
- We'll start to handle more interesting problems involving both linear and rotational motion
- We'll describe rolling motion

Quick One-Dimensional Kinematics Review

- We're going to draw explicit analogies between angular motion quantities and our old friends **position, velocity,** and **acceleration** from one-dimensional kinematics
- Time for a bit of review
  - Definitions of velocity and acceleration

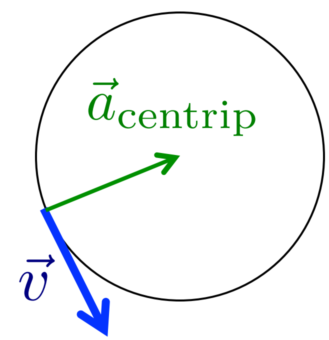
$$\text{velocity} \quad v \equiv \frac{dx}{dt} \qquad\qquad \text{acceleration} \quad a \equiv \frac{dv}{dt}$$

  - Constant acceleration motion

$$x - x_0 = v_0 t + \frac{1}{2} a t^2 \qquad\qquad v = v_0 + at$$

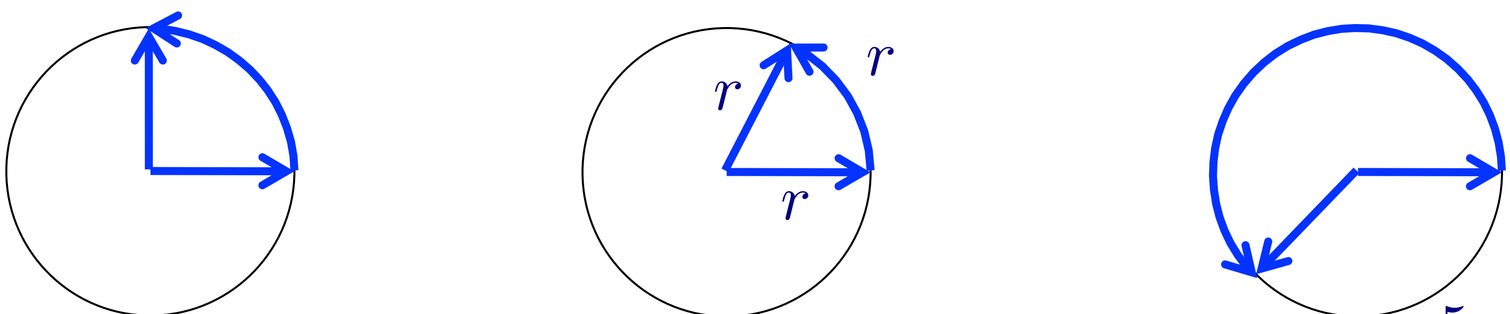
  - Centripetal acceleration related to tangential velocity

$$a_{\text{centrip}} = \frac{v^2}{r}$$



Angular Position

- What do we use for position in angular motion problems?
  - The angle that an object is from a reference angle $\theta = 0$
  - The sign convention is **usually that clockwise is positive**
  - The $\theta = 0$ location, like x=0, is usually defined by the problem
    - We care more about **angular distances**, $\Delta\theta = \theta(t_2) - \theta(t_1)$
  - We also always use **radians** where $\boxed{2\pi \text{ rad} = 360°}$
    - 1 rad is the angle where the arc length is equal to the circle radius



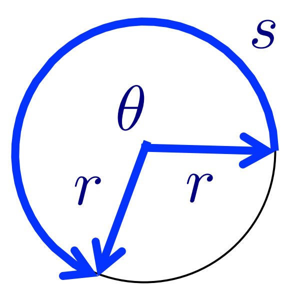
$$\Delta\theta = 90° = \frac{\pi}{2} \text{ rad} \qquad \Delta\theta = 57.3° = 1 \text{ rad} \qquad \Delta\theta = 225° = \frac{5\pi}{4} \text{ rad}$$

- A conversion example: $225° \left(\frac{2\pi \text{ rad}}{360°}\right) = \frac{5}{8}(2\pi) \text{ rad} = \frac{5\pi}{4} \text{ rad}$

Angular Position: Importance of Radians

- That radians thing? Yeah, that's important…
  - If we write angles in radians, we can write a **tremendously useful equation** that relates the actual distance around the arc s to angles and radii:

$$\boxed{s = r\theta}$$



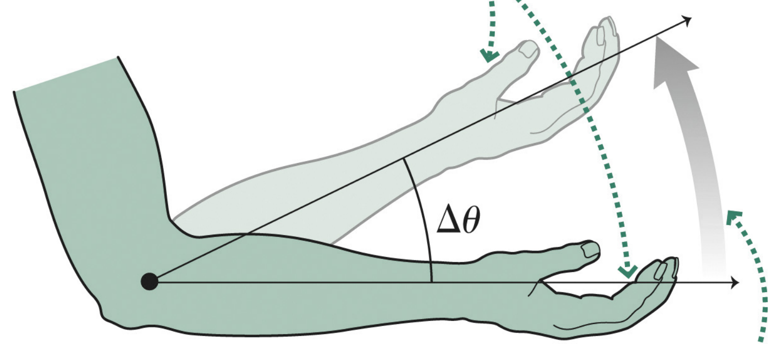
  - Here s is the distance around the arc. This formula lets us switch between real distances (like s and r, which are in distance units like meters) and angular distances (which are in radians)
  - Example: $\theta = 30^\circ = \frac{\pi}{6}\ \mathrm{rad}, \quad r = 2\ \mathrm{m} \quad \Rightarrow \quad s = \frac{\pi}{3}\ \mathrm{m} \approx 1.05\ \mathrm{m}$
- **Warning**: This equation (and most others we'll derive from now on) only work if the angle θ is in radians!
  - Since a radian is a ratio, it really is technically "dimensionless"

Angular Velocity

- **Angular velocity** ω [rad/s] is the rate of change of angular position with time

$$\text{Average :} \quad \bar{\omega} \equiv \frac{\Delta\theta}{\Delta t}$$

$$\text{Instantaneous :} \quad \omega \equiv \frac{d\theta}{dt}$$



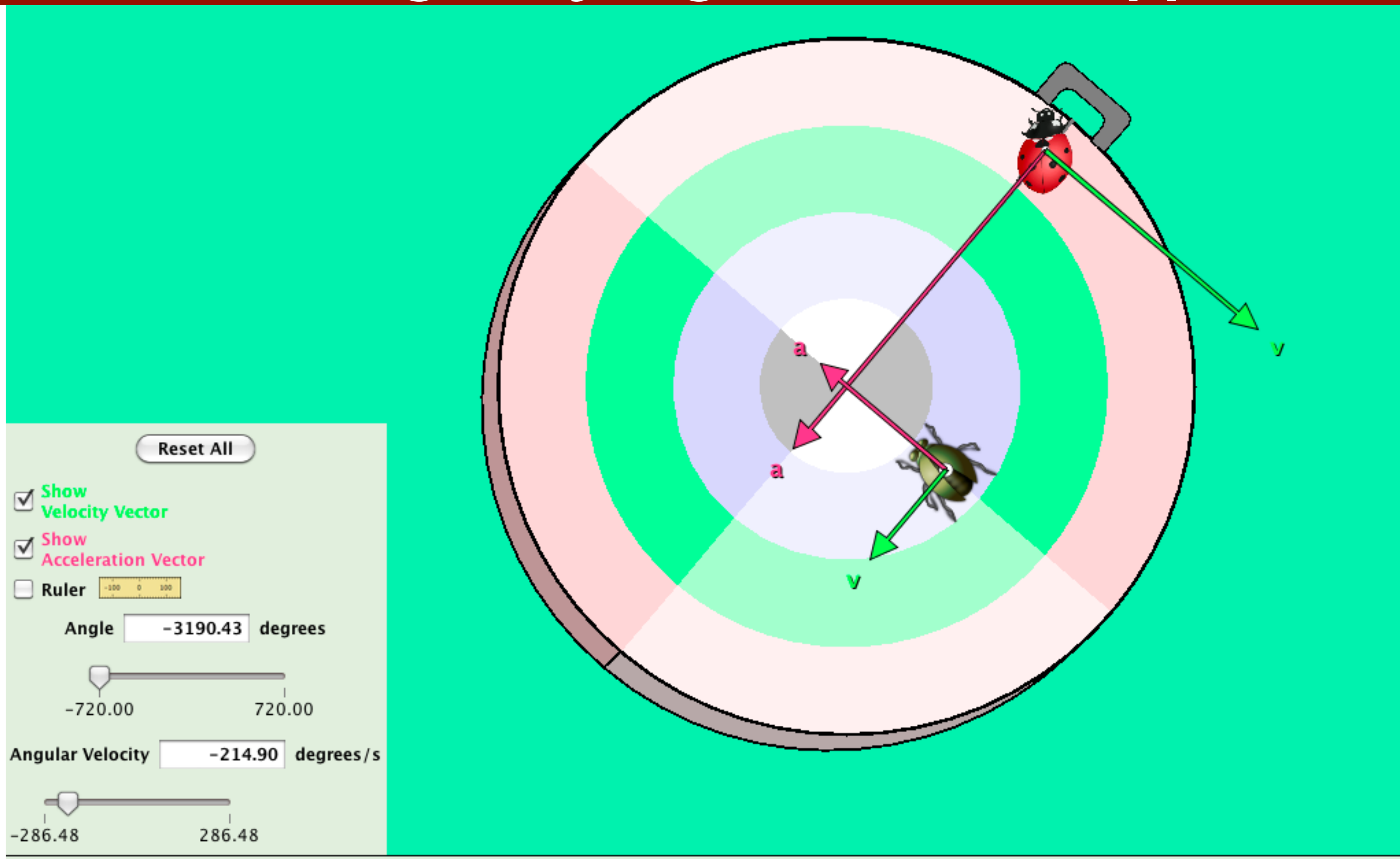
- Angular velocity ω is related to linear velocity v at a particular radius r from the rotation axis

$$\boxed{v = \omega\, r}$$

Warning: This is only true if the angular velocity distance units are radians!!!

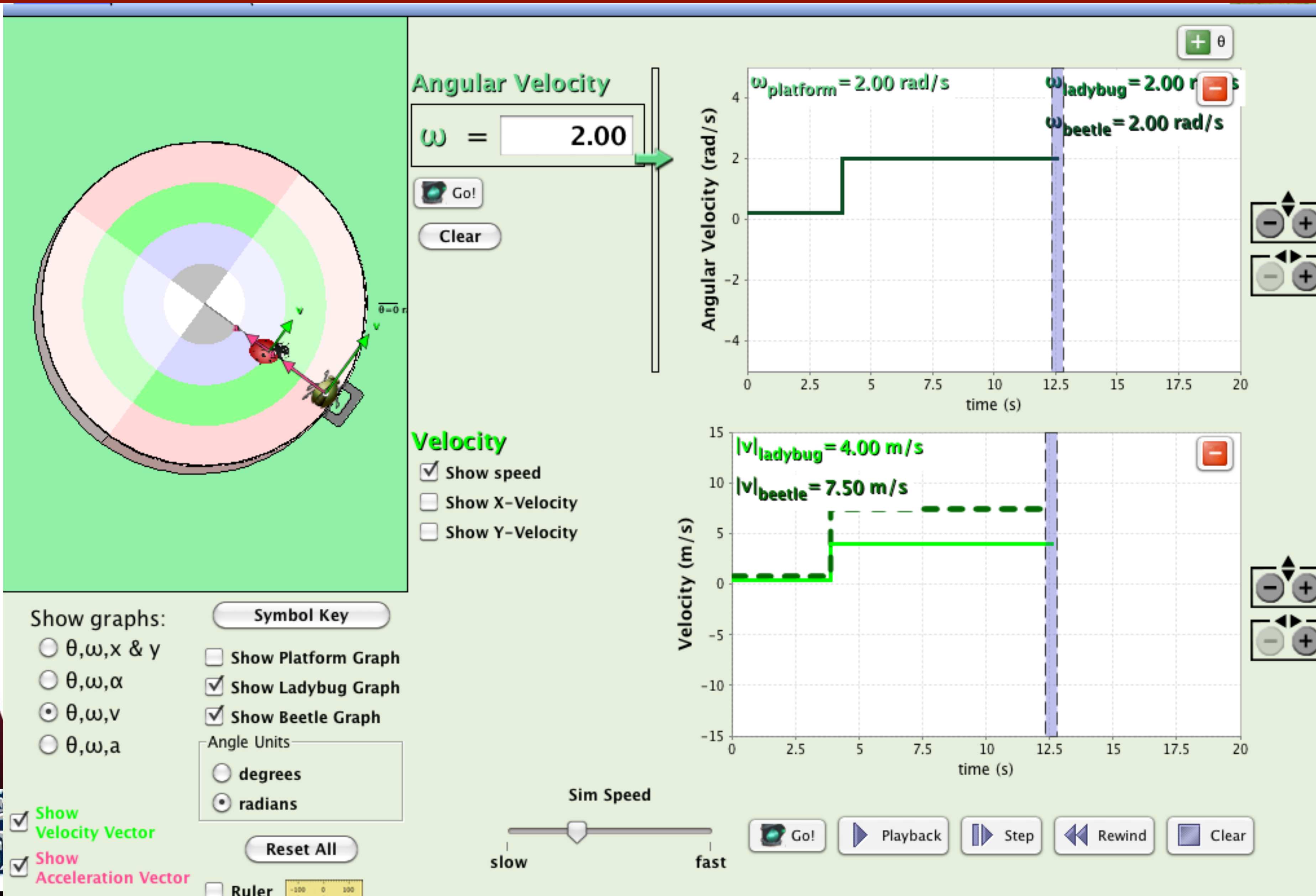
all radii rotate at 45 rpm but different v!

Revisiting Ladybug Revolution Applet



- http://phet.colorado.edu/en/simulation/rotation
- See differences in magnitudes **and** directions of **linear velocity and centripetal acceleration**
- Click on the “Rotation” tab

Revisiting Ladybug Revolution



Revisit an old Ponderable (10 minutes)

- Todd gets nostalgic and spins up his old **45 RPM** (revolutions per minute) record collection. Each record has a **7 inch total diameter**.
  - How fast is the outermost edge of the album moving in inches/sec?
  - How many "gees" of acceleration does a bug on the edge feel? ($g=32$ feet/s^2=384 inches/s^2)

Angular Acceleration

- **Angular acceleration** α [rad/s^2] is the rate of change of angular velocity with time

$$\text{Average}: \quad \bar{\alpha} \equiv \frac{\Delta\omega}{\Delta t}$$

$$\text{Instantaneous}: \quad \alpha \equiv \frac{d\omega}{dt}$$

- Angular acceleration α is related to **tangential** linear acceleration a_t at a particular radius r from the rotation axis

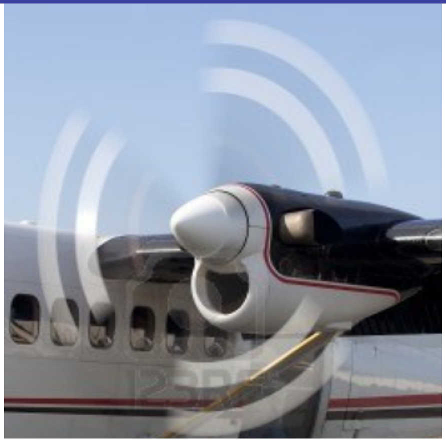
$$a_t = \alpha\, r$$

Recall there is also centripetal (radial) acceleration

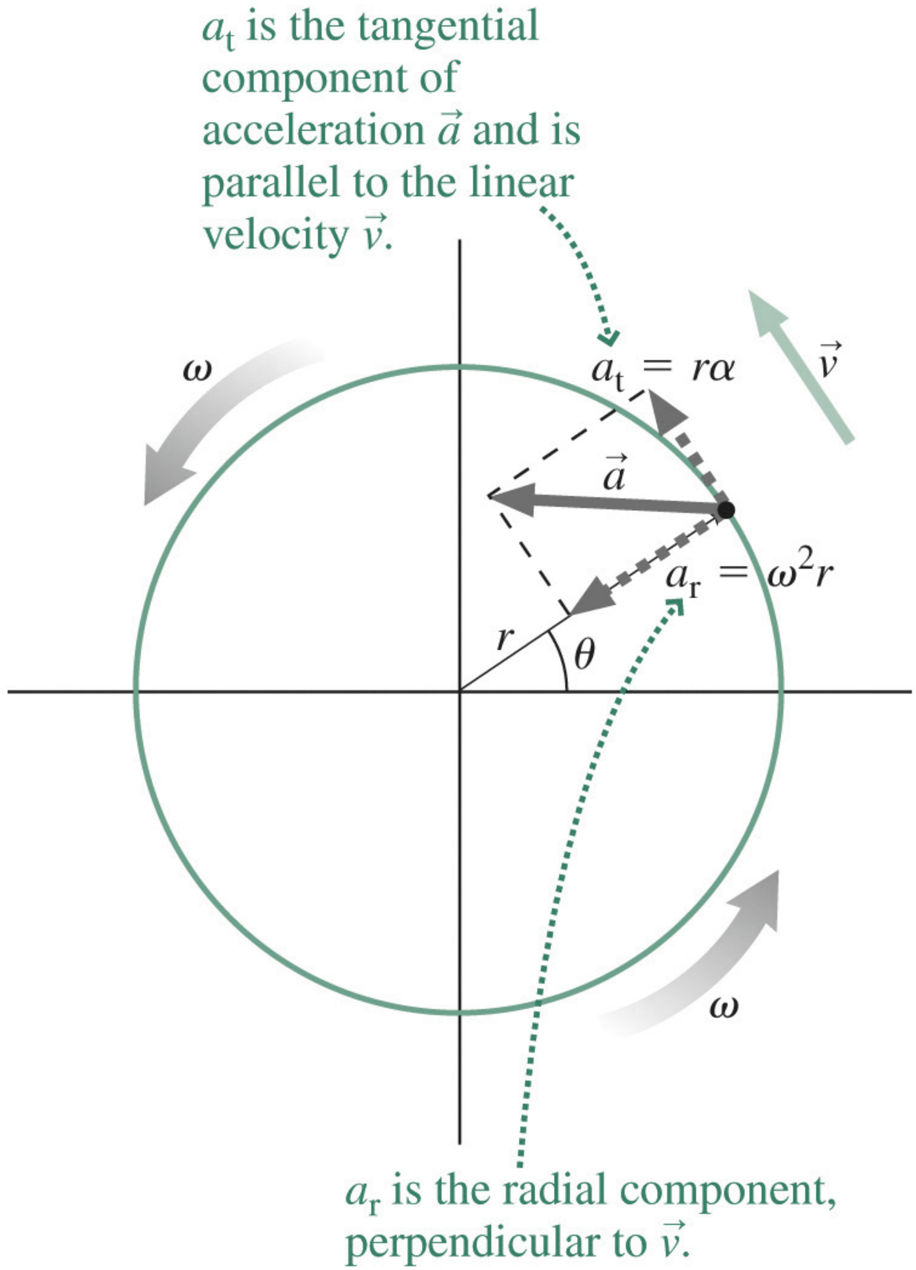
$$a_r = a_{\text{centrip}} = \frac{v^2}{r} = \omega^2 r$$

Think of an amusement park ride or propeller or engine spinning up – or spinning down.

That's angular acceleration.



Angular Acceleration Picture (from text)



$$a_t = \alpha\, r$$

$$a_r = a_{\text{centrip}} = \frac{v^2}{r} = \omega^2 r$$

Constant Angular Acceleration

- Problems with constant angular acceleration are exactly analogous to similar problems involving linear motion in one dimension.
  - The **exact** same equations apply, with $x \to \theta, \quad v \to \omega, \quad a \to \alpha$

Table 10.1 Angular and Linear Position, Velocity, and Acceleration

Linear Quantity	**Angular Quantity**
Position x	Angular position θ
Velocity $v = \dfrac{dx}{dt}$	Angular velocity $\omega = \dfrac{d\theta}{dt}$
Acceleration $a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$	Angular acceleration $\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$

Equations for Constant Linear Acceleration		**Equations for Constant Angular Acceleration**	
$\overline{v} = \frac{1}{2}(v_0 + v)$	(2.8)	$\overline{\omega} = \frac{1}{2}(\omega_0 + \omega)$	(10.6)
$v = v_0 + at$	(2.7)	$\omega = \omega_0 + \alpha t$	(10.7)
$x = x_0 + v_0 t + \frac{1}{2}at^2$	(2.10)	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	(10.8)
$v^2 = v_0^2 + 2a(x - x_0)$	(2.11)	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	(10.9)

Example Problem

- A record on a turntable is accelerated from rest to an angular velocity of 33.3 revolutions/minute in 2 secs. Find the average angular acceleration.

  - Solution: The initial angular velocity is zero.

$$\omega_0 = 0 \text{ rad/s}$$

The final angular velocity is

$$\omega = (33 \text{ rev/min}) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 3.46 \text{ rad/s}$$

Then we have

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{\Delta t} = \frac{3.46 \text{ rad/s}}{2 \text{ s}} = 1.73 \text{ rad/s}^2$$

Example Problem 2

- A fan is started from rest and after 5.0 s has reached its maximum rotational velocity of 60 rad/s. Find the average angular acceleration and how many revolutions the fan makes while accelerating, assuming constant acceleration.

Solution:

$$\omega_0 = 0 \text{ rad/s}$$
$$\omega = 60 \text{ rad/s}$$
$$\Delta t = 5.0 \text{ s}$$

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{\Delta t} = \frac{60 \text{ rad/s}}{5.0 \text{ s}} = 12 \text{ rad/s}$$

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha t^2 \qquad \text{like} \quad \Delta x = v_0 t + \frac{1}{2} a t^2$$

$$\Delta\theta = \frac{1}{2}(12 \text{ rad/s})(5.0 \text{ s})^2 = 150 \text{ rad}$$

or converting to revolutions:

$$\Delta\theta = 150 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 23.9 \text{ revolutions}$$

Torque

- Torque τ is the rotational analog of force, and results from the application of one or more forces.
- Torque is relative to a chosen rotation axis.
- Torque depends on:
  - the distance from the rotation axis to the force application point.
  - the magnitude of the force $\vec{F}$
  - the orientation of the force relative to the displacement $\vec{r}$ from axis to force application point:

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \tau = rF\sin\theta$$