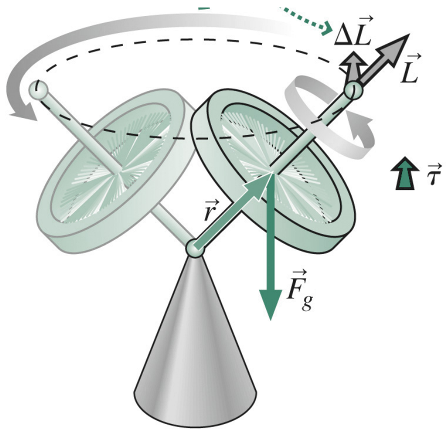

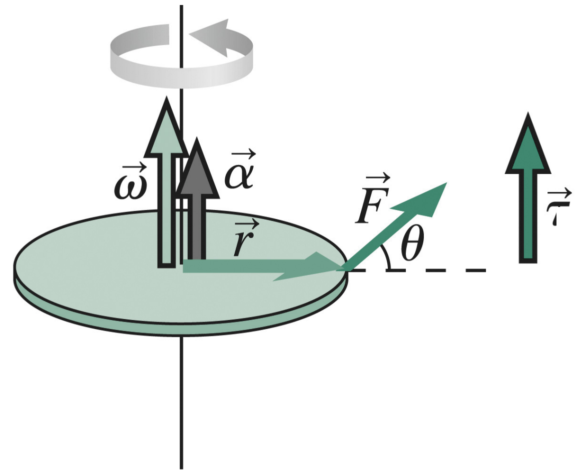

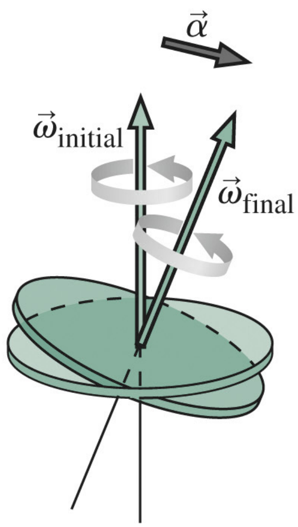

University Physics 226N/231N
Old Dominion University
More Rotational Motion

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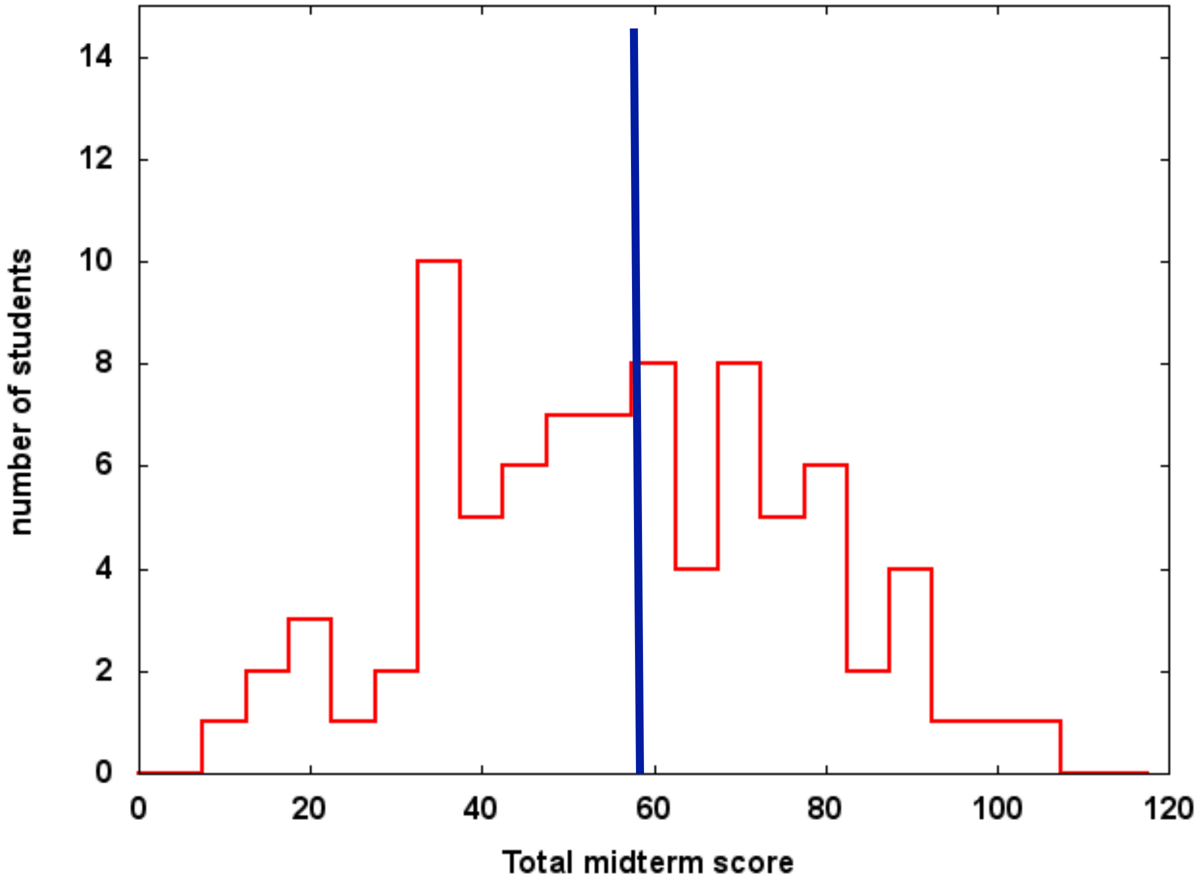
Friday October 26, 2012

Happy Birthday to Keith Urban, Jim Butcher, Natalie Merchant, and Pat Sajak!

Happy Frankenstein Friday and National Mincemeat Day!

Midterm 2 Grade Distribution

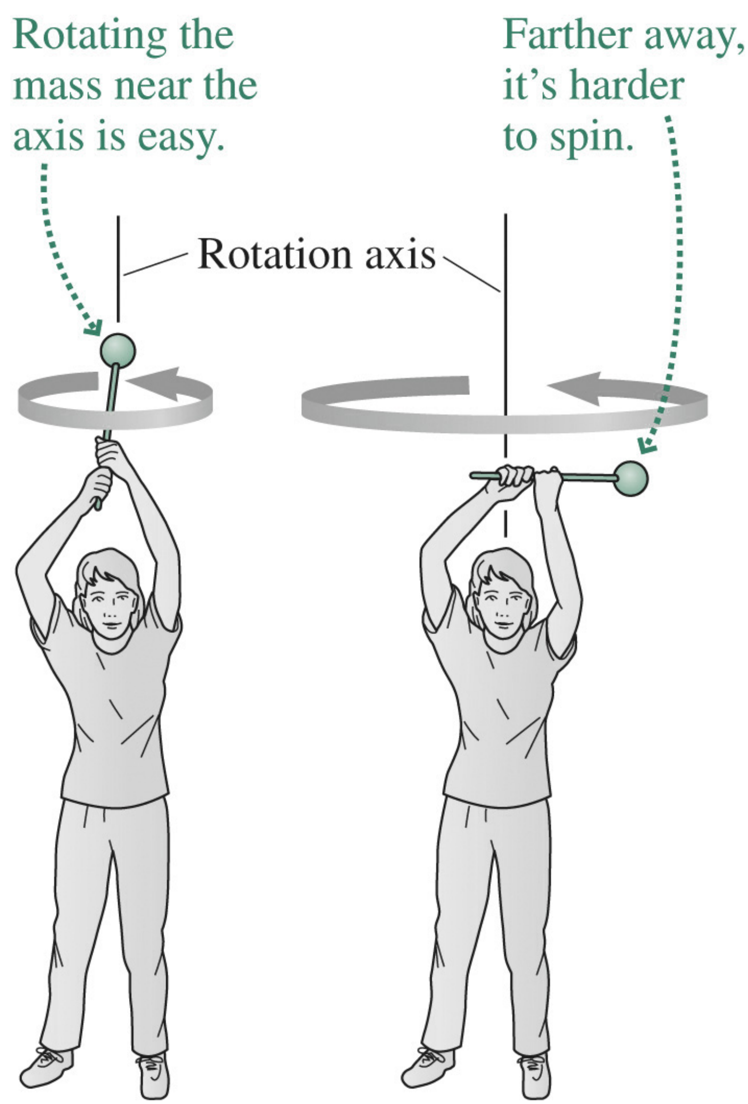
- Midterm 2 class average score: 58.2 (+/- 21.2 std deviation)



- Full statistics are in solution guide on class website
- Remember that I drop one midterm grade for final grading!

Rotational Inertia and the Analog of Newton's Law

- **Rotational inertia** I (or moment of inertia) is the rotational analog of mass.
  - Rotational inertia depends on the distribution of mass and its distance from the rotation axis, similar to center of mass.



- Rotational acceleration, torque, and rotational inertia combine to give the rotational analog of Newton's second law $F = ma$

$$\boxed{\tau = I\alpha}$$

(or, more properly with vectors)

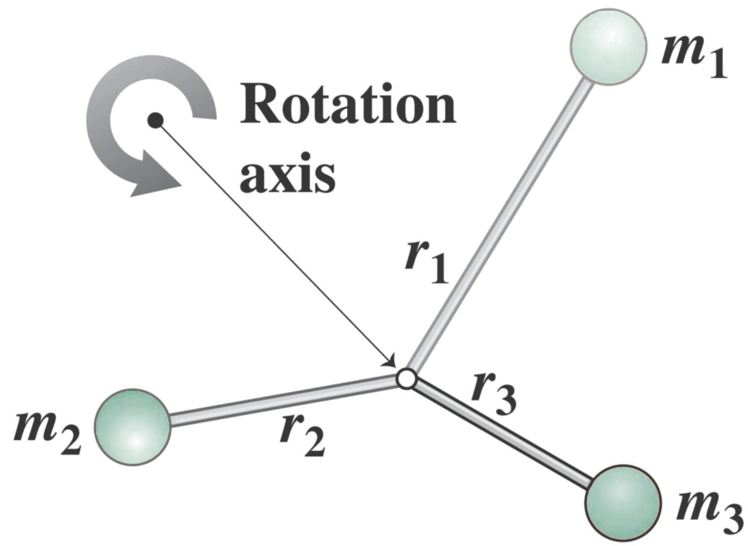
$$\vec{\tau} = I\vec{\alpha}$$

like $\vec{F} = m\vec{a}$

Calculating Rotational Inertia

- For a single point mass *m*, rotational inertia is the product of mass with the square of the distance *r* from the rotation axis:

$$I = mr^2$$

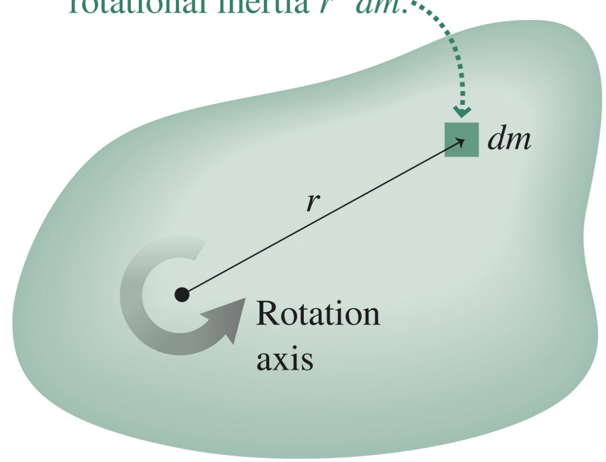


- For a system of discrete masses, the rotational inertia is the sum of the rotational inertias of the individual masses:

$$\boxed{I = \sum m_i r_i^2}$$

- For continuous matter, the rotational inertia is given by an integral over the distribution of matter:

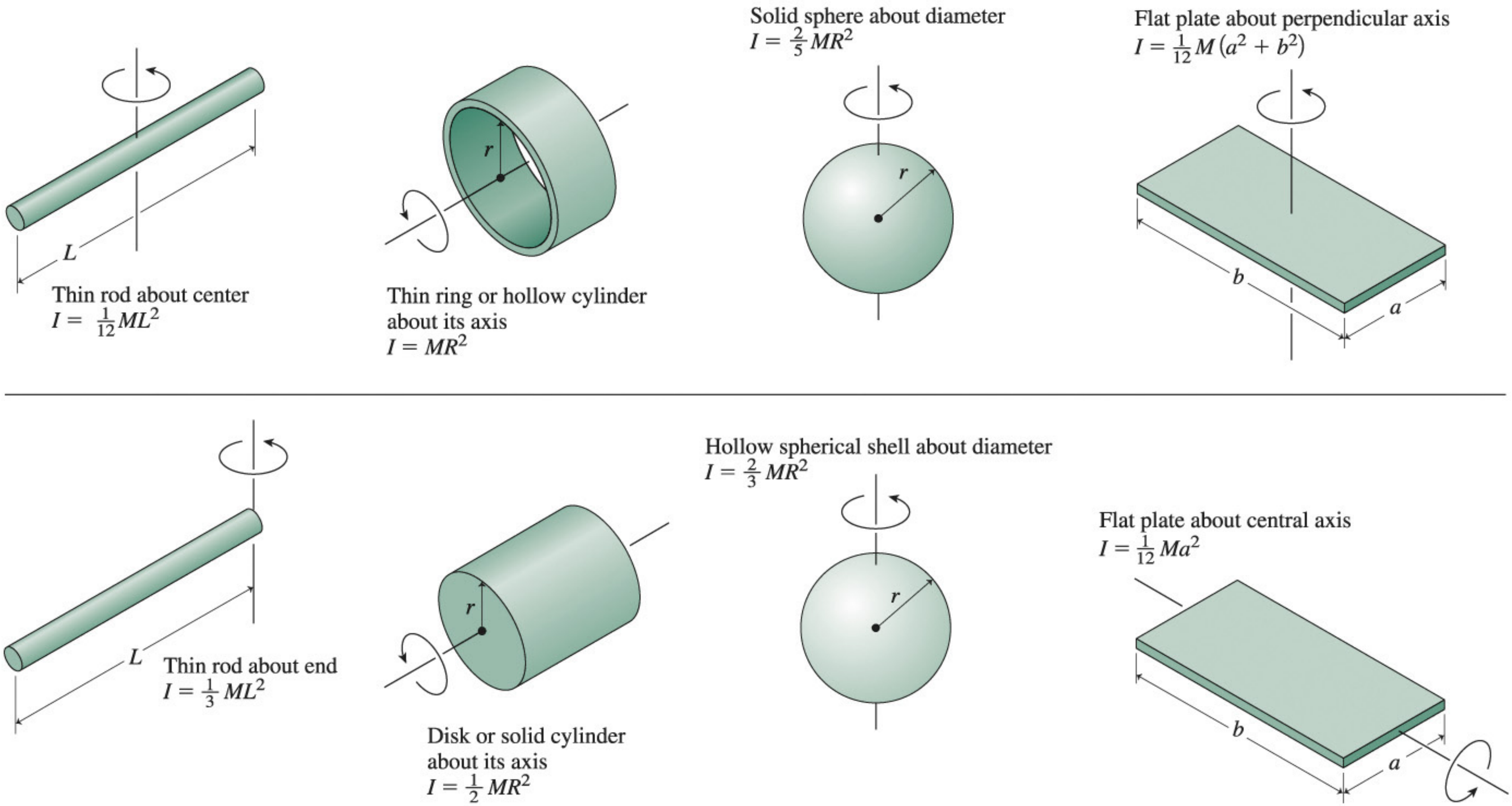
$$\boxed{I = \int r^2 \, dm}$$



Similar to center of mass: $\vec{r}_{\mathrm{cm}} = \dfrac{\int \vec{r}\, dm}{M}$

Some Rotational Inertias of Simple Objects

- We really do need to use calculus to figure out rotational inertias of most simple (three-dimensional) geometrical objects

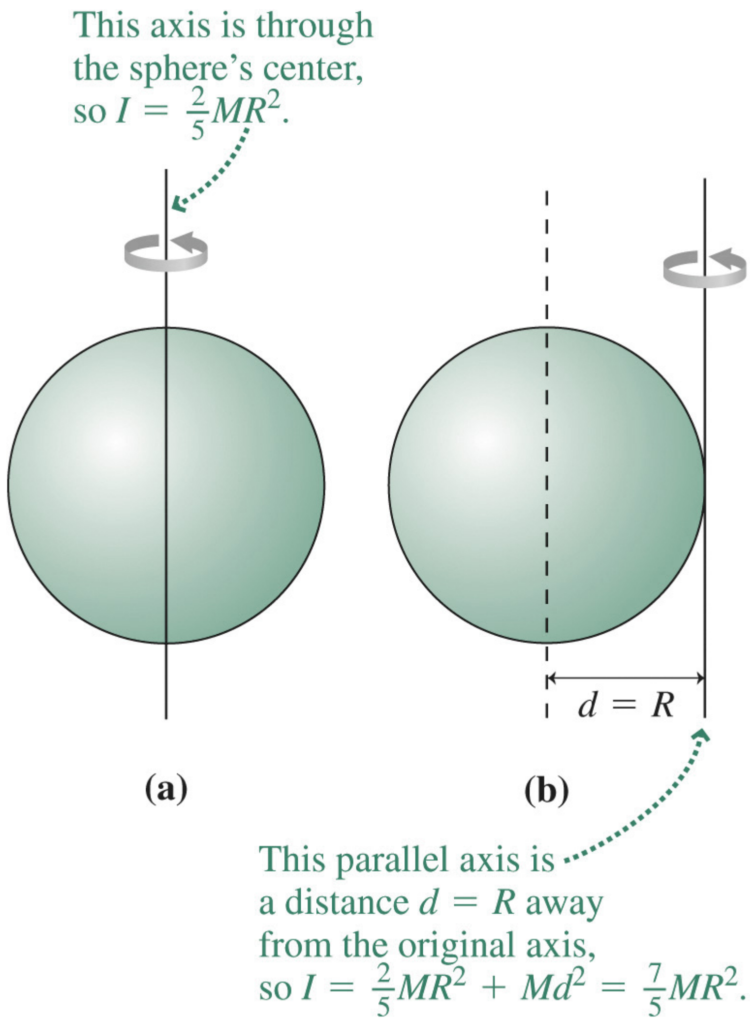


Parallel Axis Theorem

- If we know the rotational inertia I_{cm} about an axis through the center of mass of a body, the **parallel-axis theorem** allows us to calculate the rotational inertia I through any parallel axis.
- The parallel-axis theorem states that

$$\boxed{I = I_{\mathrm{cm}} + Md^2}$$

where d is the distance from the center-of-mass axis to the parallel axis and M is the total mass of the object.



Example: Hula Hoop Rotational Inertia

- What is the rotational inertia of a hula hoop of radius r and mass M around its edge?

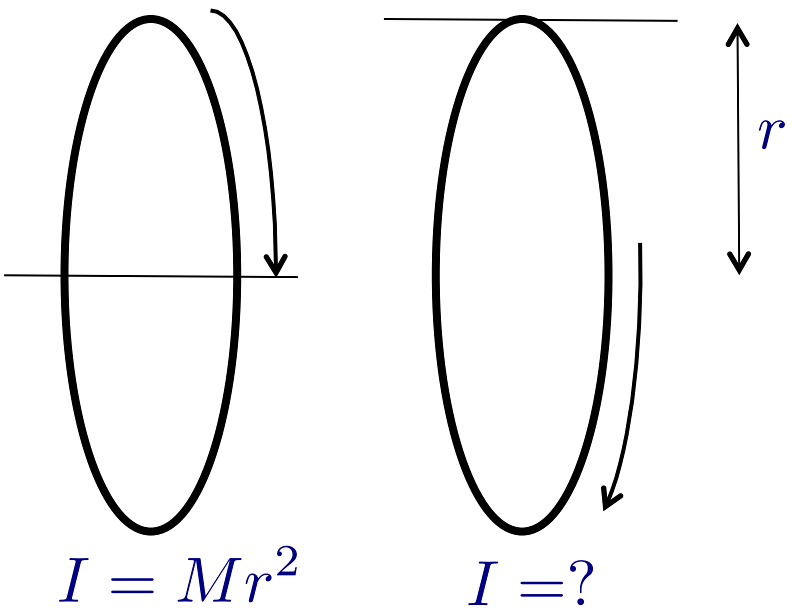
- Its center of mass is (obviously) at the center of the circle and all of its mass is at the same radius

$$I_{\rm cm} = Mr^2$$

- The parallel axis theorem gives

$$I_{\rm edge} = Mr^2 + Mr^2 = 2Mr^2$$

Interesting… It takes twice as much torque to turn a ring around its edge as it takes to turn around its center.

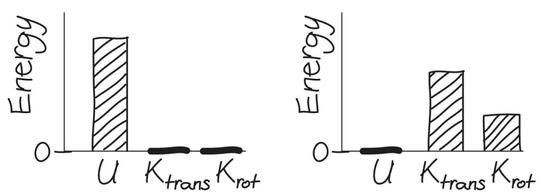


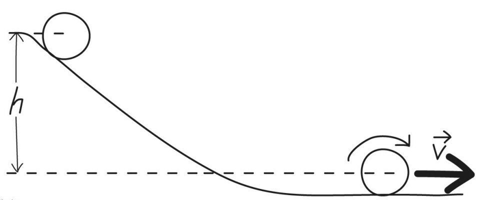
Rotational Energy

- A rotating object has kinetic energy $K_{\text{rot}} = \frac{1}{2} I\omega^2$ associated with its rotational motion alone.
  - It may also have translational kinetic energy: $K_{\text{trans}} = \frac{1}{2} Mv^2$.
- In problems involving energy conservation with rotating objects, both forms of kinetic energy must be considered.
  - For rolling objects, the two are related:
    - The relation depends on the rotational inertia.

Example: A solid ball rolls down a hill. How fast is it moving at the bottom?

Energy bar graphs





Equation for energy conservation

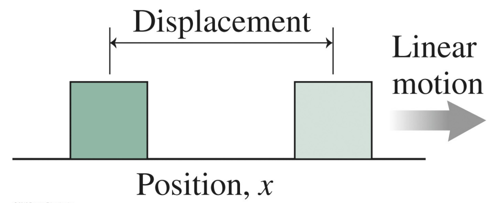
$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$
$$= \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{7}{10}Mv^2$$

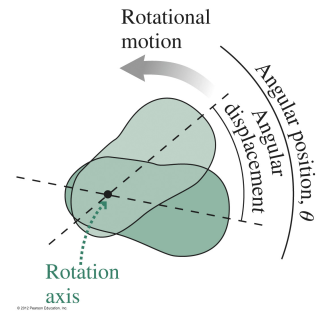
Solution:

$$v = \sqrt{\frac{10}{7}gh}$$

Summary

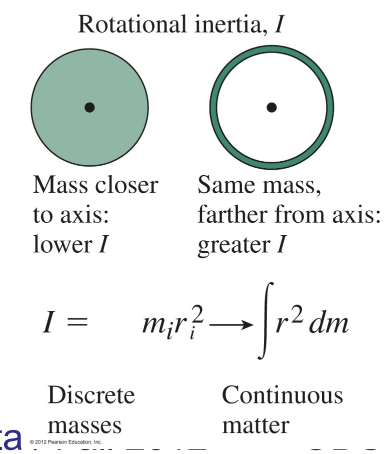
- Rotational motion in one dimension is exactly analogous to linear motion in one dimension.
  - Linear and angular motion:

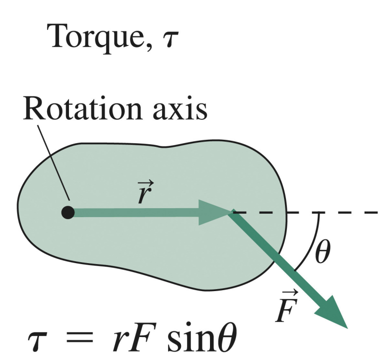




  - Analogies between rotational and linear quantities:

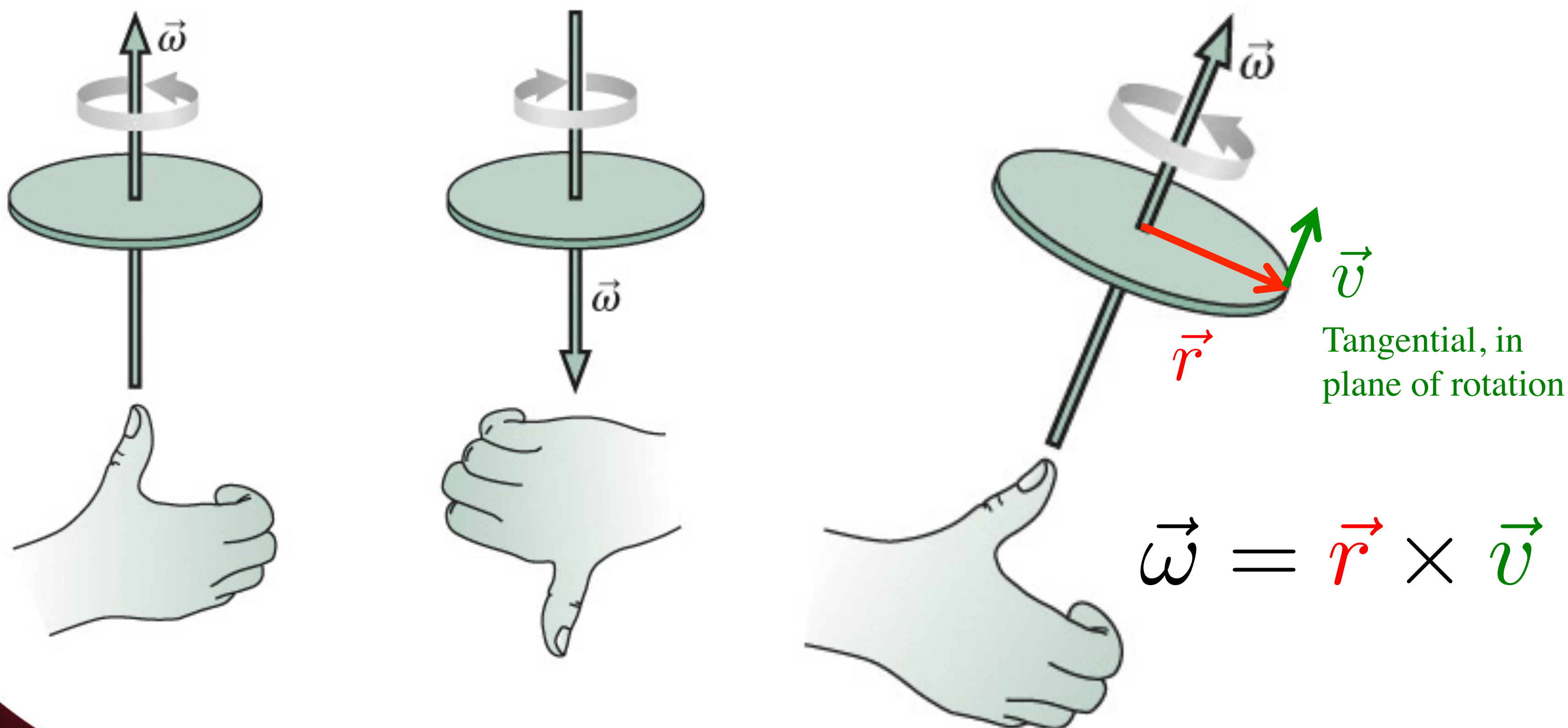
Linear Quantity or Equation	Angular Quantity or Equation	Relation Between Linear and Angular Quantities
Position x	Angular position θ	
Speed $v = dx/dt$	Angular speed $\omega = d\theta/dt$	$v = \omega r$
Acceleration a	Angular acceleration α	$a_t = \alpha r$
Mass m	Rotational inertia I	$I = \int r^2\,dm$
Force F	Torque τ	$\tau = rF\sin\theta$
Kinetic energy $K_{\text{trans}} = \frac{1}{2}mv^2$	Kinetic energy $K_{\text{rot}} = \frac{1}{2}I\omega^2$	
Newton's second law (constant mass or rotational inertia):		
$F = ma$	$\tau = I\alpha$	





Direction of the Angular Velocity Vector

- The direction of angular velocity is given by the **right-hand rule.**
  - Curl the fingers of your right hand in the direction of rotation, and your thumb points in the direction of the angular velocity vector $\vec{\omega}$



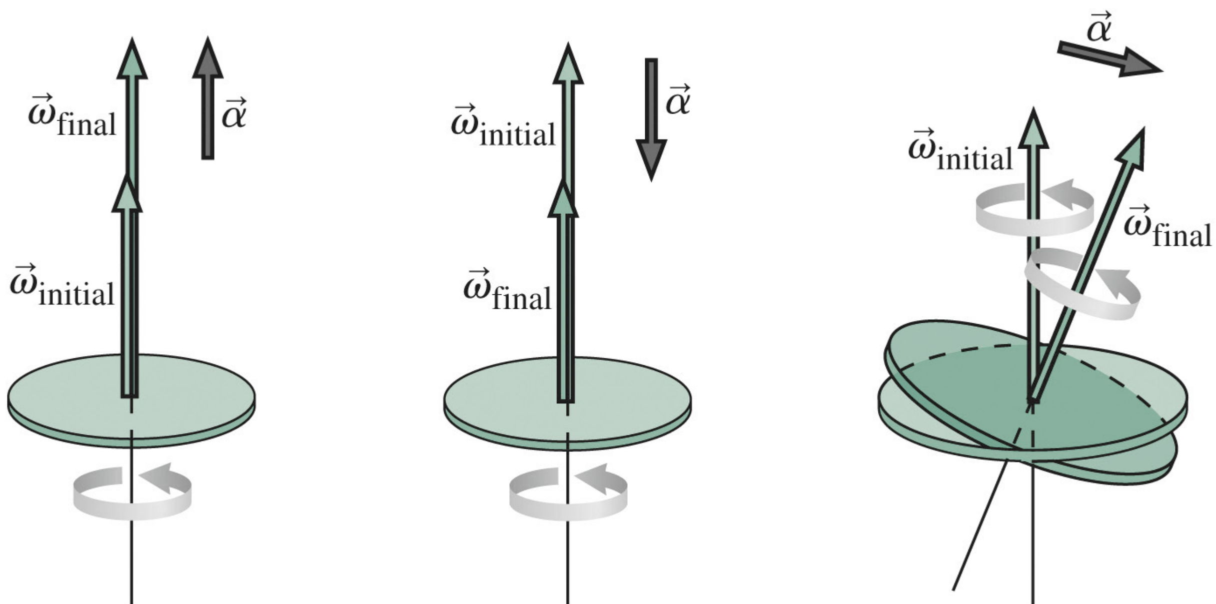
$$\vec{\omega} = \vec{r} \times \vec{v}$$

Direction of the Angular Acceleration

- Angular acceleration points in the direction of the change in the angular velocity $\Delta\vec{\omega}$:

$$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$$

  - $\vec{\alpha}$ can be in the same direction as the angular velocity $\vec{\omega}$, increasing its magnitude but not changing its direction.
  - $\vec{\alpha}$ can be opposite the angular velocity, decreasing its magnitude and possibly flipping its sign.
  - Or $\vec{\alpha}$ can be in completely different direction than $\vec{\omega}$, changing its direction **and** magnitude.



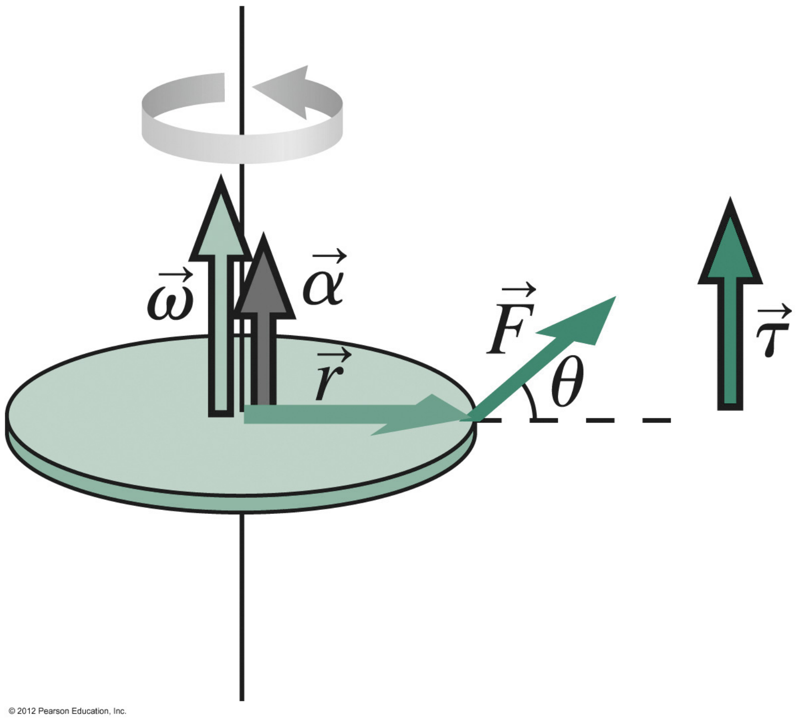
Direction of the Torque Vector

- The torque vector is perpendicular to both the force vector $\vec{F}$ and the displacement vector $\vec{r}$ from the rotation axis to the force application point.
  - The magnitude of the torque is

$$\tau = rF\sin\theta$$

  - But torque, radius, and the force are all vectors, and we can more correctly express this as another cross product:

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

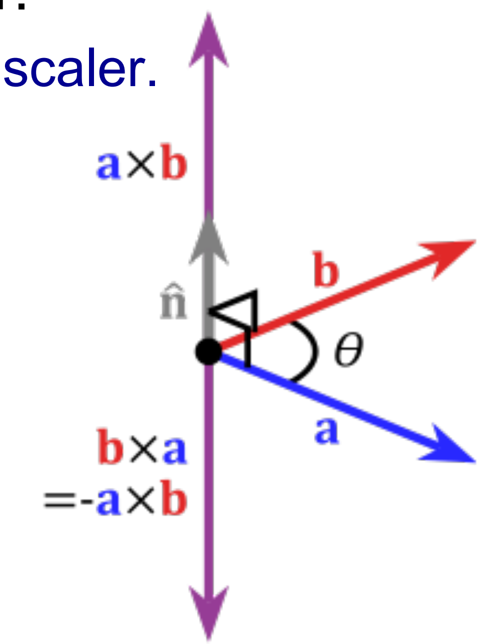


The Cross Product

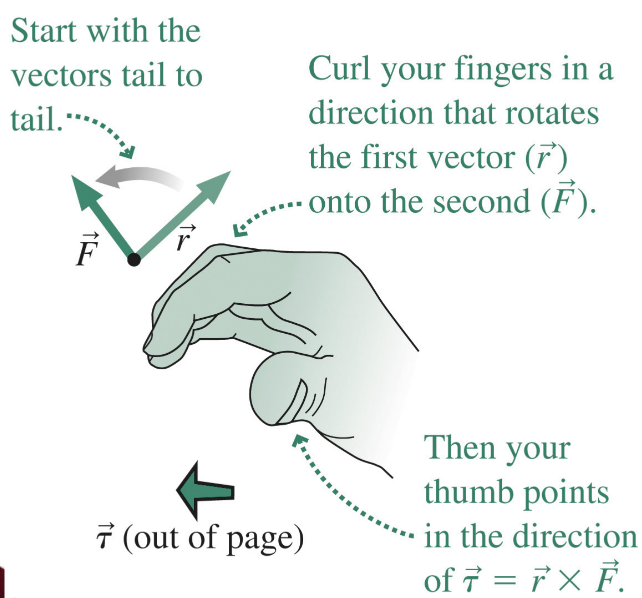
- The **cross product** or **vector product** is defined as the product of two vectors that produces another vector.
  - This is in contrast to the dot product that produces a scaler.
  - The picture at right shows $\vec{a} \times \vec{b}$ with

$$\vec{b} \times \vec{a} = -\vec{a} \times \vec{b}$$

$$\vec{a} \times \vec{b} \;\; \text{is perpendicular to } \textbf{both } \vec{a} \textbf{ and } \vec{b}$$



  - Cross products are **intrinsically** three-dimensional!



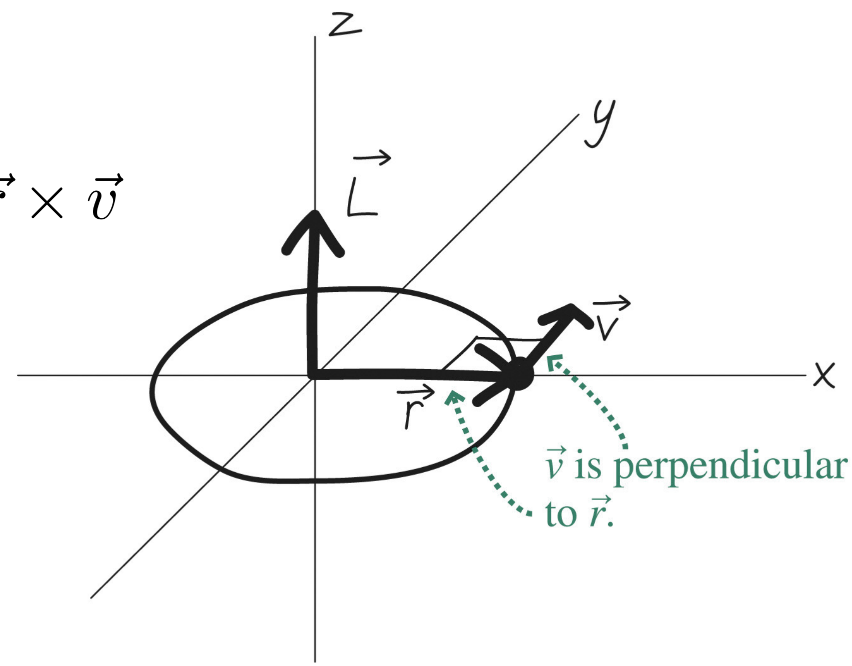
Some properties of cross products:

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$\vec{A} \times \left(\vec{B} + \vec{C}\right) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

Angular Momentum

- For a single particle, angular momentum $\vec{L}$ is a vector given by the cross product of the displacement vector $\vec{r}$ from the rotation axis with the linear momentum of the particle: $$\boxed{\vec{L} = \vec{r} \times \vec{p}}$$



- Yes, this looks a lot like $\vec{\omega} = \vec{r} \times \vec{v}$

- For solid objects where we know the rotational inertia, we have another parallel to linear motion:

$$\vec{p} = m\vec{v} \qquad \rightarrow \qquad \boxed{\vec{L} = I\vec{\omega}}$$

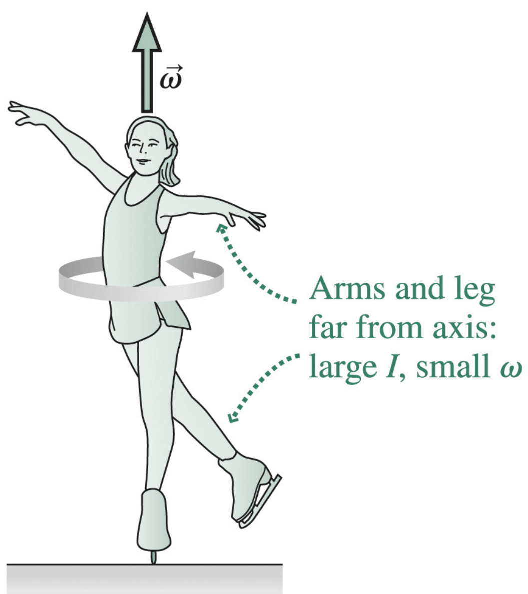
Newton's Law and Angular Momentum

- In terms of angular momentum, the rotational analog of Newton's second law is $\boxed{\vec{\tau} = \frac{d\vec{L}}{dt}}$

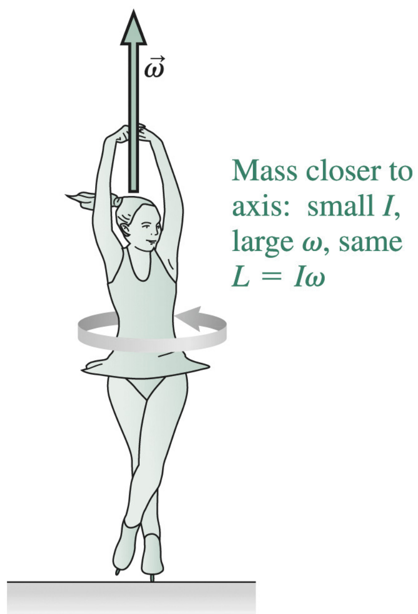
This is very general since rotational inertia I can change!

  - A system's angular momentum changes only if there's a non-zero net torque acting on the system.
  - If the net torque is zero, then **angular momentum is conserved**.

Changes in rotational inertia I then create changes in angular speed



The skater's angular momentum is conserved, so her angular speed increases when she reduces her rotational inertia.



Conservation of Angular Momentum

- The spinning wheel initially contains all the system's angular momentum.
- When the student turns the wheel upside down, she changes the direction of its angular momentum vector.
- Student and turntable rotate the other way to keep the total angular momentum unchanged.