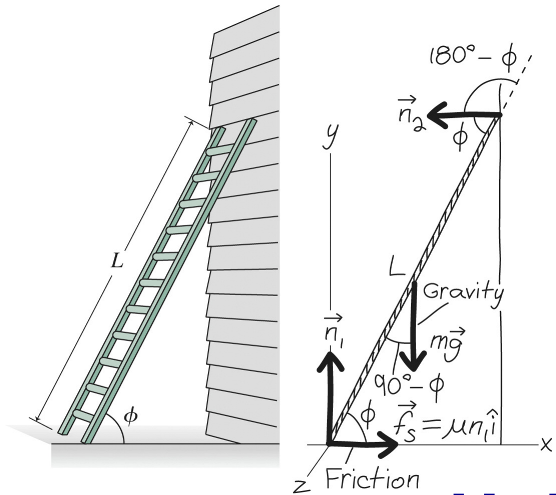
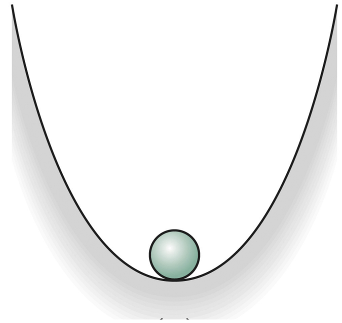
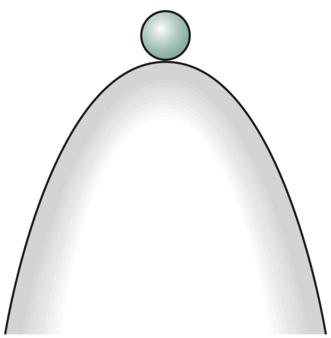
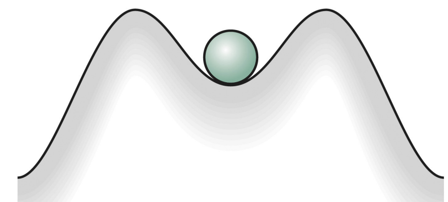


# University Physics 226N/231N
# Old Dominion University
# Static Equilibrium (Ch 12)



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Friday November 2, 2012

Happy Birthday to Richard Taylor (1990 Nobel, deep inelastic $e^-$ scattering), Marie Antoinette, and Nelly!

Happy All Souls Day, El Dia de los Muertos, and Get Out and Vote Day!

**Get your quizzes/midterm back from Dave!**

**I'll post some of next week's homework later today**

**Next exam: The Monday before Thanksgiving!**

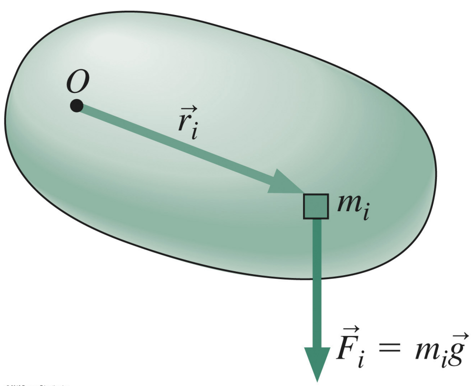
# In This Lesson You'll Learn

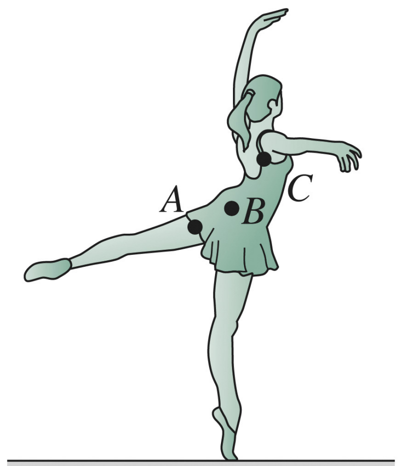
- About center of gravity and its relation to center of mass
- To describe the conditions necessary for static equilibrium
- To calculate forces and torques needed to ensure that a system is in static equilibrium
- To determine whether or not an equilibrium is stable

# Center of Gravity

- The gravitational forces acting on all parts of an object exert a torque on the object.
  - These forces act like a single force, equal to the object's weight, acting at a point called the **center of gravity.**
  - In a uniform gravitational field, the **center of gravity** coincides with the **center of mass**.

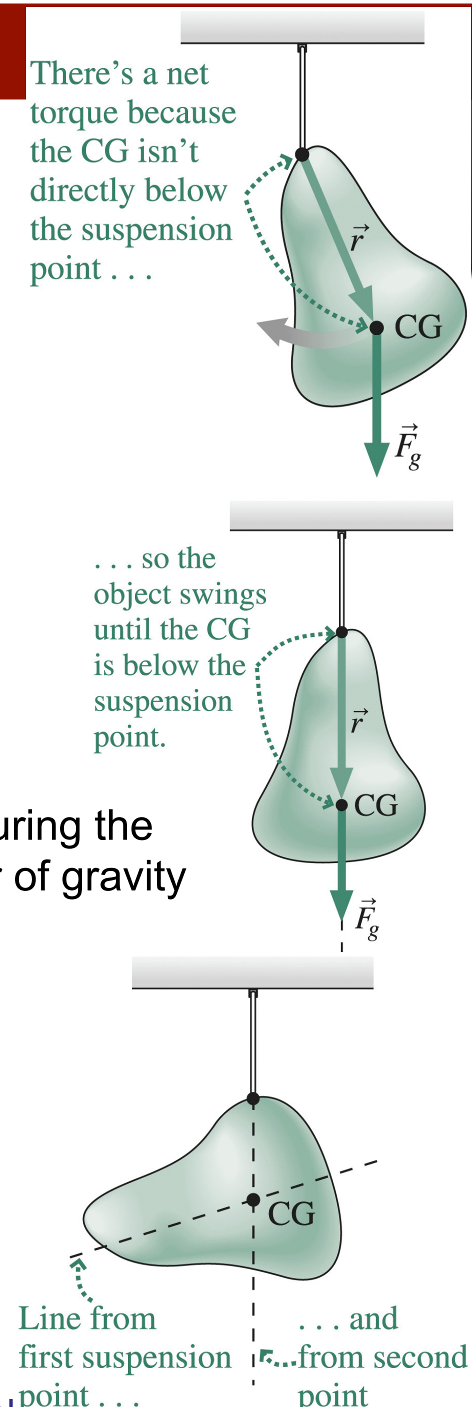
Gravitational force due to a single mass element produces a torque about $O$:



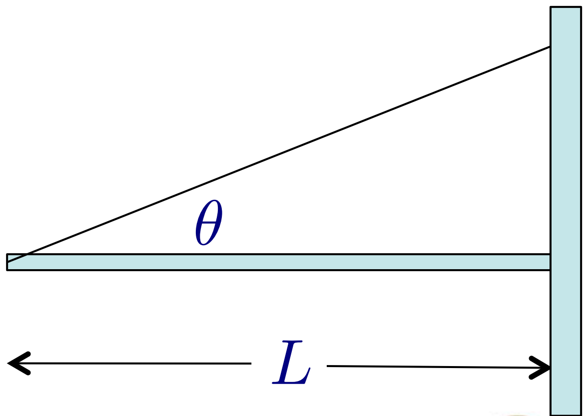

The dancer is in static equilibrium. Which point is her center of gravity?

Measuring the center of gravity

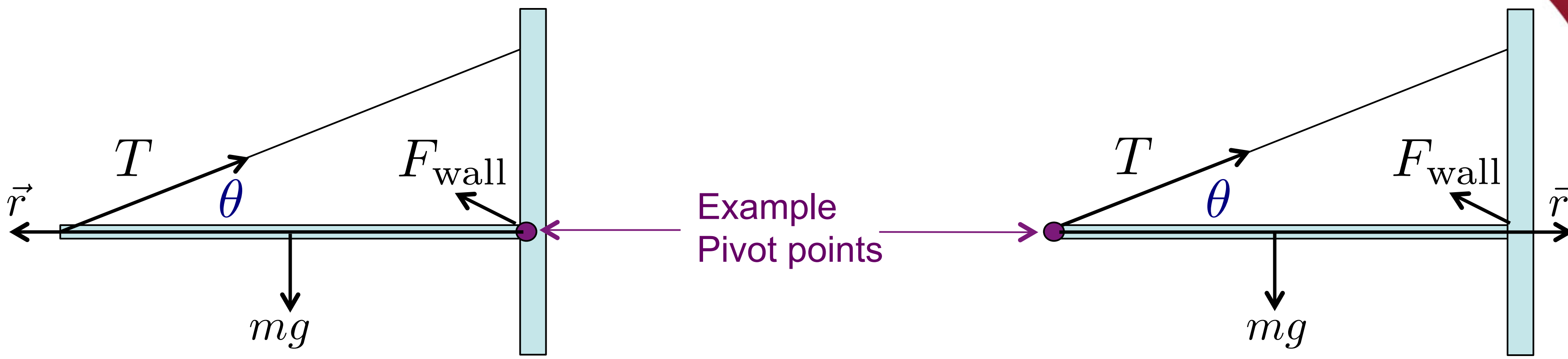


# Conditions for Static Equilibrium

- A system in static equilibrium undergoes no angular or linear acceleration.
  - Basically Newton's first law
    - Hint: A system that is moving at constant velocity is still in equilibrium since its linear and angular accelerations are zero!
- The conditions for static equilibrium are
  - No net force: $\boxed{\sum_i \vec{F}_i = \vec{0}}$ No net torque: $\boxed{\sum_i \vec{\tau} = \sum_i \vec{r}_i \times \vec{F}_i = \vec{0}}$
  - Torques can be evaluated about any convenient pivot point
    - Hint: Eliminate extra unknown forces from torque equation by choosing pivot point on line of that unknown force
  - **Example:** Rod of length L, mass m pulled onto a wall by a wire at angle $\theta$
    - What is the tension T of the wire in terms of the other values that you're given?
    - Where should you choose a pivot point?



# Example: Rod on a Wall




**Good pivot choice:**
Zero torque from extra unknown $F_{wall}$

**Bad pivot choice:**
Zero torque from our unknown T

- Clockwise torque is positive
- Torque: $\tau = rF\sin\theta$ where $\theta$ is the angle between $\vec{r}$ and $\vec{F}$

- Using torques from the good pivot choice:

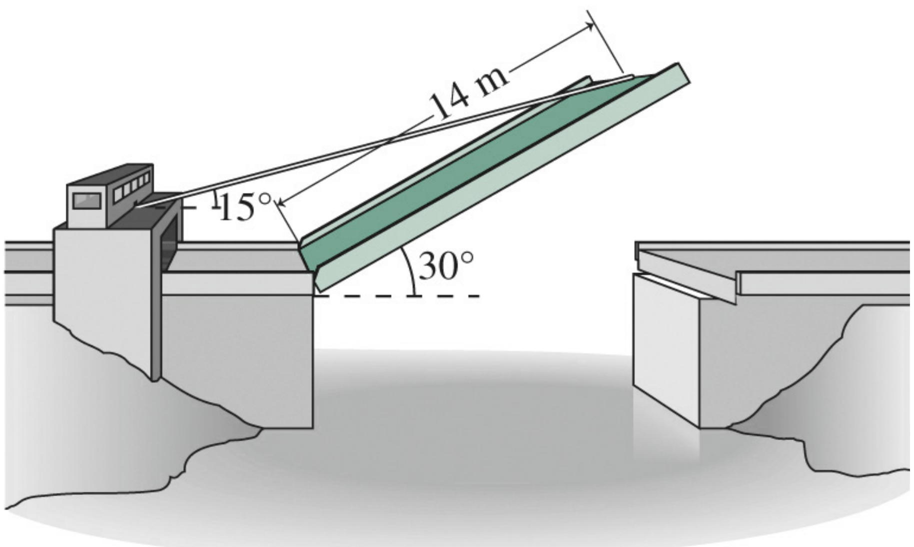
$$\text{Torque from } T: \quad \tau_T = -TL\sin(180^\circ - \theta) = -TL\sin(\theta)$$

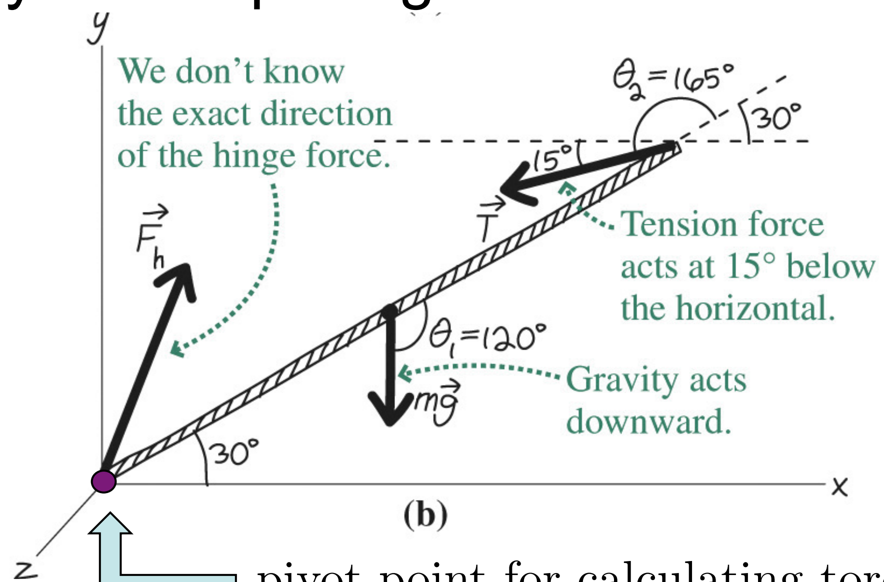
$$\text{Torque from } mg: \quad \tau_{mg} = (mg)(L/2)\sin(90^\circ) = mgL/2$$

$$\sum_i \tau_i = 0 \quad \Rightarrow \quad \tau_T + \tau_{mg} = 0 \quad \Rightarrow \quad \boxed{T = \frac{mg}{2\sin\theta}}$$

# Example: Static Equilibrium of a Drawbridge

- A drawbridge is suspended by a cord pulling from one side




pivot point for calculating torque

Torque due to tension $\vec{T}$: $\quad \tau_T = LT\sin(165°)$

Torque due to weight $m\vec{g}$: $\quad \tau_g = -\frac{L}{2}mg\sin(120°)$

Static: No net torque

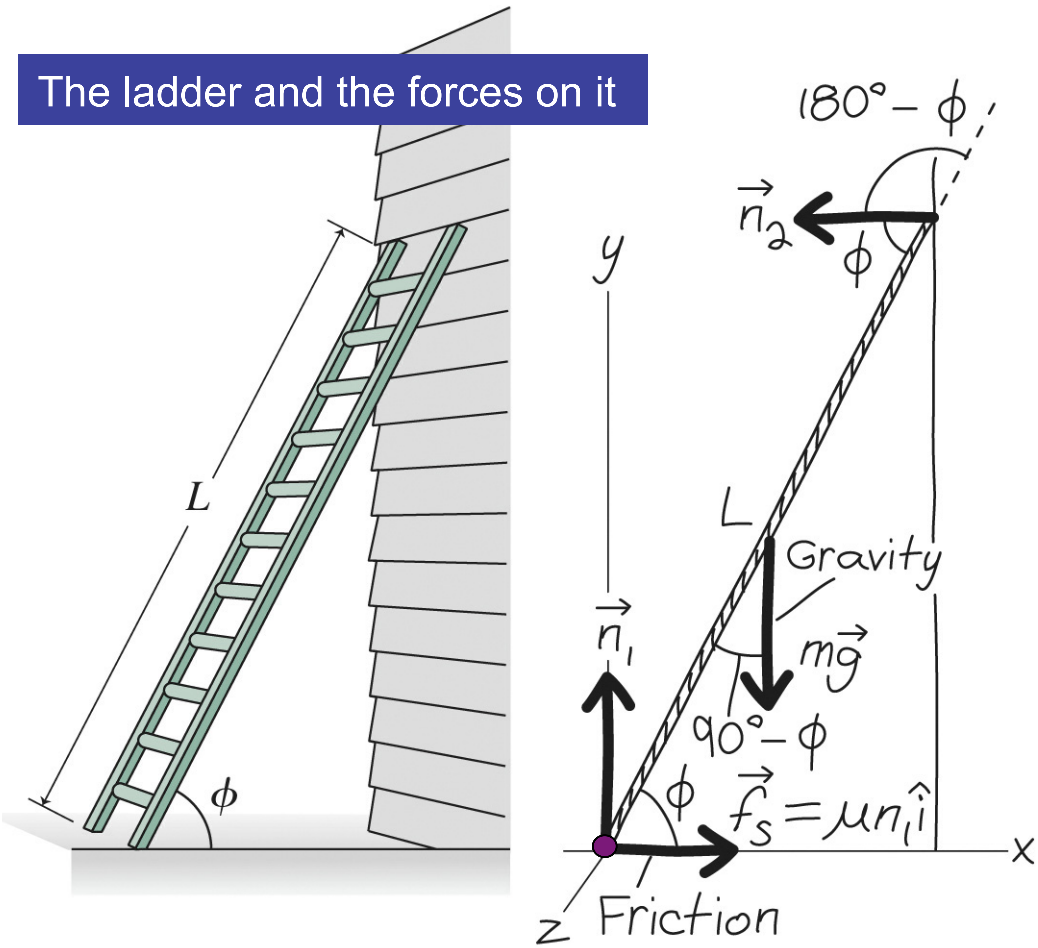
$$\sum_i \vec{\tau} = \sum_i \vec{r}_i \times \vec{F}_i = \vec{0} \qquad \sum_i \vec{\tau}_i = \vec{\tau}_T + \vec{\tau}_g = \vec{0} \quad \Rightarrow \quad T = \frac{mg}{2}\frac{\sin(120°)}{\sin(165°)}$$

Can also find hinge force components from $\sum_i \vec{F}_i = \vec{0}$

# Example: A Leaning Ladder

- At what angle will a leaning ladder slip?

The ladder and the forces on it




pivot point for calculating torque

Forces in both directions sum to zero:

$$\sum F_x = 0 \quad \Rightarrow \quad \mu n_1 - n_2 = 0$$

$$\sum F_y = 0 \quad \Rightarrow \quad n_1 - mg = 0$$

Torques are all perpendicular to the plane of the page, so there is only one torque equation:

$$\sum \tau = 0$$

$$L n_2 \sin(180^\circ - \phi) - \frac{L}{2} mg \cos\phi = 0$$

$$L n_2 \sin(\phi) - \frac{L}{2} mg \cos\phi = 0$$

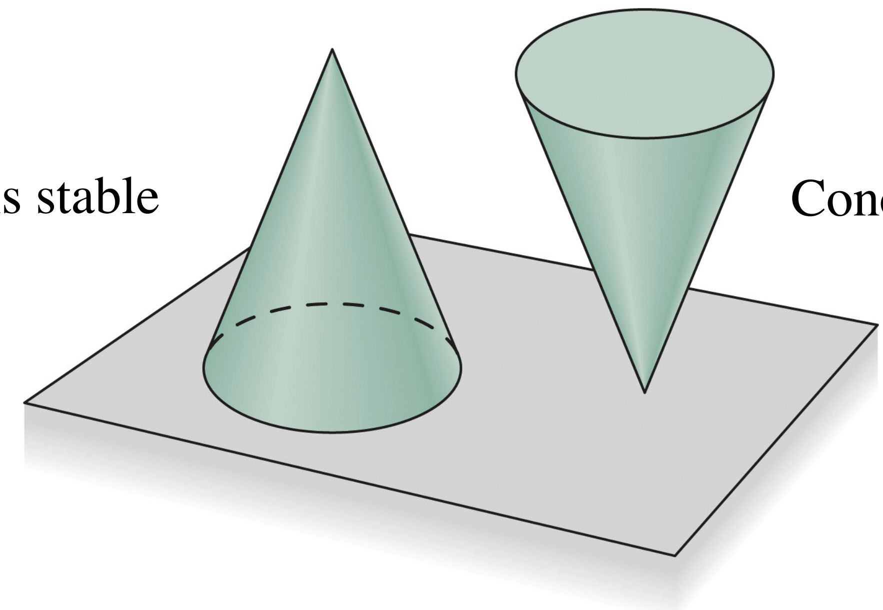
Solve the three equations to get

$$\boxed{\tan\phi = \frac{1}{2\mu}}$$

# Stability

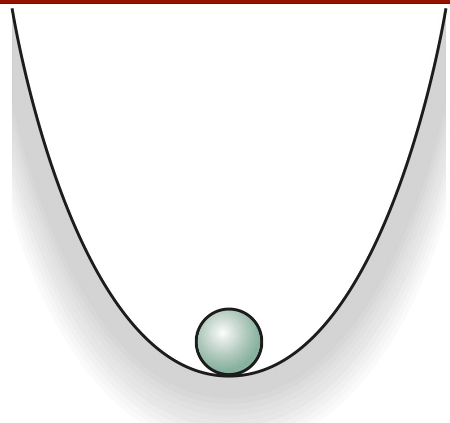
- An equilibrium is stable if a slight disturbance from equilibrium results in forces and/or torques that tend to restore the equilibrium.
- An equilibrium is unstable if a slight disturbance causes the system to move away from the original equilibrium.

Cone on its base is stable

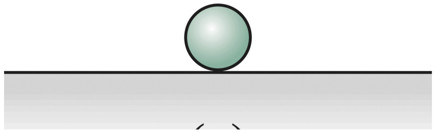


Cone on its tip is unstable

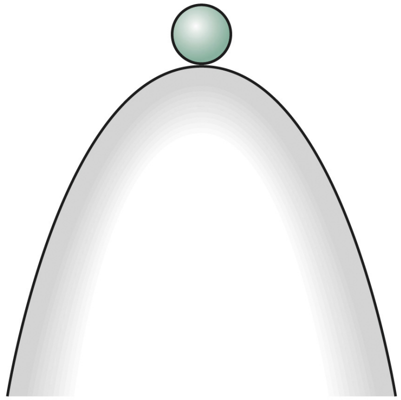
# Kinds of Stability



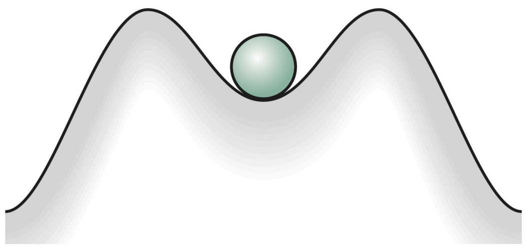
**Stable equilibrium**: disturbed ball will return to equilibrium



**Neutrally stable equilibrium**



**Unstable equilibrium**: disturbed ball will leave original equilibrium



**Metastable** or **conditionally stable equilibrium**: ball returns for small disturbances, but not for large ones

# Conditions for Equilibrium and Stability

- To be in equilibrium, there must be zero net force on an object.
  - Therefore the object must be at a maximum or minimum of its **potential energy curve** (like on top of a hill or bottom of a well)

$$\frac{dU}{dx} = 0 \quad \text{(condition for equilibrium)}$$

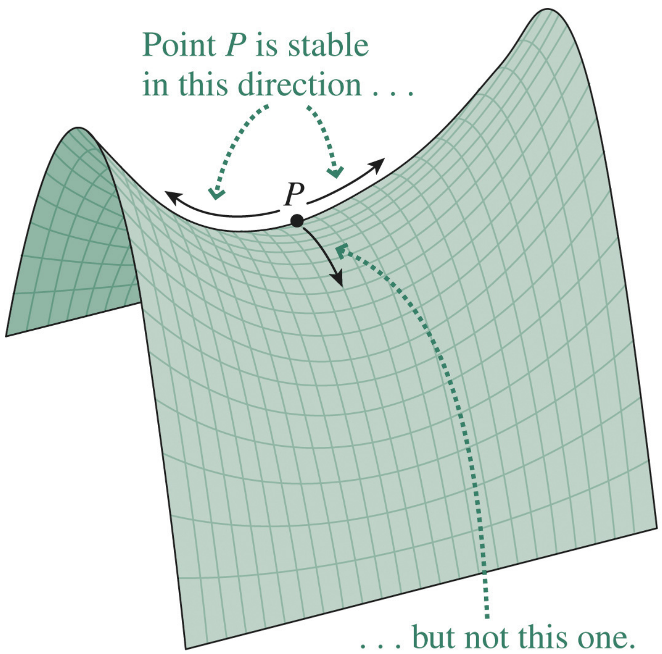
- For stable equilibrium, the object must be at a minimum:

$$\frac{d^2U}{dx^2} > 0 \quad \text{(stable equilibrium)}$$

- The condition for unstable equilibrium is

$$\frac{d^2U}{dx^2} < 0 \quad \text{(unstable equilibrium)}$$

- In two and three dimensions, an object can be stable in one direction but not another.

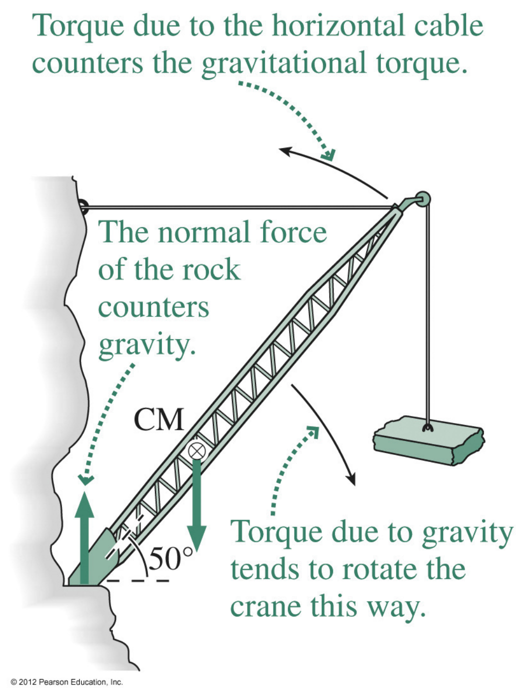


# Summary

- Static equilibrium requires zero net force and zero net torque on a system:

$$\boxed{\sum_i \vec{F}_i = \vec{0}} \qquad \boxed{\sum_i \vec{\tau} = \sum_i \vec{r}_i \times \vec{F}_i = \vec{0}}$$

Example: A crane in static equilibrium




Equilibria can be stable, unstable, neutrally stable, or metastable.