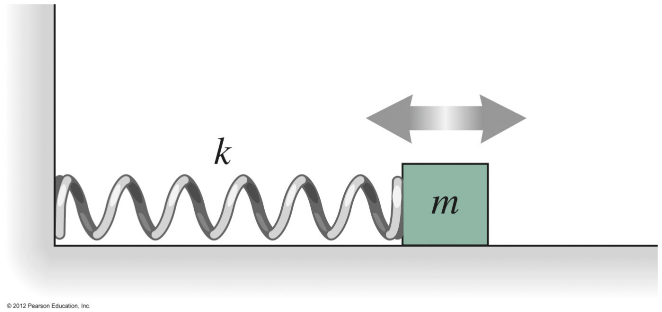
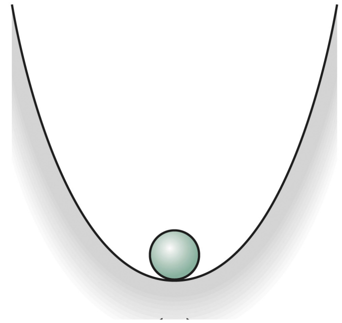




$$x(t) = A\sin(\omega t + \phi_0)$$

University Physics 226N/231N
Old Domination University
Starting Oscillatory Motion (Chap 13)

~~Dr. Todd Satogata (ODU/Jefferson Lab)~~

Dave and Shankar and Audience Participation!

http://www.toddsatogata.net/2012-ODU

Monday November 5, 2012 (Continued on Wednesday w/Dr. Godunov)

Happy Birthday to William Daniel Phillips (Nobel 1997, laser slowing), Famke Janssen, and Ella Wheeler Wilcox!

Happy National Donut Day and Guy Fawkes Night!

Get your quizzes/midterm back from Dave!

More questions added to this week's homework today

Next exam and HW journal due: The Monday before Thanksgiving!

Review: Conditions for Static Equilibrium

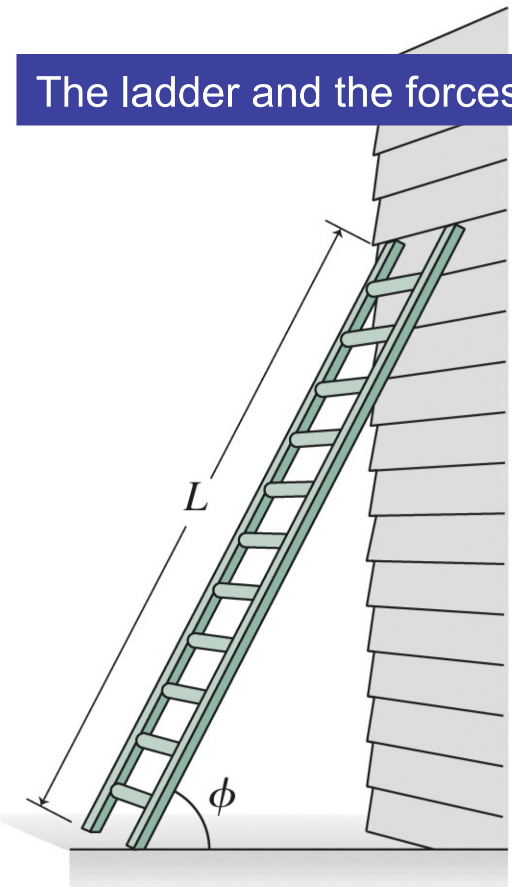
- A system in static equilibrium undergoes no angular or linear acceleration.
  - Basically Newton's first law
    - Hint: A system that is moving at constant velocity is still in equilibrium since its linear and angular accelerations are zero!
- The conditions for static equilibrium are
  - No net force: $\boxed{\sum_i \vec{F}_i = \vec{0}}$ No net torque: $\boxed{\sum_i \vec{\tau} = \sum_i \vec{r}_i \times \vec{F}_i = \vec{0}}$

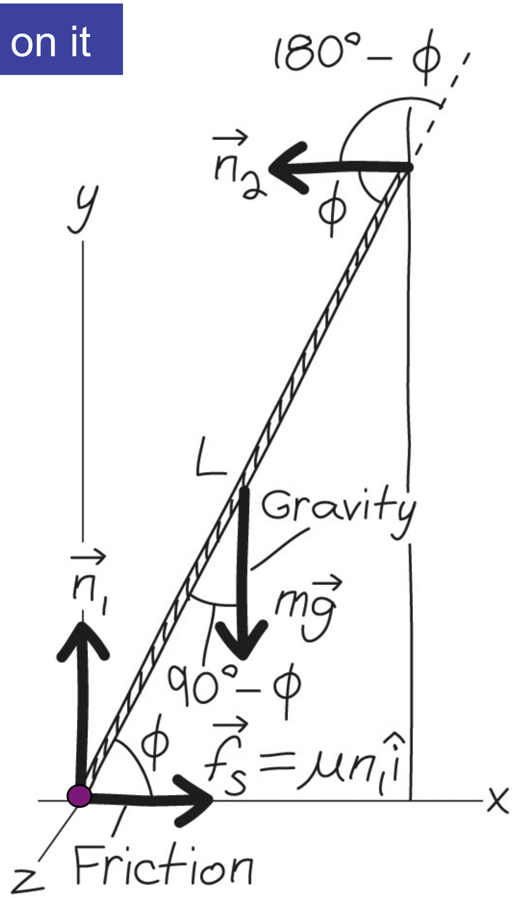
- Torques can be evaluated about **any** convenient pivot point
  - **Hint: Eliminate extra (even extraneous) unknown forces from torque equation by choosing pivot point on line of that unknown force**

Review Example: A Leaning Ladder

- At what angle will a leaning ladder slip?

The ladder and the forces on it





pivot point for calculating torque

Forces in both directions sum to zero:

$$\sum F_x = 0 \quad \Rightarrow \quad \boxed{\mu n_1 - n_2 = 0}$$

$$\sum F_y = 0 \quad \Rightarrow \quad \boxed{n_1 - mg = 0}$$

Torques are all perpendicular to the plane of the page, so there is only one torque equation:

$$\sum \tau = 0$$

$$L n_2 \sin(180^\circ - \phi) - \frac{L}{2} mg \cos\phi = 0$$

$$\boxed{L n_2 \sin(\phi) - \frac{L}{2} mg \cos\phi = 0}$$

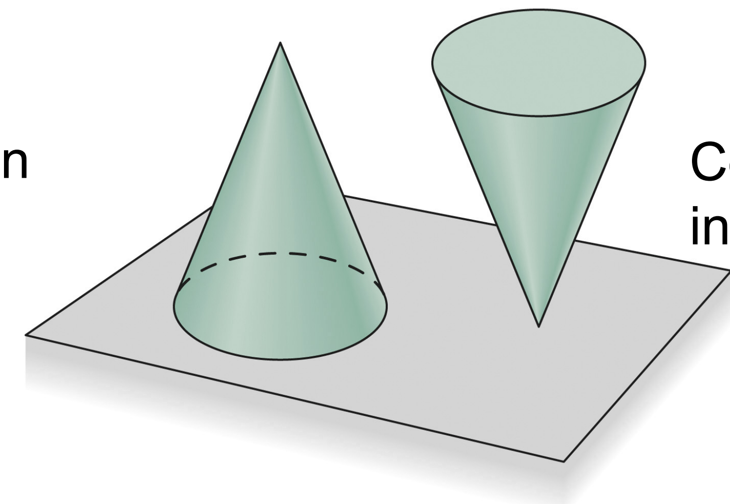
Solve the three dotted boxed equations to get

$$\boxed{\tan\phi = \frac{1}{2\mu}}$$

Stability

- An equilibrium is **stable** if a slight disturbance (a "perturbation") from equilibrium results in forces and/or torques that tend to restore the equilibrium.
- An equilibrium is **unstable** if a slight disturbance causes the system to move away from the original equilibrium.

Cone on its base is in **stable equilibrium**

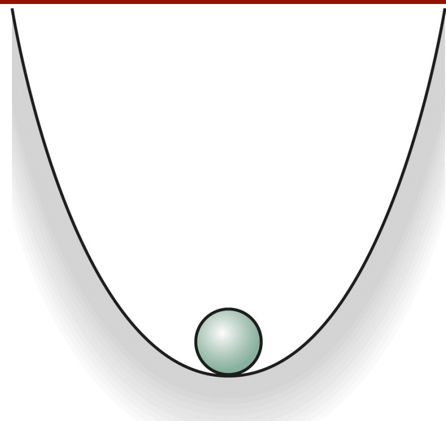


Cone balanced on its tip is in **unstable equilibrium**

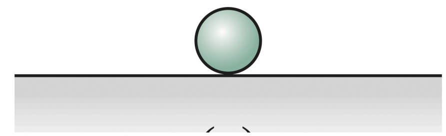
Meter stick balanced on finger is in unstable equilibrium…

Leaning ladder could be in stable or unstable equilibrium

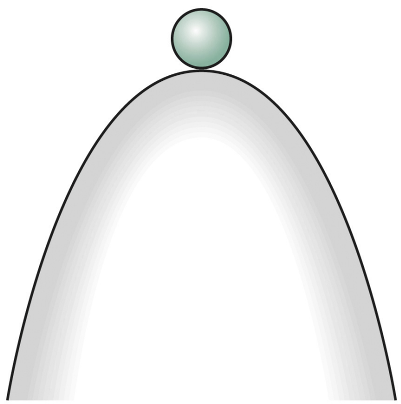
Kinds of Stability



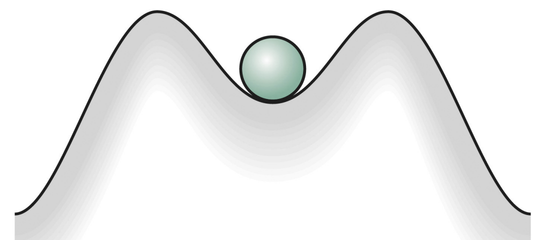
Stable equilibrium: perturbed ball will return to equilibrium



Neutrally stable equilibrium: no forces push the ball back into equilibrium **or** away from it



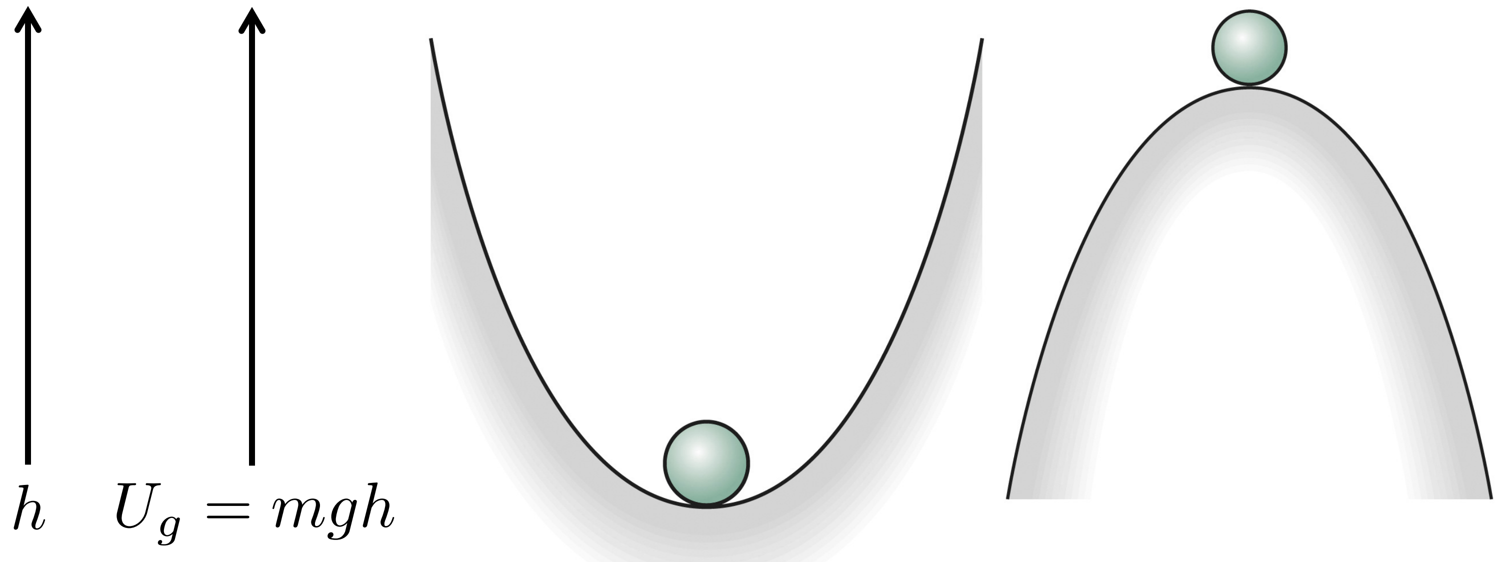
Unstable equilibrium: perturbed ball will move away from its original equilibrium



Metastable or **conditionally stable equilibrium**: ball returns for small disturbances, but not for large ones

Conditions for Equilibrium and Stability

- To be in equilibrium, there must be zero net force on an object.
  - The object must be at a maximum or minimum of its **potential energy curve** (like on top of a hill or bottom of a well)



  - But move off this maximum or minimum and the ball will experience a force
    - This force always pushes the ball **downhill** in these pictures
    - In general, an object moving on a potential energy curve feels a force that is proportional to the curve's **slope** or **derivative** $F = dU/dx$

Conditions for Equilibrium and Stability

- To be in equilibrium, there must be zero net force on an object.
  - The object must be at a maximum or minimum of its **potential energy curve** (like on top of a hill or bottom of a well)

$$\frac{dU}{dx} = 0 \quad \text{(condition for equilibrium)}$$

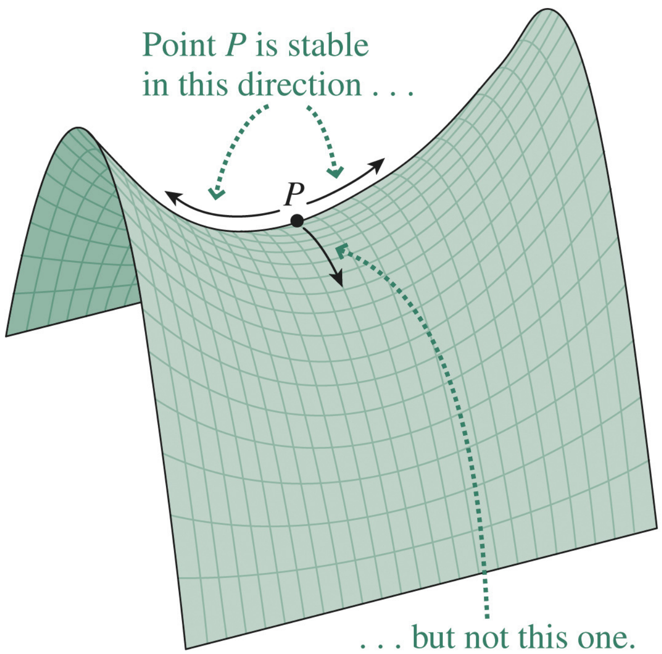
- For stable equilibrium, the object must be at a minimum:

$$\frac{d^2U}{dx^2} > 0 \quad \text{(stable equilibrium)}$$

- The condition for unstable equilibrium is

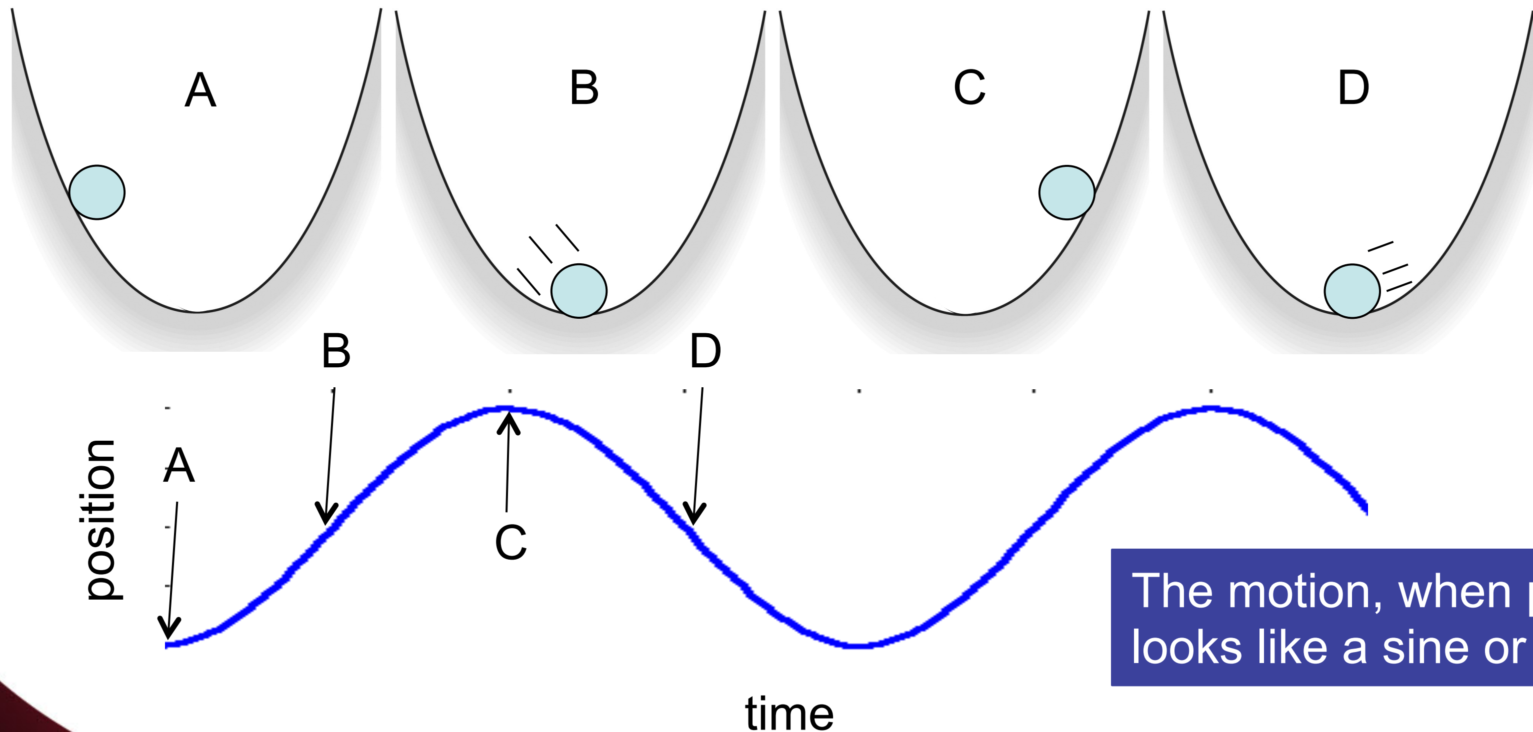
$$\frac{d^2U}{dx^2} < 0 \quad \text{(unstable equilibrium)}$$

- In two and three dimensions, an object can be stable in one direction but not another.



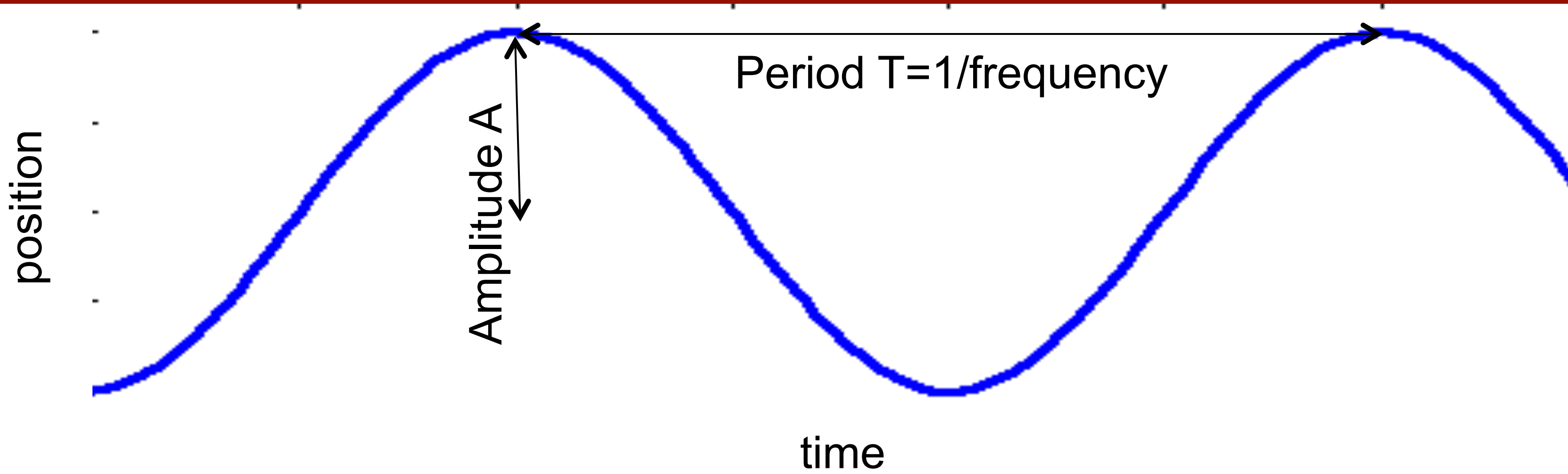
Simple Harmonic Motion

- Objects in small motions around a stable equilibrium point are ubiquitous in physics and engineering – they're everywhere!
  - In these problems, the **restoring force is proportional to displacement from the equilibrium point x=0:** $F = -kx$
    - Negative sign is there because the direction of the force F is opposite to the object's displacement x from equilibrium (at x=0)



The motion, when plotted, looks like a sine or cosine!

Period, Frequency, Amplitude, Phase



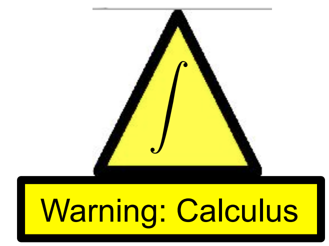
- We use standard terms to describe sine- and cosine-like curves
  - **Amplitude** A is the height of the curve below and above zero.
    - Amplitude has the same units as position
  - **Period** T is the time the curve takes for one oscillation
  - **Frequency** f=1/T (in units of Hz where 1 Hz is 1 cycle/s)
    - **Angular frequency** ω is often used where $\omega=2\pi f$
  - **Phase** ϕ_0 is phase of the curve at the time t=0
  - Then the periodic motion here is written as

$$\boxed{x = A\sin(\omega t + \phi_0)} = A\sin(2\pi f t + \phi_0) = A\sin(2\pi t/T + \phi_0)$$

Simple Harmonic Motion: Springs

- Wait, that force back there looked familiar… $F = -kx$
  - That's just **the restoring force of a spring**
  - We also know Newton's 2nd law:

$$F = ma = m\frac{dv}{dt} = m\frac{d}{dt}\left(\frac{dx}{dt}\right)$$



  - Set these equal and guess that the solution looks like a general sine or cosine-like function

$$x(t) = A\sin(\omega t + \phi_0)$$

$$\frac{dx}{dt} = A\omega\cos(\omega t + \phi_0) \qquad \text{(chain rule)}$$

$$\frac{d}{dx}\left(\frac{dx}{dt}\right) = -A\omega^2\sin(\omega t + \phi_0) = -\omega^2 x(t)$$

Simple Harmonic Motion: Springs

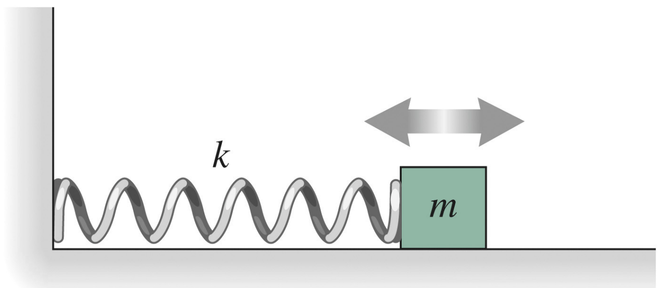
- Wait, that force equation looks familiar… $F = -kx$
  - That's just **the restoring force of a spring**
  - We also know Newton's 2nd law:

$$F = ma = m\frac{dv}{dt} = m\frac{d}{dt}\left(\frac{dx}{dt}\right)$$

$$\frac{d}{dt}\left(\frac{dx}{dt}\right) = -x_0\omega^2 \sin(\omega t + \phi_0) = -\omega^2 x(t)$$

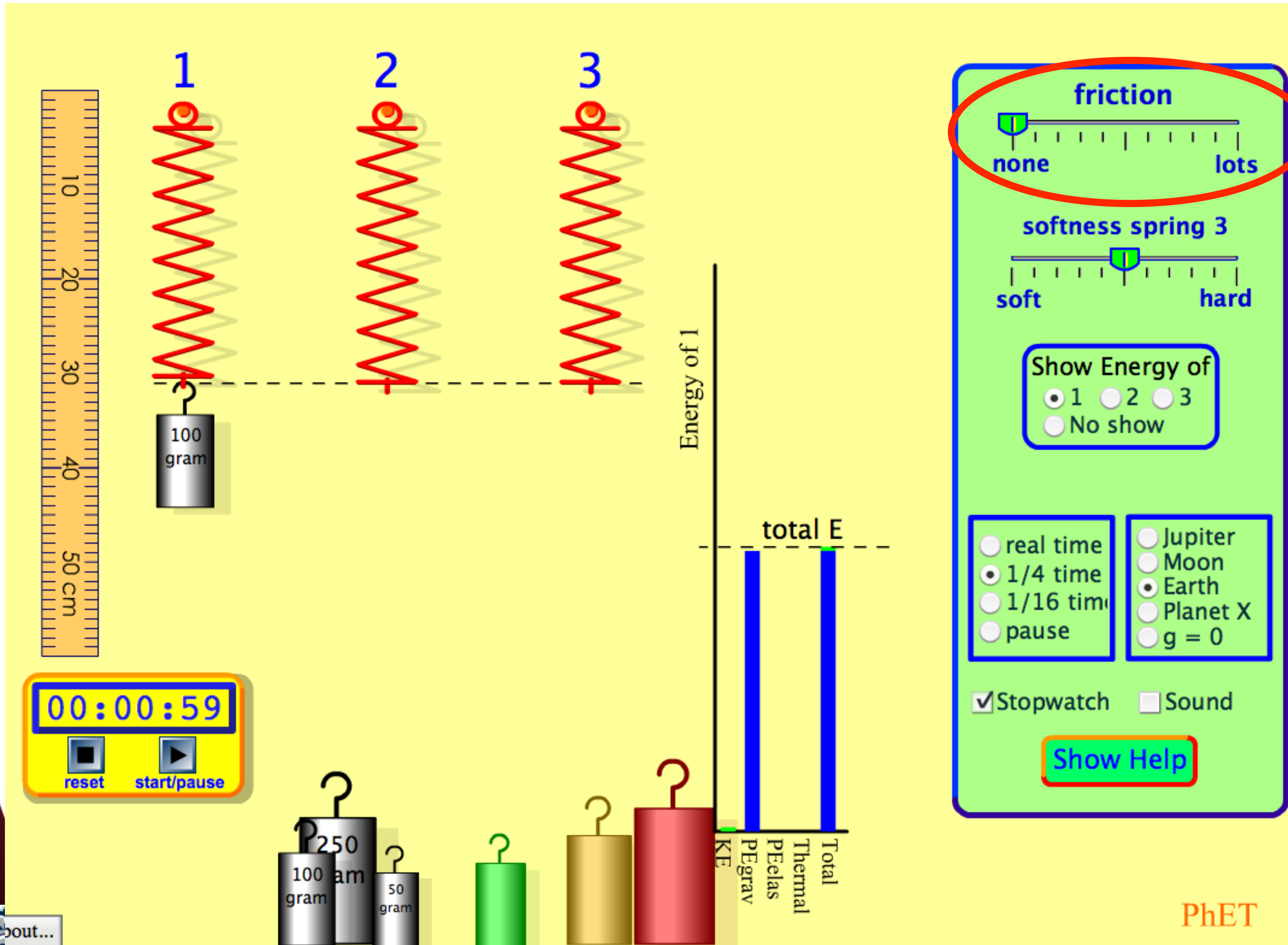
$$F = -kx = m(-\omega^2 x) \quad \Rightarrow \quad \boxed{\omega = \sqrt{\frac{k}{m}}}$$

  - This is the frequency of oscillations of a mass attached to a stretched spring



Tangible: Play with Springs!

http://phet.colorado.edu/sims/mass-spring-lab/mass-spring-lab_en.html



Turn off friction

Hang weights and time period T, frequency f, ang freq ω

How can you measure k for each spring?