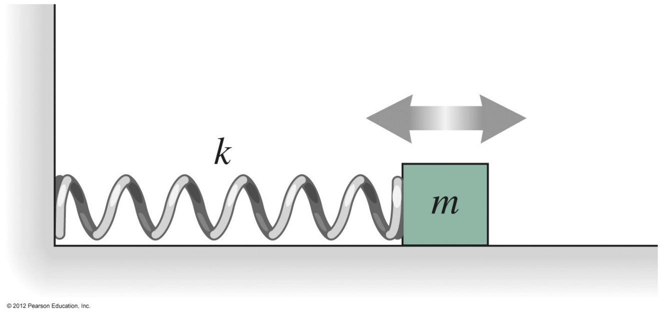
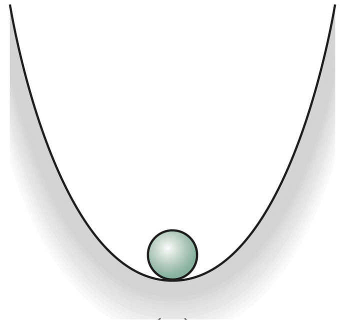


$$x(t) = A\sin(\omega t + \phi_0)$$

# University Physics 226N/231N
# Old Dominion University
# Reviewing Periodic Motion (Chapter 13)

Dr. Todd Satogata (ODU/Jefferson Lab)

http://www.toddsatogata.net/2012-ODU

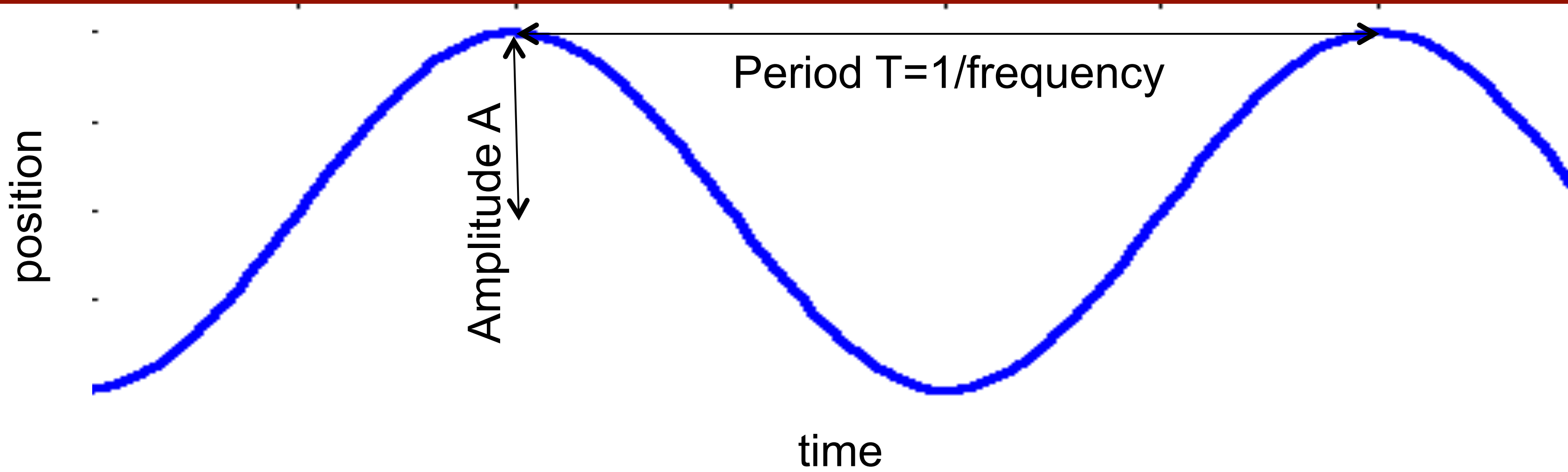
Monday November 12, 2012

Happy Birthday to Lord Rayleigh (1904 Nobel), Auguste Rodin, Anne Hathaway, and Ryan Gosling!
Happy Belated Veterans Day!

**There will be homework this week, but no quiz (Review Fri Nov 16)**

**Next exam and homework journal due: Mon Nov 19**

# Period, Frequency, Amplitude, Phase




- We use standard terms to describe sine- and cosine-like curves
  - **Amplitude** A is the height of the curve below and above zero.
    - Amplitude has the same units as position
  - **Period** T is the time the curve takes for one oscillation
  - **Frequency** f=1/T (in units of Hz where 1 Hz is 1 cycle/s)
    - **Angular frequency** $\omega$ is often used where $\omega=2\pi f$
  - **Phase** $\phi_0$ is phase of this periodic motion at the time t=0
  - Then the periodic motion here is written as

$$\boxed{x = A\sin(\omega t + \phi_0)} = A\sin(2\pi f t + \phi_0) = A\sin(2\pi t/T + \phi_0)$$

# Periodic Motion Refresher

- We had described periodic motion in one dimension with the equation

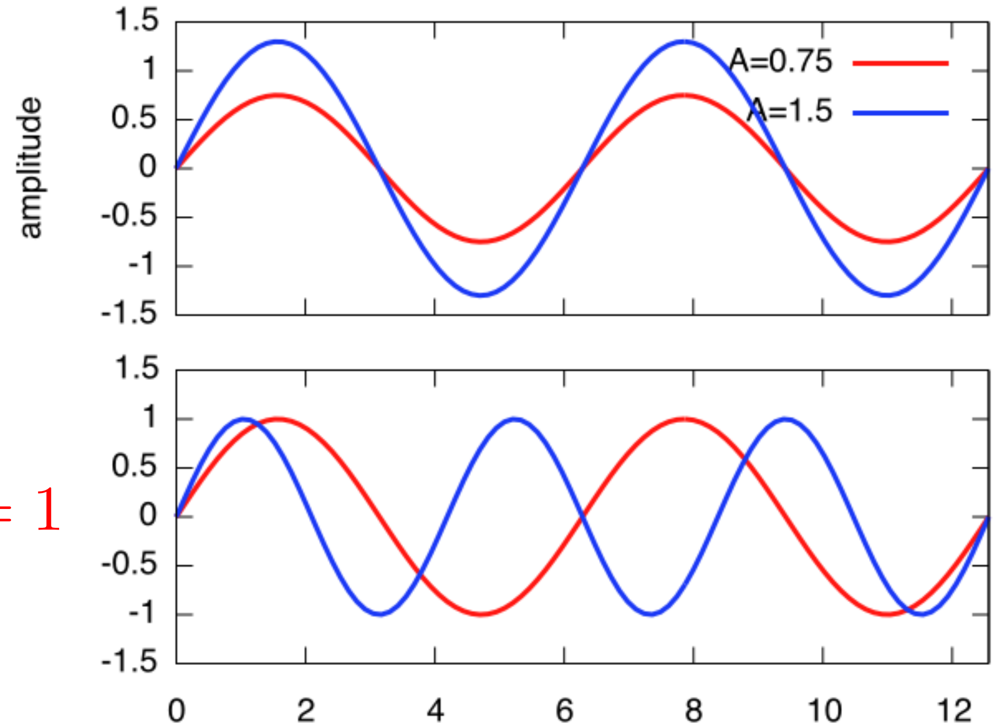
$$\boxed{x = A\sin(\omega t + \phi_0)} = A\sin(2\pi f t + \phi_0) = A\sin(2\pi t/T + \phi_0)$$

  - Sometimes this is written with a cosine which is okay – that's just like the sine but with a different phase $\phi_0$ at time t=0.

- Examples

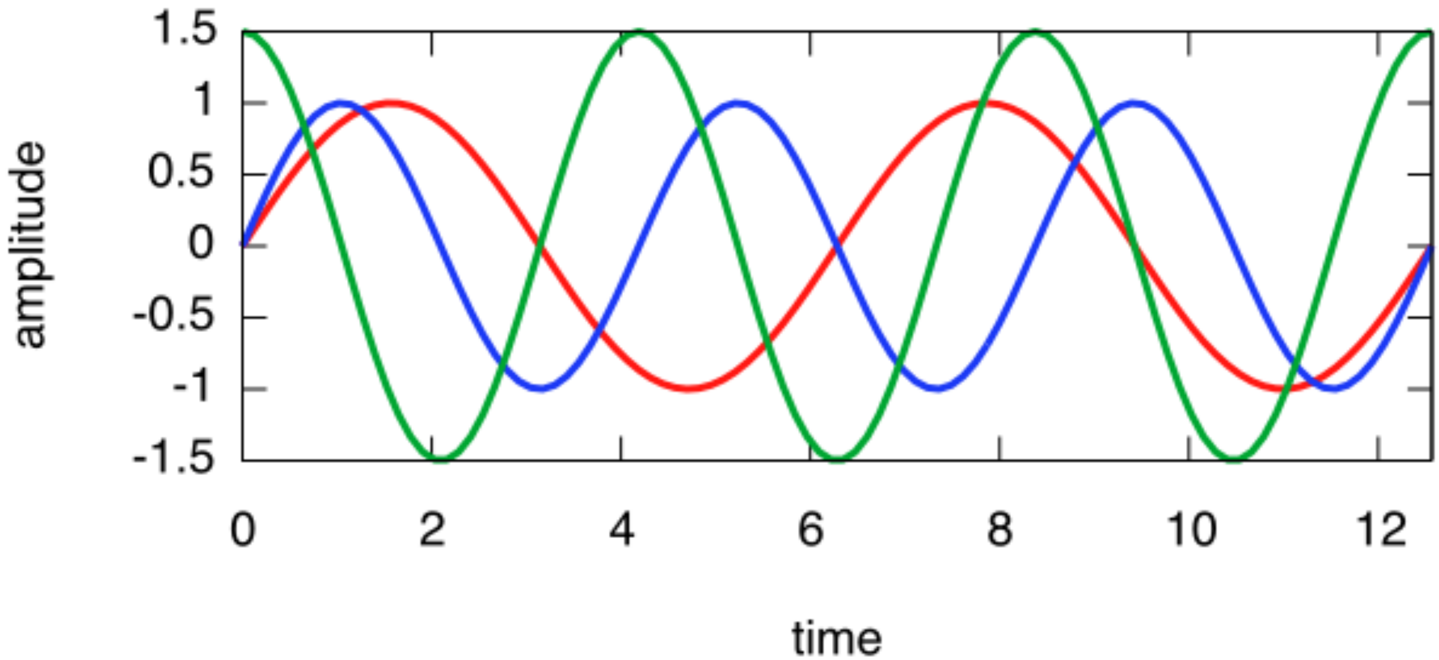
$A = 1.5,\ A = 0.75$

$\omega = 1.5,\ \omega = 1$



# Another example

$$\boxed{x = A\sin(\omega t + \phi_0)} = A\sin(2\pi f t + \phi_0) = A\sin(2\pi t/T + \phi_0)$$




$$A = 1.0 \quad \phi_0 = 0.0 \quad \omega = 1.5$$

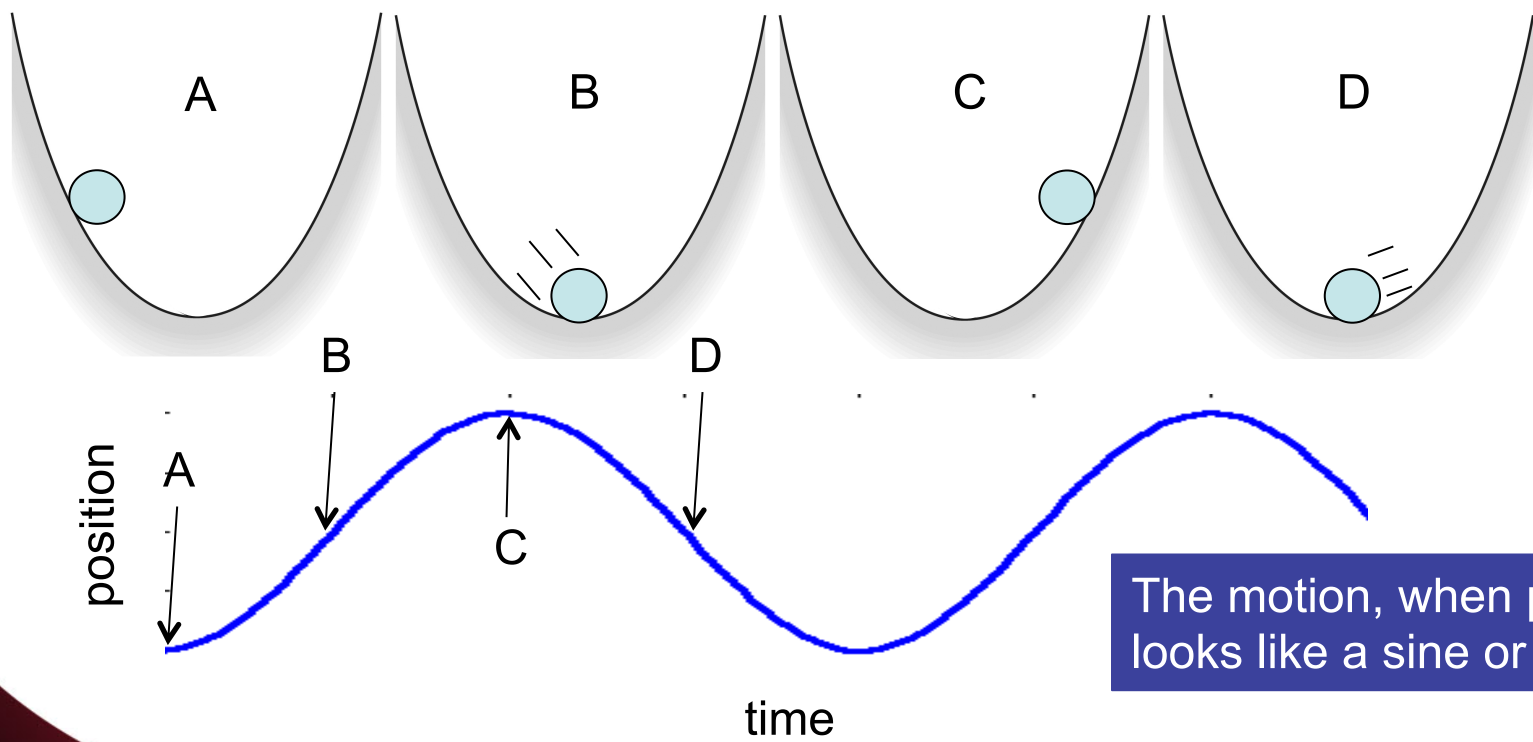
$$A = 1.0 \quad \phi_0 = 0.0 \quad \omega = 1.0$$

$$A = 1.5 \quad \phi_0 = \frac{\pi}{2} \quad \omega = 1.5$$

$$\boxed{f = \frac{1}{T} = \frac{\omega}{2\pi}}$$

# Simple Harmonic Motion

- Objects in small motions around a stable equilibrium point are ubiquitous in physics and engineering – they're everywhere!
  - In these problems, the **restoring force is proportional to displacement from the equilibrium point x=0:** $F = -kx$
    - Negative sign is there because the direction of the force F is opposite to the object's displacement x from equilibrium (at x=0)

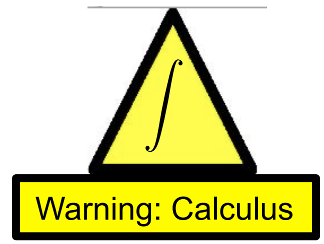



The motion, when plotted, looks like a sine or cosine!

# Simple Harmonic Motion: Springs

- Remember the restoring force of a spring… $F = -kx$
  - That's just **the restoring force of a spring**
  - We also know **Newton's 2nd law**:

$$F = ma = m\frac{dv}{dt} = m\frac{d}{dt}\left(\frac{dx}{dt}\right)$$




  - Position: $x(t) = A\sin(\omega t + \phi_0)$
  - Velocity: $\frac{dx}{dt} = A\omega\cos(\omega t + \phi_0) \qquad \text{(chain rule)}$
  - Acceleration: $\frac{d}{dx}\left(\frac{dx}{dt}\right) = -A\omega^2\sin(\omega t + \phi_0) = -\omega^2 x(t)$

# Simple Harmonic Motion: Springs

- Remember the restoring force of a spring… $F = -kx$
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Warning: Calculus

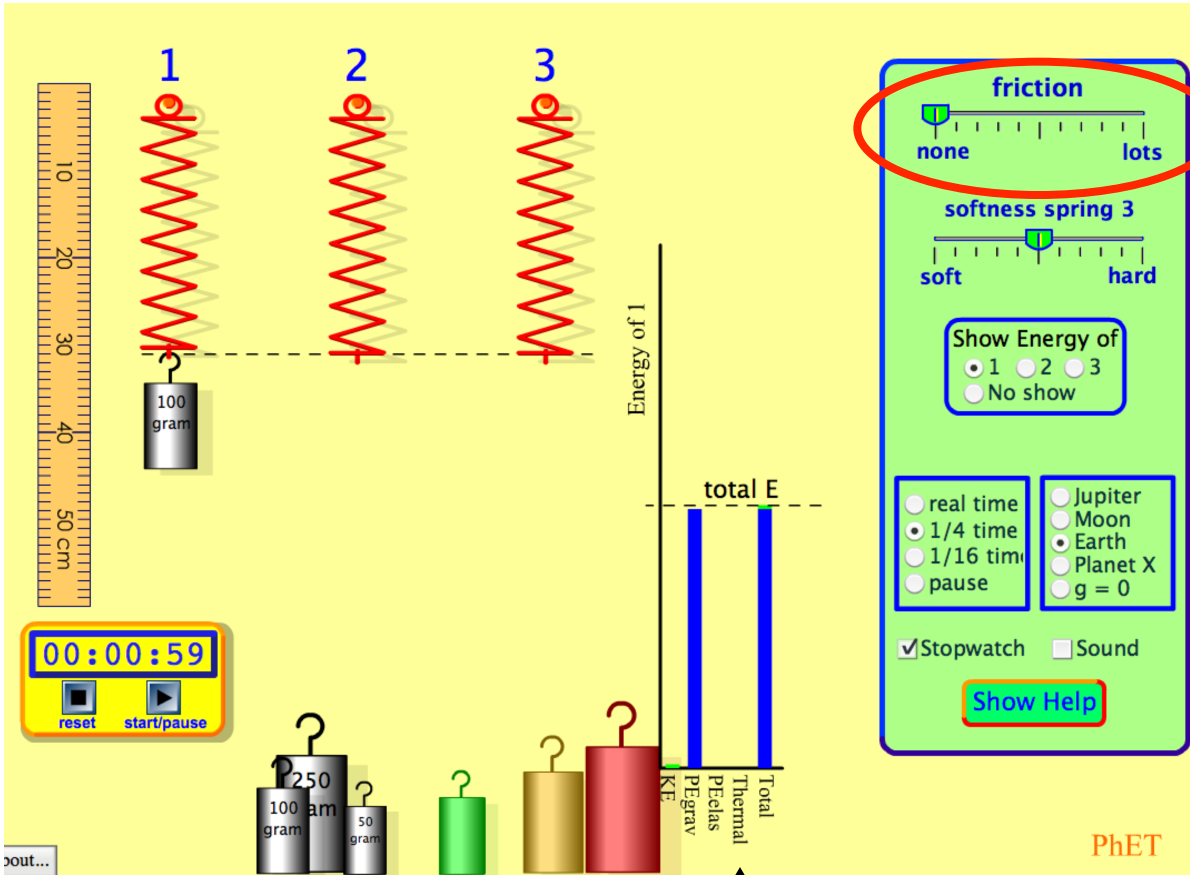
- Position: $x(t) = A\sin(\omega t + \phi_0)$
- Velocity: $\frac{dx}{dt} = A\omega\cos(\omega t + \phi_0)$ (chain rule)
- Acceleration: $\frac{d}{dx}\left(\frac{dx}{dt}\right) = -A\omega^2\sin(\omega t + \phi_0) = -\omega^2 x(t)$

$$F = ma = m(-\omega^2 x) = -m\omega^2 x = -kx$$

$$k = m\omega^2 \quad \Rightarrow \quad \boxed{\omega = \sqrt{k/m}}$$

# Tangible: Play with Springs!

http://phet.colorado.edu/sims/mass-spring-lab/mass-spring-lab_en.html




Turn off friction

Measure period T for 100 g weight, spring 1

What is spring k?

$$\omega = \sqrt{k/m}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{m/k}$$

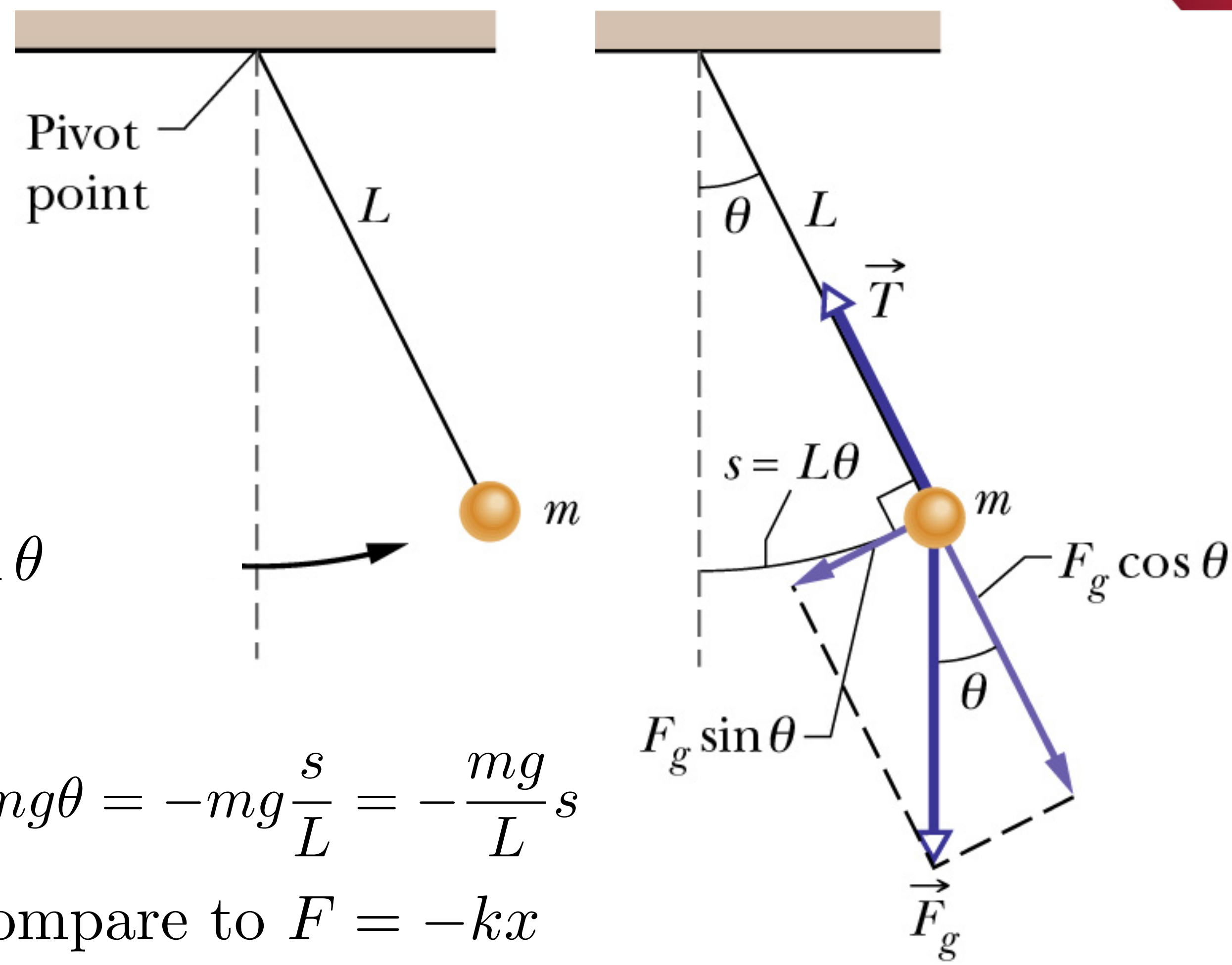
$$k = 4\pi^2 m/T^2$$

How much bigger is T for the 250 g weight?

Observe potential/kinetic energy exchange

# Simple Pendulum

Motion in one plane!




Force is $F = -mg\sin\theta$

For small angles $\theta$

Restoring force $F \approx -mg\theta = -mg\dfrac{s}{L} = -\dfrac{mg}{L}s$

Compare to $F = -kx$

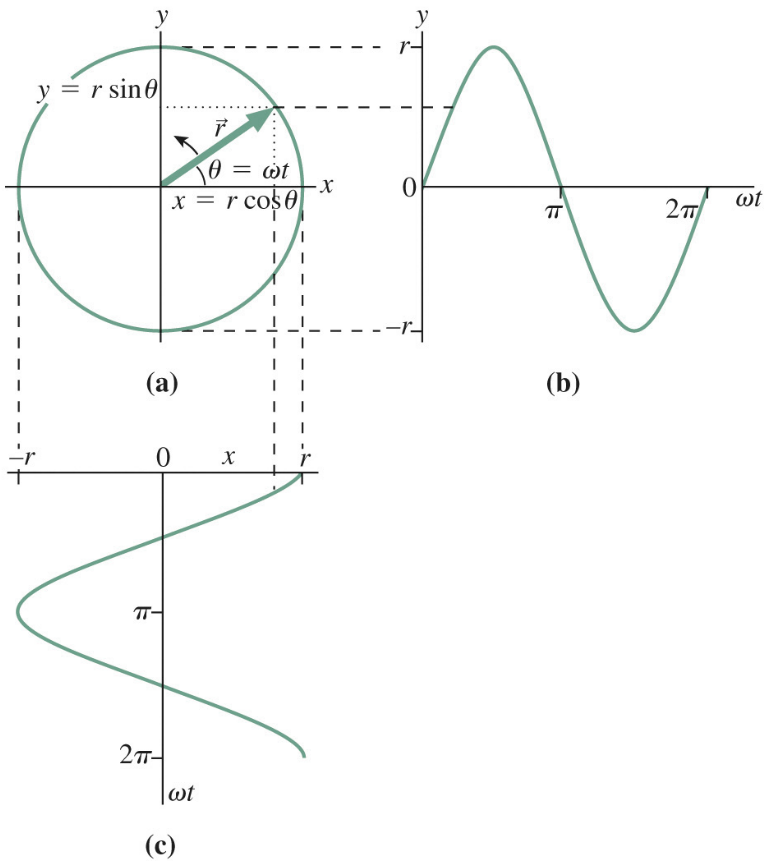
$$\boxed{k = \frac{mg}{L}} \qquad \boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$ for simple, small – angle pendulum

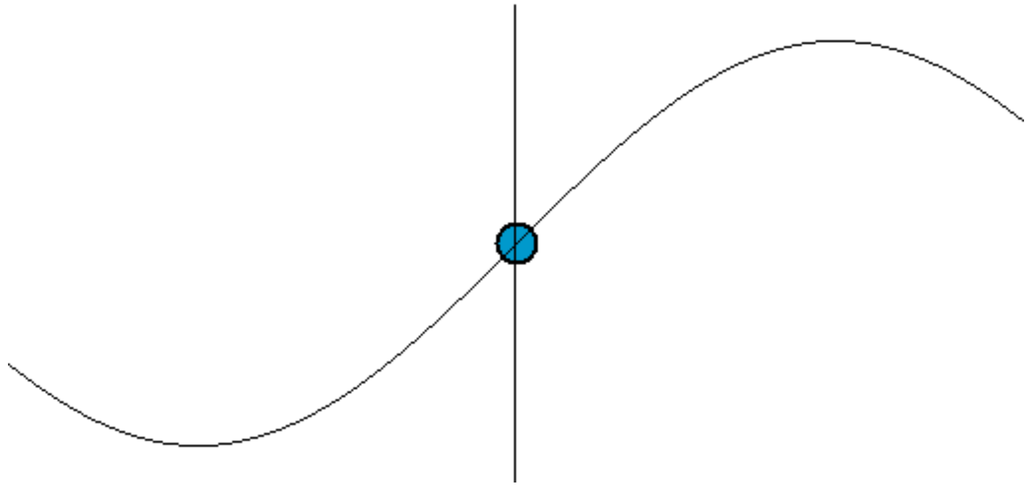
# Ponderable: Grandfather Clock

- You are asked to build a pendulum clock
  - You want the period of the pendulum of the clock to be 2 seconds (counting 1s each half-swing).
  - How long does the pendulum bob need to be to swing freely with a period of T=2 seconds?
    - Is this reasonable for, say, a 6 foot wall clock? A 1 foot mantel clock?
  - Does this depend on the pendulum bob's mass?
  - How does your answer qualitatively change if the pendulum rod has non-negligible mass?
  - (Insert Todd's pendulum demo here)

# Simple Harmonic Motion and Circular Motion

- Simple harmonic motion (SHM) can be viewed as one component (or a projected shadow) of uniform circular motion.
  - Angular frequency $\omega$ in SHM is the same as angular velocity $\omega$ in circular motion.



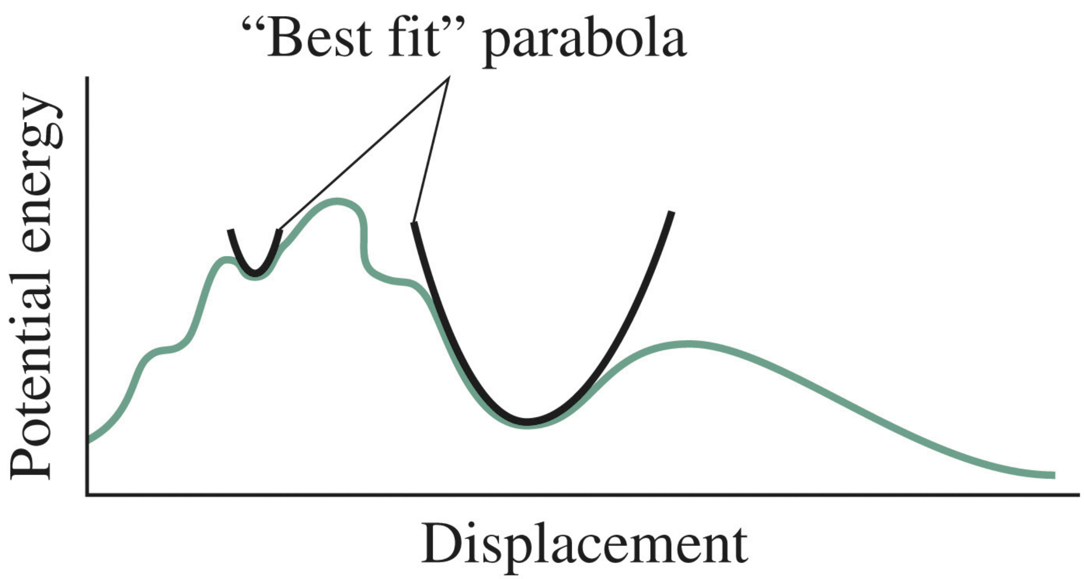



As the position vector $\vec{r}$ traces out a circle, its *x*– and *y*– components are sinusoidal functions of time.

# Simple Harmonic Motion is Everywhere!

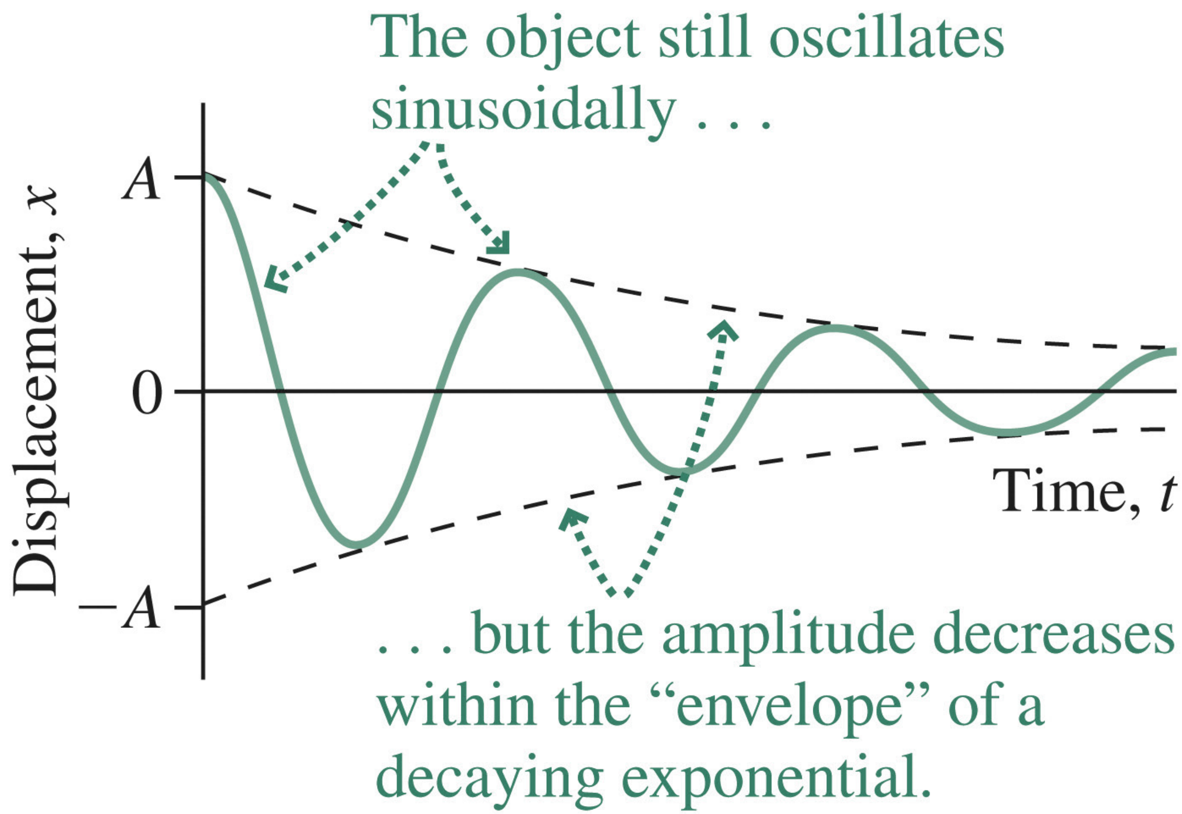
- Most systems near stable equilibrium have potential energy curves that are approximately parabolic
  - That is, restoring forces are approximately linear
- Examples include
  - Ideal springs and pendulums
  - Intermolecular forces in a solid
  - Gravitational motion near stable orbits
  - Waves (water, sound, light…) – to be covered in chapter 14

$$F \approx -kx \qquad U \approx \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$$



# Damped Harmonic Motion

- With nonconservative forces (like friction) present, the motion of a simple harmonic oscillator "dies out"
  - Energy is "drained" from the system into heat and friction
  - Oscillation amplitude goes down over time (Slinky demo)



# Damped Harmonic Motion Physics

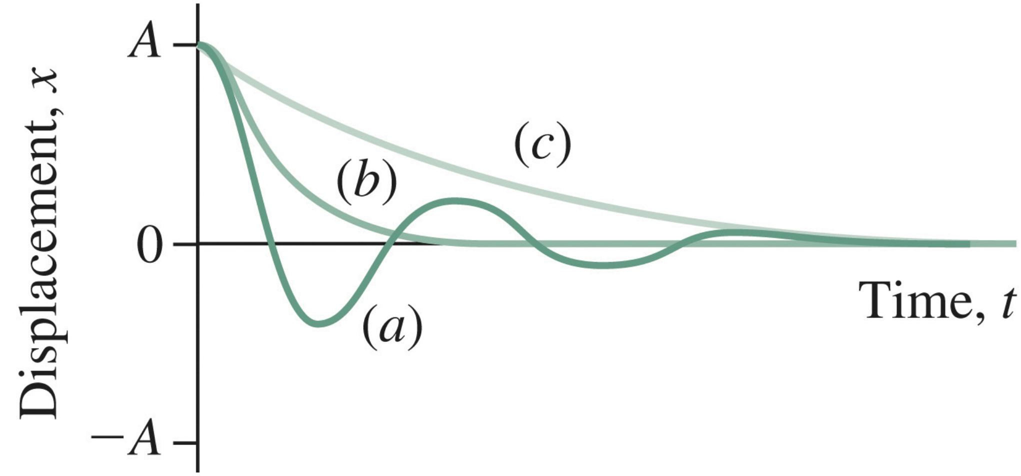
- Damping is usually because of a force that resists the object's motion
  - So the usual model is to include a force that is proportional to the object's **velocity** and acts against it
  - This makes sense: the damping force is zero when the object is not moving (or v=0)

$$a \equiv \frac{d^2x}{dt^2} \qquad v \equiv \frac{dx}{dt} \qquad F = ma = m\frac{d^2x}{dt^2} = \underbrace{-kx}_{\text{Restoring force}} - \underbrace{b\frac{dx}{dt}}_{\text{Damping force}}$$

  - For small b, the motion is like before except the amplitude declines exponentially towards zero (see pic on previous page)

$$\boxed{x(t) = A\mathrm{e}^{-bt/2m}\sin(\omega t + \phi_0)}$$

# Different Types of Damping




- (a) **Underdamped** $b < 2\sqrt{mk}$
  - Amplitude goes down slowly over many oscillations
- (c) **Overdamped** $b > 2\sqrt{mk}$
  - No oscillation, amplitude just gradually settles to equilibrium
- (b) **Critically damped** $b = 2\sqrt{mk}$
  - The fastest the oscillator can return to equilibrium without overshooting and continuing to oscillate
  - "The perfect shock absorber"

# Driven Oscillations

- Often we're interested in what happens when an external periodic force acts on our oscillatory system
  - Pumping a swing, shining laser light on an atom…
  - Here we naturally add an extra force to our Newton's 2nd law!

$$m\frac{d^2x}{dt^2} = \underbrace{-kx}_{\text{Restoring force}} - \underbrace{b\frac{dx}{dt}}_{\text{Damping force}} + \underbrace{F_0\cos(\omega_d t)}_{\text{Driving force}}$$

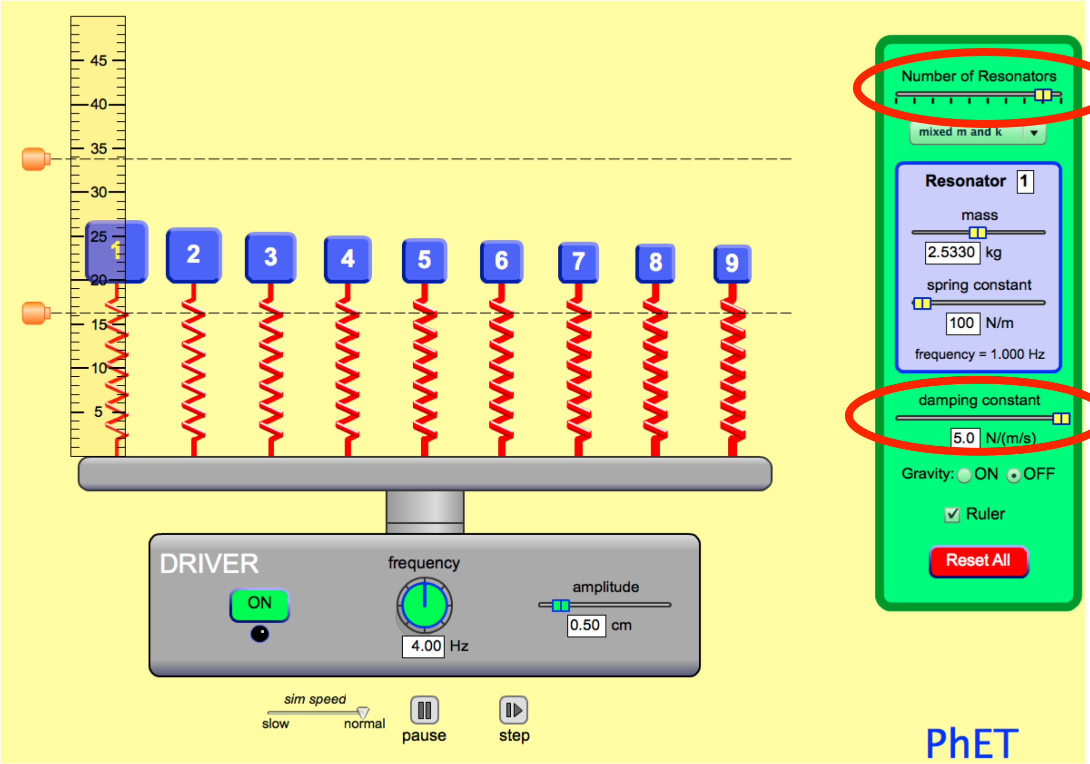
  - The system now oscillates with a rather complicated amplitude!

$$\omega_0 \equiv \sqrt{\frac{k}{m}} \qquad A(\omega) = \frac{F_0}{m\sqrt{(\omega_d^2 - \omega_0^2)^2 + b^2\omega_d^2/m^2}}$$

  - The important thing is that this amplitude is largest when $\omega_d = \omega_0$
    - That is, **when the drive frequency matches the natural frequency**

# Take-Home Tangible

http://phet.colorado.edu/sims/resonance/resonance_en.html




Max # of resonators (different m/k values)

Damping constant 1.0 N/(m/s)

- Setup and turn driver on, then adjust frequency by typing in new numbers
  - How do motions vary for different m/k values as you adjust f?
  - At f=3 Hz, which mass is moving the most? If you turn off the driver, is it underdamped or overdamped?