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University Physics 226N/231N Old Dominion University Wave Motion (Chapter 14)

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<http://www.toddsatogata.net/2012-ODU>

Wednesday November 14, 2012

Happy Birthday to Travis Barker, Josh Duhamel, and Beethoven and Mozart's fathers!
Happy Operating Room Nurse Day, Eclipse Day, and World Diabetes Day!

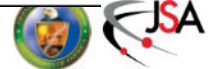
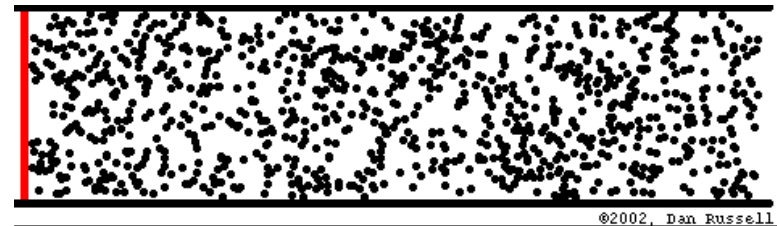
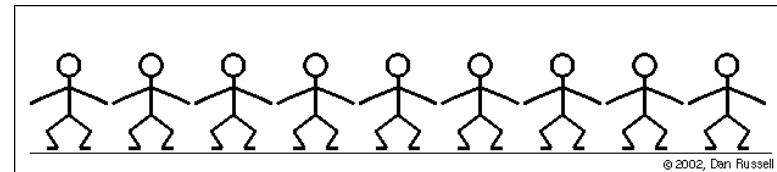
There will be homework this week, but no quiz (Review Fri Nov 16)

Next exam and homework journal due: Mon Nov 19

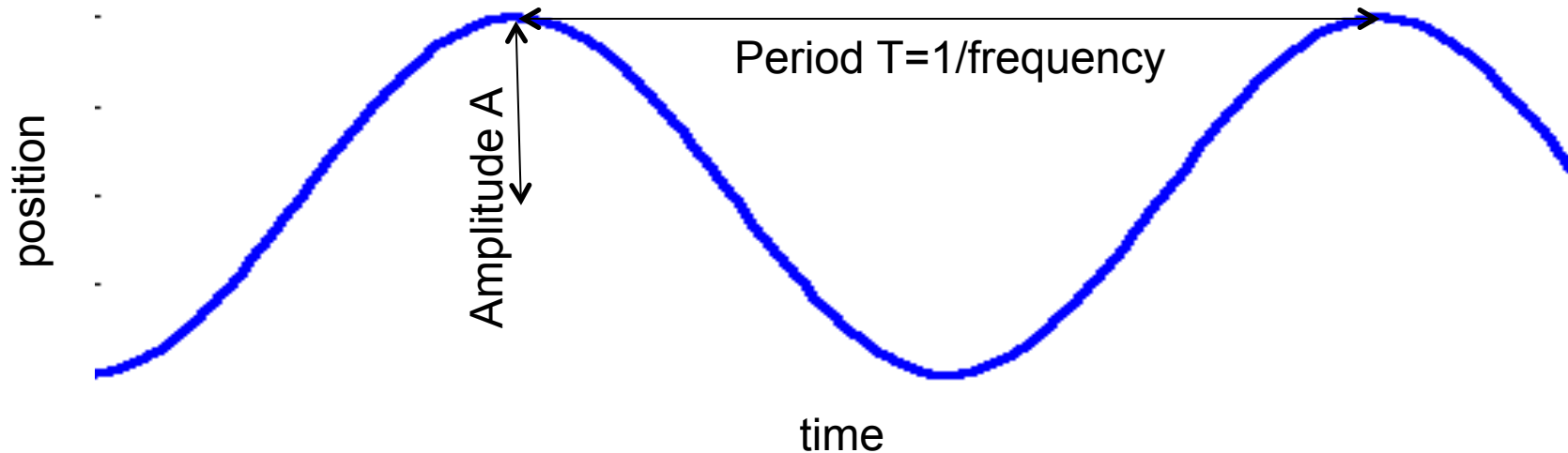


Chapter 13: Wave Motion

- Explain waves as traveling disturbances that transport **energy** but not **matter**
- Describe waves quantitatively
 - Frequency, period, wavelength, and amplitude
 - Wave number and velocity
- Describe example waves
 - Waves on strings
 - Sound waves
- Describe interference, reflection, and standing waves
- Describe the Doppler effect and shock waves



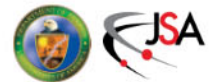
Period, Frequency, Amplitude, Phase



- We use standard terms to describe sine- and cosine-like curves
 - **Amplitude A** is the height of the curve below and above zero.
 - Amplitude has the same units as position
 - **Period T** is the time the curve takes for one oscillation
 - **Frequency $f=1/T$** (in units of Hz where 1 Hz is 1 cycle/s)
 - **Angular frequency ω** is often used where $\omega=2\pi f$
 - **Phase ϕ_0** is phase of the curve at the time $t=0$
 - Then the periodic motion here is written as

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$x = A \sin(\omega t + \phi_0) = A \sin(2\pi f t + \phi_0) = A \sin(2\pi t/T + \phi_0)$$



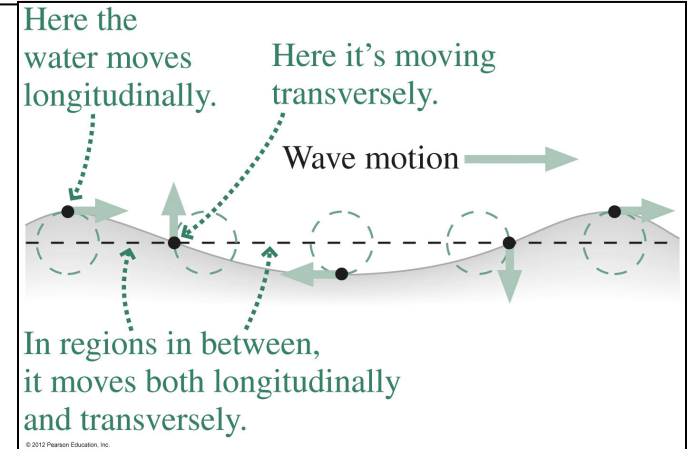
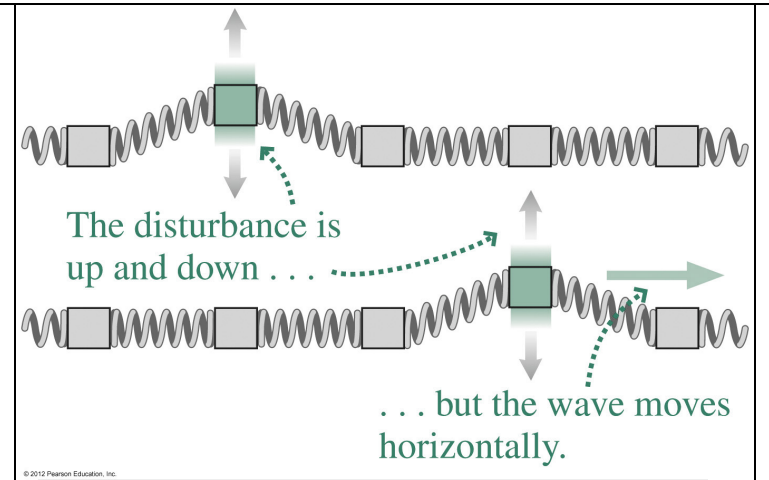
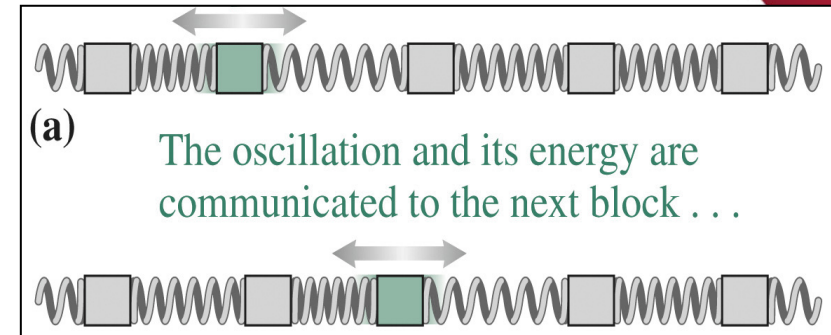
What's a Wave?

- A **wave** is a traveling disturbance that transports energy but not matter.
- **Mechanical waves** are disturbances of a **material medium**.
 - The medium moves briefly as the wave goes by, but the medium itself isn't transported any distance.
 - The wave propagates as the disturbance of the medium is communicated to adjacent parts of the medium.
 - These waves occur because of forces between particles in the medium
 - Water molecules, molecules in air, atoms in a solid, springs, etc...
- **Electromagnetic waves**, including light, do not require a medium.
 - They still share many of properties of mechanical waves.
 - There may also be **gravity waves** but these have not been observed.

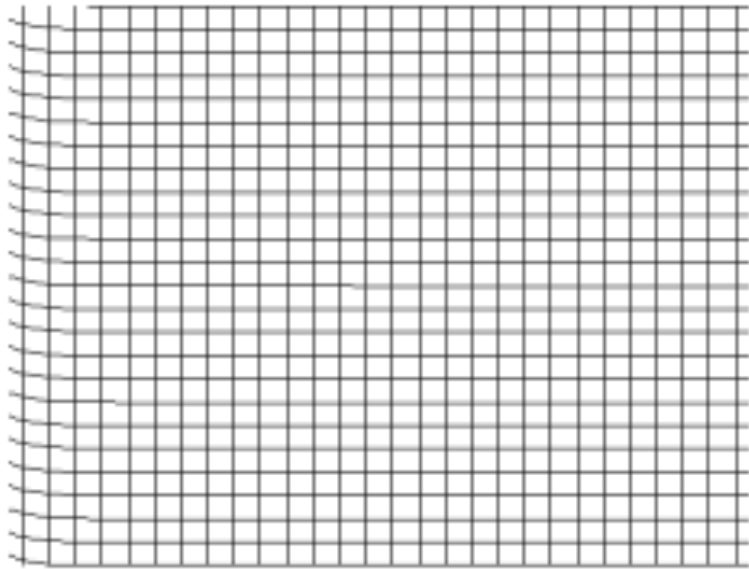


Longitudinal and Transverse Waves

- In a **longitudinal wave**, the disturbance is parallel to the wave propagation.
- In a **transverse wave**, the disturbance is perpendicular to the wave propagation.
- Some waves, like surface waves on water, involve **both** longitudinal and transverse motions.

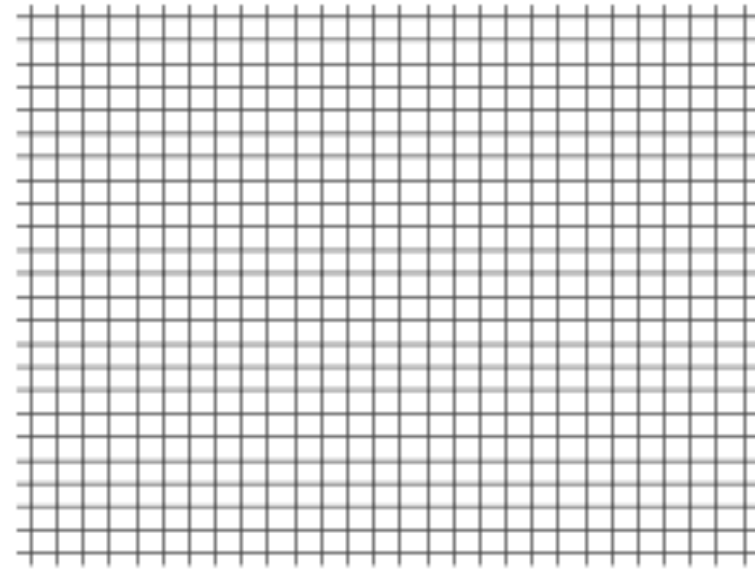


Longitudinal and Transverse Wave Animations



Transverse wave

Displacement is perpendicular
to direction of wave propagation



Longitudinal wave

Displacement is parallel
to direction of wave propagation

Both have similar velocities and wavelengths here (not always the case)

Gases cannot support pure transverse waves (shear)

Longitudinal waves can occur in any matter (compression/rarefaction)



Properties of Waves

- **Wavelength** λ is the distance over which a wave repeats in space.

- **Wave number** $k = 2\pi/\lambda$

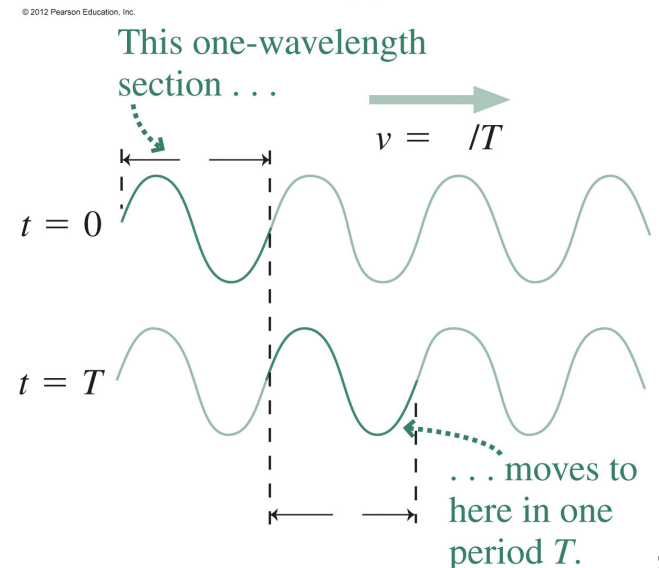
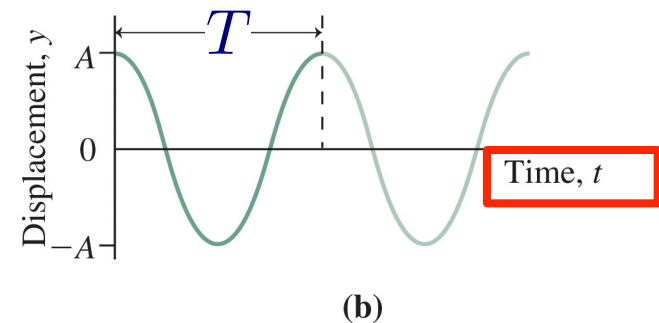
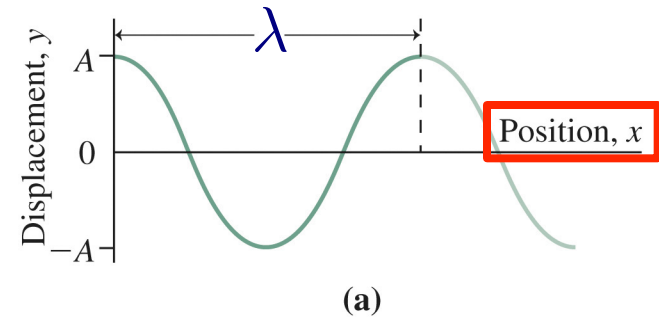
- **Period** T is the time for a complete cycle of the wave at a fixed position:

- **Frequency** $f = \frac{1}{T} = \frac{\omega}{2\pi}$

- **Amplitude** A is the peak value of the wave disturbance.

- **Wave speed** is the rate at which the wave propagates:

$$v = \frac{\lambda}{T} = \lambda f$$



Simple Harmonic Waves

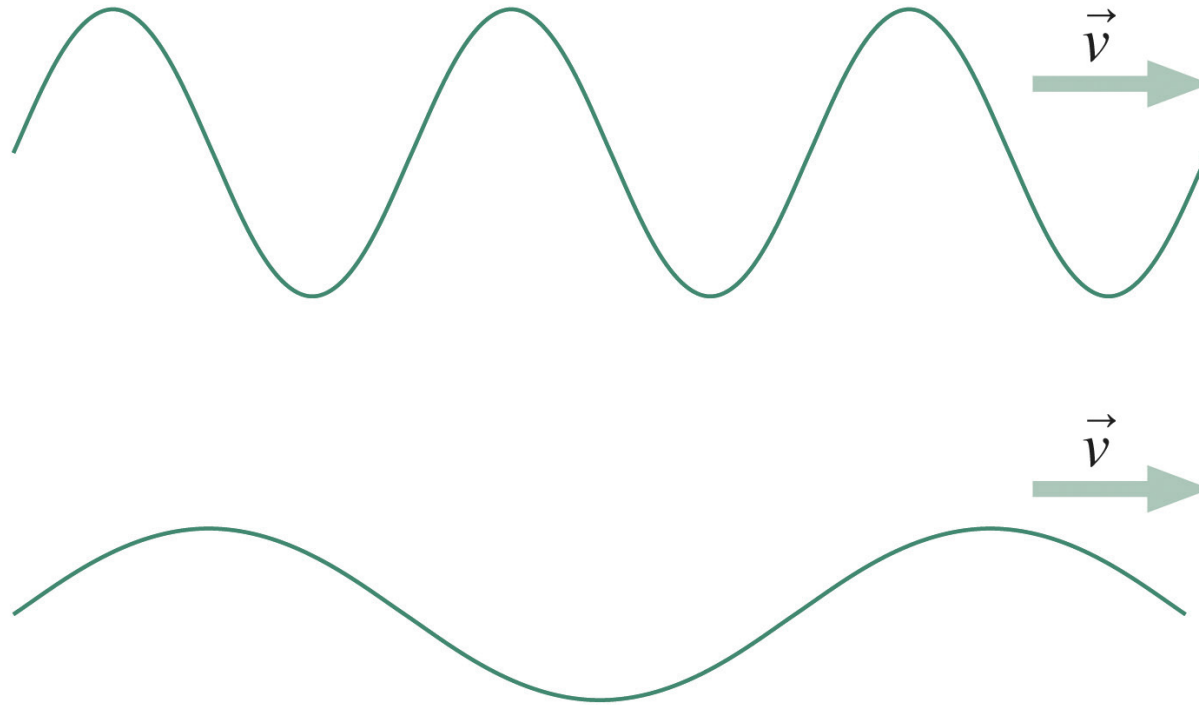
- Waves have dependency on **both** time t and location x
- A **simple harmonic wave** has a sinusoidal shape:

$$y(x, t) = A \cos(kx - \omega t)$$

- y measures the wave disturbance at position x and time t .
- $k = 2\pi/\lambda$ is the **wave number**, a measure of the rate at which the wave varies in *space*.
- $\omega = 2\pi f = 2\pi/T$ is the **angular frequency**, a measure of the rate at which the wave varies in *time*.
- The **wave speed**, as mentioned before, is $v = \lambda f = \omega/k$.
- This is describing one “simple” wave in space and time
 - There may be many waves all interacting at once!



Comparison



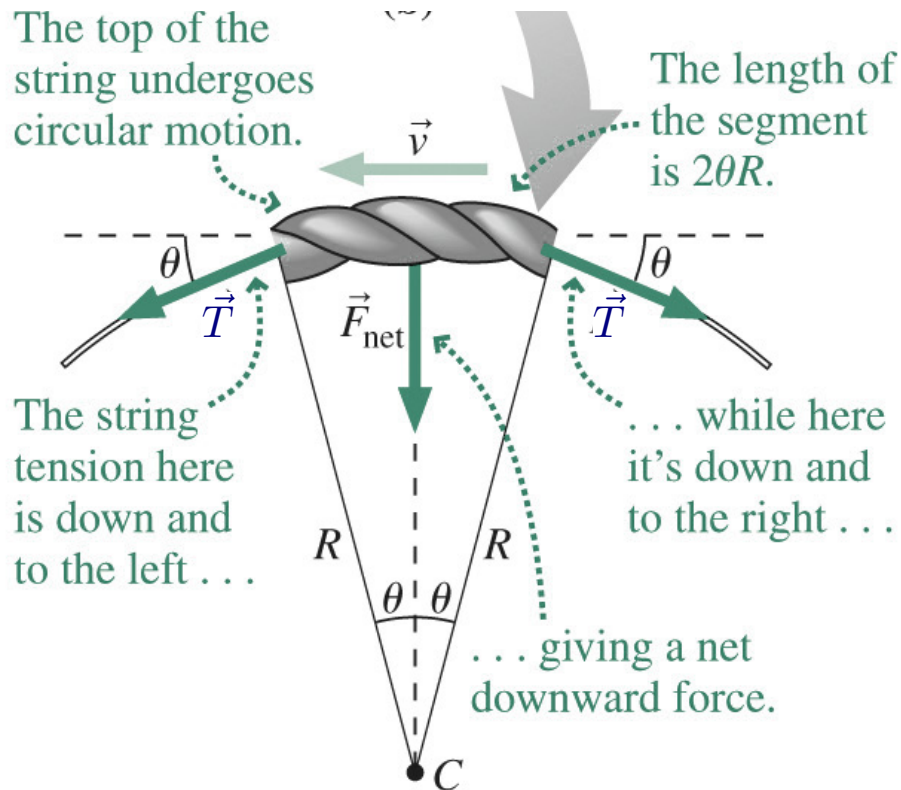
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- Consider two waves shown above at time $t=0$. Both have the same velocity v .
 - Which has the larger wavelength, λ ?
 - Which has the larger wave number, k ?
 - Which has the larger period, T ?
 - Which has the larger angular frequency, ω ?



Waves on Strings

- A classic example of wave motion is a transverse disturbance traveling along a rope of tension T
 - We also need the “mass per unit length”, $\mu = M/L$
 - This μ is **not** related to coefficients of friction!!!



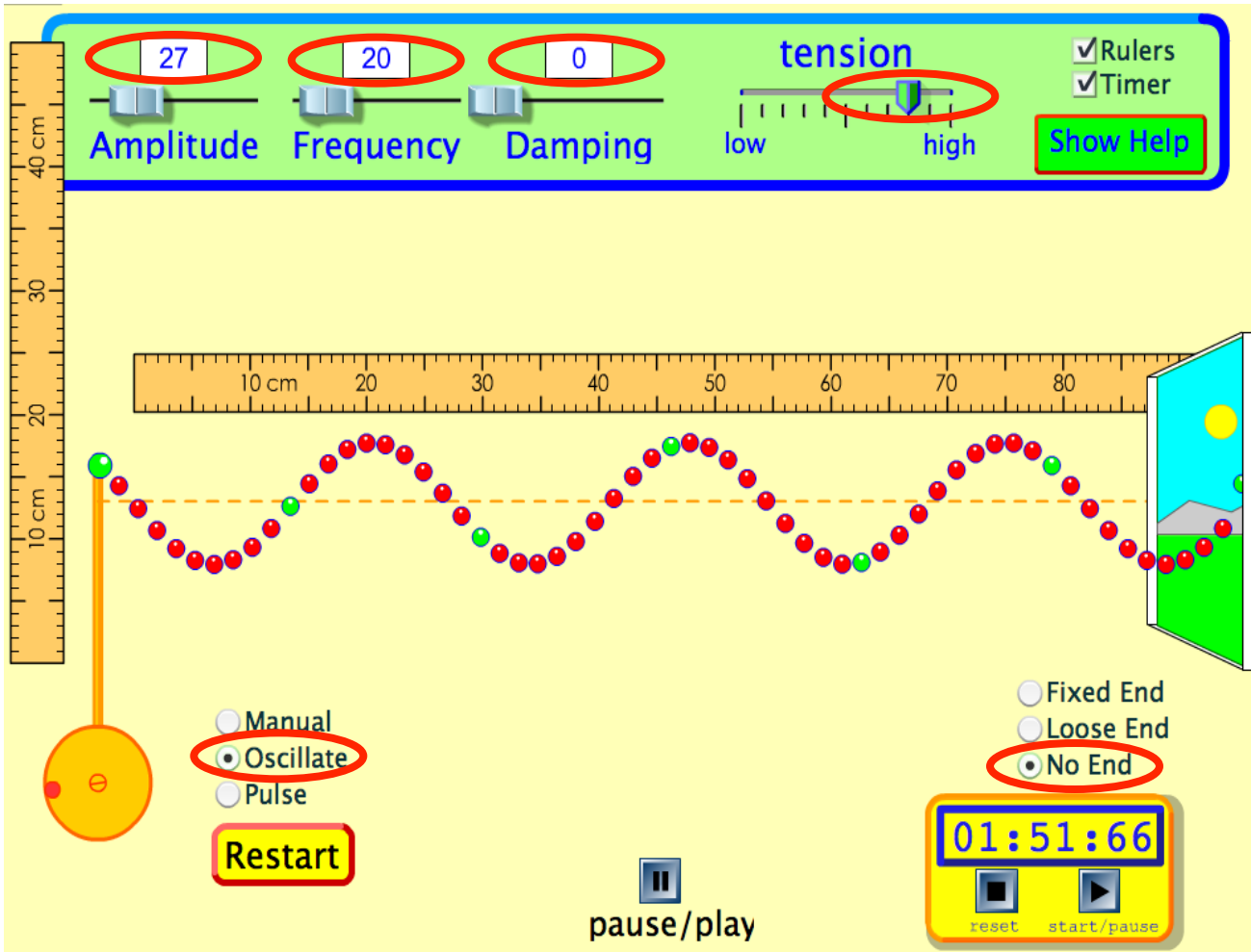
T provides a restoring force that makes a rope section go around the edge of the wave using our old equations for centripetal acceleration

We can find the velocity of the wave propagation, v

$$v = \sqrt{\frac{T}{\mu}}$$

Tangible: Waves on Strings

http://phet.colorado.edu/sims/wave-on-a-string/wave-on-a-string_en.html



Set up as shown

What are

- ...period T ?
- ...frequency f ?
- ...angular freq ω ?
- ...wavelength λ ?
- ...wave number k ?
- ...wave velocity v ?

Lower the “tension” – what happens to v ?

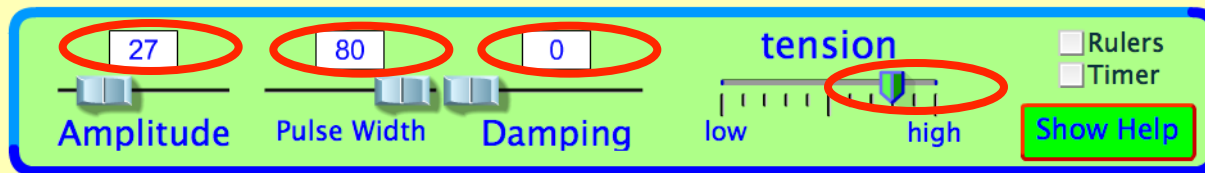
$$v = \sqrt{\frac{T}{\mu}} = \lambda f$$

Observe that each particle only moves transversely! This is a **transverse wave**



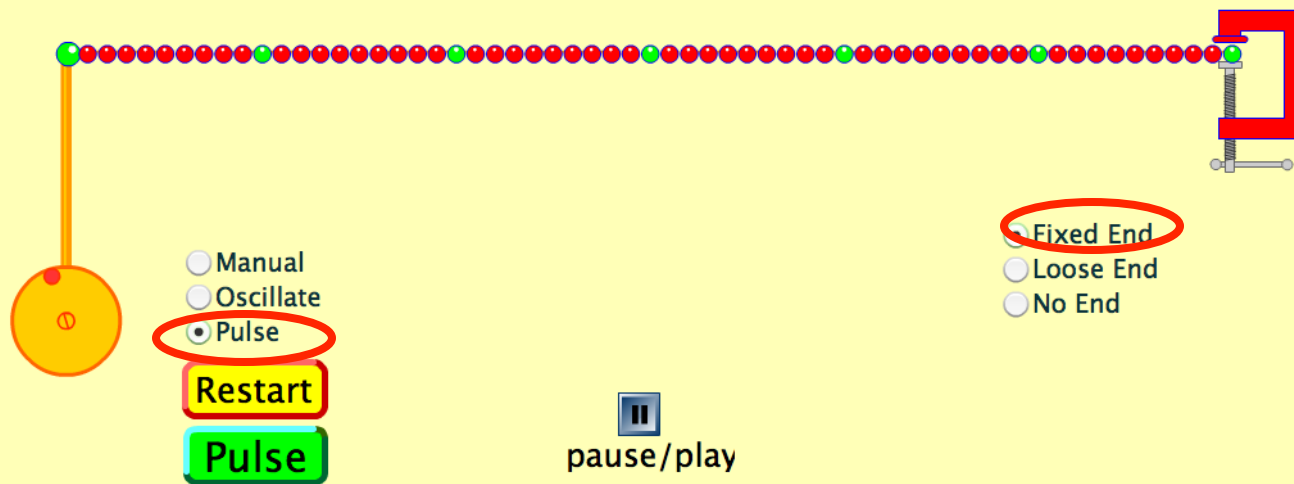
Tangible: Waves on Strings 2

http://phet.colorado.edu/sims/wave-on-a-string/wave-on-a-string_en.html



Set up as shown

Click “pulse” and watch how the pulse moves



What happens when it reaches the clamp?

What happens if you change to “loose end”?

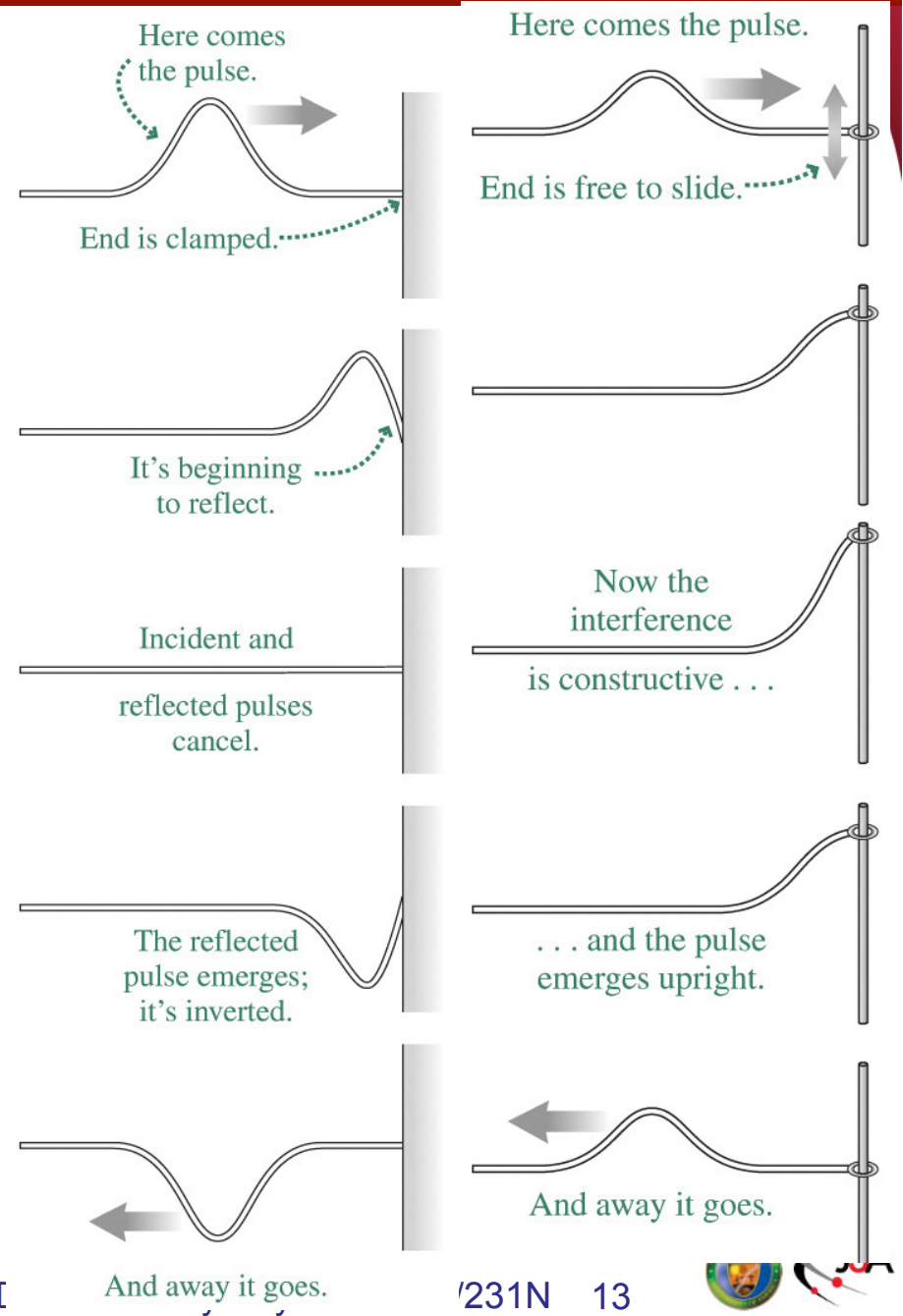
Pulse multiple times; how do the waves interact?

The interaction of multiple waves with each other is called **interference**



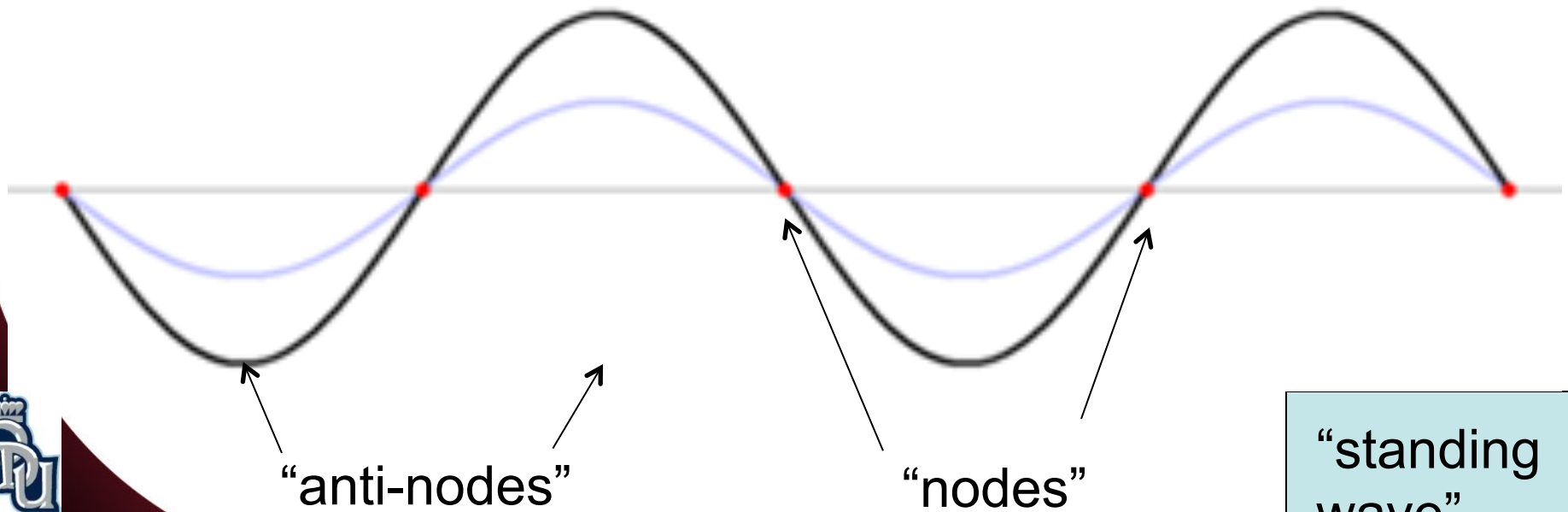
Wave Reflection

- Waves reflect at an interface with a different medium.
 - The outgoing wave interferes with the incoming wave.
 - The reflected wave is inverted, depending on properties of the second medium.
 - The diagram shows waves on a string reflecting at clamped and free ends.
- More generally, waves are partially reflected and partially transmitted at an interface between different media.



Wave Interference

- Unlike particles, multiple waves can intersect in space/time.
- When they are, they **interfere**.
 - In most cases, the waves **superpose**, or simply add.
 - When wave crests coincide, the interference is **constructive**.
 - When crests coincide with troughs, the interference is **destructive**.
 - Here the **red** and blue waves add up to the black wave
 - Sometimes they cancel each other out (destructively interfere) and sometimes they add up (constructively interfere)



Standing Waves: Fixed at Both Ends

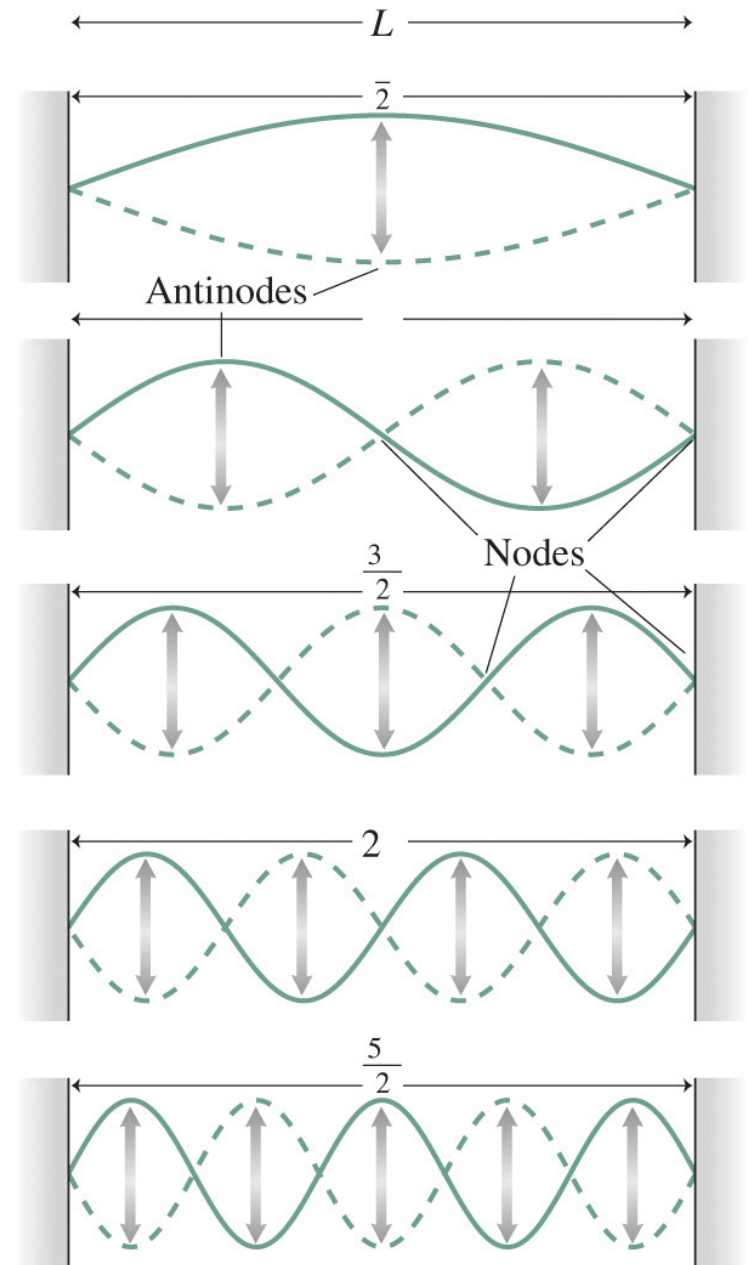
- Waves on a confined medium reflect (with flip) at both ends.
 - An example of **boundary conditions**: nodes at both ends!
 - The result is **standing waves** that oscillate but don't propagate.
 - The length of the medium restricts allowed wavelengths and frequencies to specific, discrete values.

On a string clamped at **both ends**, the string length **must** be an integer multiple of a half-wavelength

$$L = \frac{m\lambda}{2}$$

with m an integer (1,2,3,...)

Examples: Guitar/piano strings, drum heads



Standing Waves: Fixed at One End

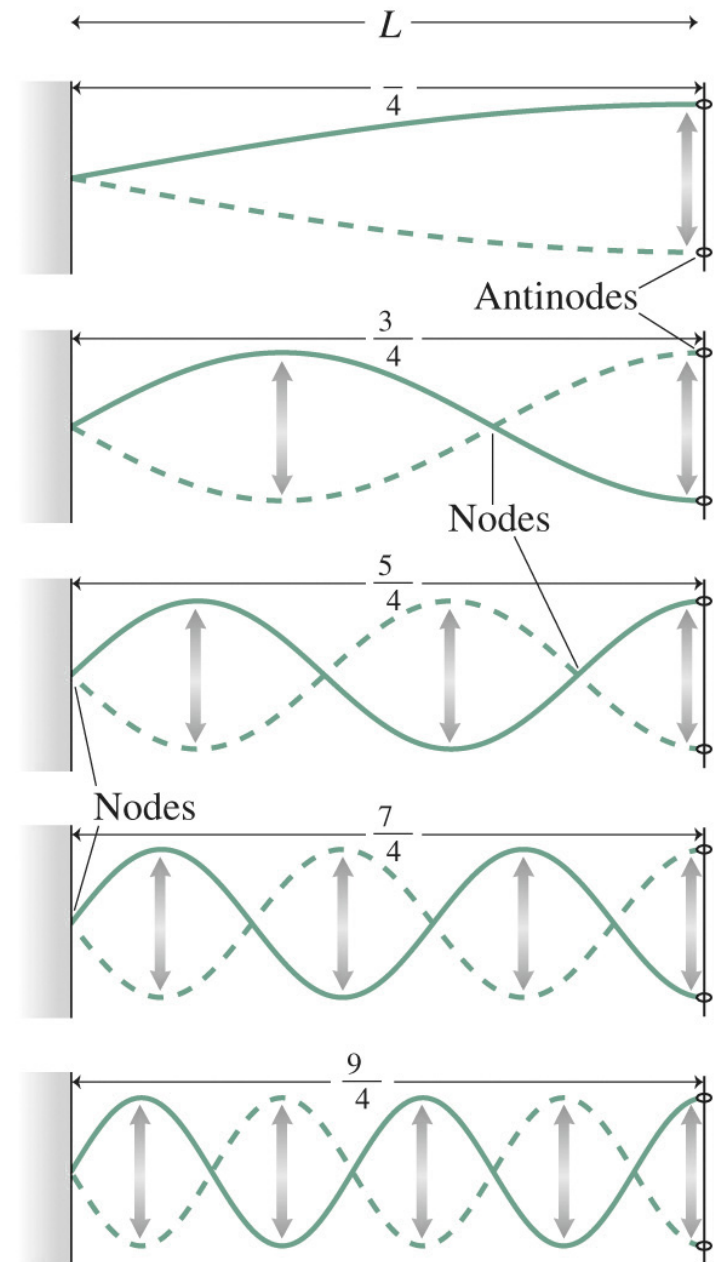
- Waves on a confined medium reflect at both ends here too
 - But with different boundary conditions
 - Node at one end, anti-node at the other
 - The length of the medium also restricts allowed wavelengths and frequencies to specific, discrete values here

On a string clamped at **one end**, the string length **must** be an odd integer multiple of a quarter-wavelength

$$L = \frac{m\lambda}{4}$$

with m an odd integer (1,3,5,...).

Examples: Reed woodwinds, organ pipes



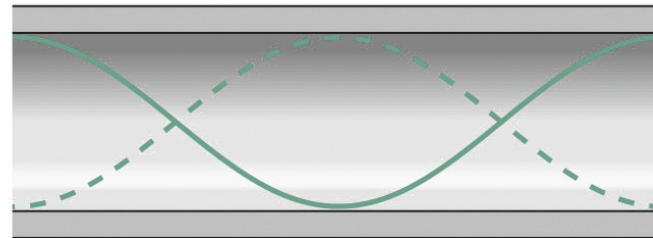
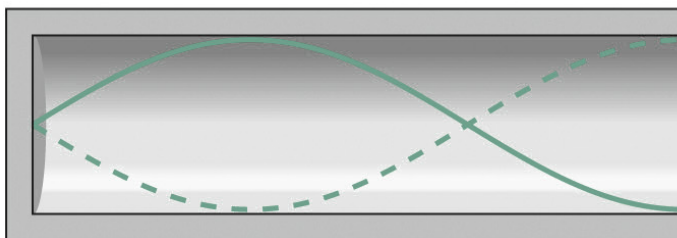
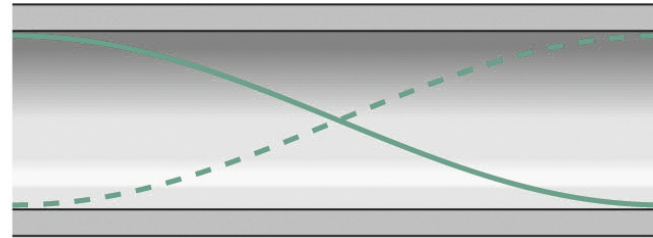
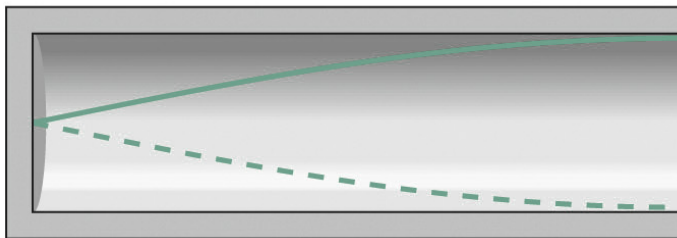
Ponderable

- A string 1 m long is clamped down tightly at one end and is free to slide up and down at the other. Which one of the following values is a possible wavelength for this string?
 - A. $4/3$ m
 - B. $3/2$ m
 - C. 2 m
 - D. 3 m



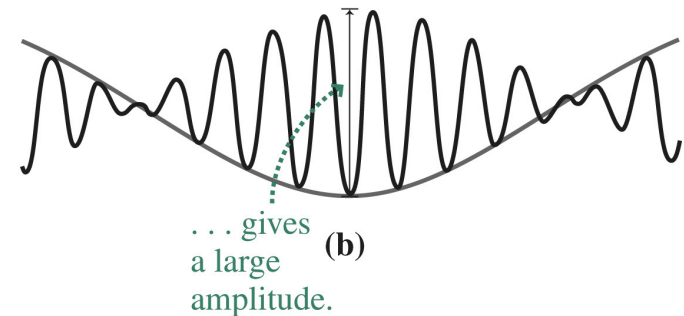
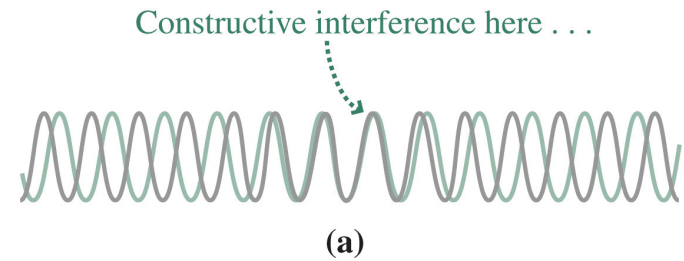
Standing Waves in Musical Instruments

- **Stringed instruments** are analogous to the strings fixed at both ends
 - The string length and density determines the allowed wavelengths and, together with the wave speed, the allowed frequencies.
 - Shorter instrument => shorter strings => higher pitch
- **Wind instruments** are analogous to the strings fixed at one end, with sound waves in their air columns.
 - Wind instruments are typically open at one end **or** both (e.g. flute).



Interference Phenomena

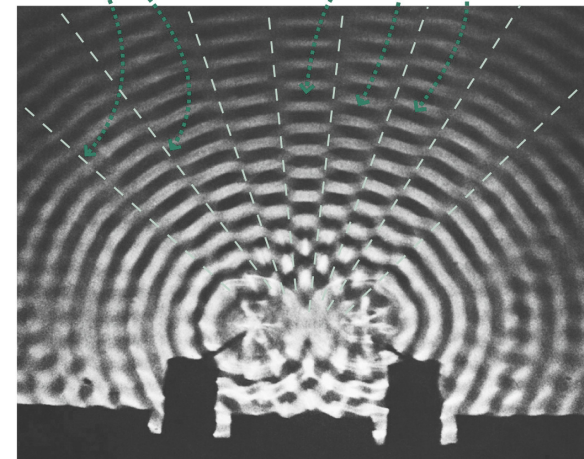
- When waves of slightly different frequencies interfere, they alternate between constructive and destructive interference.
 - This results in **beats** at the difference of their frequencies.
- Interference from two closely spaced sources results in patterns of high- and low-amplitude waves.
 - The photo shows such an interference pattern with water waves.



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Nodal lines:
destructive
interference

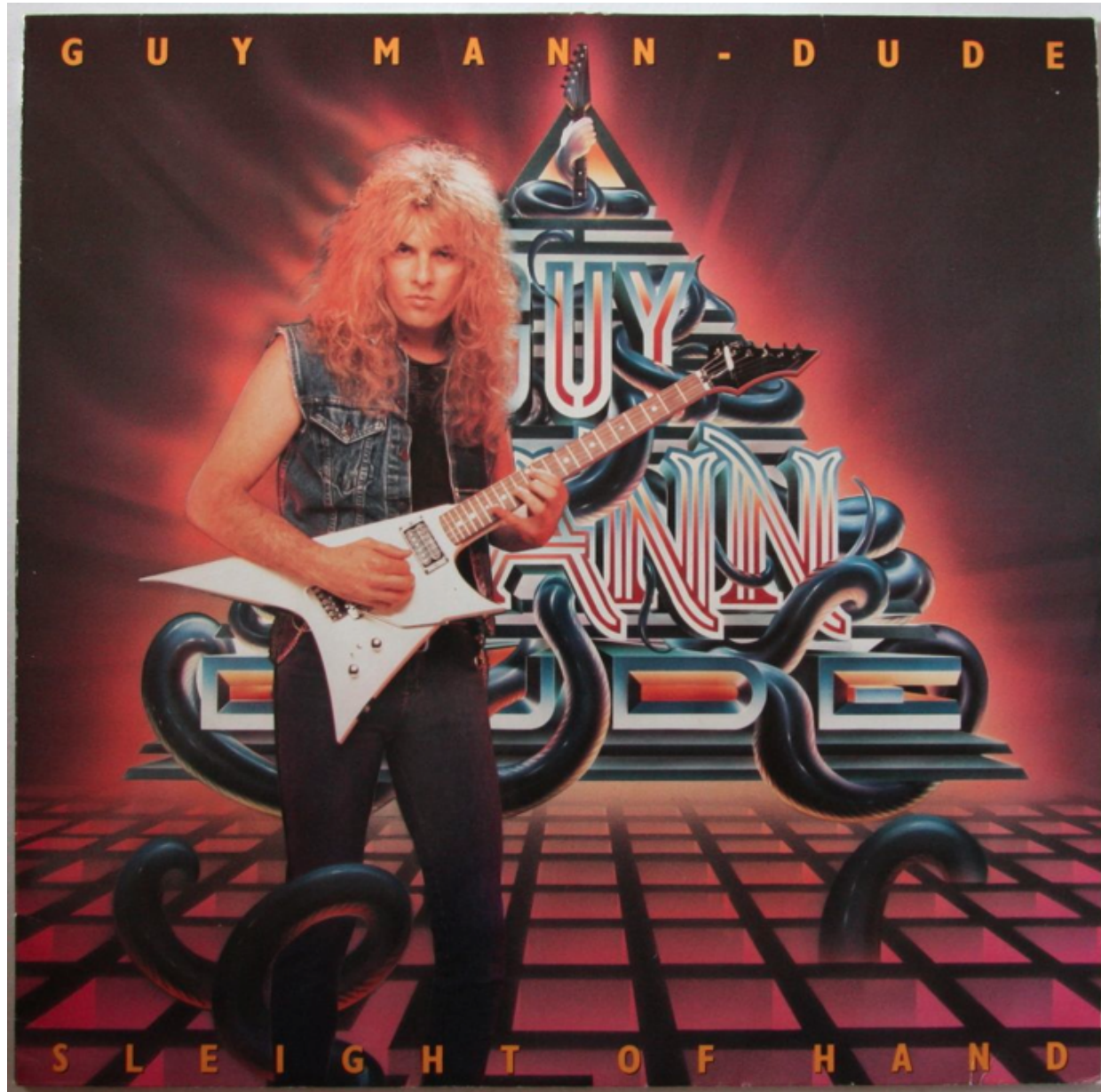
Large amplitude:
constructive
interference



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Musical Interlude



(Yes, your professor actually has this album)



Wave Power and Intensity

- The **power** carried by a wave is proportional to the wave speed and to the square of the wave amplitude.

- Details depend on the type of wave; for waves on a string, the average power is $\bar{P} = \frac{1}{2} \mu \omega^2 A^2 v$

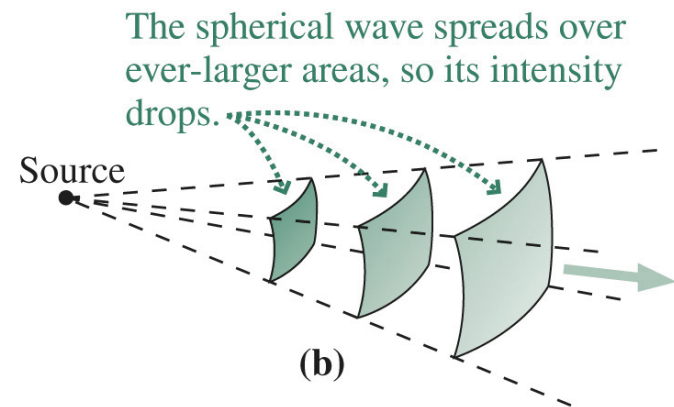
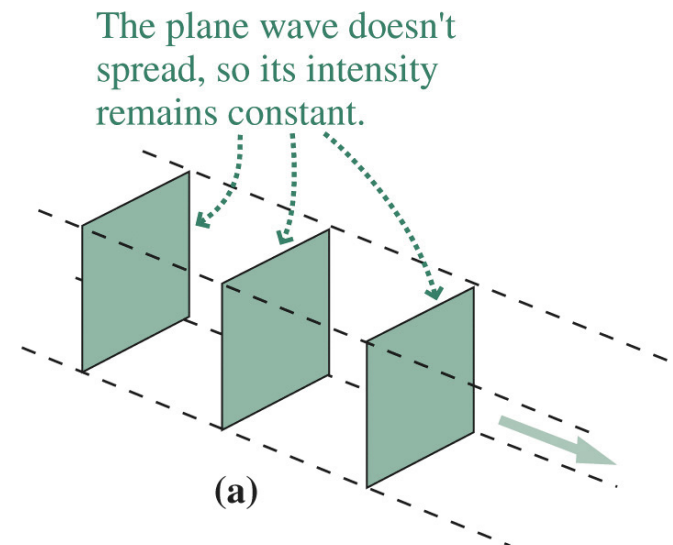
- Wave **intensity** is the power per unit area.

- In a **plane wave**, the intensity remains constant.

- The plane wave is a good approximation to real waves far from their source.

- A **spherical wave** spreads in three dimensions, so its intensity drops as the inverse square of the distance from its source:

$$I = \frac{\text{Power}}{\text{Area}} = \frac{\text{Power}}{4\pi r^2}$$



The Doppler Effect

- When a wave source moves through the wave medium, a stationary observer experiences a shift in wavelength and frequency.
 - The frequency increases for an approaching source.
 - The frequency decreases for a receding source.



Higher frequency
heard when source
is approaching

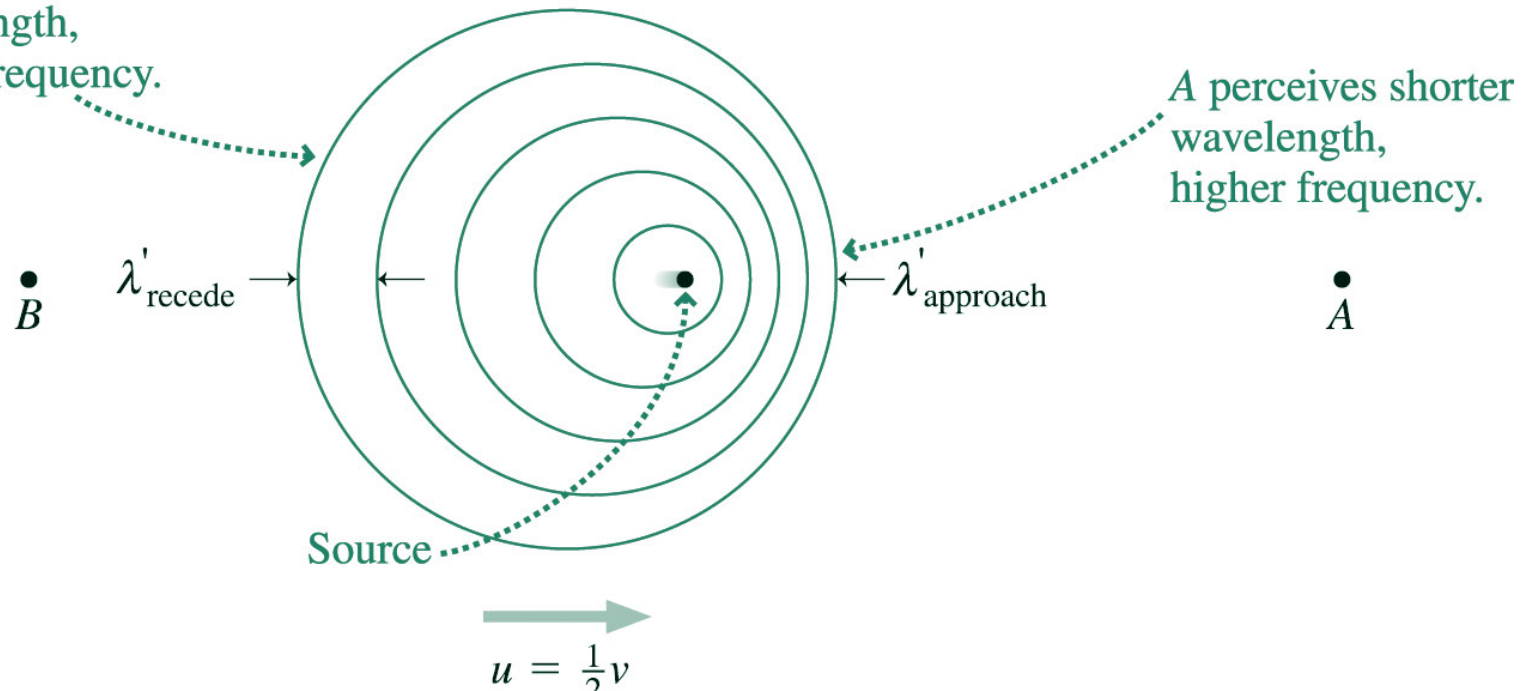
Lower frequency
heard when source
is receding



The Doppler Effect

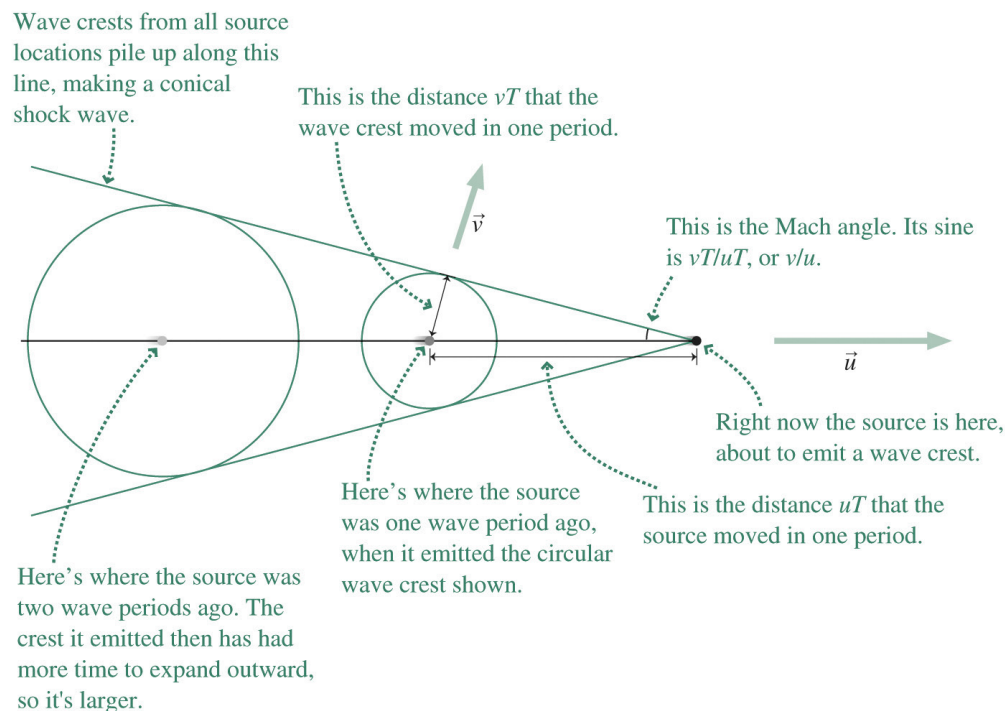
- The shifted frequencies are given by $f' = f/(1 \pm u/v)$
 - Here u is the source speed and v is the wave speed.
 - A similar effect occurs for a moving observer, but there's no wavelength shift.
 - The Doppler effect for light is similar but slightly different because light has no medium. The formula above applies only for speeds much less than light.

B perceives longer wavelength, lower frequency.



Shock Waves

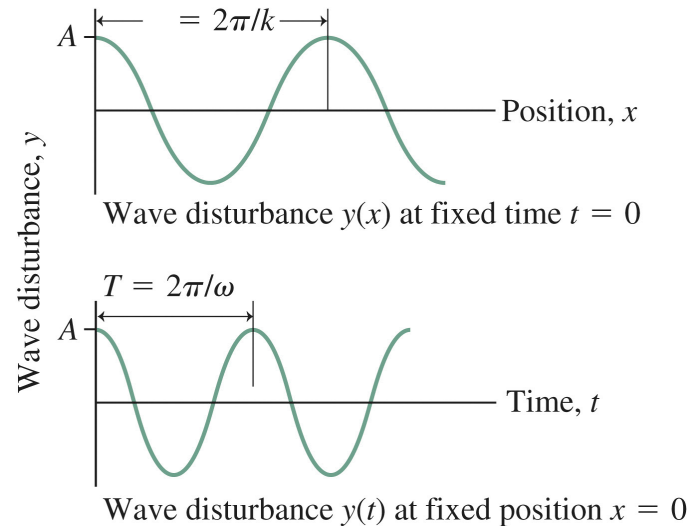
- **Shock waves** occur when a wave source moves through the medium at a speed u greater than the wave speed v .
 - The ratio u/v is called the **Mach number**.
 - **Mach angle** θ is defined by $\sin\theta = u/v$.
 - Examples include sonic booms from aircraft, wakes of boats, and astrophysical bodies moving through interplanetary and interstellar gas.



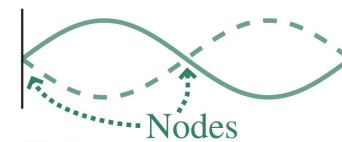
Summary

- A **wave** is a traveling disturbance that carries energy but not matter.
 - **Mechanical waves** involve the disturbance of a material medium.
 - These include sound waves.
 - **Electromagnetic waves**, including light, have no medium.
 - **Simple harmonic waves** are sinusoidal in shape.

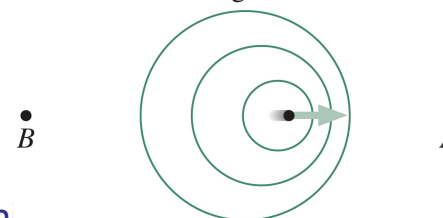
$$y(x,t) = A \cos(kx - \omega t)$$



- The speed of a wave follows from its frequency and wavelength or from its angular frequency and wavenumber: $v = \lambda f = \omega / k$.
- Important wave phenomena include
 - Reflection and refraction
 - Interference
 - Standing waves
 - The Doppler effect
 - Shock waves



Moving source



Ponderable

- A boat bobs up and down on a water wave moving a vertical distance of 2 m in 1 s. A wave crest moves a horizontal distance of 10 m in 2 s. What is magnitude of the wave speed?

A. 2.0 m/s
B. 5.0 m/s
C. 7.5 m/s
D. 10 m/s



Ponderable

- A traveling wave is described by

$$y(x, t) = 4.0 \cos(15x - 30t).$$

What is the wave speed?

- A. 4.0 m/s
- B. 2.0 m/s
- C. 0.5 m/s
- D. 120 m/s
- E. 60 m/s



Ponderable

- If the tension in a violin string is doubled, the fundamental frequency of the string is _____ times the original frequency.
 - A. 4
 - B. 2
 - C. $\sqrt{2}$
 - D. $1/\sqrt{2}$
 - E. $1/2$



Ponderable

- The displacement of a wave is described by

$$y(x, t) = 4.0 \sin(10\pi x + 30\pi t),$$

where x and y are in meters and t in seconds.
What is the wavelength?

- A. 5.0 m
- B. 0.20 m
- C. 3.0 m
- D. 0.33 m
- E. $(10\pi)^{-1}$ m

