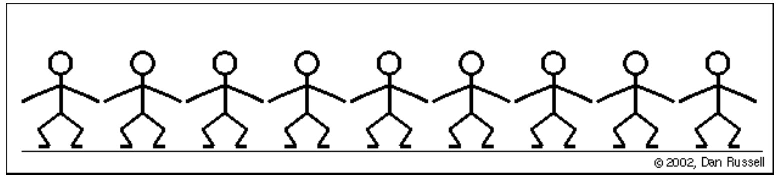
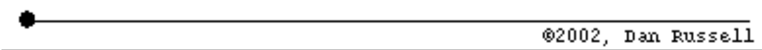
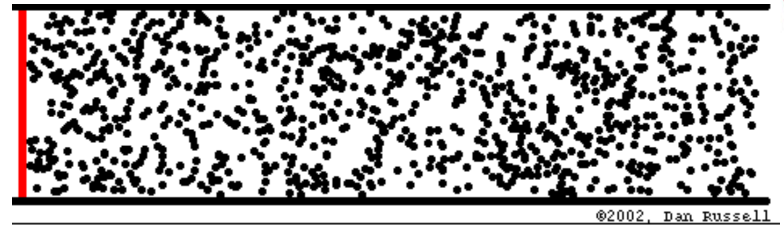






University Physics 226N/231N
Old Dominion University
Wave Motion, Interference, Reflection (Chapter 14)

Dr. Todd Satogata (ODU/Jefferson Lab)

http://www.toddsatogata.net/2012-ODU

Monday November 26, 2012

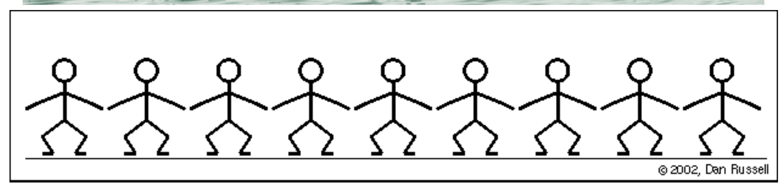
Happy Birthday to Bach, Norbert Wiener, Charles Schulz, Fred Pohl, and Tina Turner!
Happy Cyber Monday!

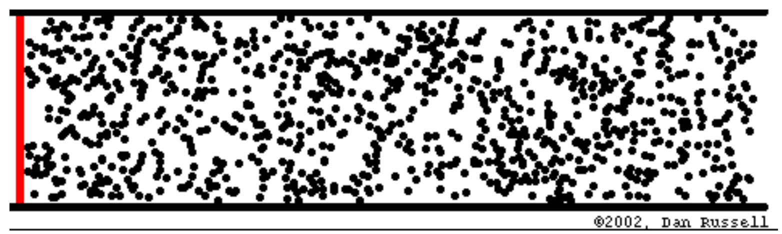
Midterm 3 will be returned Wednesday

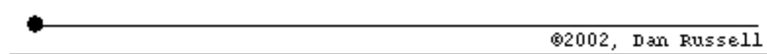
We have homework due Friday, and a quiz on Friday

Review of Wave Motion

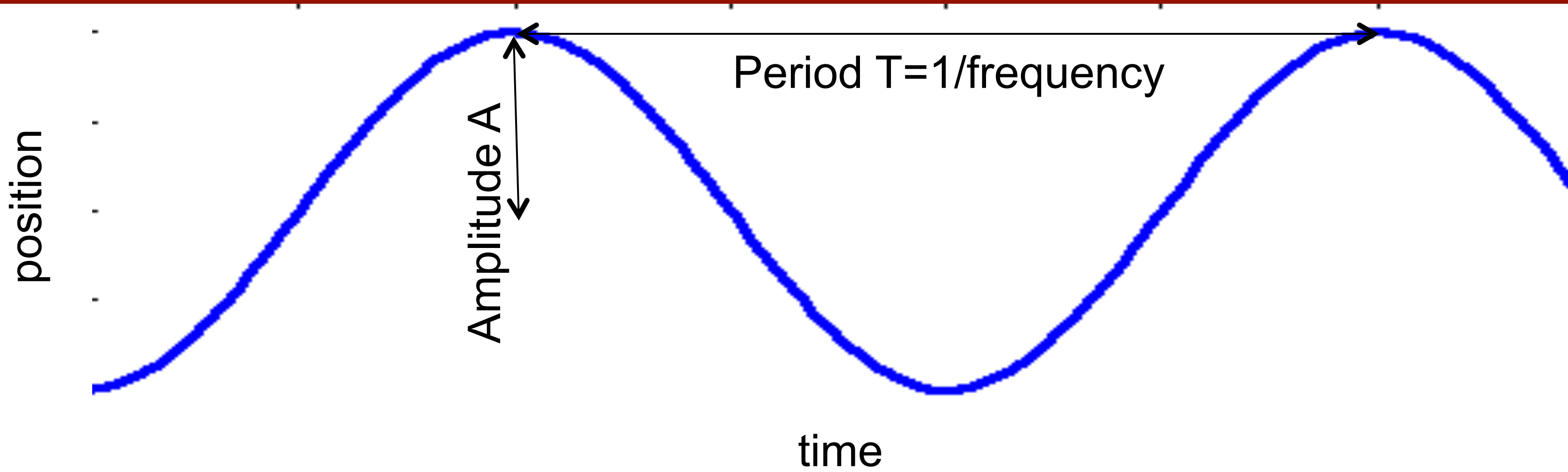
- Explain waves as traveling disturbances that transport **energy** but not **matter**
- Describe waves quantitatively
  - Frequency, period, wavelength, and amplitude
  - Wave number and velocity
- Describe example waves
  - Waves on strings
  - Sound waves
- Describe interference, reflection, and standing waves
- Describe the Doppler effect and shock waves







Period, Frequency, Amplitude, Phase



- We use standard terms to describe sine- and cosine-like curves
  - **Amplitude** A is the height of the curve below and above zero.
    - Amplitude has the same units as position
  - **Period** T is the time the curve takes for one oscillation
  - **Frequency** f=1/T (in units of Hz where 1 Hz is 1 cycle/s)
    - **Angular frequency** ω is often used where $\omega=2\pi f$
  - **Phase** ϕ_0 is phase of the curve at the time t=0
  - Then the periodic motion here is written as

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\boxed{x = A\sin(\omega t + \phi_0)} = A\sin(2\pi f t + \phi_0) = A\sin(2\pi t/T + \phi_0)$$

Simple Harmonic Waves

- Waves have dependency on **both** time t and location x

- A **simple harmonic wave** has a sinusoidal shape:

$$\boxed{y(x,t) = A\cos(kx - \omega t)}$$

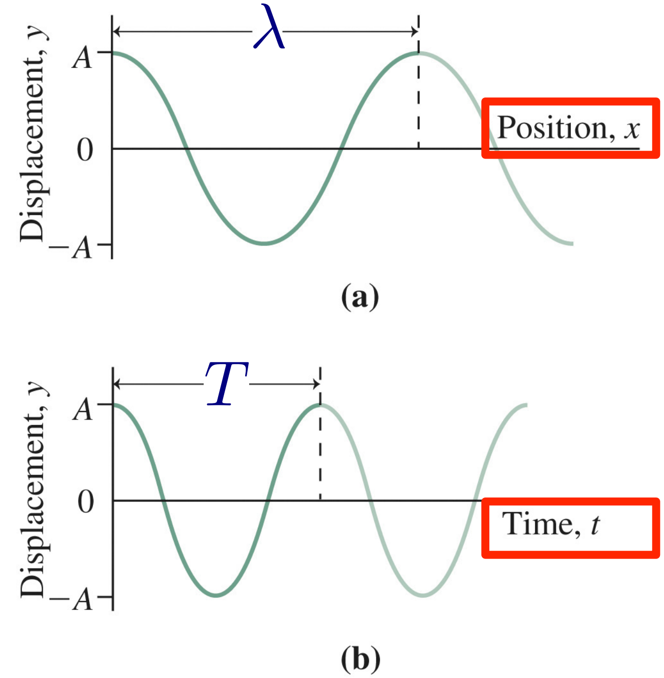
  - *y* measures the wave disturbance at position *x* and time *t.*
  - $k = 2\pi/\lambda$ is the **wave number**, a measure of the rate at which the wave varies in *space.*
  - $\omega = 2\pi f = 2\pi/T$ is the **angular frequency**, a measure of the rate at which the wave varies in *time.*
  - The **wave speed**, as mentioned before, is $v = \lambda f = \omega/k$.

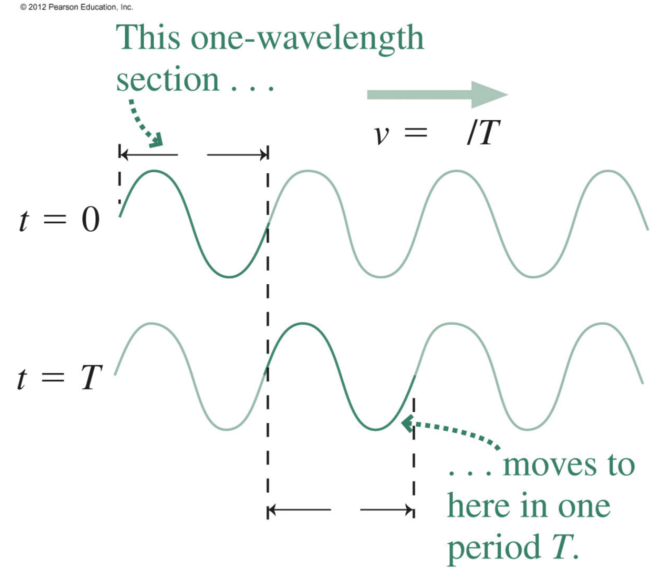
  - This is describing one "simple" wave in space and time
    - There may be many waves all interacting at once!

Properties of Waves

- **Wavelength** λ is the distance over which a wave repeats in **space**.
  - **Wave number** $\boxed{k = 2\pi/\lambda}$
- **Period** T is the **time** for a complete cycle of the wave at a fixed position:
  - **Frequency** $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$
- **Amplitude** A is the peak value of the wave disturbance.
- **Wave speed** is the rate at which the wave propagates:

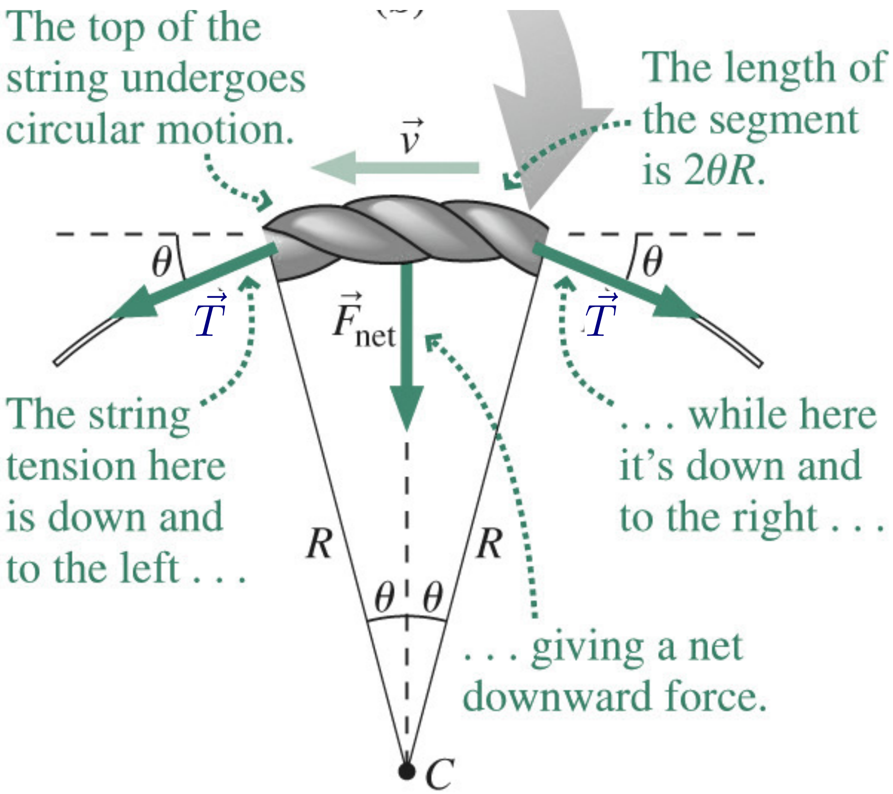
$$\boxed{v = \frac{\lambda}{T} = \lambda f}$$





Waves on Strings

- A classic example of wave motion is a transverse disturbance traveling along a rope of tension T
  - We also need the "mass per unit length", $\mu = M/L$
    - This μ is **not** related to coefficients of friction!!!



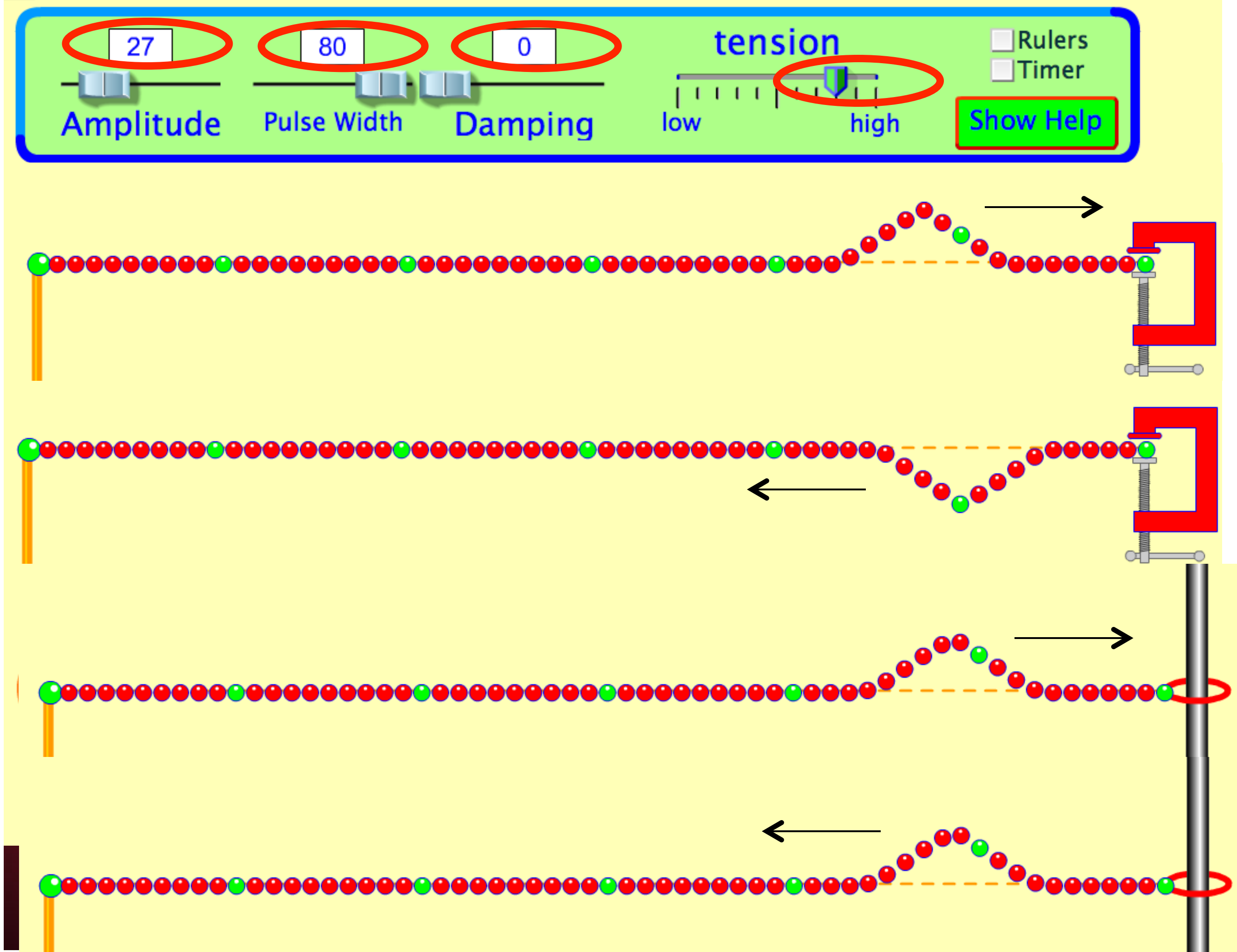
T provides a restoring force that makes a rope section go around the edge of the wave using our old equations for centripetal acceleration

We can find the velocity of the wave propagation, *v*

$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$

Tangible Reminder: Waves on Strings

http://phet.colorado.edu/sims/wave-on-a-string/wave-on-a-string_en.html



Set up as shown

Click “pulse” and watch how the pulse moves

What happens when it reaches the clamp?

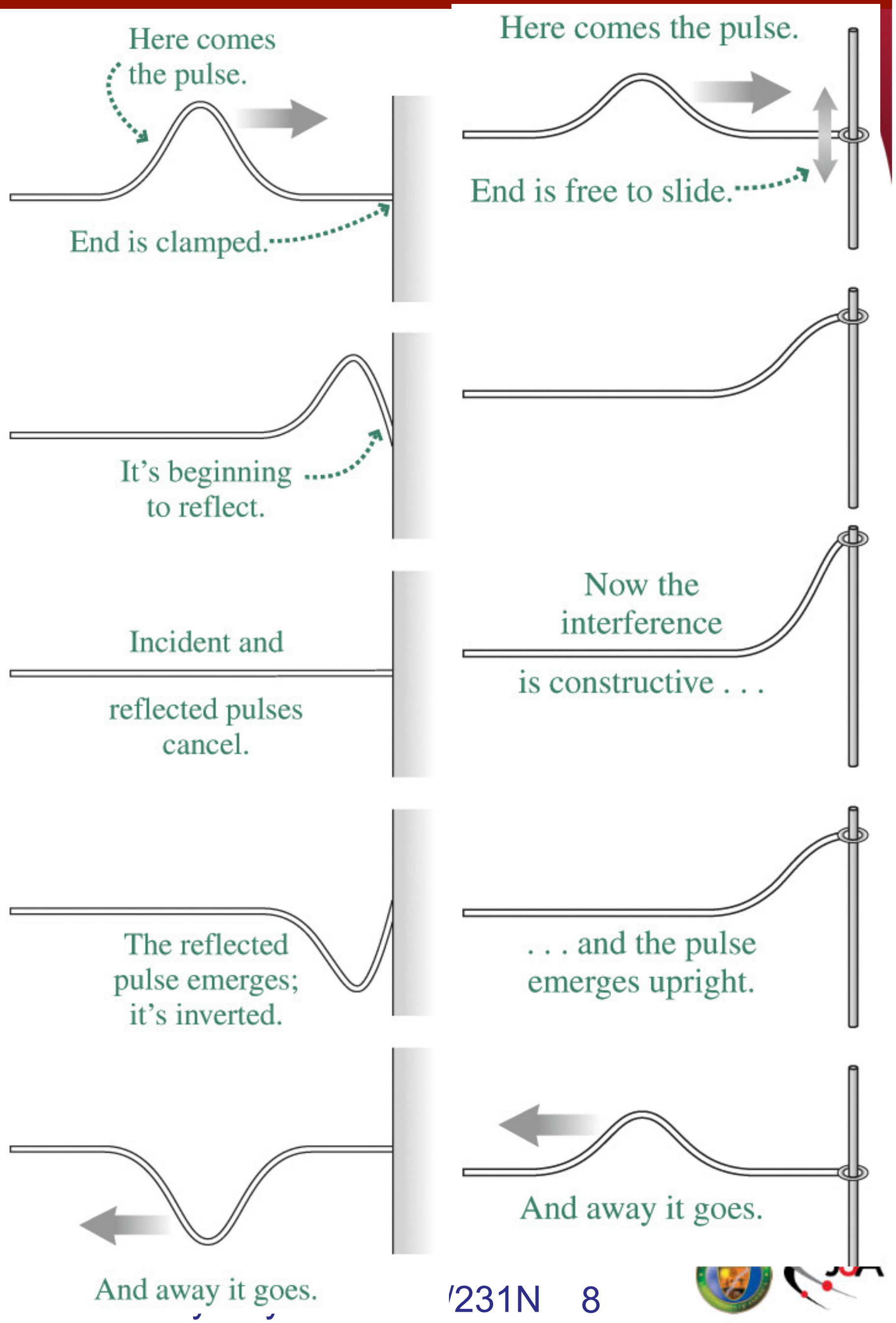
What happens if you change to “loose end”?

Pulse multiple times; how do the waves interact?

The interaction of multiple waves with each other is called **interference**

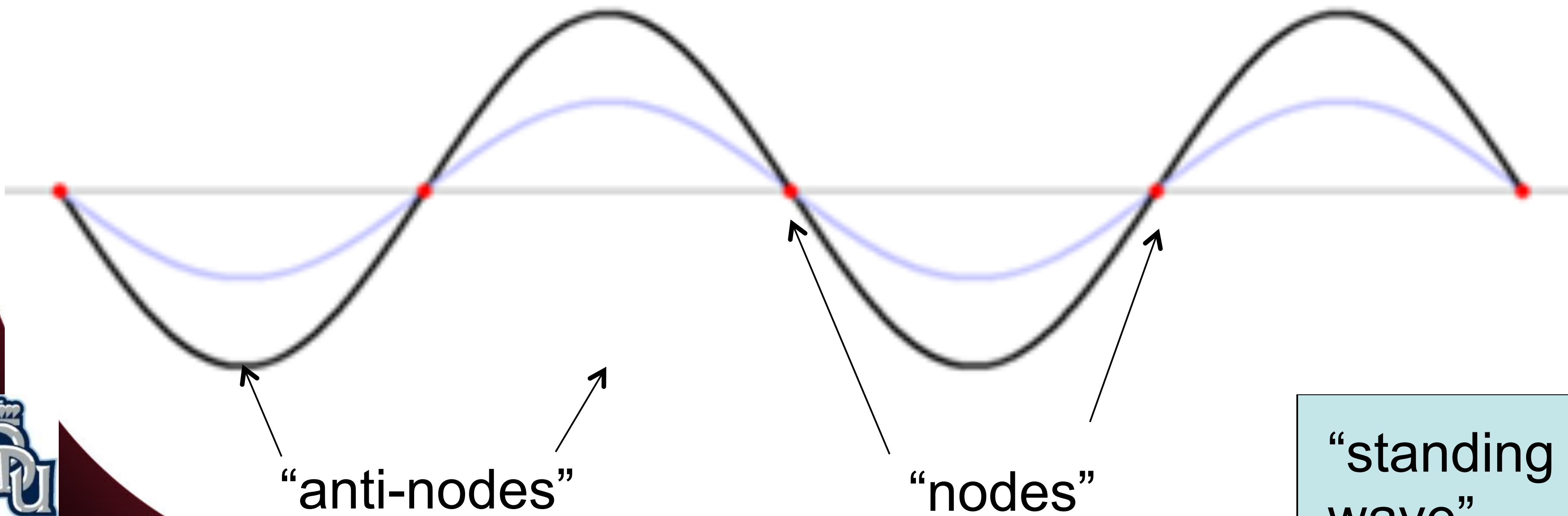
Wave Reflection

- Waves reflect at an interface with a different medium.
  - The outgoing wave interferes with the incoming wave.
  - The reflected wave is inverted, depending on properties of the second medium.
  - The diagram shows waves on a string reflecting at clamped and free ends.
- – More generally, waves are partially reflected and partially transmitted at an interface between different media.



Wave Interference

- Unlike particles, multiple waves can intersect in space/time.
- When they are, they **interfere.**
  - In most cases, the waves **superpose**, or simply add.
    - When wave crests coincide, the interference is **constructive.**
    - When crests coincide with troughs, the interference is **destructive.**
  - Here the red and blue waves add up to the black wave
    - Sometimes they cancel each other out (destructively interfere) and sometimes they add up (constructively interfere)



"standing wave"

Standing Waves: Fixed at Both Ends

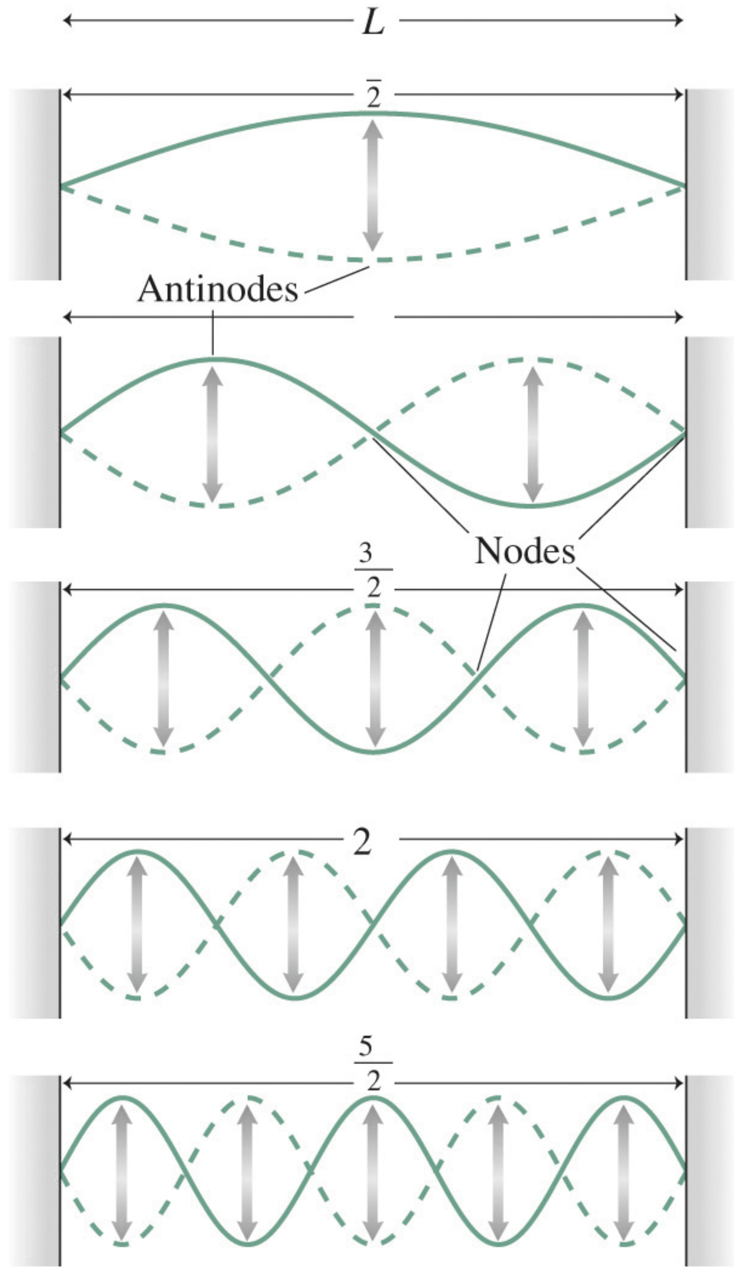
- Waves on a confined medium reflect (with flip) at both ends.
  - An example of **boundary conditions**: nodes at both ends!
  - The result is **standing waves** that oscillate but don't propagate.
  - The length of the medium restricts allowed wavelengths and frequencies to specific, discrete values.

On a string clamped at **both ends**, the string length **must** be an integer multiple of a half-wavelength

$$L = \frac{m\lambda}{2}$$

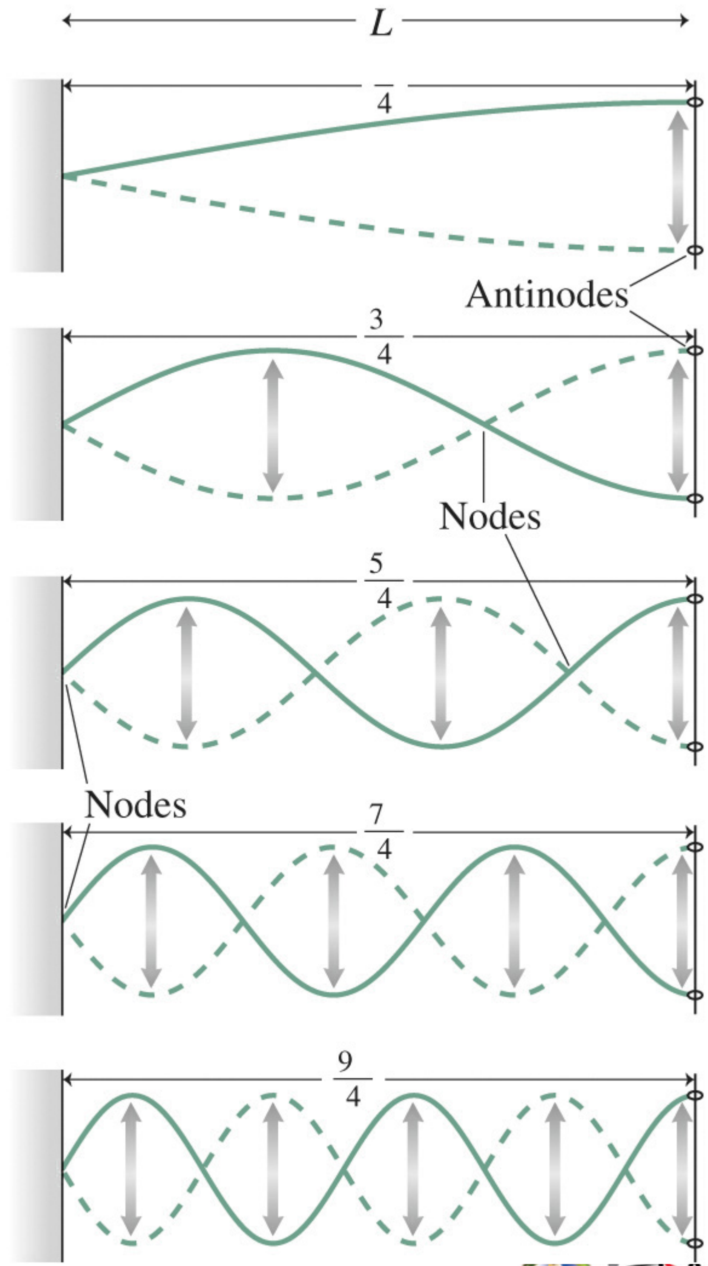
with *m* an integer (1,2,3,…)

Examples: Guitar/piano strings, drum heads



Standing Waves: Fixed at One End

- Waves on a confined medium reflect at both ends here too
  - But with different boundary conditions
  - Node at one end, anti-node at the other
  - The length of the medium also restricts allowed wavelengths and frequencies to specific, discrete values here



On a string clamped at **one end**, the string length **must** be an odd integer multiple of a quarter-wavelength

$$L = \frac{m\lambda}{4}$$

with *m* an odd integer (1,3,5,…).

Examples: Reed woodwinds, organ pipes

Ponderable

- A string 1 m long is clamped down tightly at one end and is free to slide up and down at the other. Which one of the following values is a possible wavelength λ for this string?

A. 4/3 m

B. 3/2 m

C. 2 m

D. 3 m

$$L = \frac{m\lambda}{4} \quad \text{where } m = 1, 3, 5, 7, \cdots$$

Ponderable: Answer

- A string 1 m long is clamped down tightly at one end and is free to slide up and down at the other. Which one of the following values is a possible wavelength λ for this string?

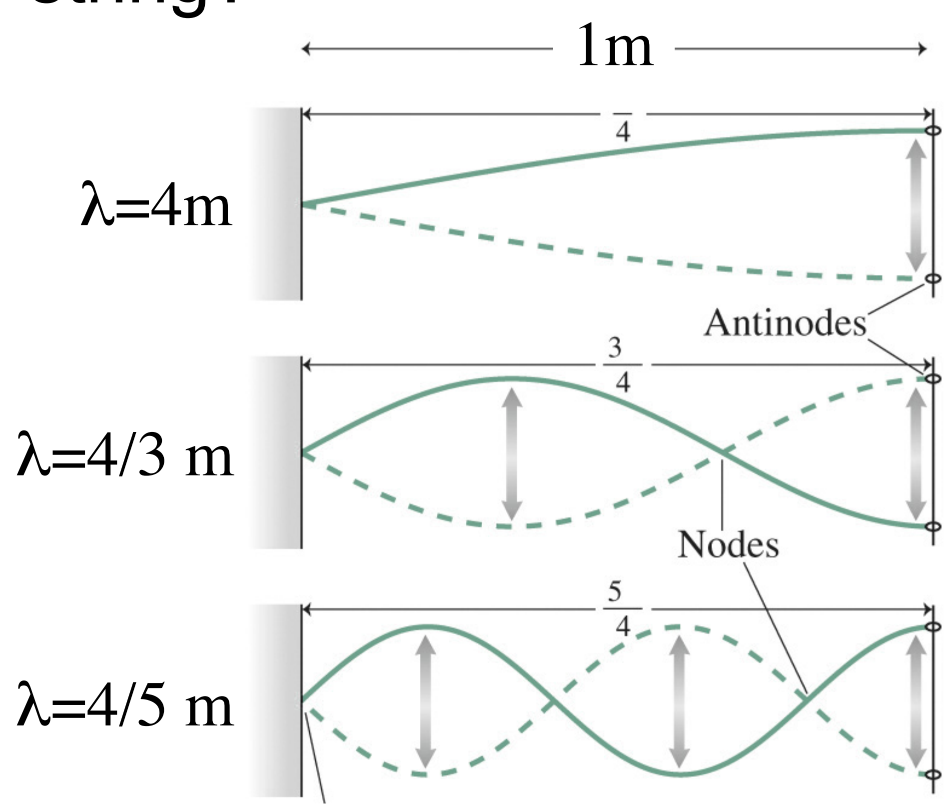
A. 4/3 m

B. 3/2 m

C. 2 m

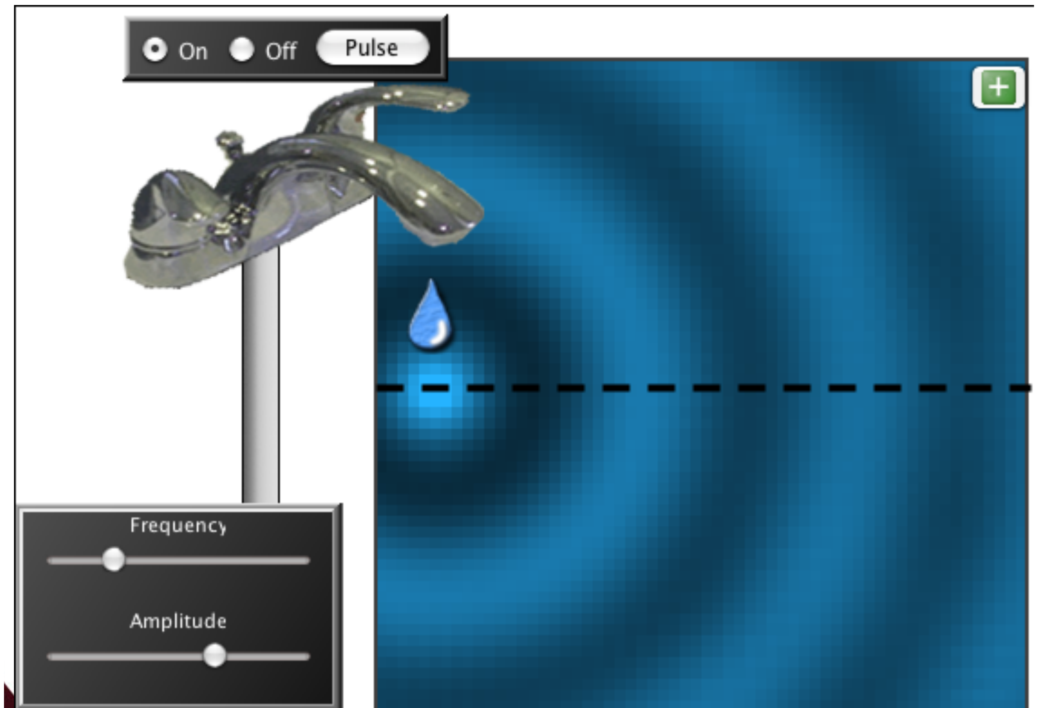
D. 3 m

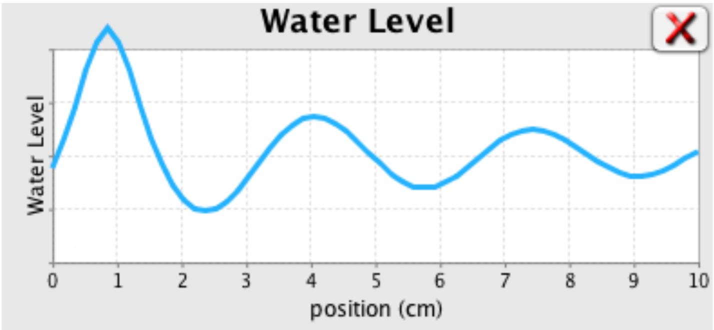
$$L = \frac{m\lambda}{4} \qquad \lambda = \frac{4L}{m}$$



2D Water Waves

- Wave motion we've discussed up to now is **one-dimensional**
- But waves also travel and spread through space
  - Examples: Sound waves, water waves, light from a point source
  - These waves travel outwards and dissipate as they spread





Height drops as wave front expands

http://phet.colorado.edu/en/simulation/wave-interference

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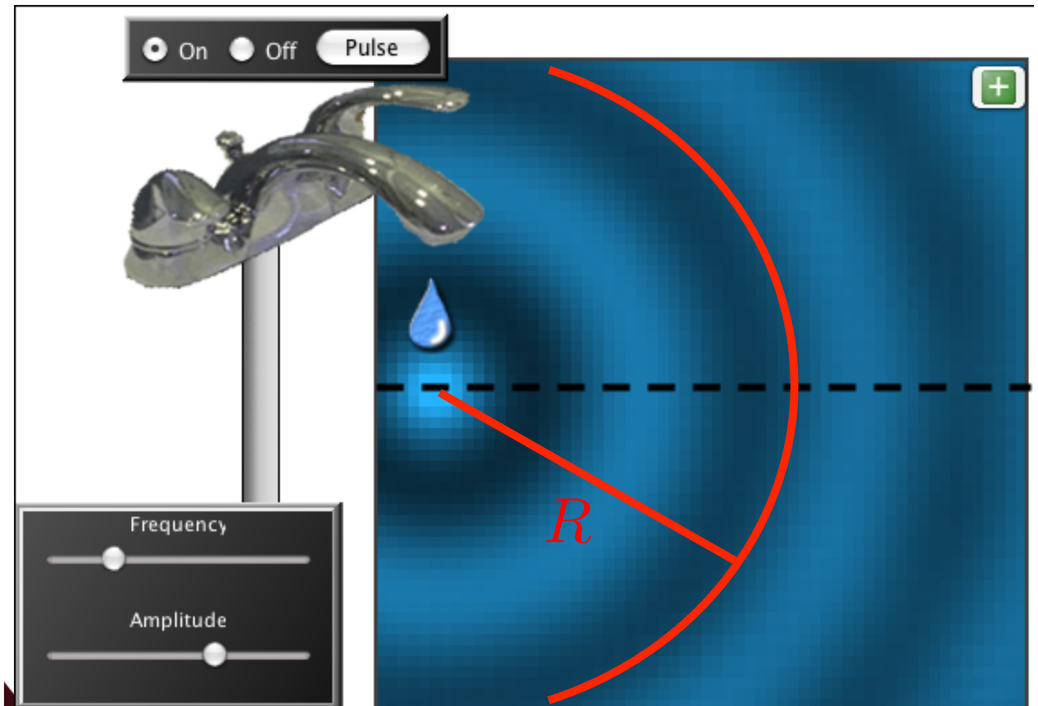
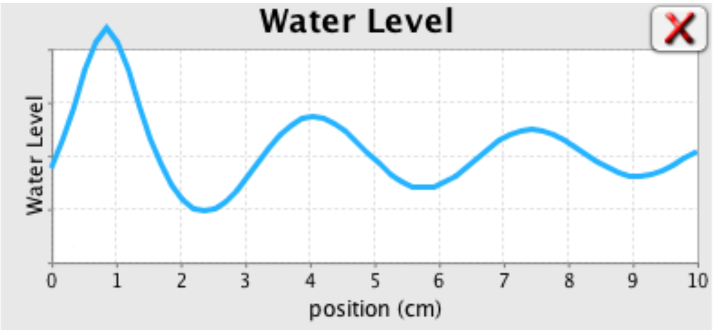




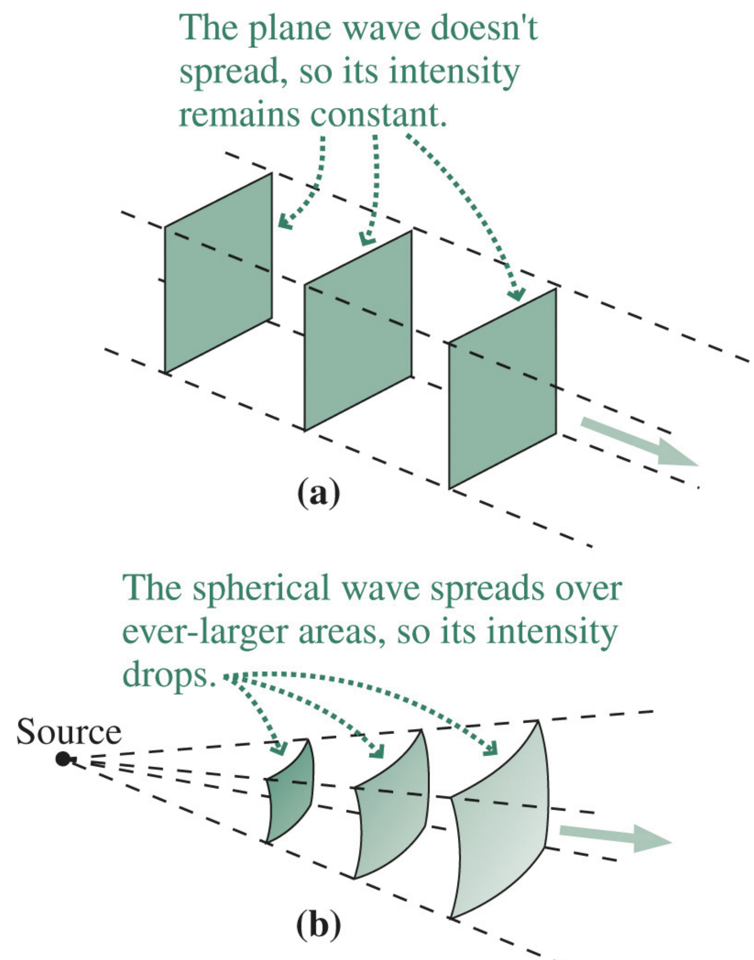
Height drops as wave front expands

wave front length $2\pi R$

http://phet.colorado.edu/en/simulation/wave-interference

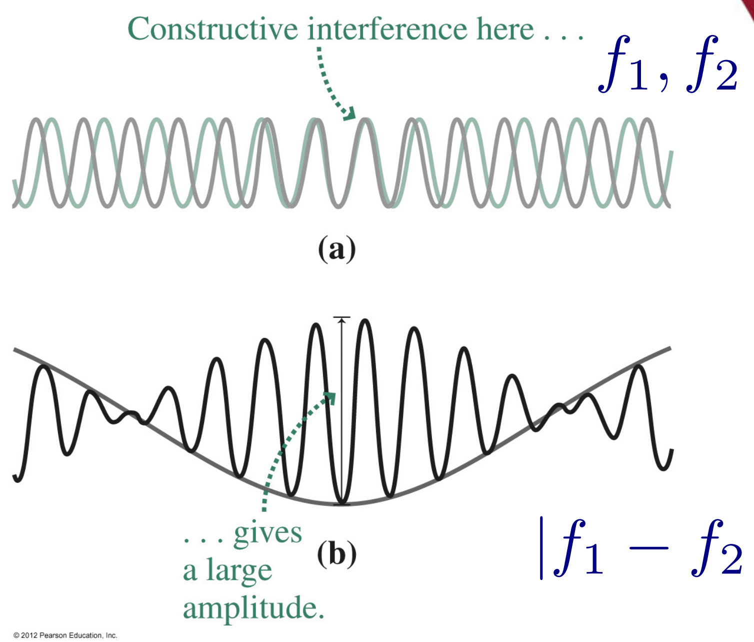
Wave Power and Intensity

- The **power** carried by a wave is proportional to the wave speed and to the square of the wave amplitude.
  - Details depend on the type of wave; for waves on a string, the average power is $\bar{P} = \frac{1}{2}\mu\omega^2 A^2 v$
- Wave **intensity** is the power per unit area.
  - In a **plane wave**, the intensity remains constant.
    - The plane wave is a good approximation to real waves far from their source.
  - A **spherical wave** spreads in three dimensions, so its intensity drops as the inverse square of the distance from its source: $$I = \frac{\text{Power}}{\text{Area}} = \frac{\text{Power}}{4\pi r^2}$$

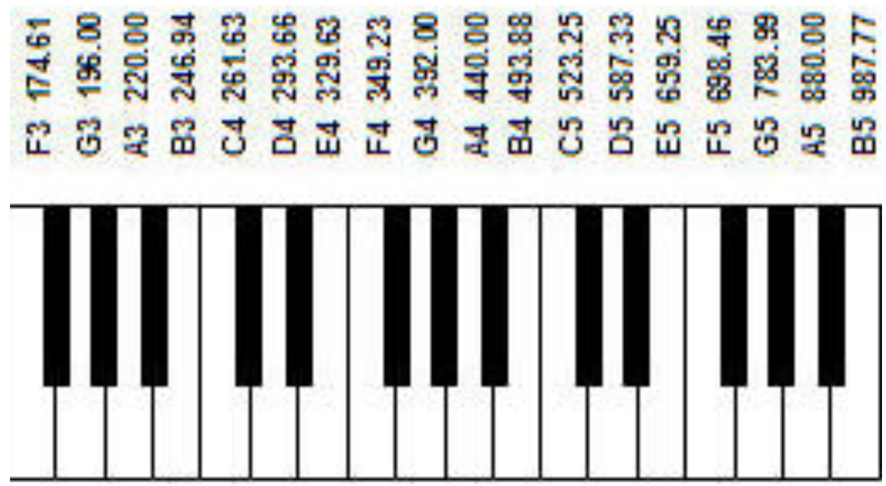


Interference Phenomena

- When waves of slightly different frequencies interfere, they alternate between constructive and destructive interference.
  - This gives **beats** at the difference of the frequencies. $f_{\mathrm{beat}} = |f_1 - f_2|$
  - Recall the guitar tuning waveform beat pattern



- Octave: doubling of frequency
- Human hearing: 20 Hz – 20 kHz
- Piano keyboard: 27.5 Hz to 4186 Hz (7+ octaves)



2D Wave Interference

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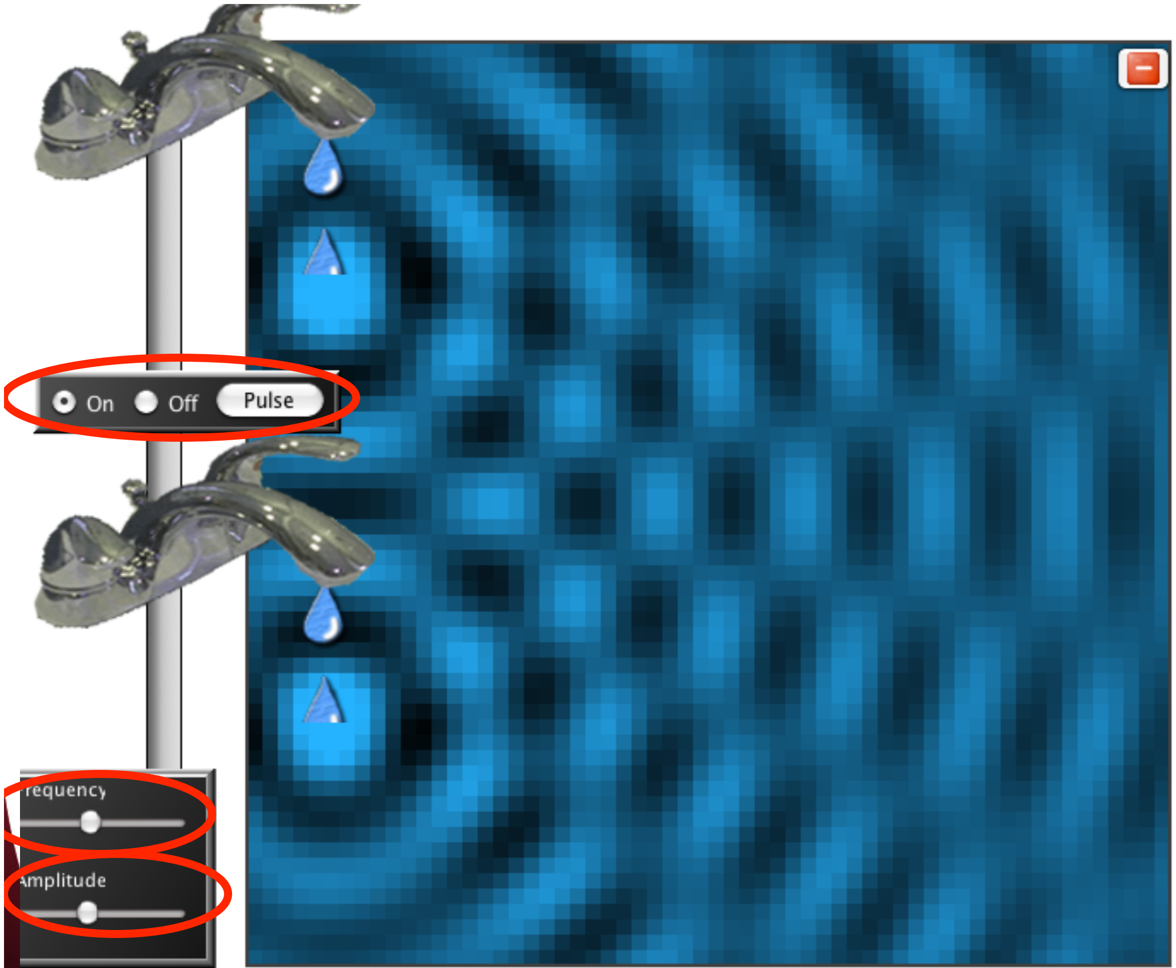

- Waves can also interfere in 2D and 3D
- Can create nodes and anti-nodes in certain directions
  - Directions/angles depend on wavelengths and spacing between sources
- Always symmetric
  - Center line is always an anti-node (constructive interference)

- Experiment with interference patterns made by one or two slits, and wave reflections from walls

2D Wave Interference

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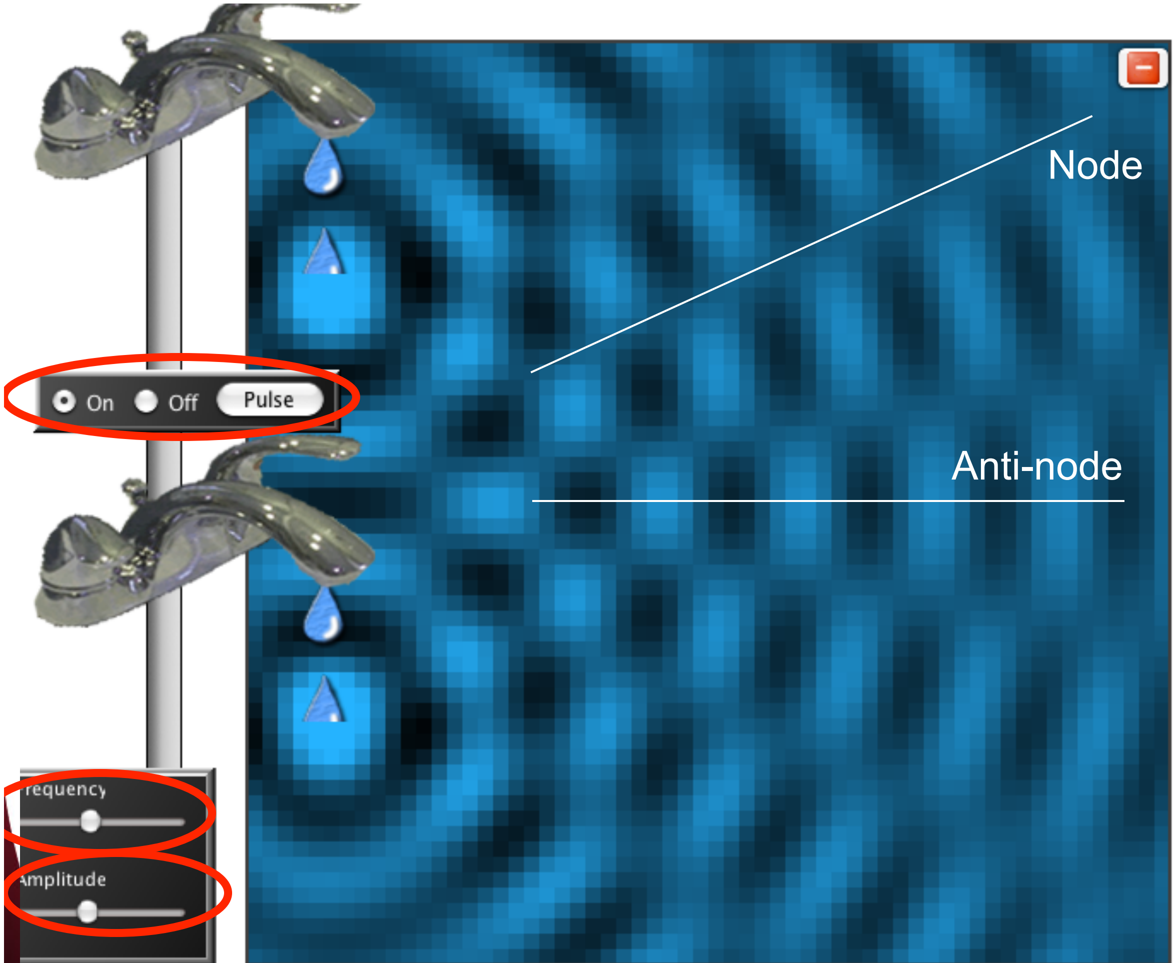

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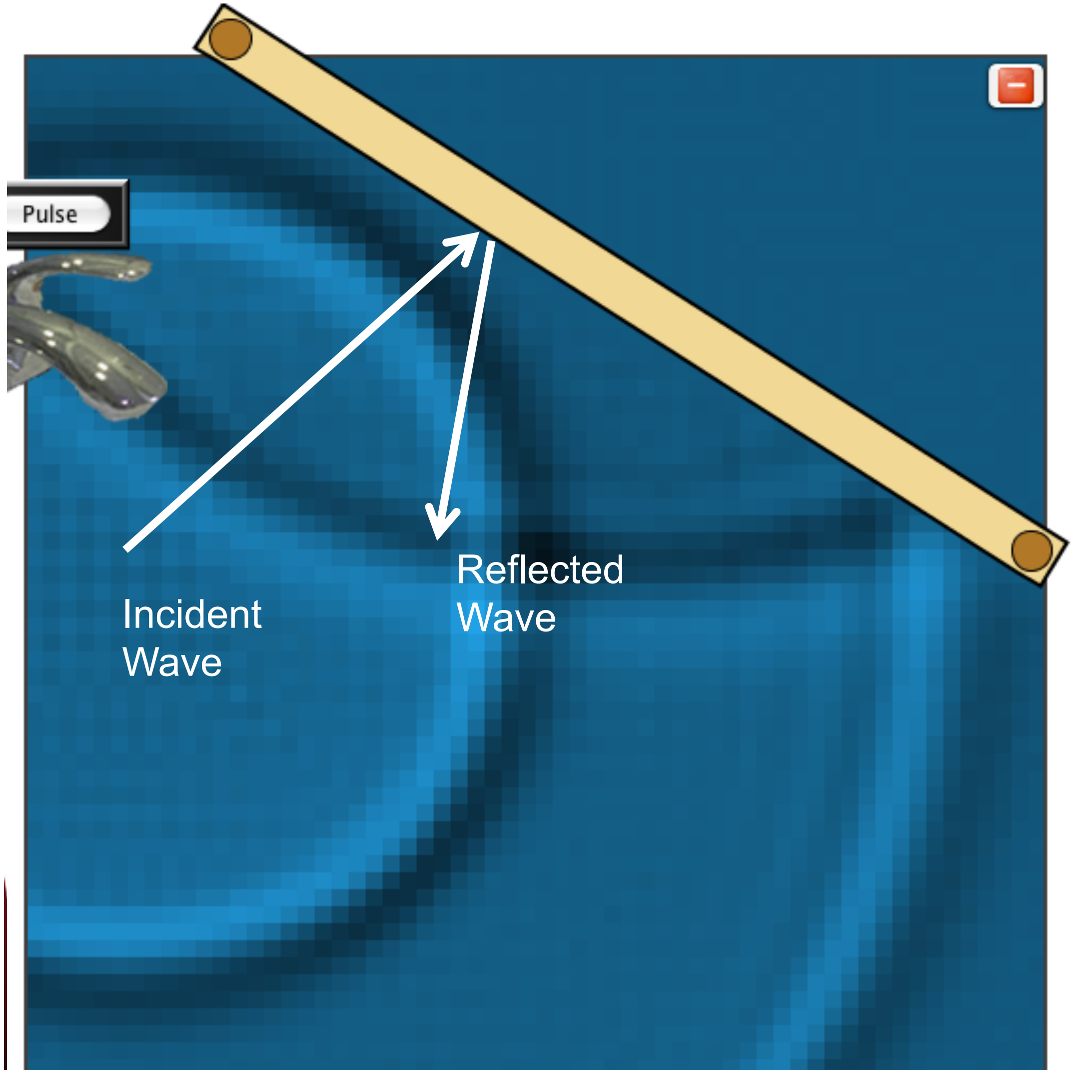
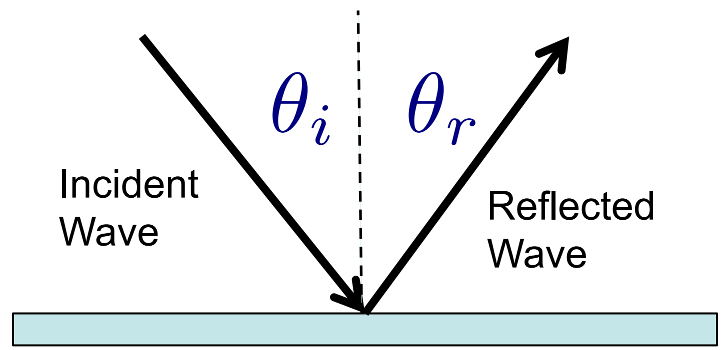
2D Wave Reflection: Basic Optics



- Waves (such as light) reflect off surfaces at the same angle of incidence to the normal direction to the surface



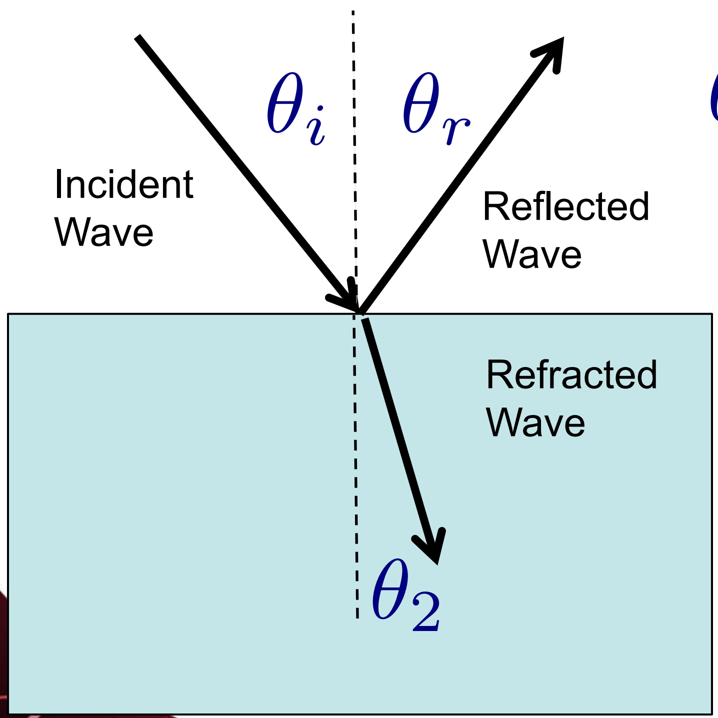
$$\theta_i = \theta_r$$

- A perfect mirror produces 100% reflection

But no mirror is perfect…

2D Wave Refraction: Basic Optics

- We know that water "bends" light
- In general, waves change angle when they enter a new medium
  - Due to difference in wave speed between the two media
  - This phenomena is called **refraction**



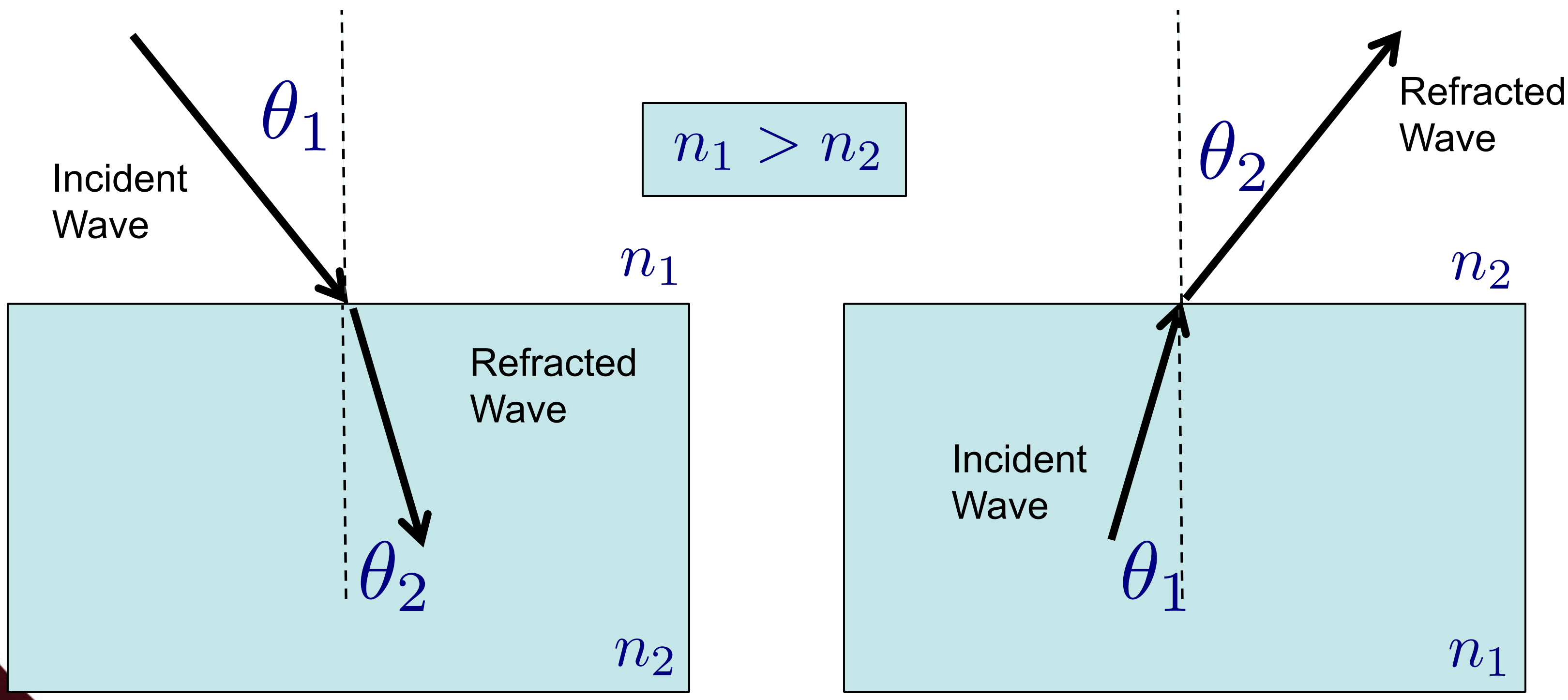
$$\theta_i = \theta_r$$

Wave angles **towards normal** when entering a **denser** medium (with lower wave speed)

2D Wave Refraction: Snell's Law

- Each wave medium is characterized by an **index of refraction** n
  - n is inversely proportional to the speed of the wave in a medium
- **Snell's law** of refraction:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$



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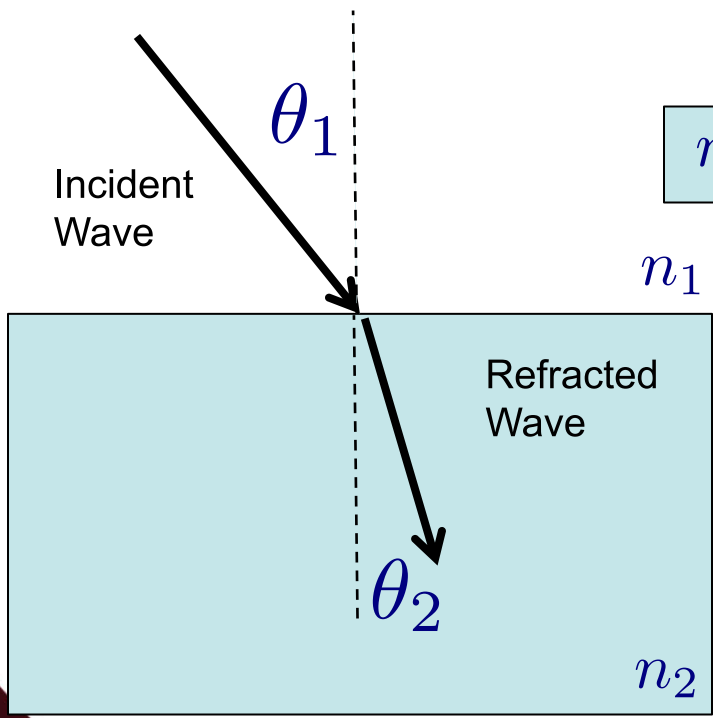
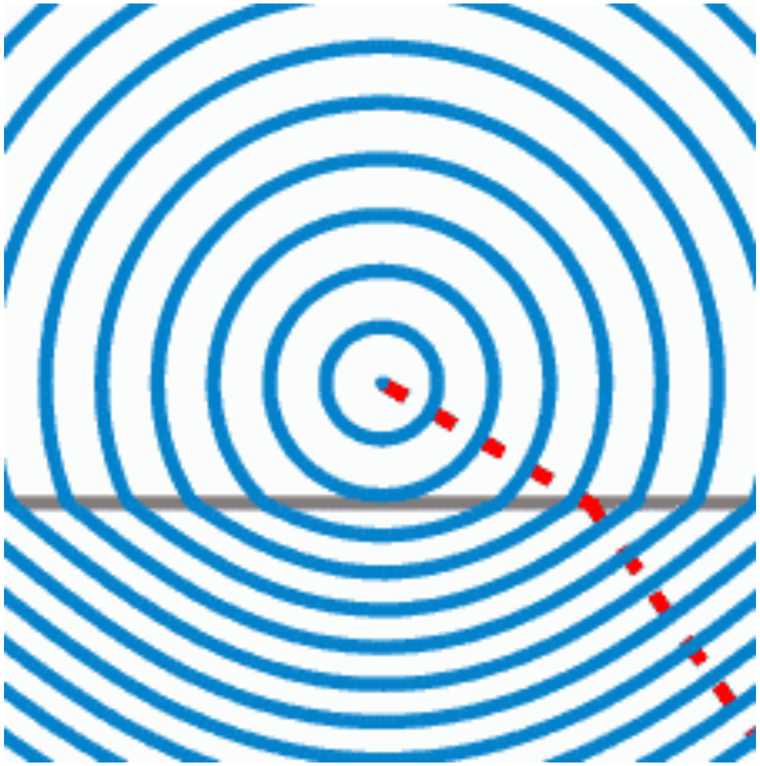
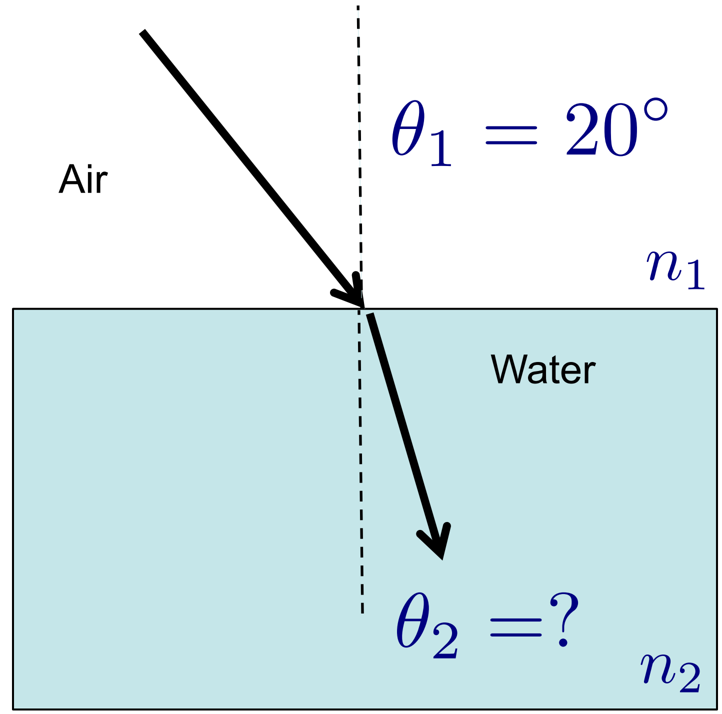


$$n_1 > n_2$$



2D Wave Refraction: Indices of Refraction

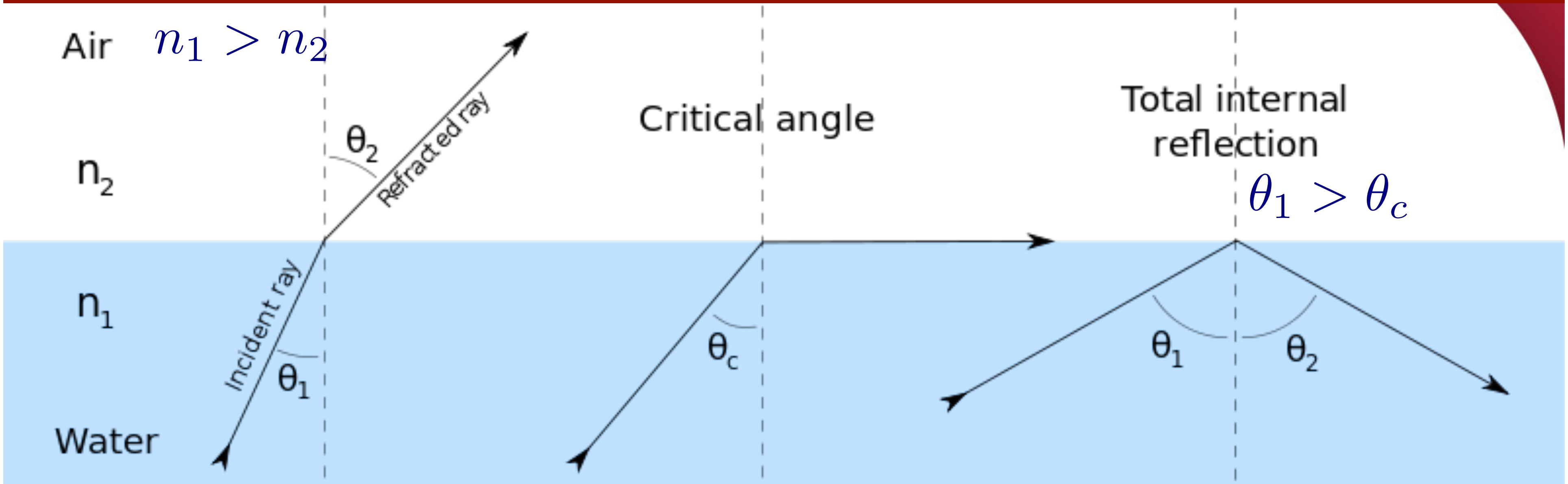
- Some common indices of refraction
  - Air/vacuum: ~1
  - Water: 1.333
  - Acrylic: 1.49
  - Pyrex glass: 1.470
  - Germanium: 4 (!)



$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} \qquad \sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = \frac{\sin(20^\circ)}{1.333} = 0.2566$$

$$\theta_2 = 14.87^\circ$$

Total Internal Reflection: Critical Angle



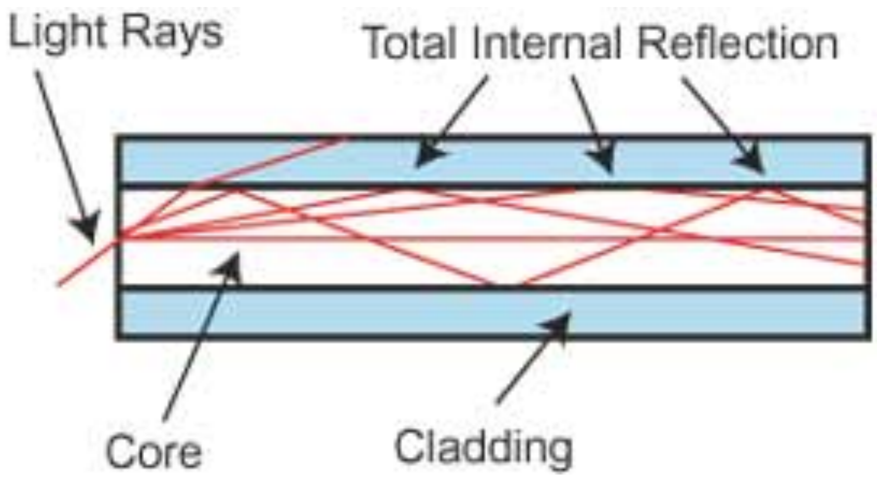
- A wave can totally reflect when interacting with a interface with a lower index of refraction
  - What happens when $\theta_2 = 90^\circ, \ \sin\theta_2 = 1$ in Snell's law?

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1} \qquad \sin\theta_1 = \boxed{\sin\theta_c = \frac{n_2}{n_1}}$$

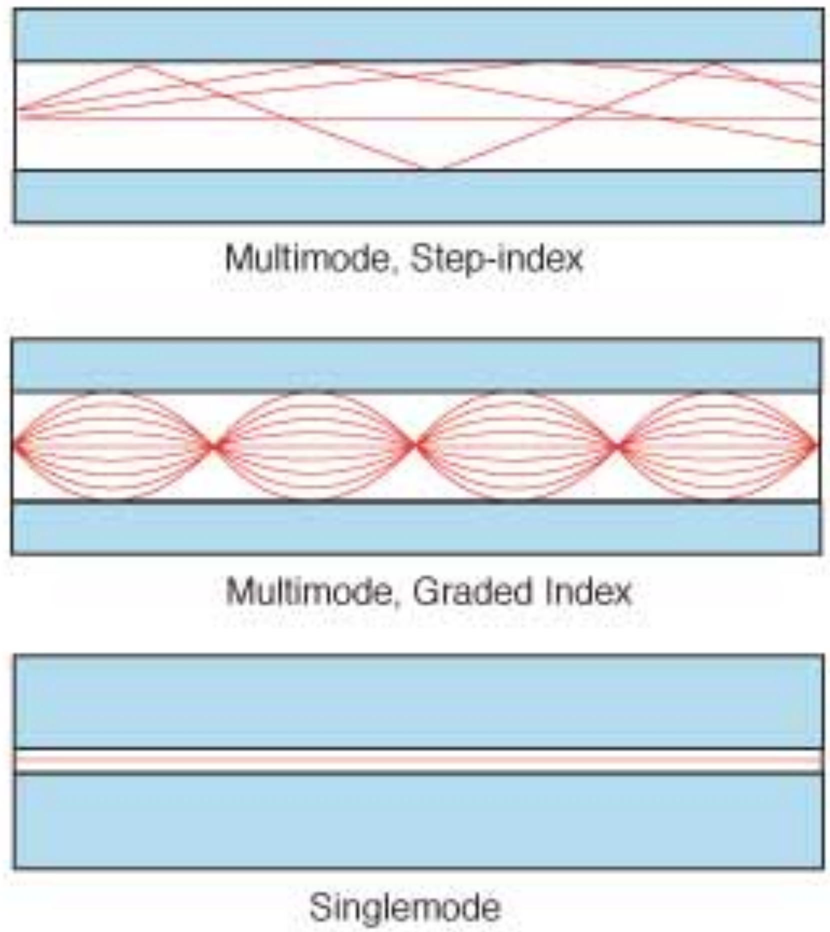
  - Example: Air n=1, water n=1.333 $\Rightarrow \quad \theta_c = 48.6^\circ$

Total Internal Reflection: Underwater Surface Mirror

Total Internal Reflection: Fiber Optics



Types of Fiber Optics



- The basis of most modern telecommunications
- **High bandwidth**: few GHz up to several hundred thousand GHz!
- But the underlying principles are based on **total internal reflection** of different wavelengths of light in the fiber

http://www.jimhayes.com/lennielw/fiber.html