Jefferson Lab GSPDA Summer Lecture Series

Introduction to Accelerators (and Applied Relativity and E&M)

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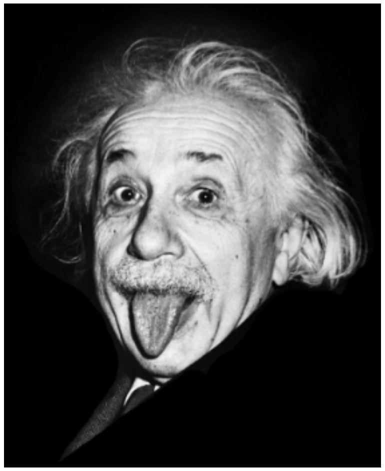
Applied Special Relativity



- Accelerators: applied special relativity
  - Subatomic particles traveling near light speed
  - Kinetic energy >> rest mass energy
- Relativistic parameters:

$$\beta \equiv \frac{v}{c} \qquad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \sqrt{1-1/\gamma^2}$$

  - γ=1 (classical mechanics) to ~2.05x10^5 (to date) (where??)
    v=0.99999999997 c
- Total energy U, momentum p, and kinetic energy W

$$U = \gamma mc^2 \qquad p = (\beta\gamma)mc = \beta\left(\frac{U}{c}\right) \qquad W = (\gamma - 1)\,mc^2$$

Relative Relativity

WolframAlpha

LEP energy

Input interpretation:

LEP (Large Electron Positron Collider) | ce

Result:

208 GeV (gigaelectronvolts)

Unit conversions:

0.208 TeV (teraelectronvolts)

2.08×10^{11} eV (electronvolts)

0.03333 μJ (microjoules)

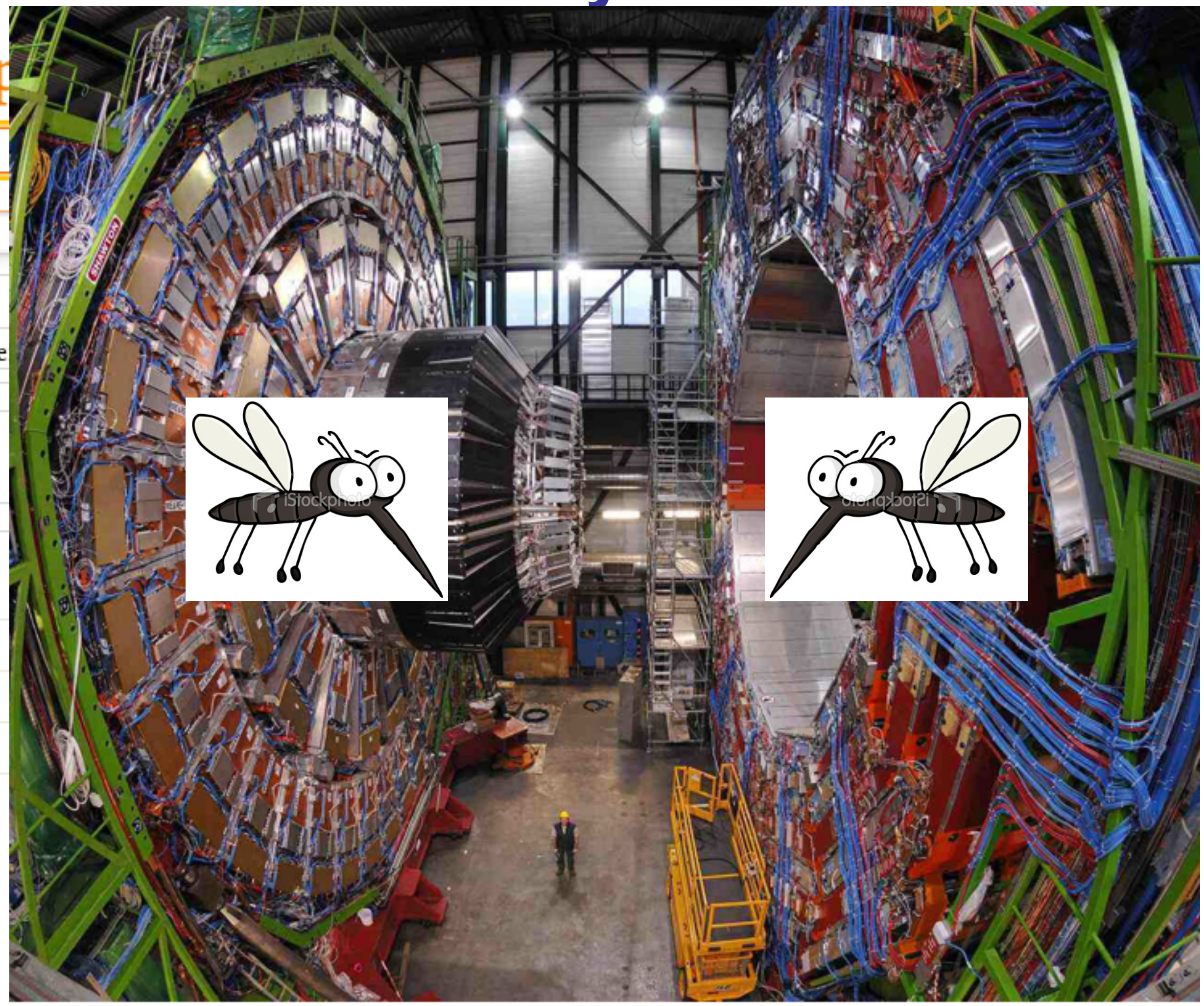
3.333×10^{-8} J (joules)

0.3333 ergs (unit officially deprecated)

Comparisons as energy:

$\approx$ (0.21 $\approx$ 1/5) $\times$ approximate kinetic energy of a flying mosquito ($\approx 1.6 \times 10^{-7}$ J)

$\approx$ 2.2 $\times$ mass-energy equivalent of a Z boson ($\approx 1.5 \times 10^{-8}$ J)



“Convenient” Units

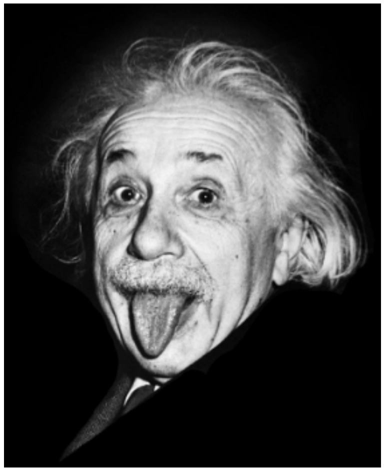
$$1\,\mathrm{eV} = (1.602 \times 10^{-19}\,\mathrm{C})(1\,\mathrm{V}) = 1.602 \times 10^{-19}\,\mathrm{J}$$
$$1\,\mathrm{MeV} = 1.602 \times 10^{-13}\,\mathrm{J}$$
$$1\,\mathrm{GeV} = 1.602 \times 10^{-10}\,\mathrm{J}$$

- How much is a TeV?
  - Energy to raise 1g about 16 μm against gravity
  - Energy to power 100W light bulb 1.6 ns
- But many accelerators have 10^{10-12} particles
  - Single bunch “instantaneous power” of tens of **Tera**watts
- Highest energy observed cosmic ray
  - ~300 EeV (3×10^{20} eV or 3×10^{8} TeV!)

(125 g hamster at 100 km/hr)

OMG particle

Applied Special Relativity



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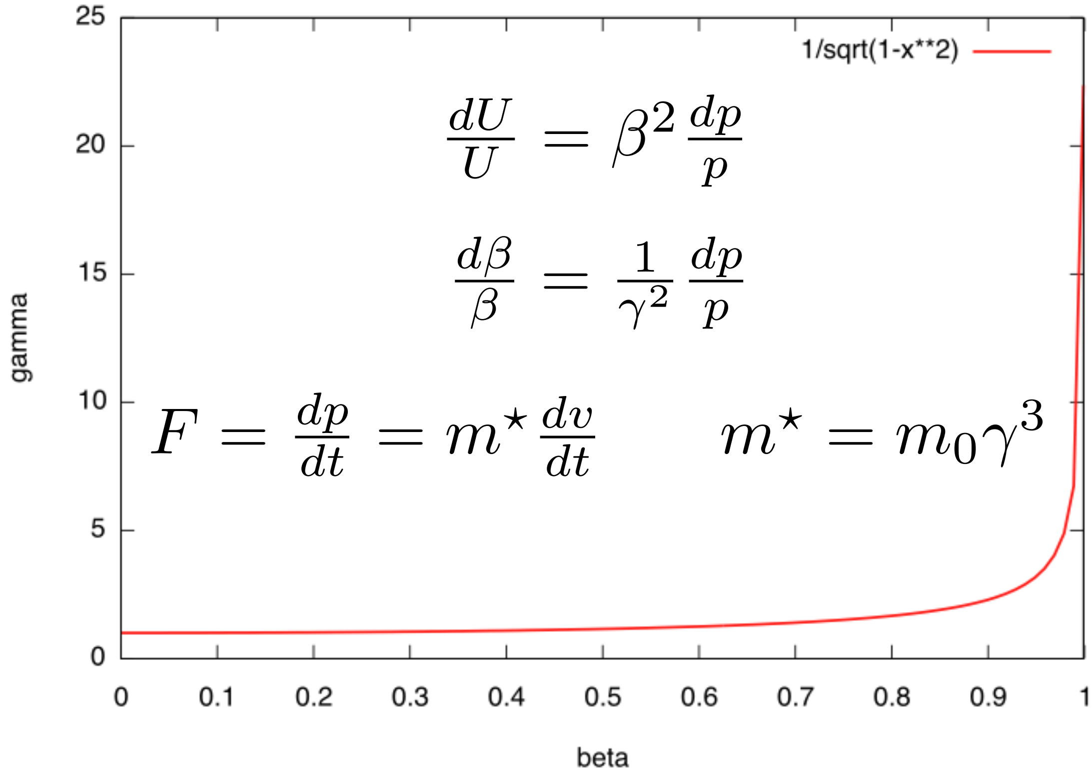
Convenient Relativity Relations



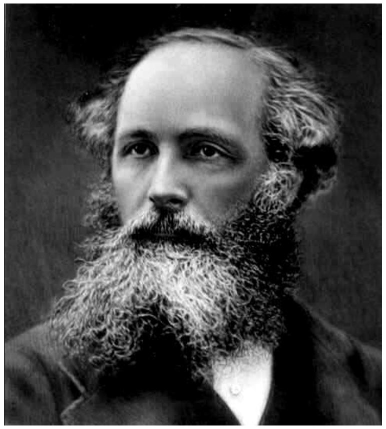
$$\frac{dU}{U} = \beta^2 \frac{dp}{p}$$

$$\frac{d\beta}{\beta} = \frac{1}{\gamma^2} \frac{dp}{p}$$

$$F = \frac{dp}{dt} = m^\star \frac{dv}{dt} \qquad m^\star = m_0 \gamma^3$$

- All derived in the text, hold for all γ
- In "ultra" relativistic limit $\beta \approx 1$
  - Usually must be careful below $\gamma \approx 5$ or $U \approx 5\ mc^2$
  - Many accelerator physics phenomena scale with γ^k or $(\beta\gamma)^k$

Applied Electricity and Magnetism



- Accelerators: applied E&M
  - Charged subatomic particles affected by controlled electric and magnetic fields
- Lorentz force: forces from **fields** on a charged particle

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(\gamma m \vec{v})}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

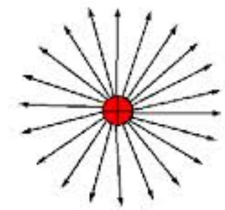
  - True even for relativistically-moving particles

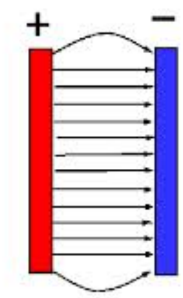
$$\vec{E} : \text{Electric field}$$

$$\vec{B} : \text{Magnetic (Induction) field}$$

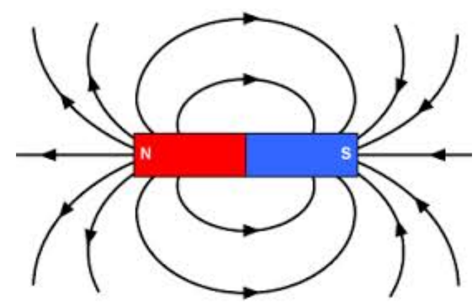
  - Only electric fields can change particle speed!
    - Magnetic fields only change particle direction

Electric and Magnetic Fields





- Electric fields
  - Created by distributions of electric charge
  - Measured in Volts/m and proportional to charge
  - ε_0 : "permittivity of free space", 8.854×10^{-12} $C^2/(J\text{-}m)$

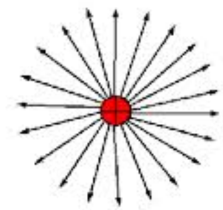


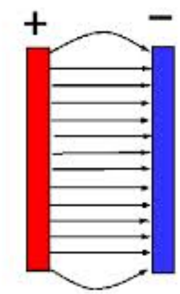
- Magnetic fields
  - Created by moving charges, currents
    - (magnetic dipole moments)
  - Measured in Tesla = 10^4 Gauss = N/(A-m)
  - μ_0 : "permeability of free space", $4\pi \times 10^{-7}$ $N\text{-}s^2/C^2$

- Since Joule=N-m, $1/(\varepsilon_0\mu_0)$ has units of m^2/s^2
  - In fact,

$$1/(\varepsilon_0\mu_0) = c^2$$

Electric and Magnetic Fields





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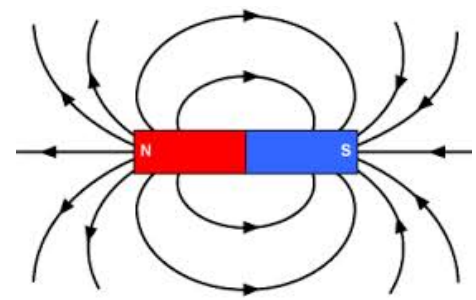

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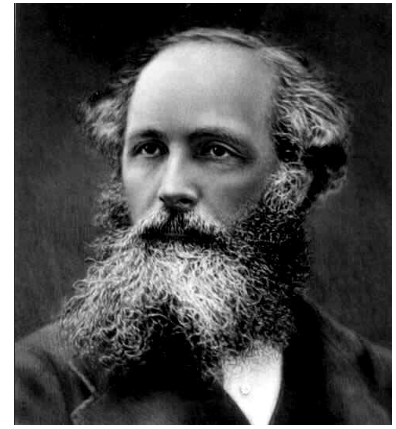
- (Also a band…)

Maxwell's Equations



- (Vector) EM fields must obey Maxwell's eqns
  - First order linear differential equations

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

Electric field is generated by electric charges

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Changing magnetic fields generate electric fields (Faraday's Law)

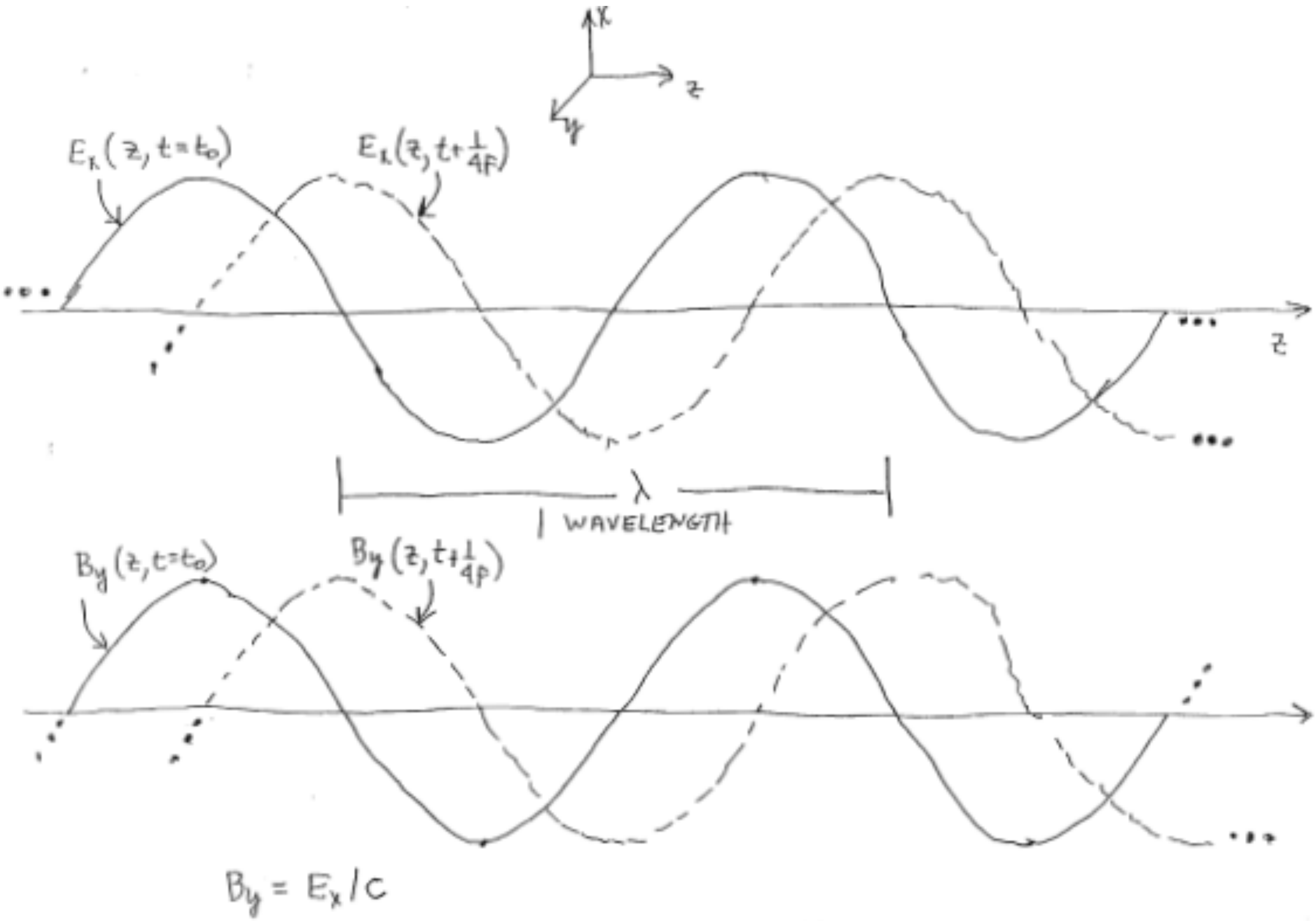
$$\vec{\nabla} \cdot \vec{B} = 0$$

No magnetic charges (monopoles) and magnetic field lines must close

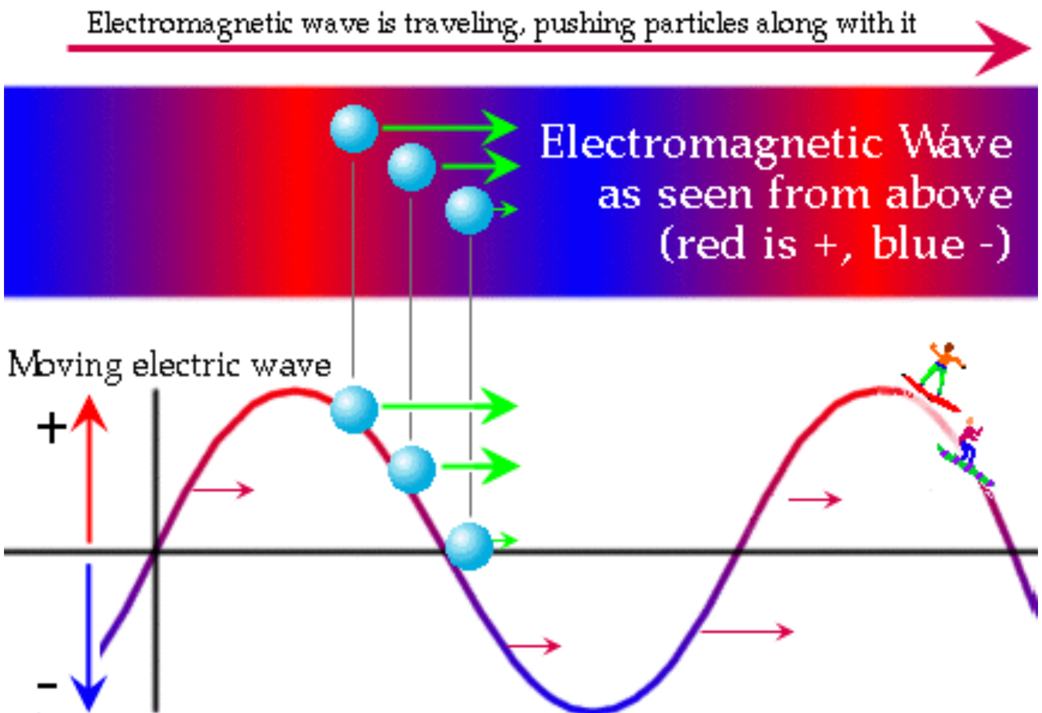
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Magnetic fields are generated by moving electric charges (currents) and changing electric fields

Electromagnetic Waves (non-artist's conception)



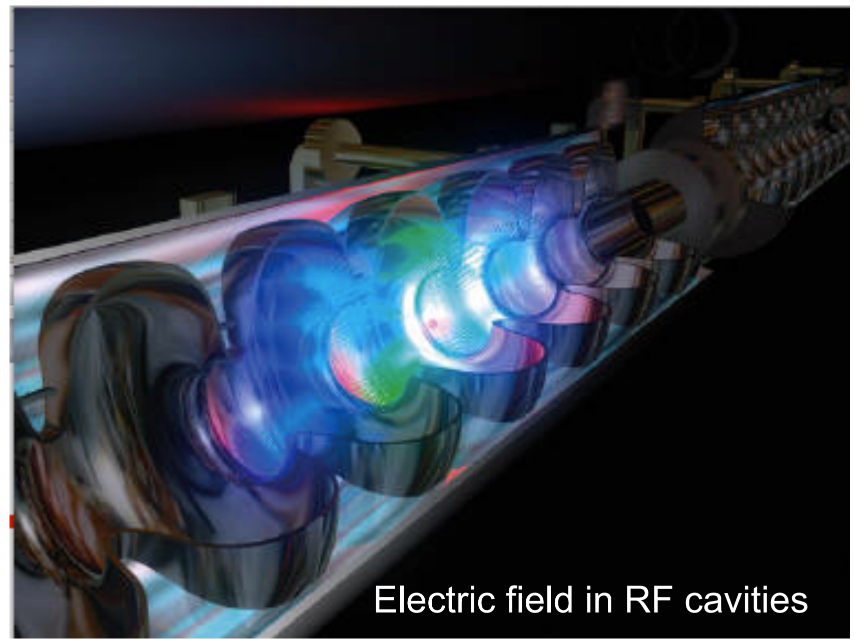
Wave Riding in RF Cavities at CEBAF



Positively charged particles ( ) close to the crest of the E-M wave experience the most force forward; those closer to the center experience less of a force. The result is that the particles tend to move together with the wave.

Original CEBAF RF

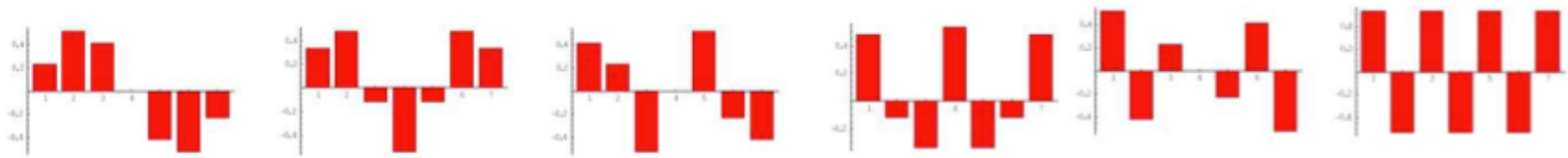
12 GeV Upgrade CEBAF RF



Electric field in RF cavities

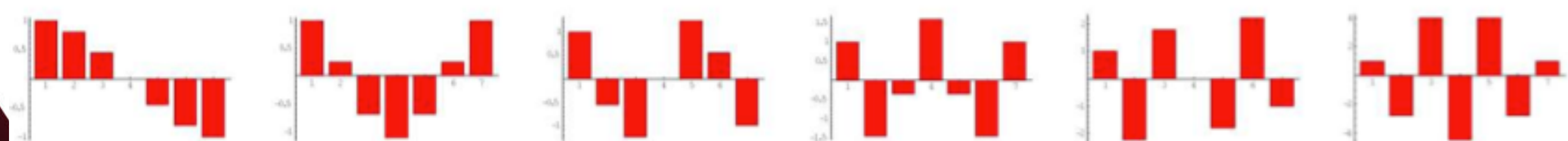
RF Cavities: Coupled Harmonic Oscillators

magnetic boundaries (left and right) → no 0-mode



2π/7 3π/7 4π/7 5π/7 6π/7 π

electric boundaries (left and right) → no π-mode



π/7 2π/7 3π/7 4π/7 5π/7 6π/7

Numerical Examples

$$\lambda f = c \qquad f = c / \lambda$$

$$f_{CEBAF} = 1497 \text{ MHz} = 1.497 \times 10^9 \sec^{-1} \rightarrow \lambda_{CEBAF} = 20 \text{ cm}$$

$$I = \frac{P}{A} = \frac{E_x B_y}{\mu_0} = \frac{E_x^2}{\mu_0 c} \text{ or on average } \frac{c\varepsilon_0 E_{x,amp}^2}{2}$$

1. Light Bulb

$10 \text{ W @ } 1 \text{ m} \rightarrow E_{x,amp} : 24.5 \text{ V/m or } 12.2 \text{ V/m @ } 2 \text{ m}$

2. Radio Station

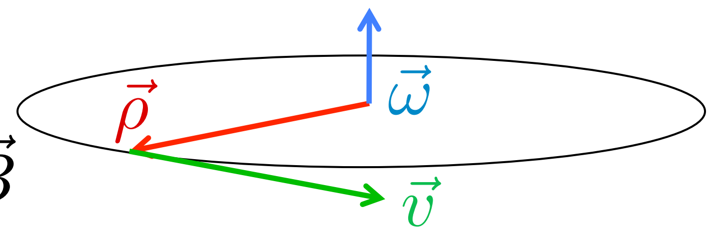
$50 \text{ kW @ } 10{,}000 \text{ m} \rightarrow E_{x,amp} : 0.17 \text{ V/m or } 0.09 \text{ V/m @ } 20 \text{ km}$

3. CEBAF Waveguide

$400 \text{ W in } .1 \text{ m} \times .2 \text{ m} \rightarrow E_{x,amp} : 3.9 \text{ kV/m}$

Constant Magnetic Field (Zero Electric Field)

- In a constant magnetic field, charged particles move in circular arcs of radius ρ with constant angular velocity ω:



$$\vec{F} = \frac{d}{dt}(\gamma m \vec{v}) = \gamma m \frac{d\vec{v}}{dt} = q\vec{v} \times \vec{B}$$

$$\vec{v} = \vec{\omega} \times \vec{\rho} \qquad \Rightarrow \qquad q\vec{v} \times \vec{B} = \gamma m \vec{\omega} \times \frac{d\vec{\rho}}{dt} = \gamma m \vec{\omega} \times \vec{v}$$

- For $\vec{B} \perp \vec{v}$ we then have

$$qvB = \frac{\gamma m v^2}{\rho} \qquad p = \gamma m(\beta c) = q(B\rho) \qquad \frac{p}{q} = (B\rho)$$

$$\omega = \frac{v}{\rho} = \frac{qB}{\gamma m}$$

Rigidity: Bending Radius vs Momentum

Beam

$$\frac{p}{q} = (B\rho)$$

Accelerator (magnets, geometry)

- This is such a useful expression in accelerator physics that it has its own name: **rigidity**
- Ratio of momentum to charge
  - How hard (or easy) is a particle to deflect?
  - Often expressed in [T-m] (easy to calculate B)
  - Be careful when q≠e!!
- A very useful expression

$$\frac{p[\mathrm{GeV/c}]}{q[e]} \approx 0.3\, B[\mathrm{T}]\, \rho[\mathrm{m}]$$

Cyclotron Frequency

$$\omega = \frac{v}{\rho} = \frac{qB}{\gamma m}$$

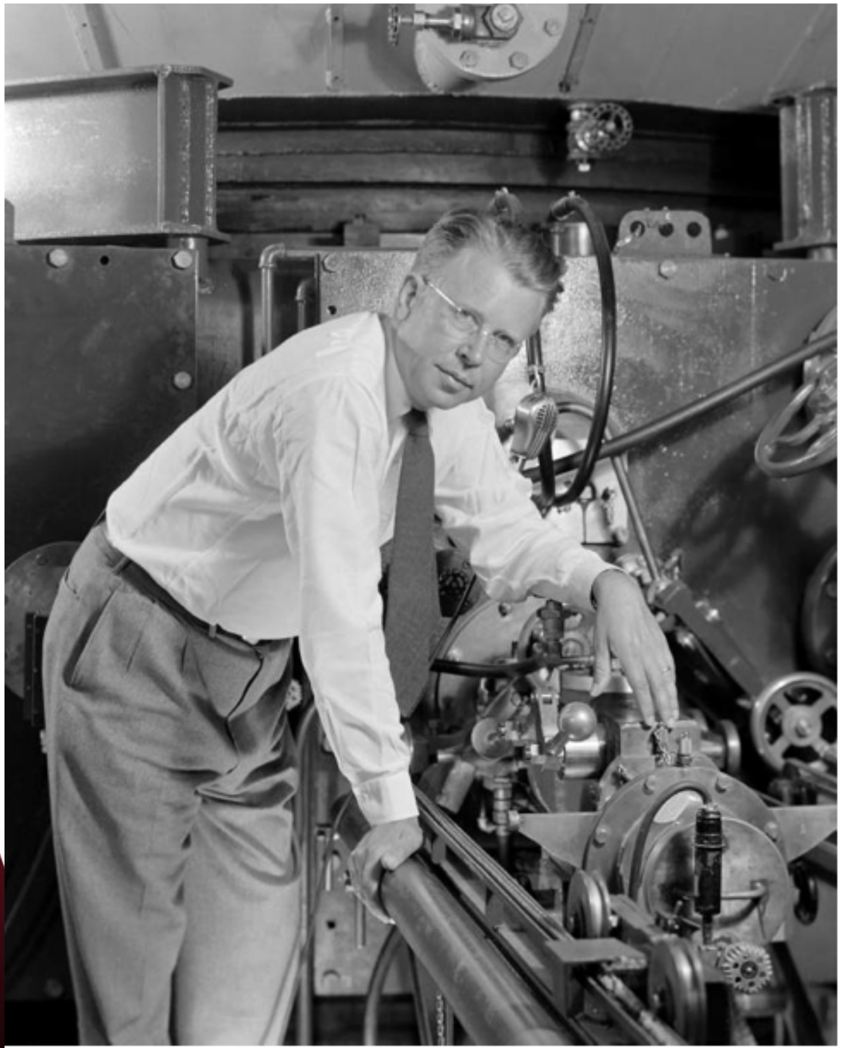
- Another very useful expression for particle angular frequency in a constant field: **cyclotron frequency**

- In the nonrelativistic approximation

$$\omega_{\mathrm{nonrelativistic}} \approx \frac{qB}{m}$$

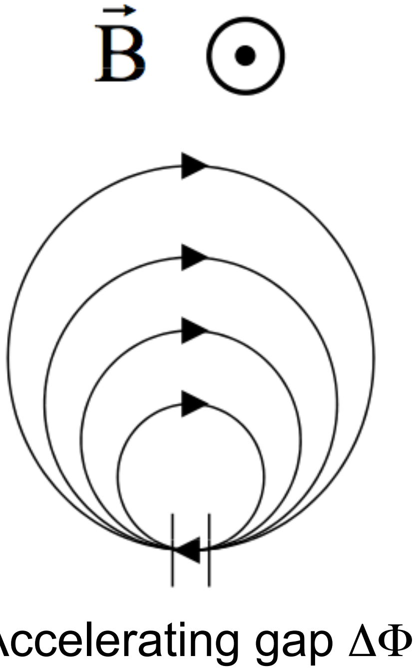
Revolution frequency is independent of radius or energy!

Lawrence and the Cyclotron



Ernest Orlando Lawrence

- Can we repeatedly spiral and accelerate particles through the same potential gap?



Cyclotron Frequency Again

- Recall that for a constant B field

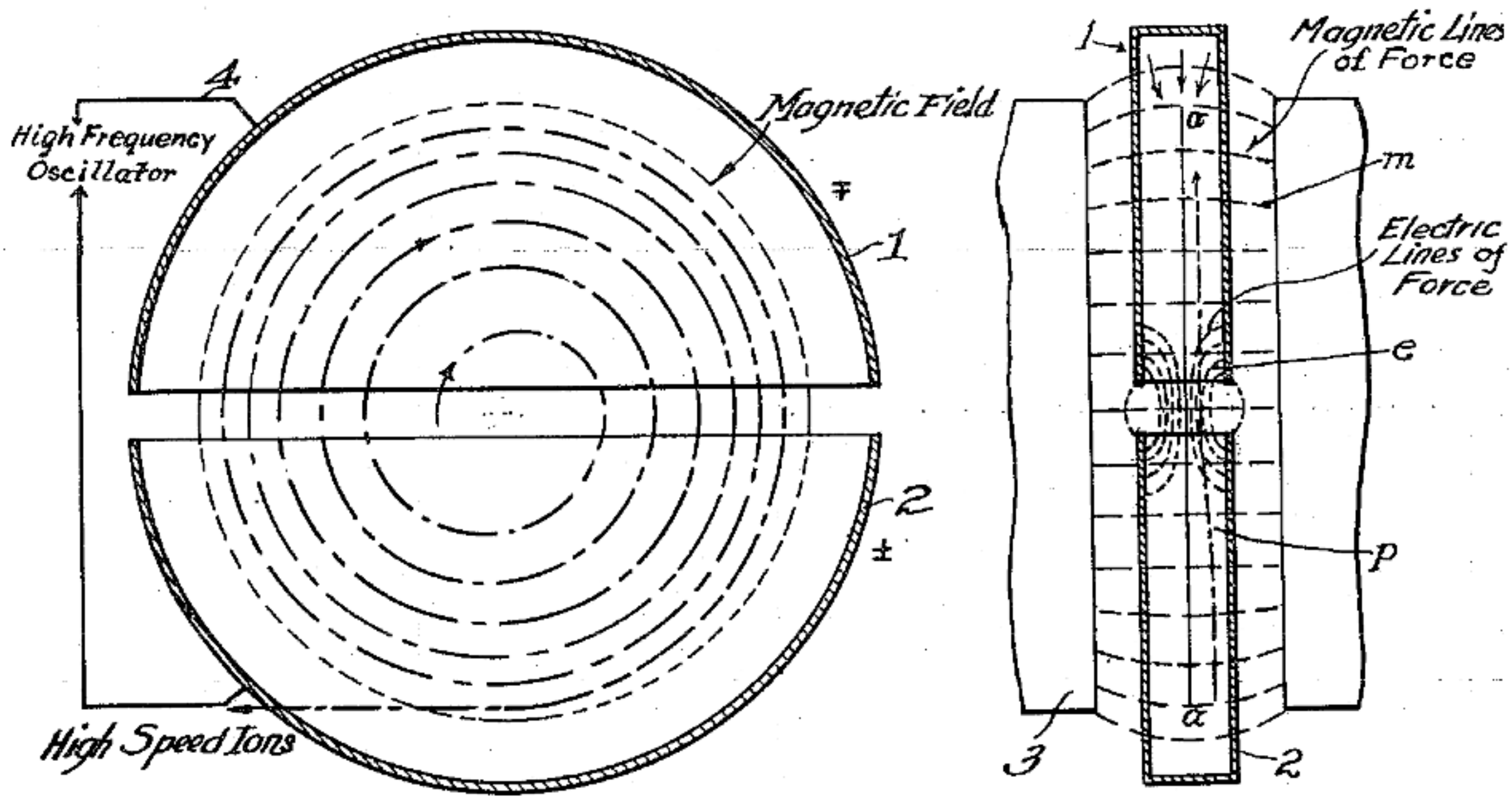
$$p = \gamma m v = q(B\rho) \quad \Rightarrow \quad \rho = \left(\frac{\gamma m}{qB}\right) v$$

  - Radius/circumference of orbit scale with velocity
    - Circulation time (and frequency) are independent of v
  - Apply AC electric field in the gap at frequency f_{rf}
    - Particles accelerate until they drop out of resonance

$$\omega = \frac{v}{\rho} = \frac{qB}{\gamma m} \qquad f_{\mathrm{rf}} = \frac{\omega}{2\pi} = \frac{qB}{2\pi\gamma m}$$

    - Note a first appearance of "bunches", not DC beam
    - Works best with heavy particles (hadrons, not electrons)

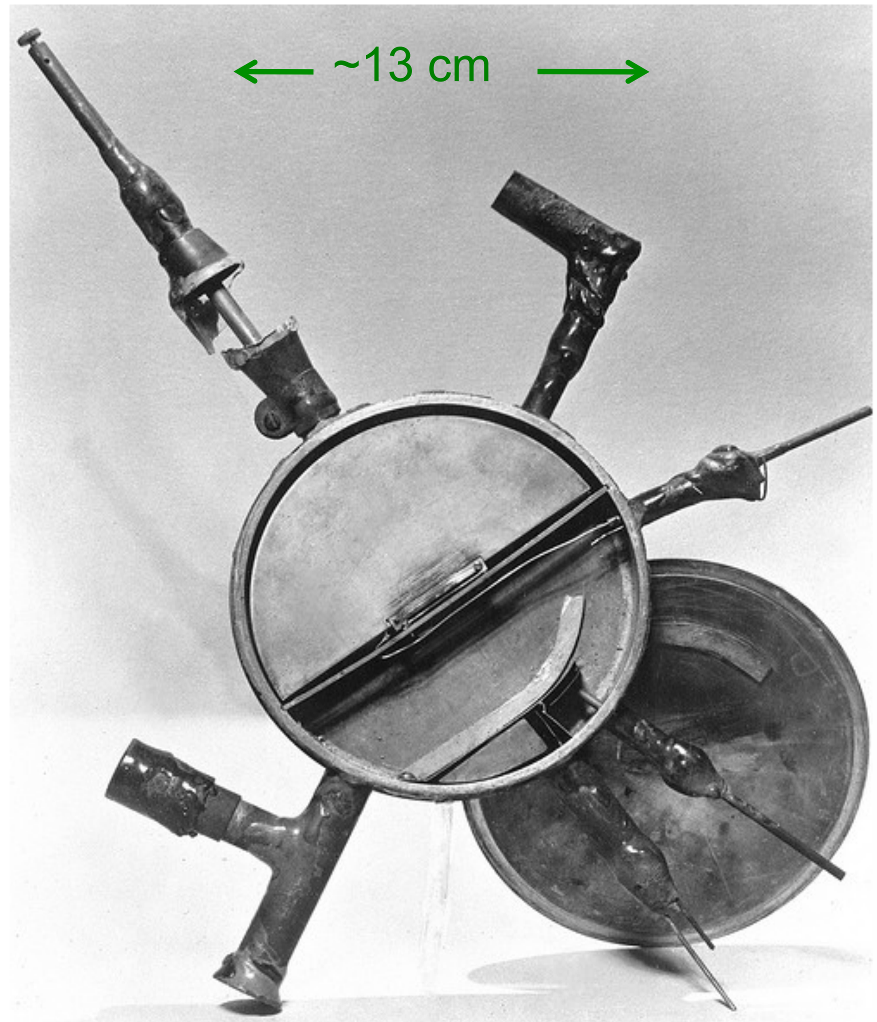
A Patentable Idea



- 1934 patent 1948384
  - Two accelerating gaps per turn!

All The Fundamentals of an Accelerator

- Large static magnetic fields for guiding (~1T)
  - But no vertical focusing
- HV RF electric fields for accelerating
  - (No phase focusing)
  - (Precise f control)
- p/H source, injection, extraction, vacuum
- 13 cm: 80 keV
- 28 cm: 1 MeV
- 69 cm: ~5 MeV
- … 223 cm: ~55 MeV



(Berkeley)

Livingston, Lawrence, 27”/69 cm Cyclotron

M.S. Livingston and E.O. Lawrence, 1934

The Joy of Physics

- Describing the events of January 9, 1932, Livingston is quoted saying:

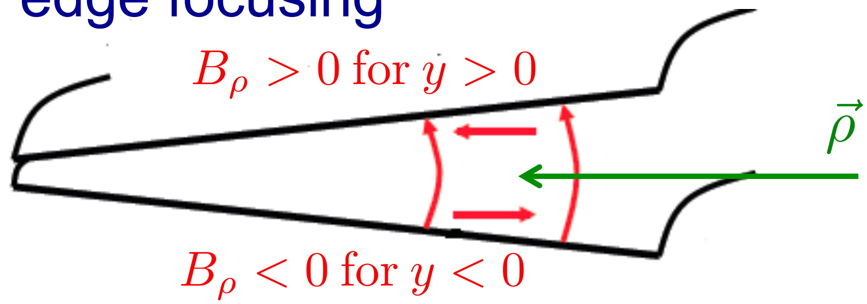
> "I recall the day when *I had adjusted the oscillator to a new high frequency*, and, with *Lawrence looking over my shoulder*, tuned the magnet through resonance. As the galvanometer spot swung across the scale, indicating that protons of 1-MeV energy were reaching the collector, *Lawrence literally danced around the room with glee*. The news quickly spread through the Berkeley laboratory, and we were busy all that day demonstrating million-volt protons to eager viewers."

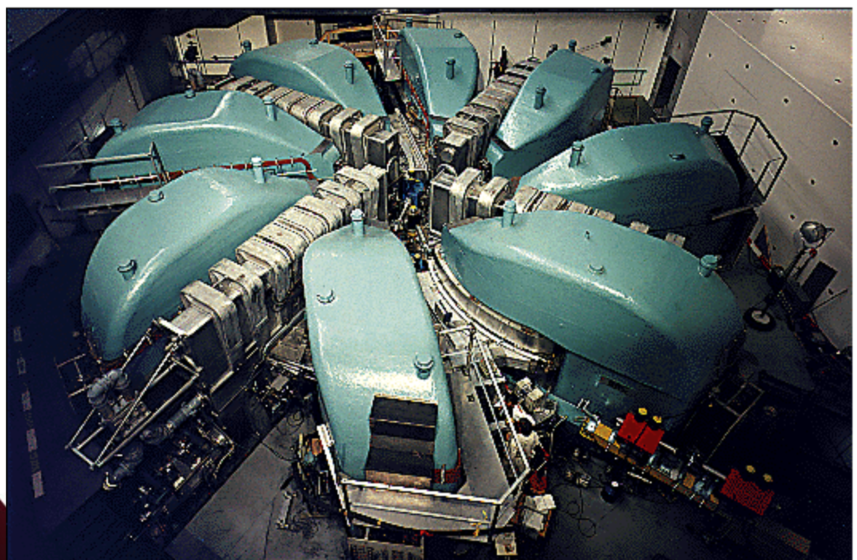
APS Physics History, Ernest Lawrence and M. Stanley Livingston

Modern Isochronous Cyclotrons

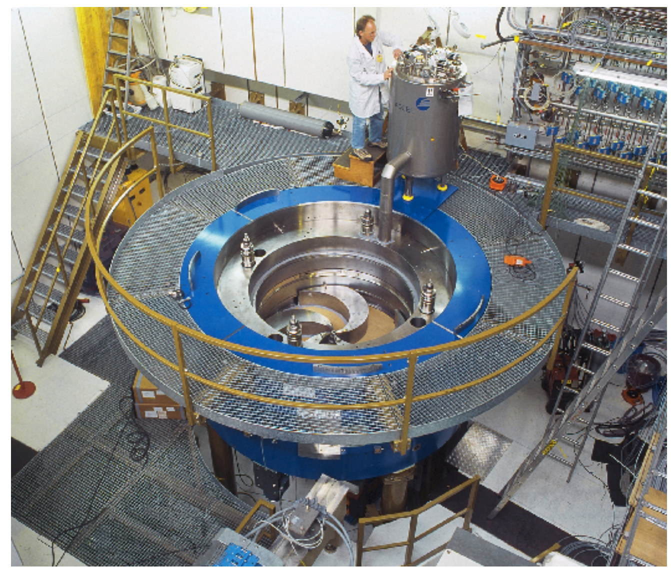
- Higher bending field at higher energies
  - But also introduces vertical defocusing
  - Use bending magnet "edge focusing"

$$f_{\text{rf}} = \frac{qB(\rho)}{2\pi\gamma(\rho)m}$$





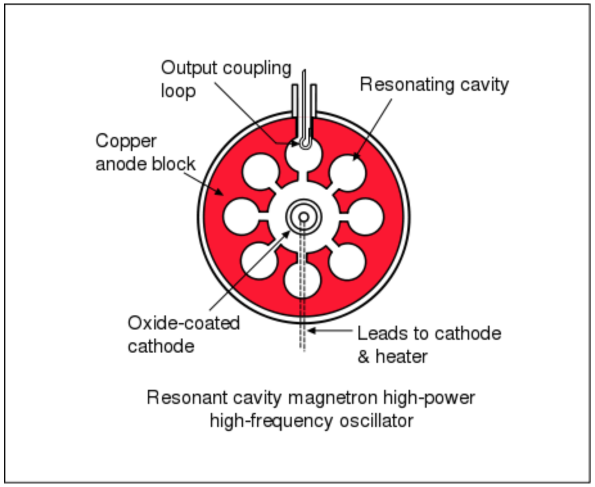
590 MeV PSI Isochronous Cyclotron (1974)

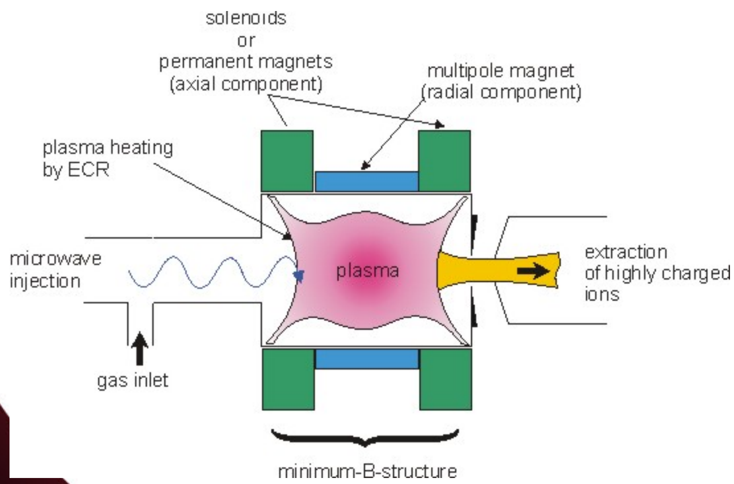


250 MeV PSI Isochronous Cyclotron (2004)

Electrons, Magnetrons, ECRs

Radar/microwave magnetron





ECR plasma/ion source

- Cyclotrons aren't good for accelerating electrons
  - Very quickly relativistic!
- But narrow-band response has advantages and uses
  - Magnetrons

    generate resonant high-power microwaves from circulating electron current
  - ECRs
    - generate high-intensity ion beams and plasmas by resonantly stripping electrons with microwaves

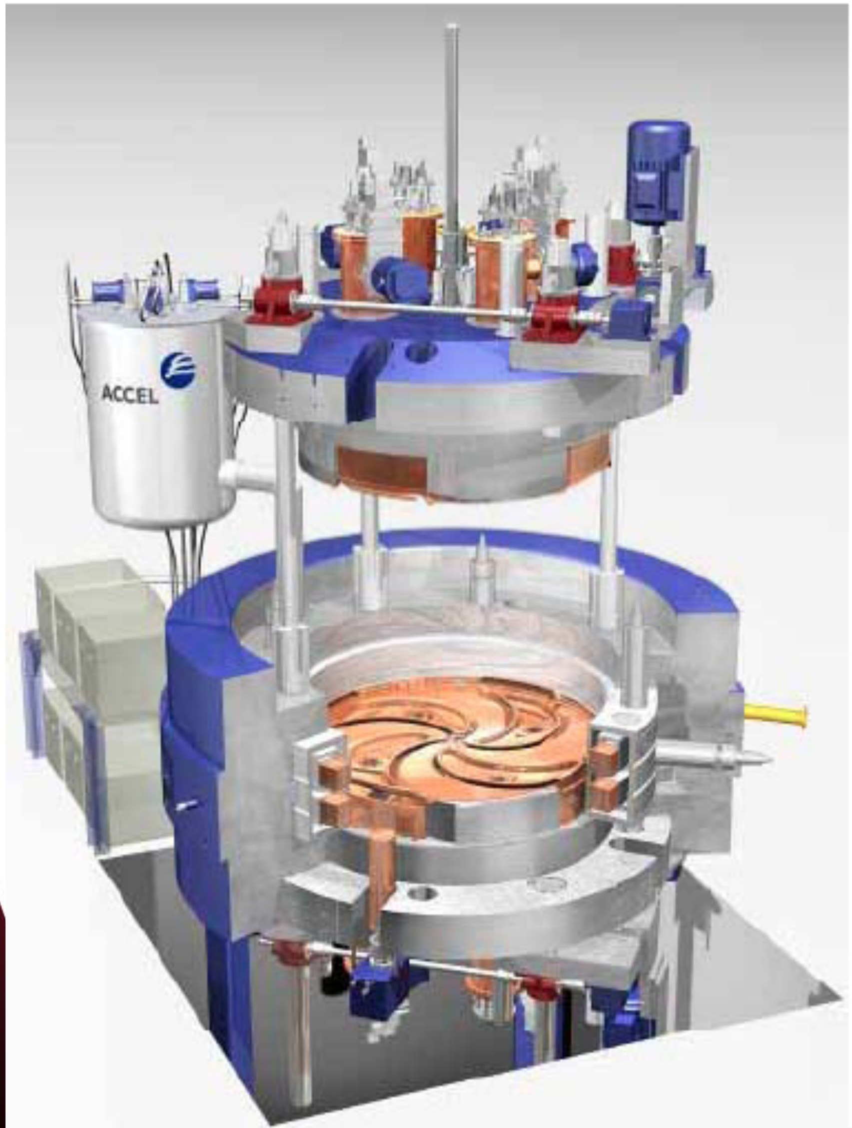
Cyclotrons Today

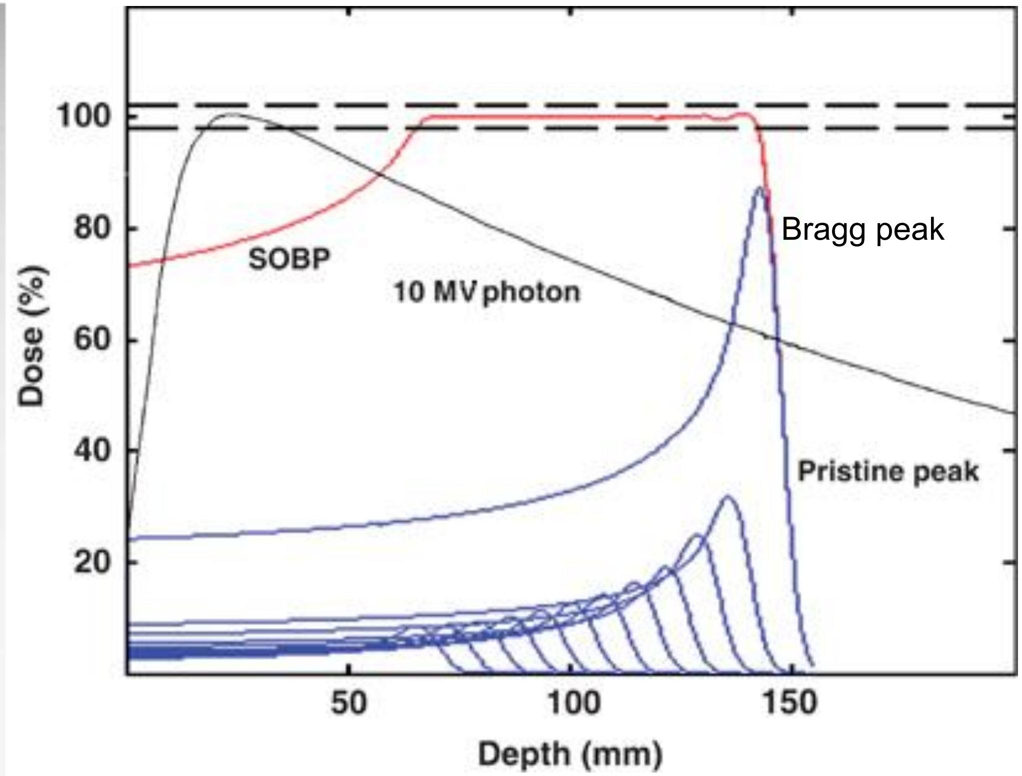
- Cyclotrons continue to evolve
  - Many contemporary developments
    - Superconducting cyclotrons
    - Synchrocyclotrons (FM modulated RF)
    - Isochronous/Alternating Vertical Focusing (AVF)
    - FFAGs (Fixed Field Alternating Gradient)

  - Versatile with many applications even below ~500 MeV
    - High power (>1MW) neutron production
    - Reliable (medical isotope production, ion radiotherapy)

    - Power+reliability: ~5 MW p beam for ADSR (accelerator driven subcritical reactors, e.g. Thorium reactors)

Accel Radiotherapy Cyclotron





Distinct dose localization advantage for hadrons over X-rays

Also present work on proton and carbon radiotherapy fast-cycling synchrotrons

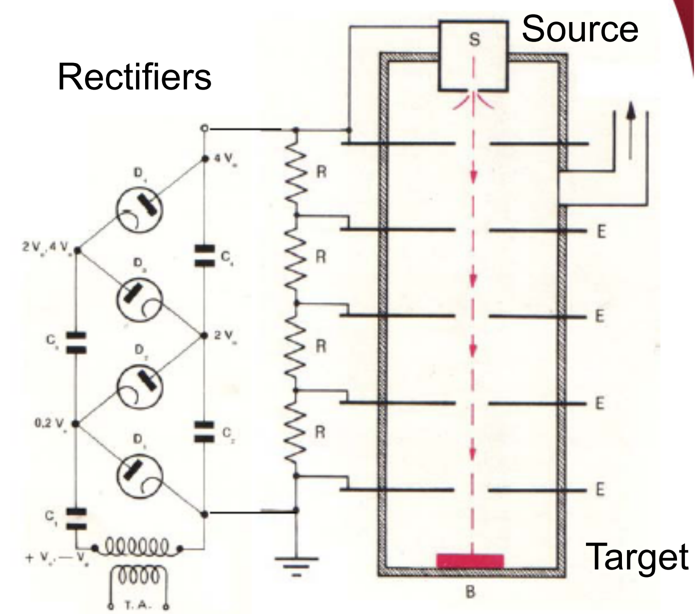
(Brief) Survey of Accelerator Concepts

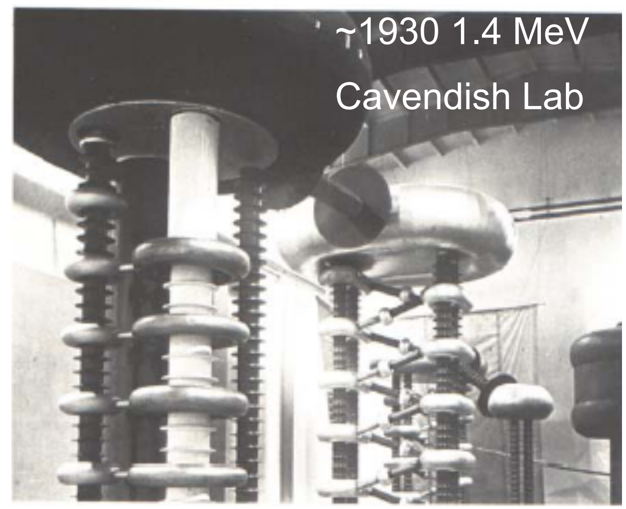
- Producing accelerating gaps and fields (DC/AC)
- Microtrons and their descendants
- Betatrons (and betatron motion)
- Synchrotrons
  - Fixed Target Experiments
  - Colliders and Luminosity (Livingston Plots)
  - Light Sources (FELs, Compton Sources)

- Others include
  - Medical Applications (radiotherapy, isotope production)
  - Spallation Sources (SNS, ESS)
  - Power Production (ADSR)

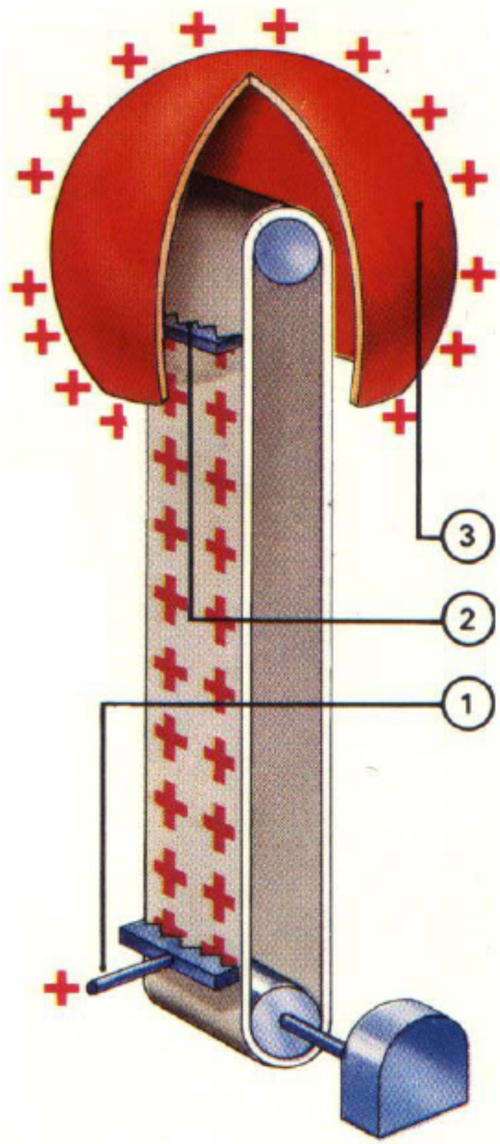
DC Accelerating Gaps: Cockcroft-Walton

- Accelerates ions through successive electrostatic voltages
  - First to get protons to >MeV
  - Continuous HV applied through intermediate electrodes
  - Rectifier-multipliers (voltage dividers)
  - Limited by HV sparking/breakdown
  - FNAL still uses a 750 kV C-W
- Also example of early ion source
  - H gas ionized with HV current
  - Provides high current DC beam





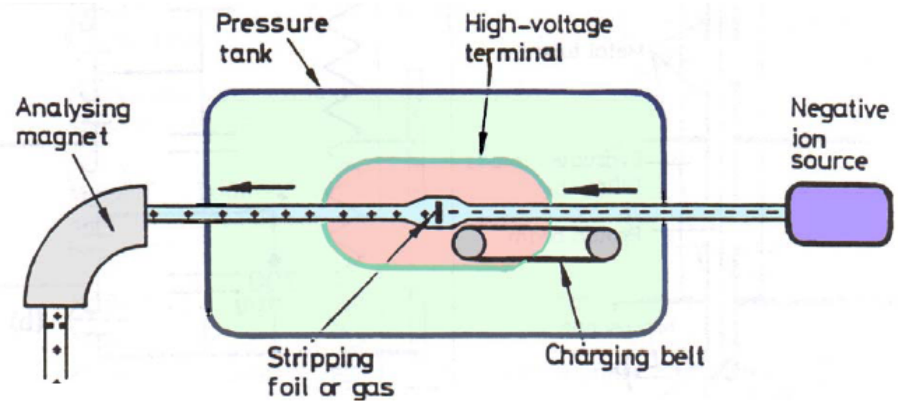
DC Accelerating Gaps: Van de Graaff



- How to increase voltage?
  - R.J. Van de Graaff: charge transport
  - Electrode (1) sprays HV charge onto insulated belt
  - Carried up to spherical Faraday cage
  - Removed by second electrode and distributed over sphere
- Limited by discharge breakdown
  - ~2MV in air
  - Up to 20+ MV in SF_6!
  - Ancestors of Pelletrons (chains)/ Laddertrons (stripes)

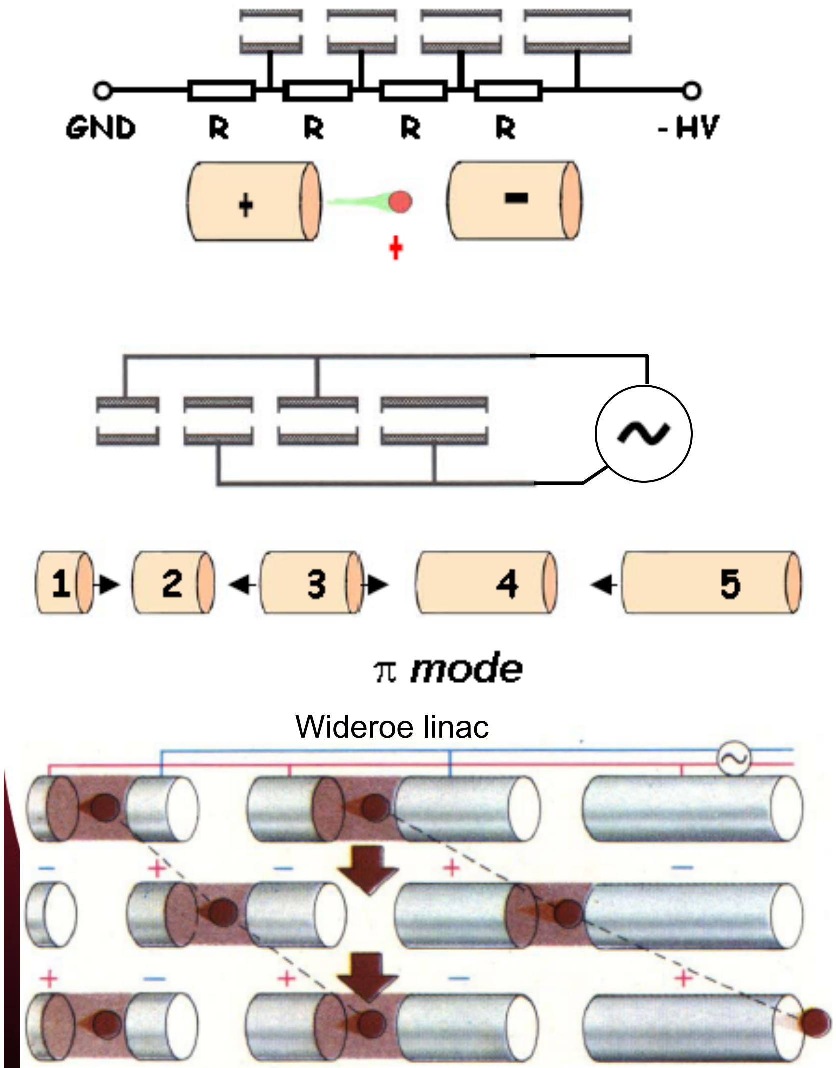
Van de Graaff Popularity

DC Accelerating Gaps: Tandem Van de Graaff



- Reverse ion charge state in middle of Van de Graaff allows over twice the energy gain
  - Source is at ground
- This only works for negative ions
- However, stripping need not be symmetric
  - Second stage accelerates more efficiently
- BNL: two Tandems (1970, 14 MV, 24m)
  - Au^{-1} to $Au^{+10}/Au^{+11}/Au^{+12}$ to Au^{+32} for RHIC
  - About a total of 0.85 MeV/nucleon total energy

From Electrostatic to RF Acceleration

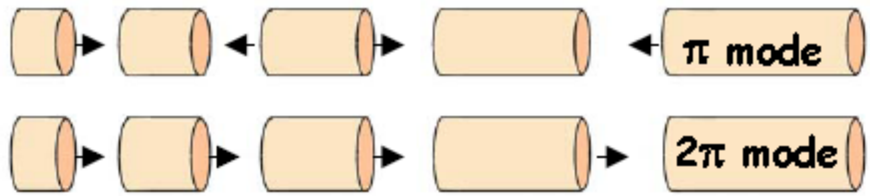


Pagani and Mueller 2002

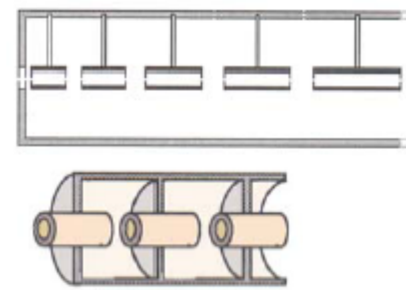
- Cockcroft-Waltons and Van de Graaffs have DC voltages, E fields
- What about putting on AC voltage?
  - Attach consecutive electrodes to opposite polarities of ACV generator
  - Electric fields between successive electrodes vary sinusoidally
  - Consecutive electrodes are 180 degrees out of phase (π mode)
- At the right drive frequency, particles are accelerated in each gap
  - While polarity change occurs, particles are shielded in drift tubes
  - To stay in phase with the RF, drift tube length or RF frequency must increase at higher energies

Resonant Linac Structures

- Wideroe linac: π mode
- Alvarez linac: 2π mode



- Need to minimize excess RF power (heating)
  - Make drift tubes/gaps resonant to RF frequency
  - In 2π mode, currents in walls separating two subsequent cavities cancel; tubes are passive
  - RF motion is much like a pendulum equation…



Wideroe linac

ALICE HI injector, IPN Orsay

Drift tube linac

Saturne, Saclay

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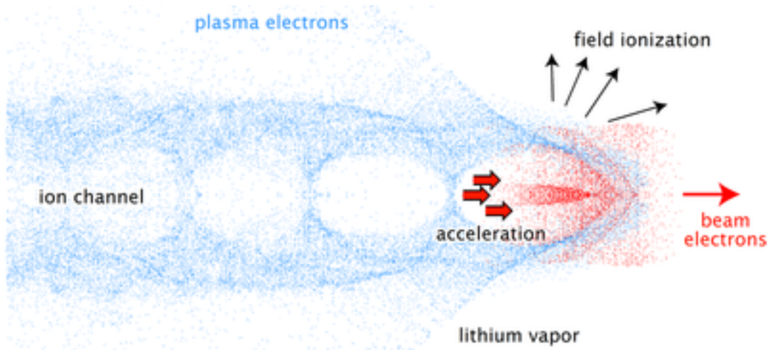
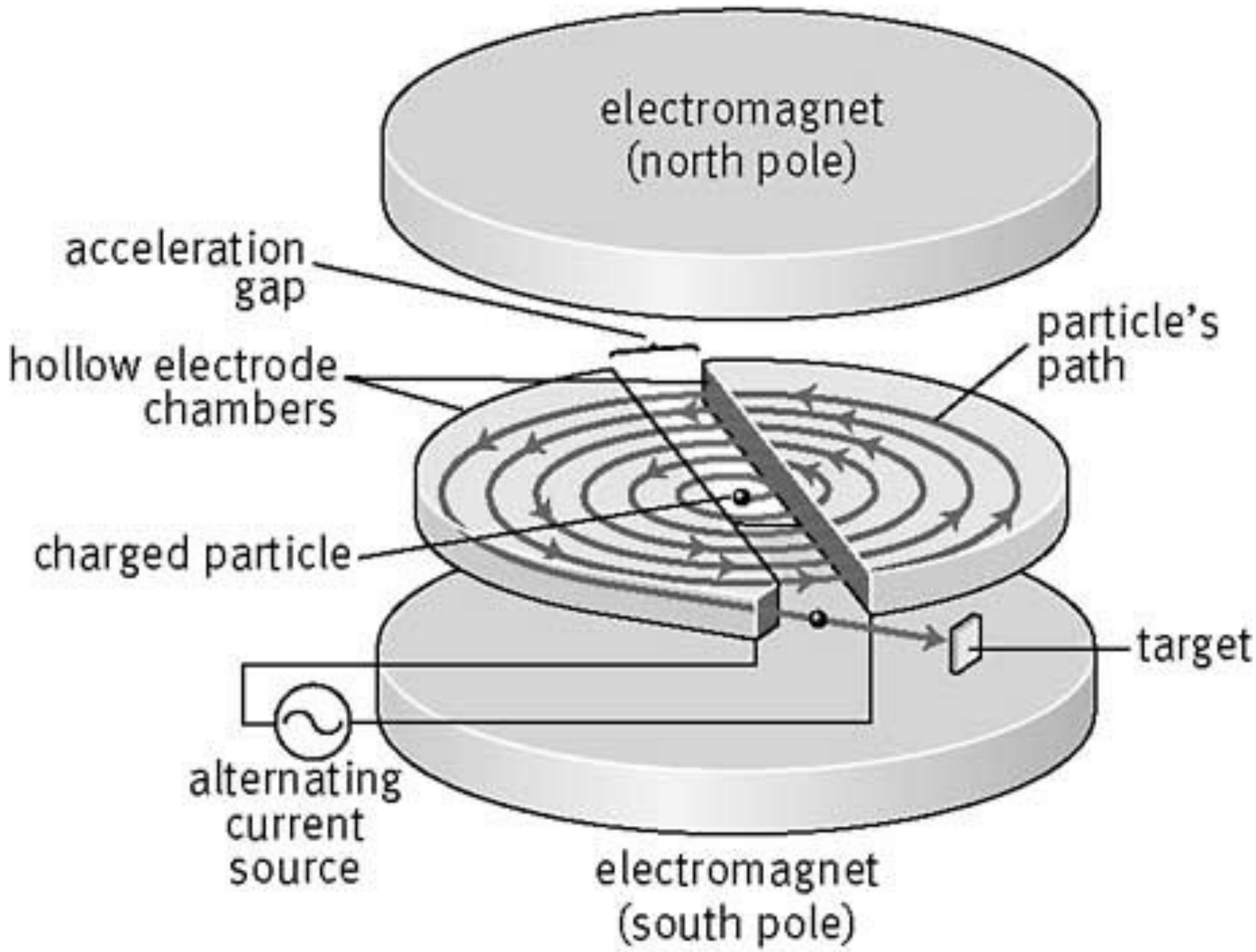
Advanced Acceleration Methods

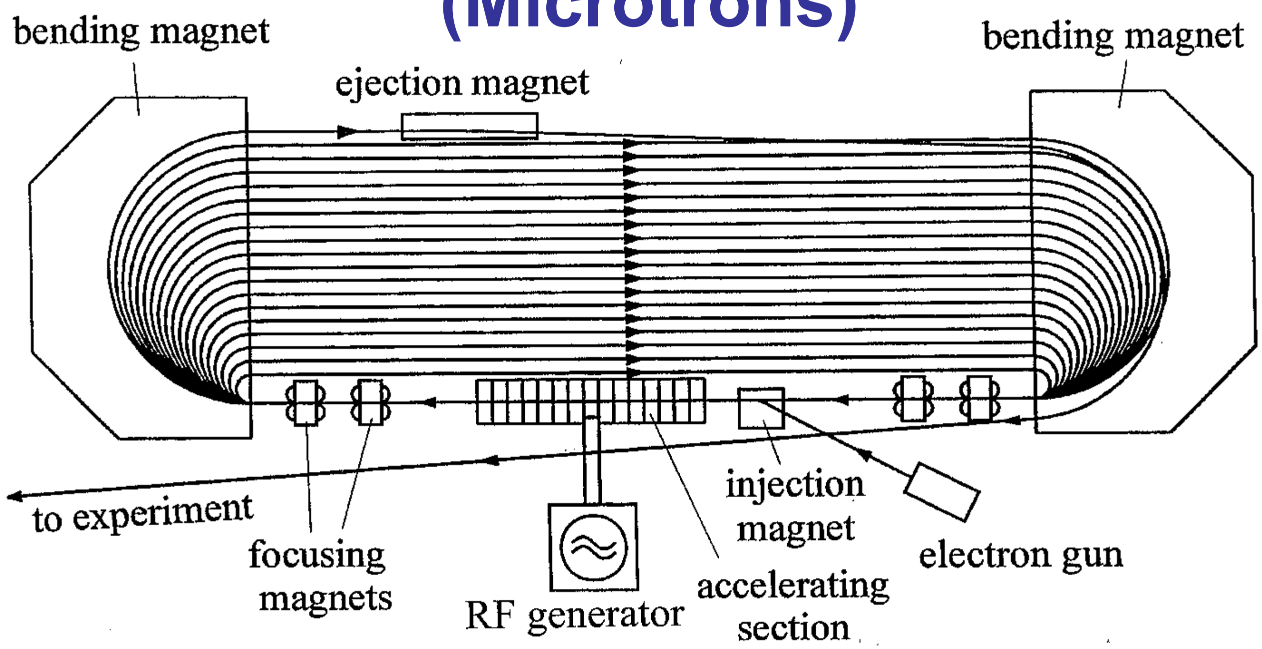
- How far do accelerating gradients go?
  - Superconducting RF acceleration: ~40 MV/m
  - CLIC: ~100 MV/m
    - Two-beam accelerator: drive beam couples to main beam
  - Dielectric wall acceleration: ~100 MV/m
    - Induction accelerator, very high gradient insulators
  - Dielectric wakefield acceleration: ~GV/m
  - Laser plasma acceleration: ~30 GV/m
    - electrons to 1 GeV in 3.3 cm
    - particles ride in wake of plasma charge separation wave



Cyclotrons (Again)

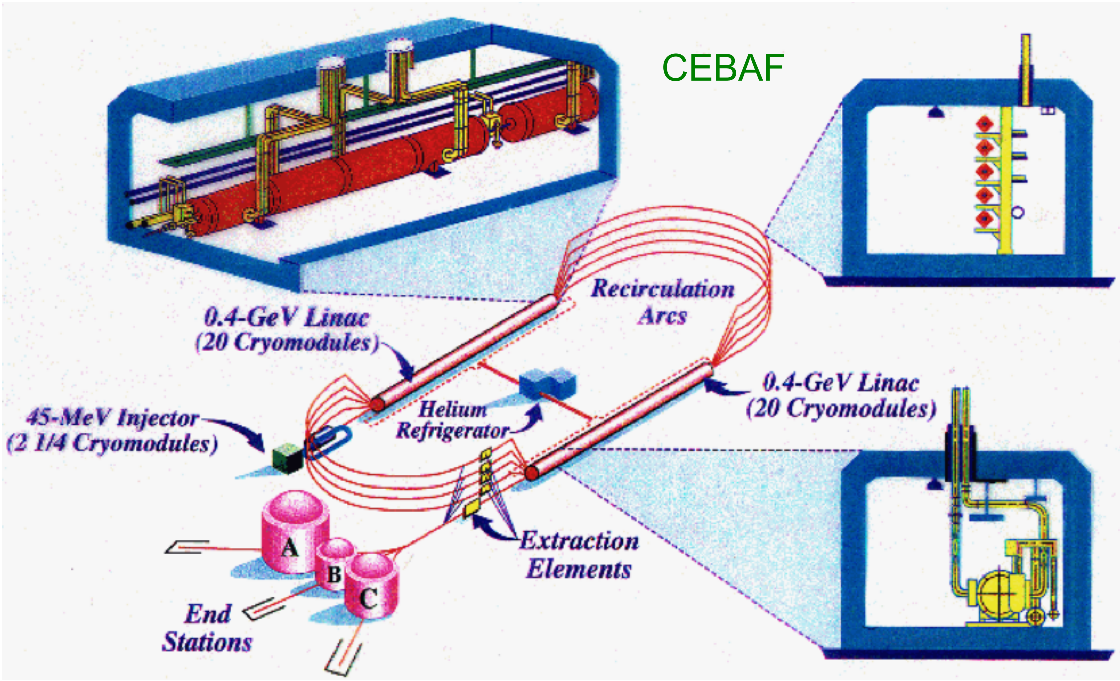


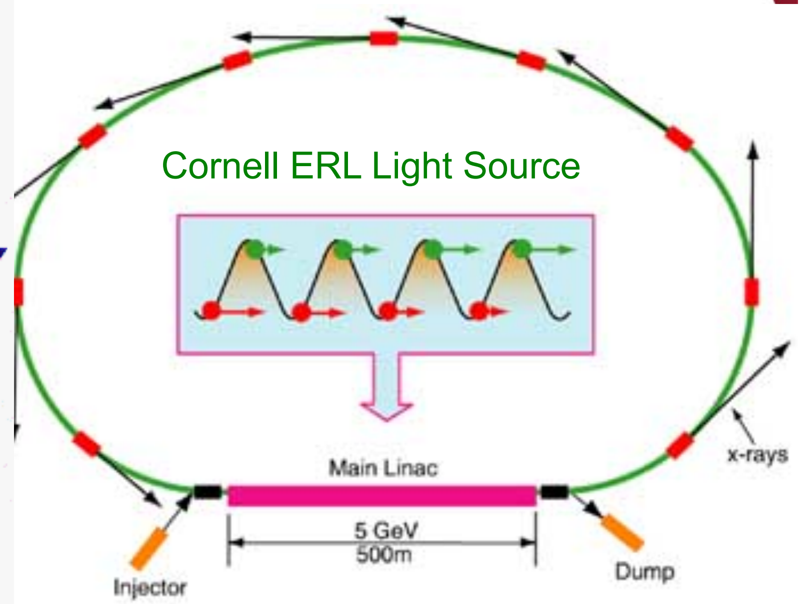
(Microtrons)



- What about electrons? Microtrons are like cyclotrons
  - but each revolution electrons "slip" by integer # of RF cycles
  - Trades off large # of revs for minimal RF generation cost
  - Bends must have large momentum aperture
  - Used for medical applications today (20 MeV, 1 big magnet)
  - Mainz MAMI: 855 MeV, used for nuclear physics

Recirculating Linacs and ERLs





- Recirculating linacs have separate arcs, longer linacs
  - CEBAF: 4->6->12 GeV polarized electrons, 2 SRF linacs
  - Higher energy at cost of more linac, separated bends
- Energy recovery linacs recirculate exactly out of phase
  - Raise energy efficiency of linac, less beam power to dump
  - Requires high-Q SRF to recapture energy efficiently