USPAS Accelerator Physics 2013
Duke University

Introductions, Relativity, E&M, Accelerator Overview

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http://www.toddsatogata.net/2013-USPAS

Introductions and Outline

- A sign-in sheet is being passed around
  - Please include any requests you have, e.g. topics you've heard about or that particularly interest you

- Introductions: Getting to know you, and us…

- Let's get it started
  - Course administrivia
  - Relativistic mechanics review
  - Relativistic E&M review, Cyclotrons
  - Survey of accelerators and accelerator concepts

Syllabus I

Day	Topic	Who	Lab?
Mon AM	Intro, Relativity, Luminosity	Todd	
Mon PM	Weak Focusing, Stability Conditions	Waldo	
Tue AM	Weak Focusing, Hamiltonians	Waldo	
Tue PM	Magnets and Field Expansions	Todd	~~Yes~~
Wed AM	Strong Focusing Theory I	Waldo	
Wed PM	Strong Focusing Theory II	Waldo	
Thu AM	Lattice Exercises I	Todd	
Thu PM	Lattice Exercises II	Waldo	Yes
Fri AM	Lattice Design I	Todd	
Fri PM	Lattice Design II	Todd	Yes

- First week: Mostly transverse linear optics
  - Fundamentals and equations of motion
  - Magnet design, fields, descriptions
  - Linear transverse optics
  - Magnetic lattices and lattice design

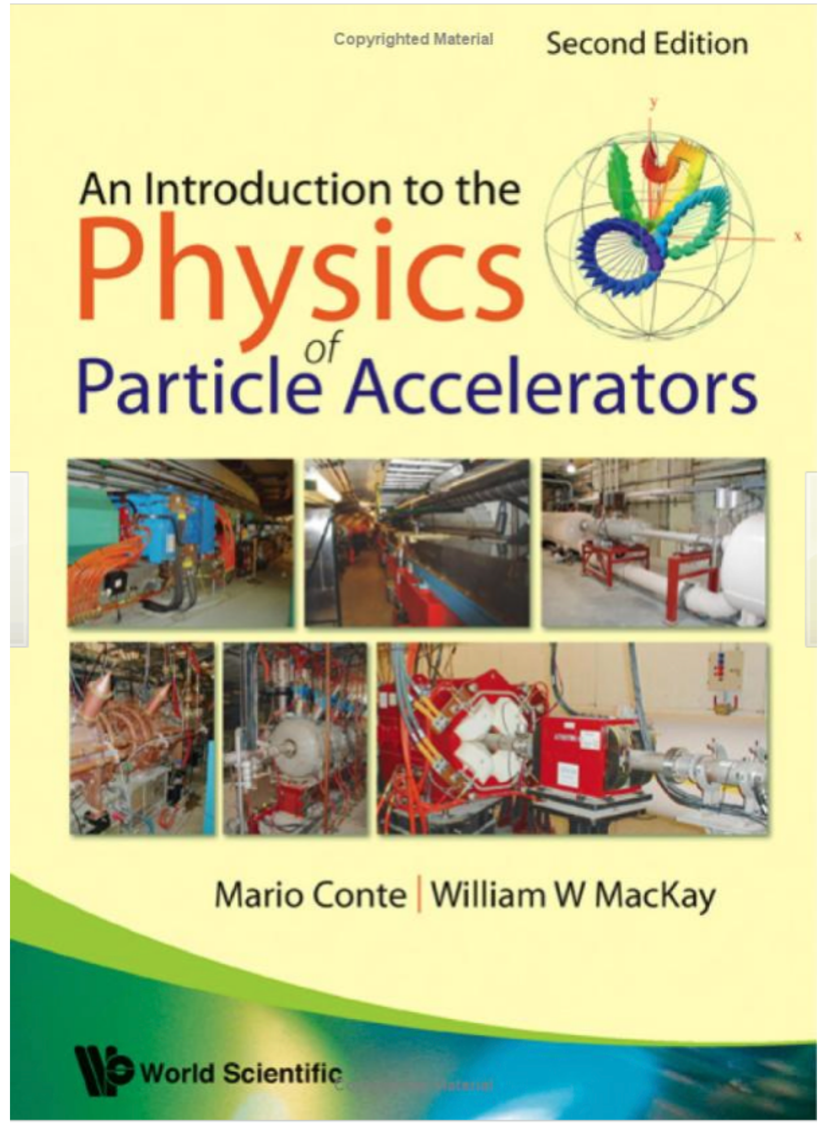
Syllabus II

Day	Topic	Who	Lab?
Mon AM Mon PM	Longitudinal Motion (Synchrotron) Longitudinal Motion (Linac)	Waldo Todd	
Tue AM Tue PM	Synchrotron Radiation Synchrotron Light Facility Lattices	Waldo Todd	Yes
Wed AM Wed PM	Resonances and Nonlinear Dynamics I Nonlinear Dynamics II	Waldo Todd	
Thu AM Thu PM	Space Charge and Beam-Beam Measurement Methods	Todd Todd	Yes (Exam)
Fri AM	Polarization and Spin Dynamics	Waldo	

- Second week: Everything else ☺
  - Longitudinal dynamics
  - Synchrotron radiation and cooling
  - Nonlinear dynamics and collective effects
  - Measurements and instrumentation

Text

- Conte and MacKay
  - 2^{nd} edition
- We will cover quite a bit of this text
- You have the advantage of being able to nag one of the authors about its contents



Homework and Schedule

- Homework is nearly half your grade!! (40%)
  - Tim is grading – be nice to him ☺
  - Collected at start of every morning class
  - Tim's homework is to get it back to you the next day
  - Lectures/lab times will run 09:00-12:00, 13:30-16:30(ish)
- Collaboration is encouraged! (Except on the final)
  - In fact, it's a good part of the reason why you're here!
  - Waldo and I will be available to work with you most evenings
- Cite references, contributions of teammates, etc
  - But everyone must hand in **individual copies** of homework

Relativity Review

- Accelerators: applied special relativity
- Relativistic parameters:

$$\beta \equiv \frac{v}{c} \qquad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \sqrt{1-1/\gamma^2}$$

  - Later β and γ will also be used for other quantities, but the context should usually make them clear
  - γ=1 (classical mechanics) to ~2.05x10^5 (to date) (where??)

- Total energy U, momentum p, and kinetic energy W

$$U = \gamma mc^2 \qquad p = (\beta\gamma)mc = \beta\left(\frac{U}{c}\right) \qquad W = (\gamma - 1)\, mc^2$$

Relative Relativity

WolframAlp

LEP energy

Input interpretation:

LEP (Large Electron Positron Collider) | ce

Result:

208 GeV (gigaelectronvolts)

Unit conversions:

0.208 TeV (teraelectronvolts)

2.08×10^{11} eV (electronvolts)

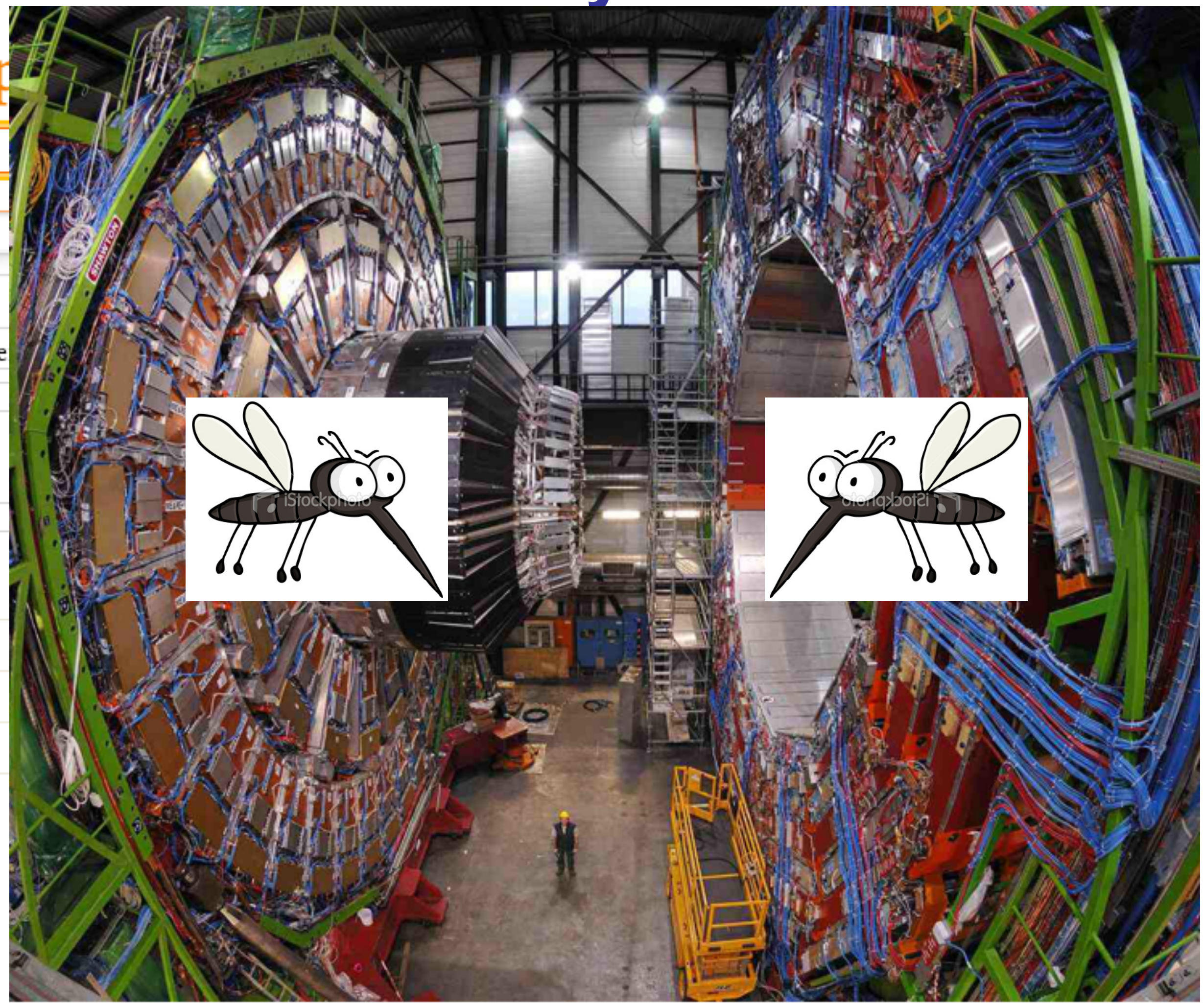
0.03333 μJ (microjoules)

3.333×10^{-8} J (joules)

0.3333 ergs
(unit officially deprecated)

Comparisons as energy:

≈ (0.21 ≈ 1/5) ×



approximate kinetic energy of a flying mosquito ($\approx 1.6 \times 10^{-7}$ J)

≈ 2.2 × mass-energy equivalent of a Z boson ($\approx 1.5 \times 10^{-8}$ J)

Convenient Units

$$1\,\mathrm{eV} = (1.602 \times 10^{-19}\,\mathrm{C})(1\,\mathrm{V}) = 1.602 \times 10^{-19}\,\mathrm{J}$$

$$1\,\mathrm{MeV} = 1.602 \times 10^{-13}\,\mathrm{J}$$

$$1\,\mathrm{GeV} = 1.602 \times 10^{-10}\,\mathrm{J}$$

- How much is a TeV?
  - Energy to raise 1g about 16 μm against gravity
  - Energy to power 100W light bulb 1.6 ns
- But many accelerators have $10^{10\text{-}12}$ particles
  - Single bunch "instantaneous power" of tens of Terawatts
- Highest energy cosmic ray
  - ~300 EeV (3×10^{20} eV or 3×10^{8} TeV!) OMG particle

(125 g hamster at 100 km/hr)

Relativity Review (Again)

- Accelerators: applied special relativity
- Relativistic parameters:

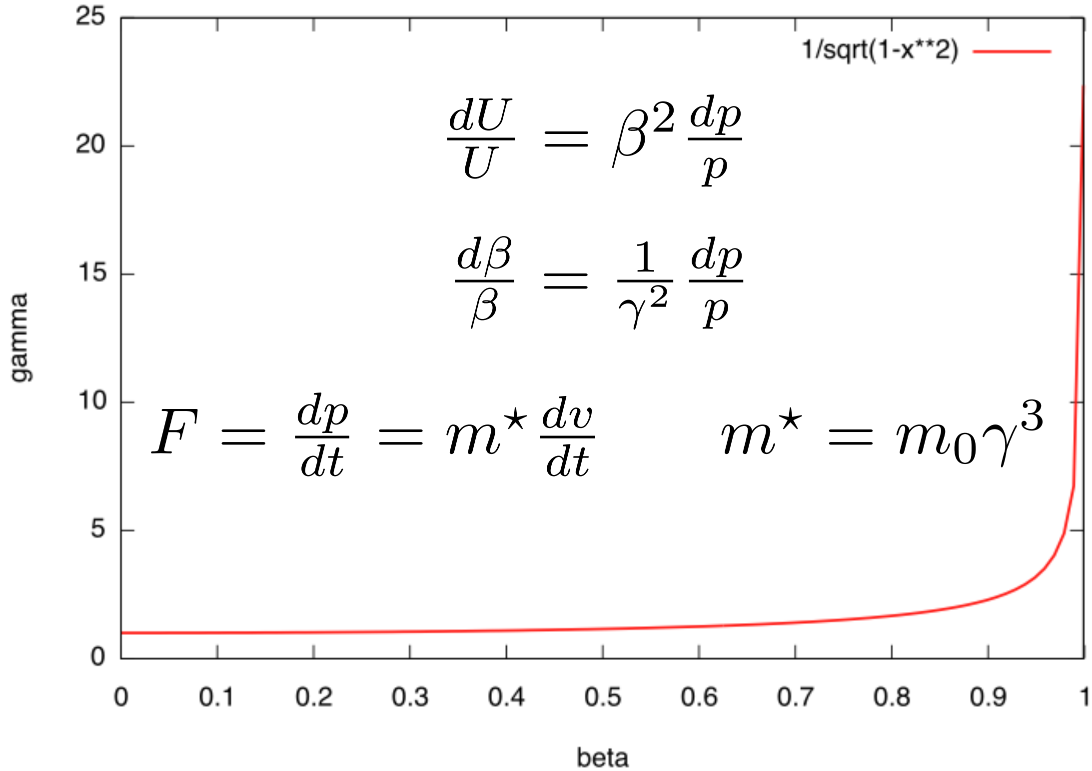
$$\beta \equiv \frac{v}{c} \qquad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \sqrt{1 - 1/\gamma^2}$$

  - Later β and γ will also be used for other quantities, but the context should usually make them clear
  - γ=1 (classical mechanics) to ~2.05x10^5 (oh yeah, at LEP)

- Total energy U, momentum p, and kinetic energy W

$$U = \gamma mc^2 \qquad p = (\beta\gamma)mc = \beta\left(\frac{U}{c}\right) \qquad W = (\gamma - 1)\, mc^2$$

Convenient Relativity Relations



$$\frac{dU}{U} = \beta^2 \frac{dp}{p}$$

$$\frac{d\beta}{\beta} = \frac{1}{\gamma^2} \frac{dp}{p}$$

$$F = \frac{dp}{dt} = m^\star \frac{dv}{dt} \qquad m^\star = m_0 \gamma^3$$

- All derived in the text, hold for all γ
- In "ultra" relativistic limit $\beta \approx 1$
  - Usually must be careful below $\gamma \approx 5$ or $U \approx 5$ mc^2
  - Many accelerator physics phenomena scale with γ^k or $(\beta\gamma)^k$

(Frames and Lorentz Transformations)

- The lab frame will dominate most of our discussions
  - But not always (synchrotron radiation, space charge…)

- Invariance of space-time interval (Minkowski)

$$(ct')^2 - x'^2 - y'^2 - z'^2 = (ct)^2 - x^2 - y^2 - z^2$$

- Lorentz transformation of four-vectors
  - For example, time/space coordinates in z velocity boost

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

(Four-Velocity and Four-Momentum)

- The proper time interval $d\tau = dt/\gamma$ is Lorentz invariant
- So we can make a velocity 4-vector

$$cu^\alpha \equiv \left(\frac{dct}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau} \right) = c\gamma(1, \beta_x, \beta_y, \beta_z)$$

$$\text{Metric } g^{\mu\nu} = g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$$

- We can also make a 4-momentum

$$p^\alpha \equiv mcu^\alpha = mc\gamma(1, \beta_x, \beta_y, \beta_z)$$

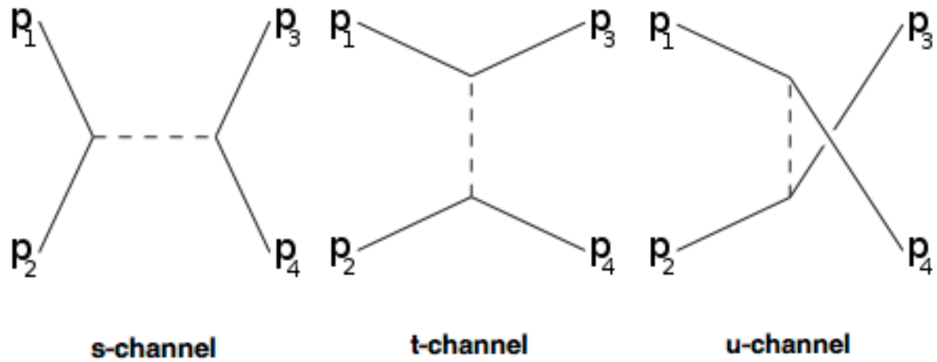
- Double-check that Minkowski norms are invariant

$$u^\alpha u_\alpha = u^\alpha g_{\alpha\beta} u^\beta = \gamma^2(1 - \beta^2) = 1$$

$$p^\alpha p_\alpha = m^2 c^2 u^\alpha u_\alpha = m^2 c^2$$

(Mandelstam Variables)

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$
$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$$
$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2$$



$$s + t + u = (m_1^2 + m_2^2 + m_3^2 + m_4^2)c^2$$

- Lorentz-invariant two-body kinematic variables
  - $p_{1\text{-}4}$ are four-momenta
- √s is the total available center of mass energy
  - Often quoted for colliders
- Used in calculations of other two-body scattering processes
  - Moller scattering (e-e), Compton scattering (e-γ)

(Relativistic Newton)

$$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$$

- But now we can define a four-vector force in terms of four-momenta and proper time:

$$F^\alpha \equiv \frac{dp^\alpha}{d\tau}$$

- We are primarily concerned with electrodynamics so now we must make the classical electromagnetic Lorentz force obey Lorentz transformations

$$\boxed{\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)}$$

Relativistic Electromagnetism

- Classical electromagnetic potentials can be shown to combine to a four-potential (with c=1):

$$A^\alpha \equiv (\Phi, \vec{A})$$

- The field-strength tensor is related to the four-potential

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

  - E/B fields Lorentz transform with factors of γ, ($\beta\gamma$)

(Lorentz Lie Group Generators)

- Lorentz transformations can be described by a Lie group where a general Lorentz transformation is

$$A = e^{L} \qquad \det A = e^{\mathrm{Tr}\, L} = +1$$

where L is 4x4, real, and traceless. With metric g, the matrix gL is also antisymmetric, so L has the general six-parameter form

$$L = \begin{pmatrix} 0 & L_{01} & L_{02} & L_{03} \\ L_{01} & 0 & L_{12} & L_{13} \\ L_{02} & -L_{12} & 0 & L_{23} \\ L_{03} & -L_{13} & -L_{23} & 0 \end{pmatrix}$$

Deep and **profound** connection to EM tensor $F^{\alpha\beta}$

J.D. Jackson, Classical Electrodynamics 2nd Ed, Section 11.7

Relativistic Electromagnetism II

- The relativistic electromagnetic force equation becomes

$$\frac{dp^\alpha}{d\tau} = m\frac{du^\alpha}{d\tau} = \frac{q}{c}F^{\alpha\beta}u_\beta$$

- Thankfully we can write this in somewhat simpler terms

$$\frac{d(\gamma m\vec{v})}{dt} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

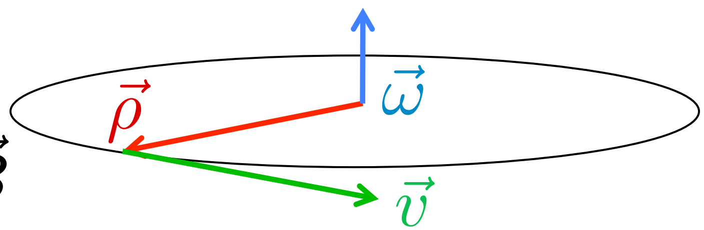
- That is, "classical" E&M force equations hold if we treat the momentum as relativistic, $\vec{p} = \gamma m\vec{v} = \gamma\vec{\beta}mc$
- If we dot in the velocity, we get energy transfer

$$\frac{d\gamma}{dt} = \frac{q\vec{E}\cdot\vec{v}}{mc^2}$$

- Unsurprisingly, we can only get energy changes from electric fields, not (conservative) magnetic fields

Constant Magnetic Field (Zero Electric Field)

- In a constant magnetic field, charged particles move in circular arcs of radius ρ with constant angular velocity ω:



$$\vec{F} = \frac{d}{dt}(\gamma m \vec{v}) = \gamma m \frac{d\vec{v}}{dt} = q\vec{v} \times \vec{B}$$

$$\vec{v} = \vec{\omega} \times \vec{\rho} \qquad \Rightarrow \qquad q\vec{v} \times \vec{B} = \gamma m \vec{\omega} \times \frac{d\vec{\rho}}{dt} = \gamma m \vec{\omega} \times \vec{v}$$

  - For $\vec{B} \perp \vec{v}$ we then have

$$qvB = \frac{\gamma m v^2}{\rho} \qquad p = \gamma m(\beta c) = q(B\rho) \qquad \boxed{\frac{p}{q} = (B\rho)}$$

$$\boxed{\omega = \frac{v}{\rho} = \frac{qB}{\gamma m}}$$

Rigidity: Bending Radius vs Momentum

Beam

$$\boxed{\frac{p}{q} = (B\rho)}$$

Accelerator (magnets, geometry)

- This is such a useful expression in accelerator physics that it has its own name: **rigidity**
- Ratio of momentum to charge
  - How hard (or easy) is a particle to deflect?
  - Often expressed in [T-m] (easy to calculate B)
  - Be careful when q≠e!!
- A very useful expression

$$\boxed{\frac{p[\mathrm{GeV/c}]}{q[e]} \approx 0.3\, B[\mathrm{T}]\, \rho[\mathrm{m}]}$$

Cyclotron Frequency

$$\boxed{\omega = \frac{v}{\rho} = \frac{qB}{\gamma m}}$$

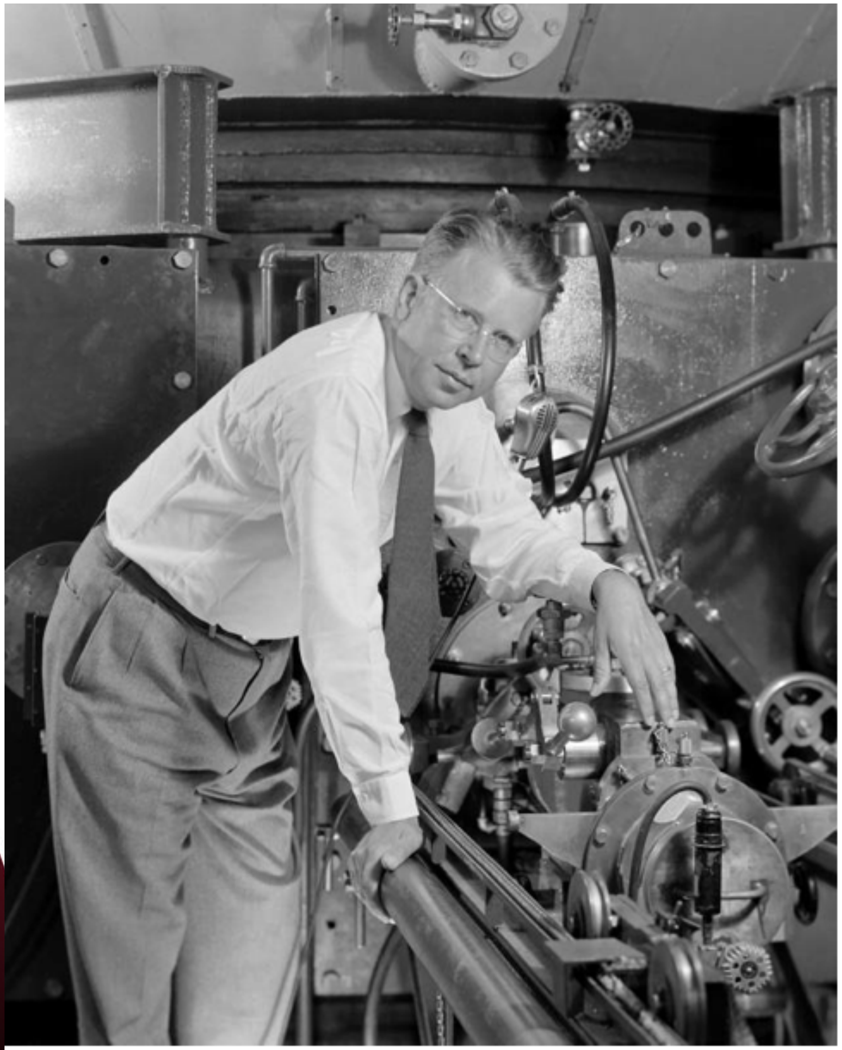
- Another very useful expression for particle angular frequency in a constant field: **cyclotron frequency**

- In the nonrelativistic approximation

$$\omega_{\text{nonrelativistic}} \approx \frac{qB}{m}$$

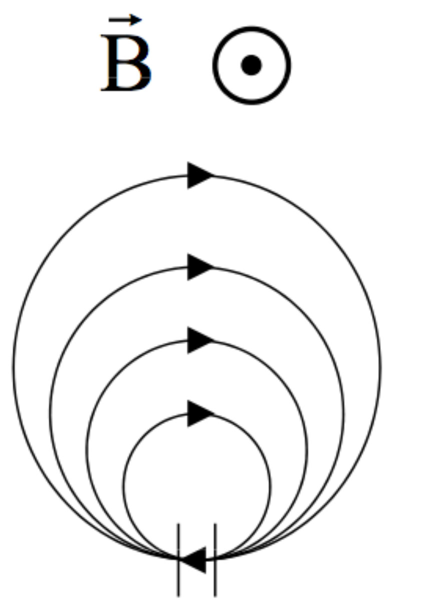
Revolution frequency is independent of radius or energy!

Lawrence and the Cyclotron



Ernest Orlando Lawrence

- Can we repeatedly spiral and accelerate particles through the same potential gap?



Cyclotron Frequency Again

- Recall that for a constant B field

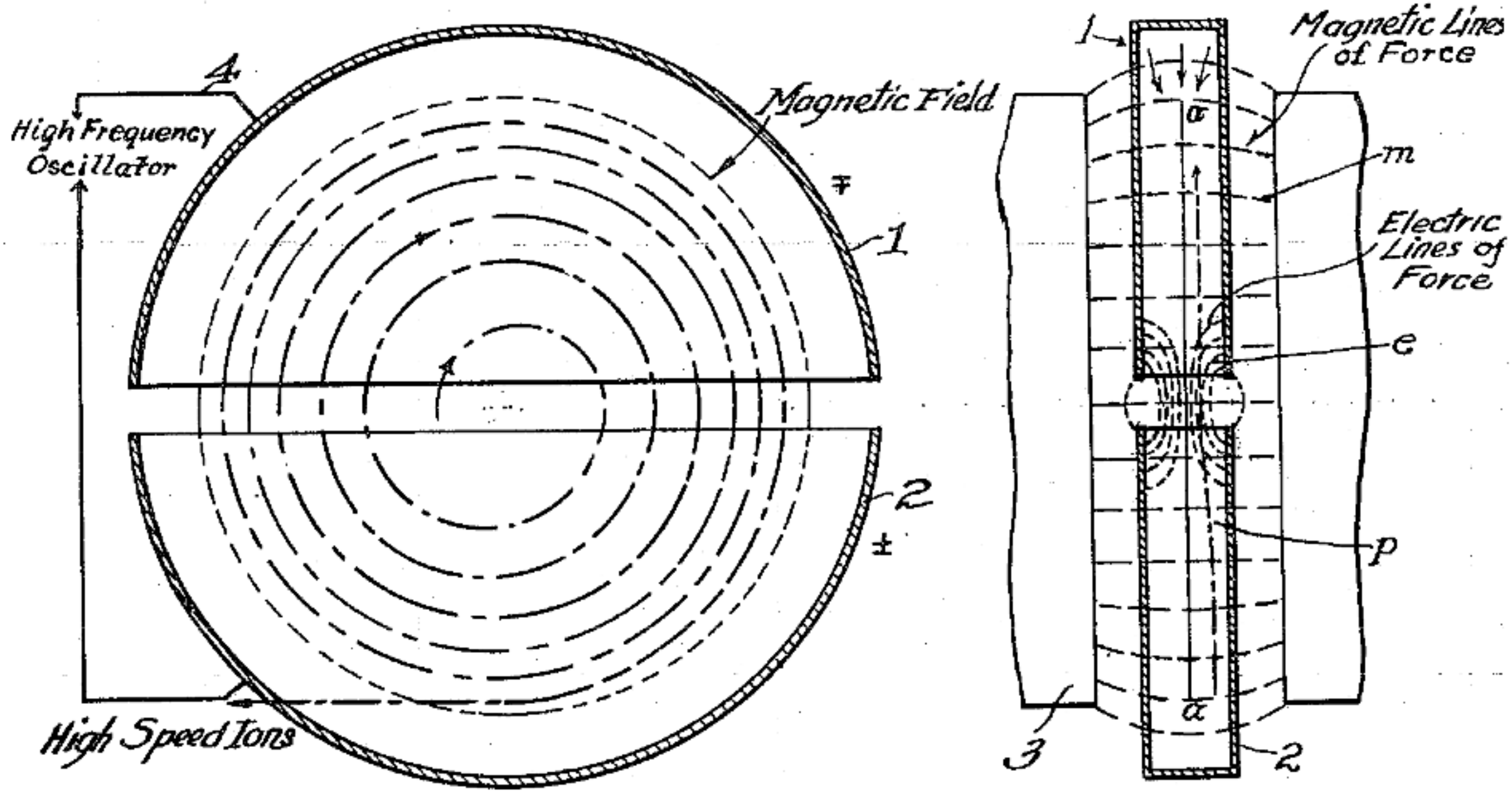
$$p = \gamma m v = q(B\rho) \quad \Rightarrow \quad \rho = \left(\frac{\gamma m}{qB}\right) v$$

  - Radius/circumference of orbit scale with velocity
    - Circulation time (and frequency) are independent of v
  - Apply AC electric field in the gap at frequency f_{rf}
    - Particles accelerate until they drop out of resonance

$$\omega = \frac{v}{\rho} = \frac{qB}{\gamma m} \qquad f_{\mathrm{rf}} = \frac{\omega}{2\pi} = \frac{qB}{2\pi\gamma m}$$

    - Note a first appearance of "bunches", not DC beam
    - Works best with heavy particles (hadrons, not electrons)

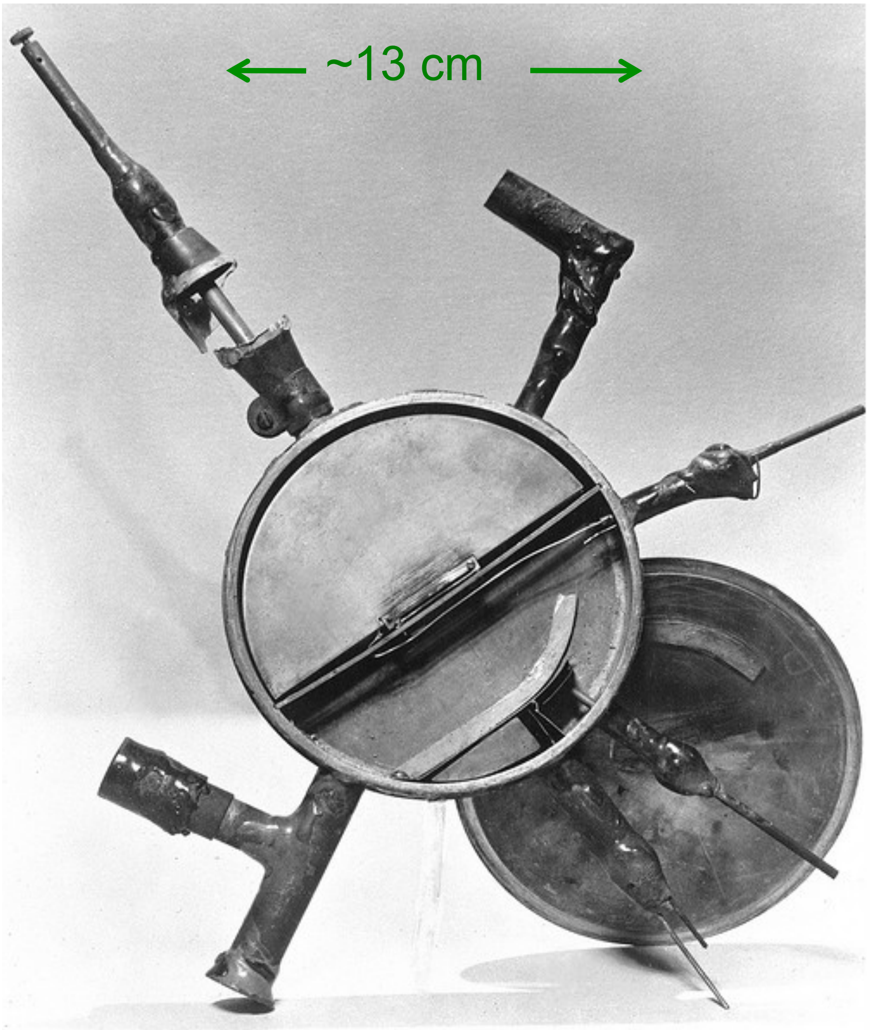
A Patentable Idea



- 1934 patent 1948384
  - Two accelerating gaps per turn!

All The Fundamentals of an Accelerator

- Large static magnetic fields for guiding (~1T)
  - But no vertical focusing
- HV RF electric fields for accelerating
  - (No phase focusing)
  - (Precise f control)
- p/H source, injection, extraction, vacuum
- 13 cm: 80 keV
- 28 cm: 1 MeV
- 69 cm: ~5 MeV
- … 223 cm: ~55 MeV



(Berkeley)

Livingston, Lawrence, 27"/69 cm Cyclotron

M.S. Livingston and E.O. Lawrence, 1934

The Joy of Physics

- Describing the events of January 9, 1932, Livingston is quoted saying:

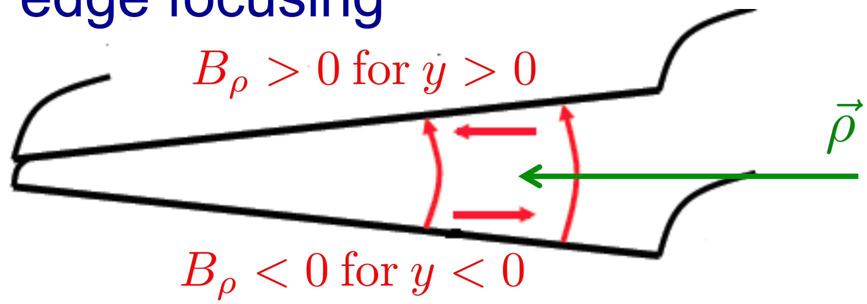
"I recall the day when *I had adjusted the oscillator to a new high frequency*, and, with *Lawrence looking over my shoulder*, tuned the magnet through resonance. As the galvanometer spot swung across the scale, indicating that protons of 1-MeV energy were reaching the collector, *Lawrence literally danced around the room with glee*. The news quickly spread through the Berkeley laboratory, and we were busy all that day demonstrating million-volt protons to eager viewers."

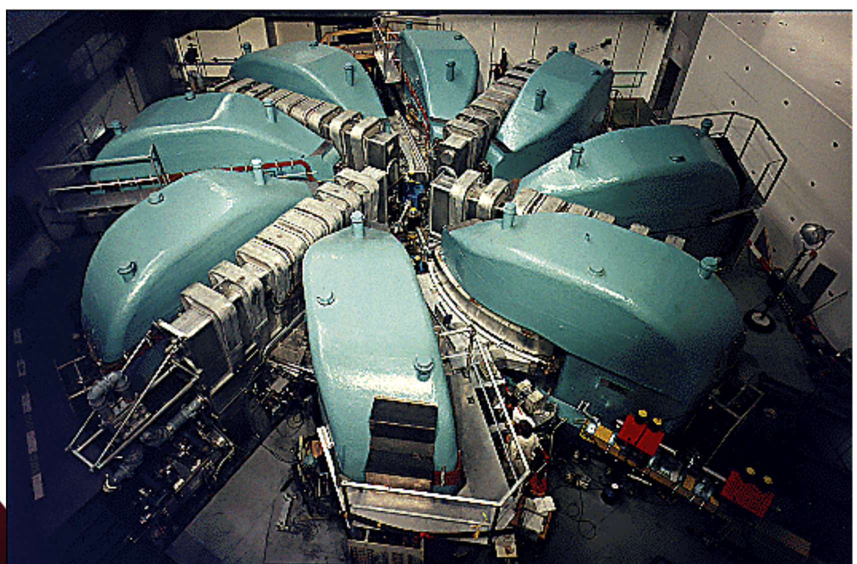
APS Physics History, Ernest Lawrence and M. Stanley Livingston

Modern Isochronous Cyclotrons

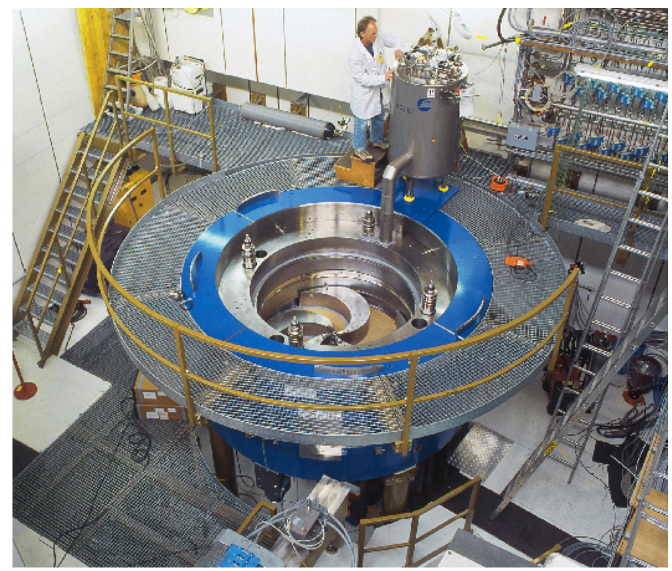
- Higher bending field at higher energies
  - But also introduces vertical defocusing
  - Use bending magnet "edge focusing"
    (Tuesday magnet lecture)

$$f_{\rm rf} = \frac{qB(\rho)}{2\pi\gamma(\rho)m}$$





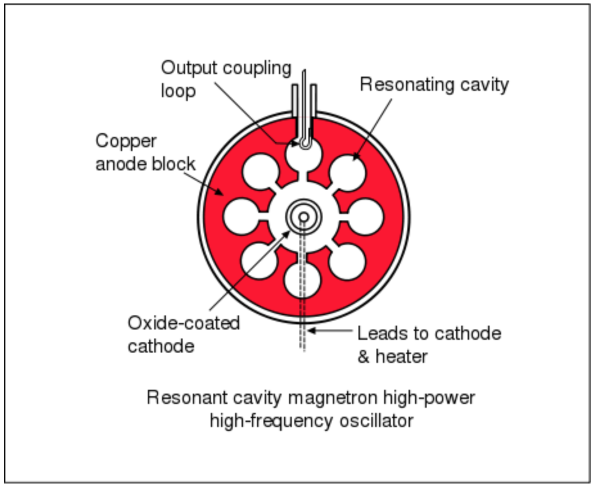
590 MeV PSI Isochronous Cyclotron (1974)



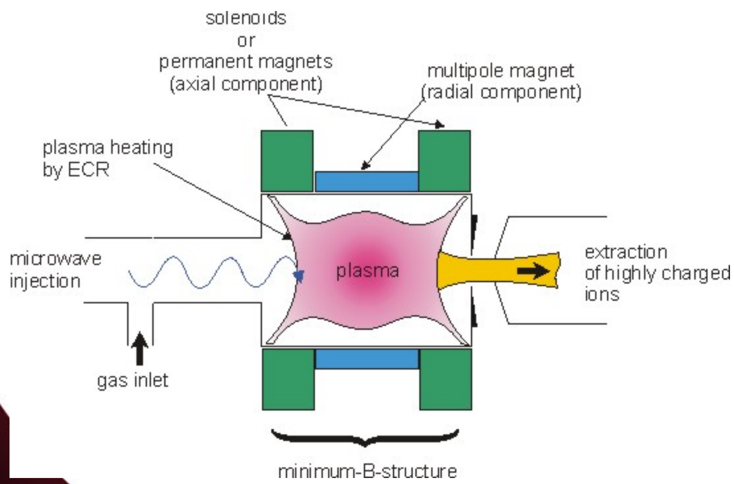
250 MeV PSI Isochronous Cyclotron (2004)

Electrons, Magnetrons, ECRs

Radar/microwave magnetron



Resonant cavity magnetron high-power high-frequency oscillator



ECR plasma/ion source

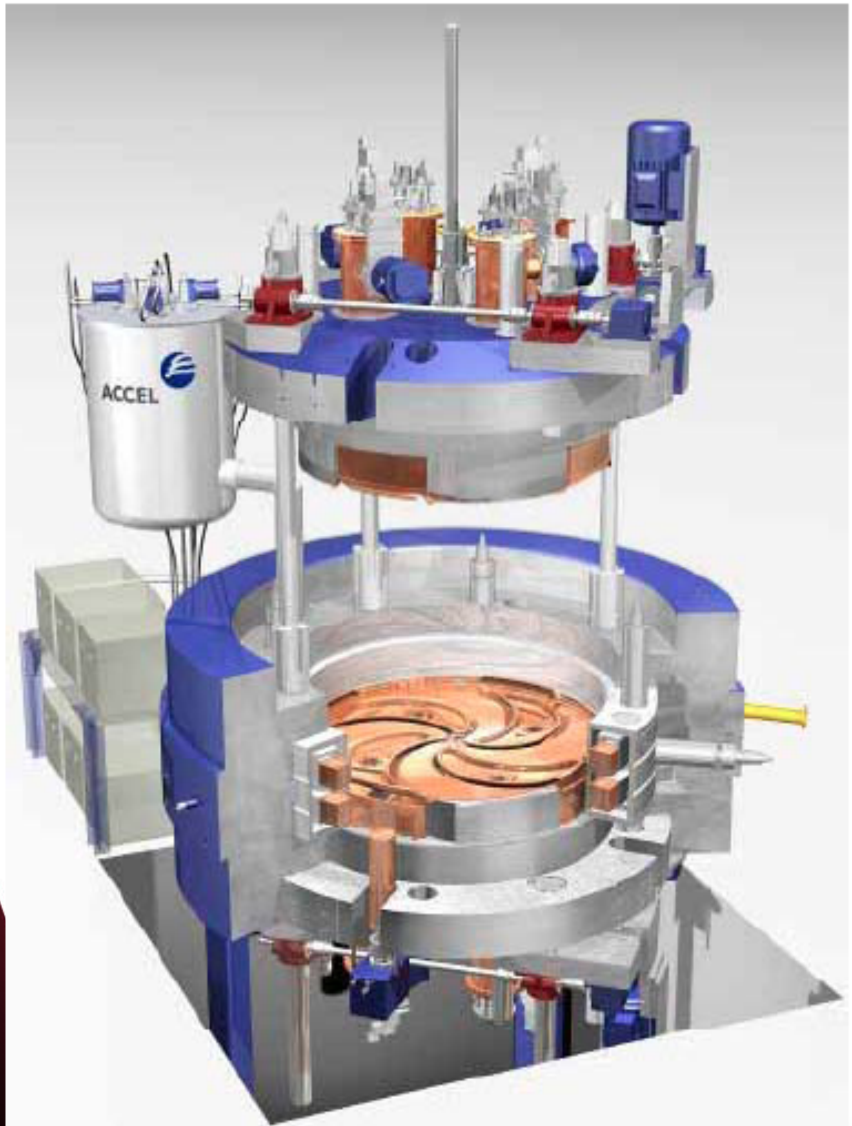
- Cyclotrons aren't good for accelerating electrons
  - Very quickly relativistic!
- But narrow-band response has advantages and uses
  - Magnetrons

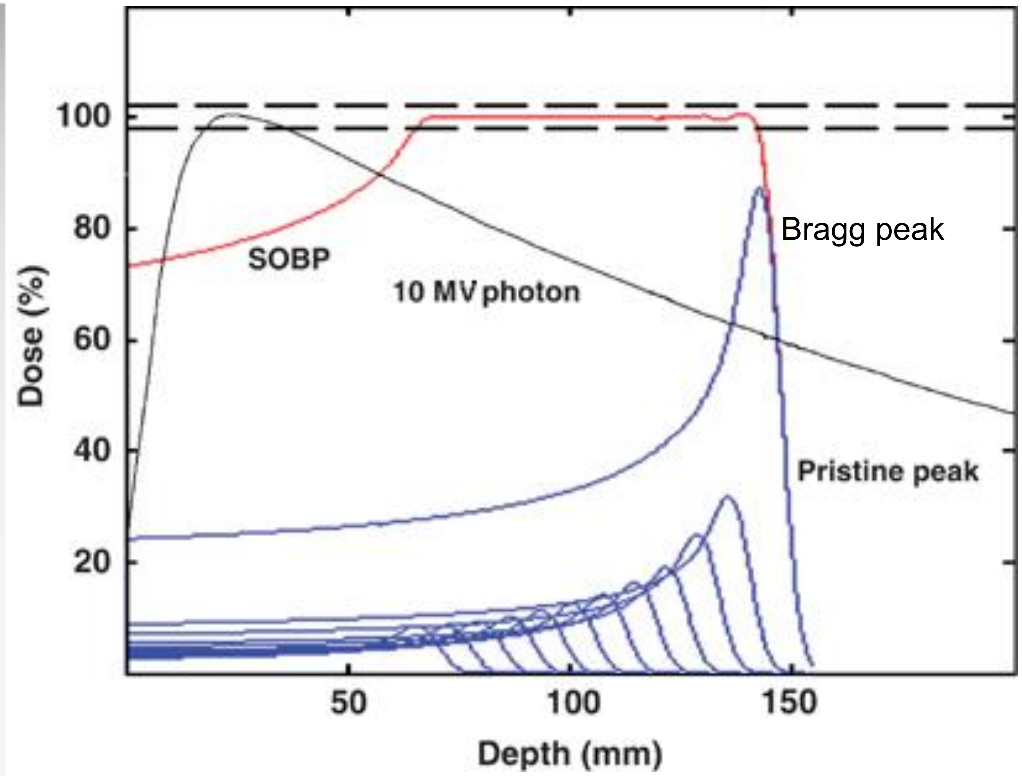
    generate resonant high-power microwaves from circulating electron current
  - ECRs
    - generate high-intensity ion beams and plasmas by resonantly stripping electrons with microwaves

Cyclotrons Today

- Cyclotrons continue to evolve
  - Many contemporary developments
    - Superconducting cyclotrons
    - Synchrocyclotrons (FM modulated RF)
    - Isochronous/Alternating Vertical Focusing (AVF)
    - FFAGs (Fixed Field Alternating Gradient)
  - Versatile with many applications even below ~500 MeV
    - High power (>1MW) neutron production
    - Reliable (medical isotope production, ion radiotherapy)
    - Power+reliability: ~5 MW p beam for ADSR (accelerator driven subcritical reactors, e.g. Thorium reactors)

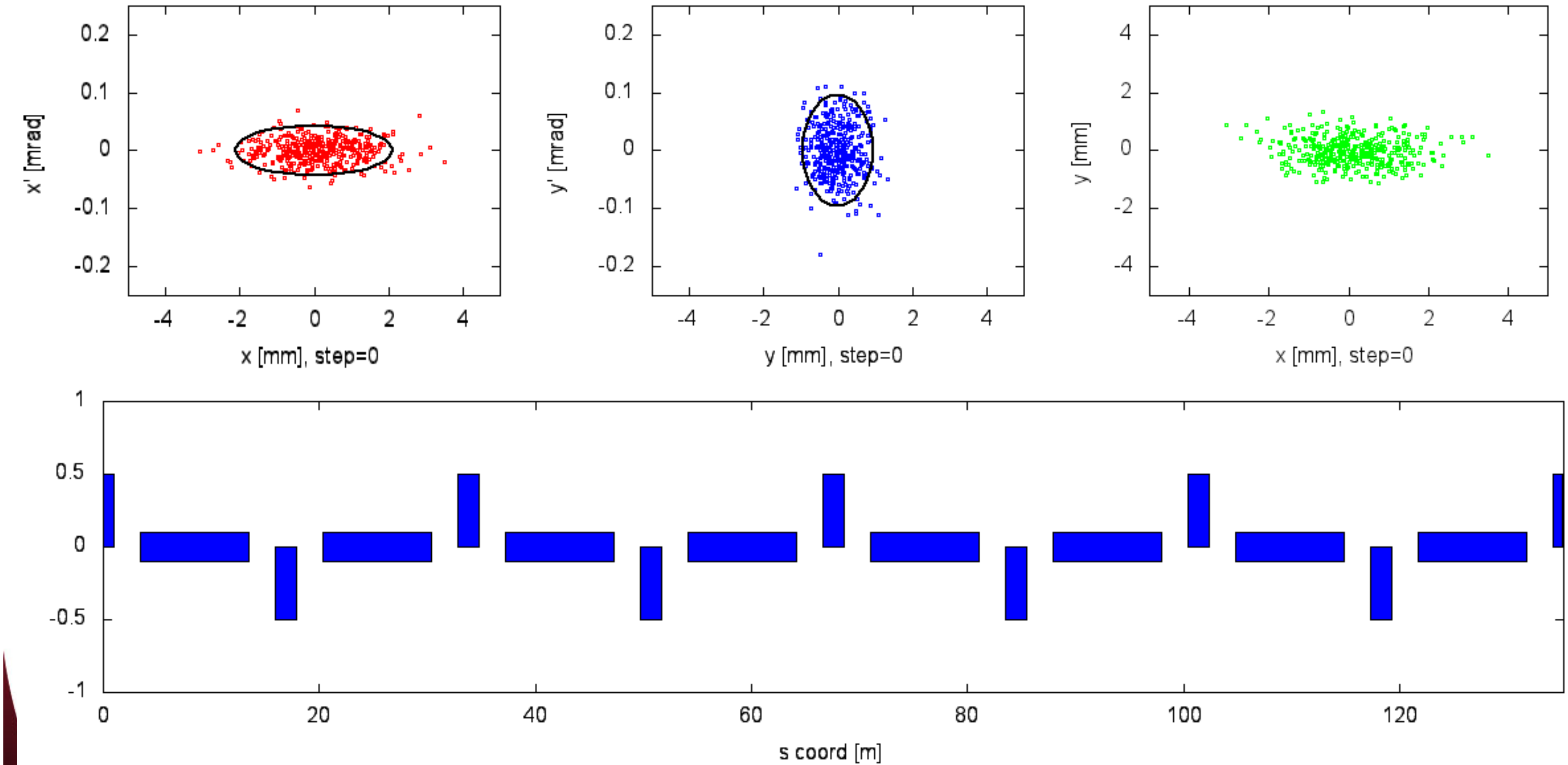
Accel Radiotherapy Cyclotron





- Distinct dose localization advantage for hadrons over X-rays
- Also present work on proton and carbon radiotherapy fast-cycling synchrotrons

=========== Break ===========



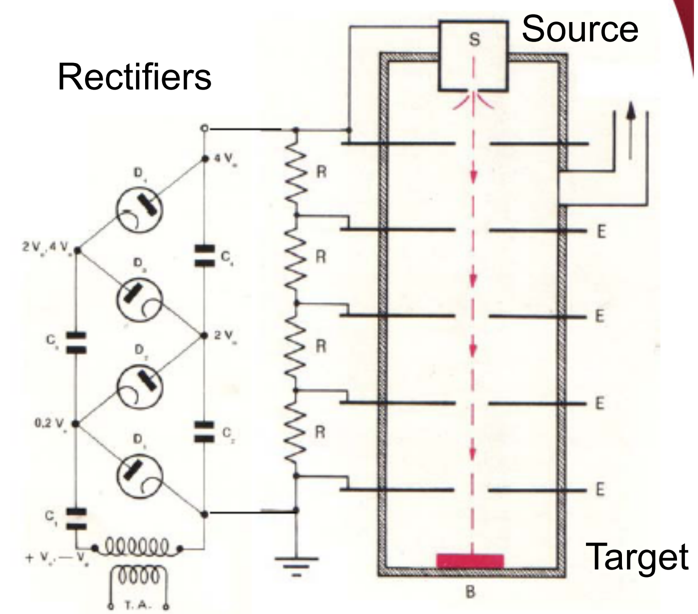
(Brief) Survey of Accelerator Concepts

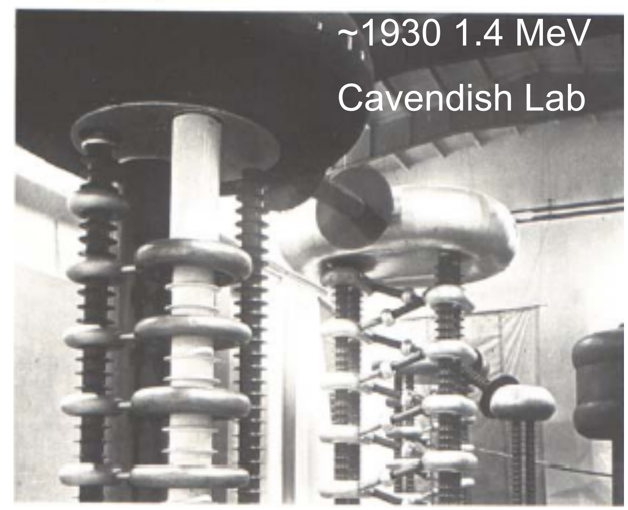
- Producing accelerating gaps and fields (DC/AC)
- Microtrons and their descendants
- Betatrons (and betatron motion)
- Synchrotrons
  - Fixed Target Experiments
  - Colliders and Luminosity (Livingston Plots)
  - Light Sources (FELs, Compton Sources)

- Others include
  - Medical Applications (radiotherapy, isotope production)
  - Spallation Sources (SNS, ESS)
  - Power Production (ADSR)

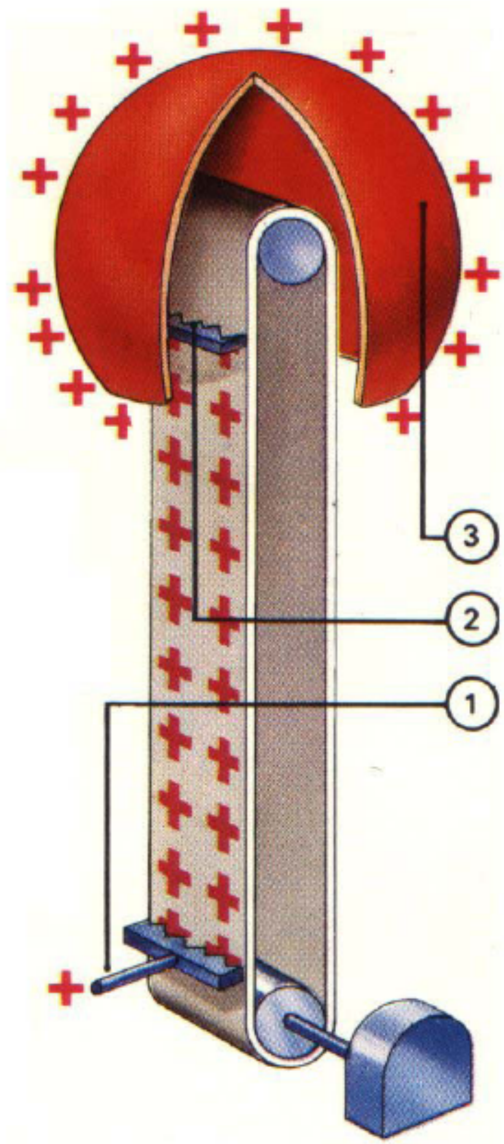
DC Accelerating Gaps: Cockcroft-Walton

- Accelerates ions through successive electrostatic voltages
  - First to get protons to >MeV
  - Continuous HV applied through intermediate electrodes
  - Rectifier-multipliers (voltage dividers)
  - Limited by HV sparking/breakdown
  - FNAL still uses a 750 kV C-W
- Also example of early ion source
  - H gas ionized with HV current
  - Provides high current DC beam





DC Accelerating Gaps: Van de Graaff

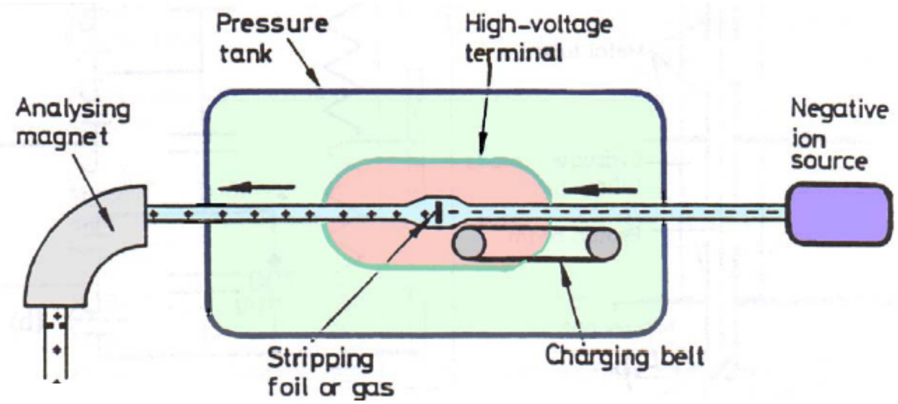


- How to increase voltage?
  - R.J. Van de Graaff: charge transport
  - Electrode (1) sprays HV charge onto insulated belt
  - Carried up to spherical Faraday cage
  - Removed by second electrode and distributed over sphere
- Limited by discharge breakdown
  - ~2MV in air
  - Up to 20+ MV in SF_6!
  - Ancestors of Pelletrons (chains)/ Laddertrons (stripes)

Van de Graaff Popularity

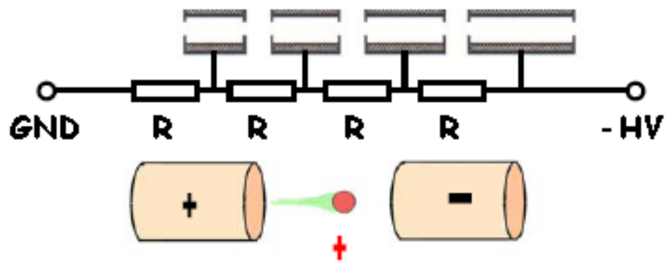
DC Accelerating Gaps: Tandem Van de Graaff

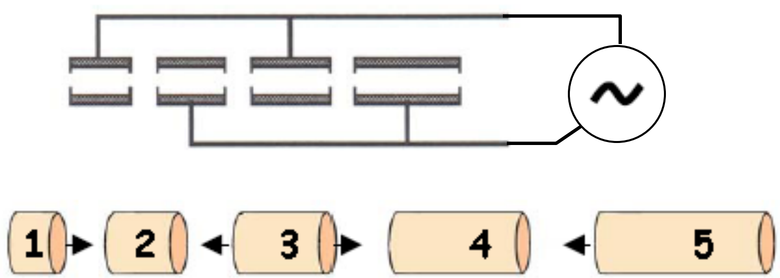
- Reverse ion charge state in middle of Van de Graaff allows over twice the energy gain
  - Source is at ground



- This only works for negative ions
- However, stripping need not be symmetric
  - Second stage accelerates more efficiently
- BNL: two Tandems (1970, 14 MV, 24m)
  - Au^{-1} to $Au^{+10}/Au^{+11}/Au^{+12}$ to Au^{+32} for RHIC
  - About a total of 0.85 MeV/nucleon total energy

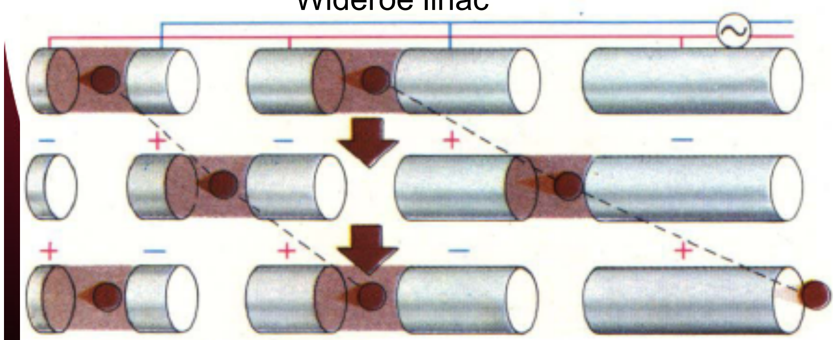
From Electrostatic to RF Acceleration





π *mode*

Wideroe linac

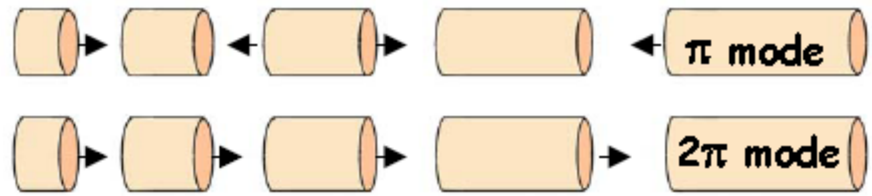


Pagani and Mueller 2002

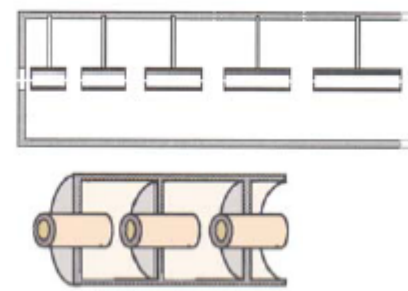
- Cockcroft-Waltons and Van de Graaffs have DC voltages, E fields
- What about putting on AC voltage?
  - Attach consecutive electrodes to opposite polarities of ACV generator
  - Electric fields between successive electrodes vary sinusoidally
  - Consecutive electrodes are 180 degrees out of phase (π mode)
- At the right drive frequency, particles are accelerated in each gap
  - While polarity change occurs, particles are shielded in drift tubes
  - To stay in phase with the RF, drift tube length or RF frequency must increase at higher energies

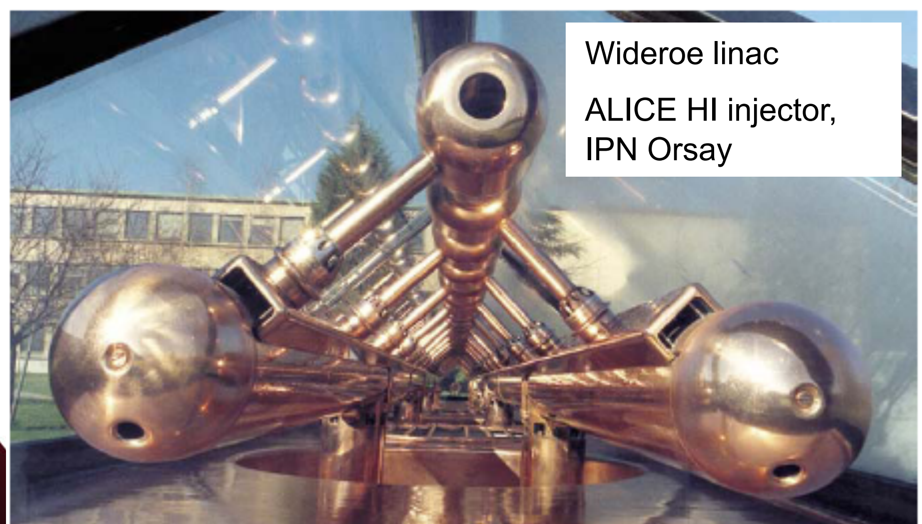
Resonant Linac Structures

- Wideroe linac: π mode
- Alvarez linac: 2π mode



- Need to minimize excess RF power (heating)
  - Make drift tubes/gaps resonant to RF frequency
  - In 2π mode, currents in walls separating two subsequent cavities cancel; tubes are passive
  - We'll cover RF and longitudinal motion next week…





Wideroe linac

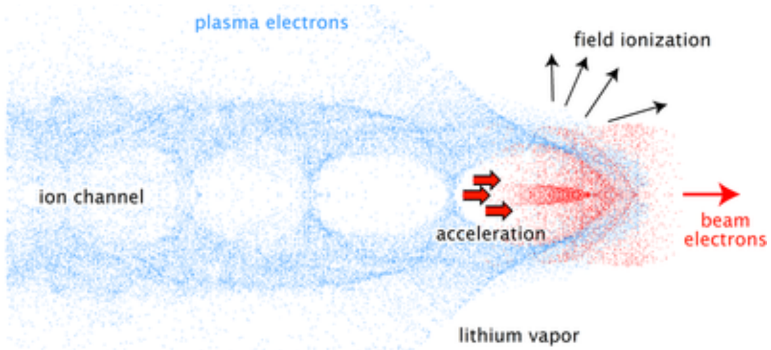
ALICE HI injector, IPN Orsay

Drift tube linac

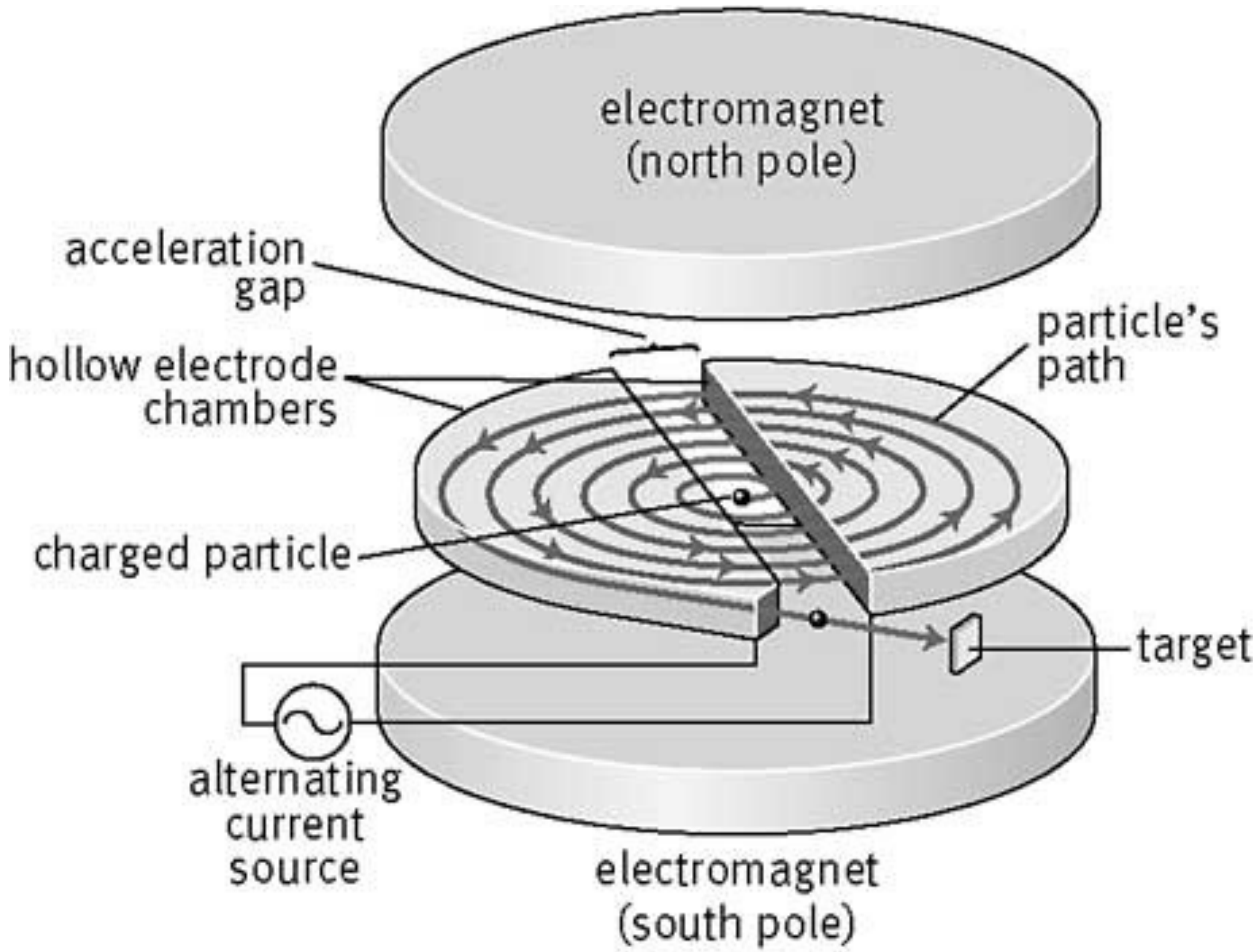
Saturne, Saclay

Advanced Acceleration Methods

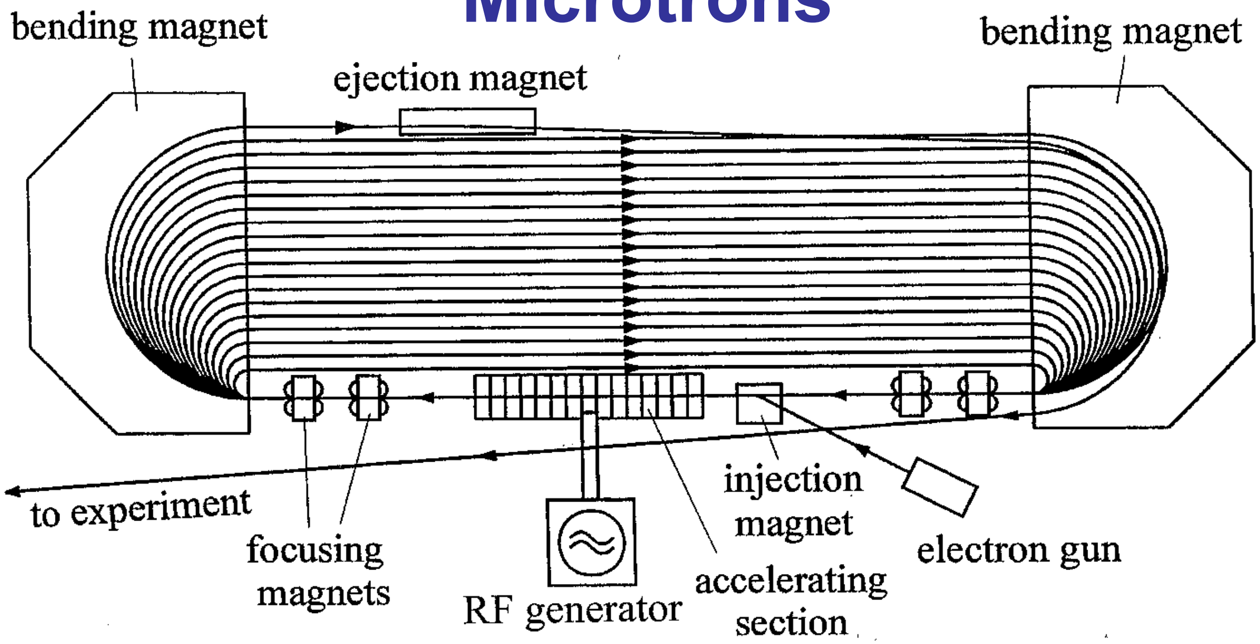
- How far do accelerating gradients go?
  - Superconducting RF acceleration: ~40 MV/m
  - CLIC: ~100 MV/m
    - Two-beam accelerator: drive beam couples to main beam
  - Dielectric wall acceleration: ~100 MV/m
    - Induction accelerator, very high gradient insulators
  - Dielectric wakefield acceleration: ~GV/m
  - Laser plasma acceleration: ~30 GV/m
    - electrons to 1 GeV in 3.3 cm
    - particles ride in wake of plasma charge separation wave



Cyclotrons (Again)

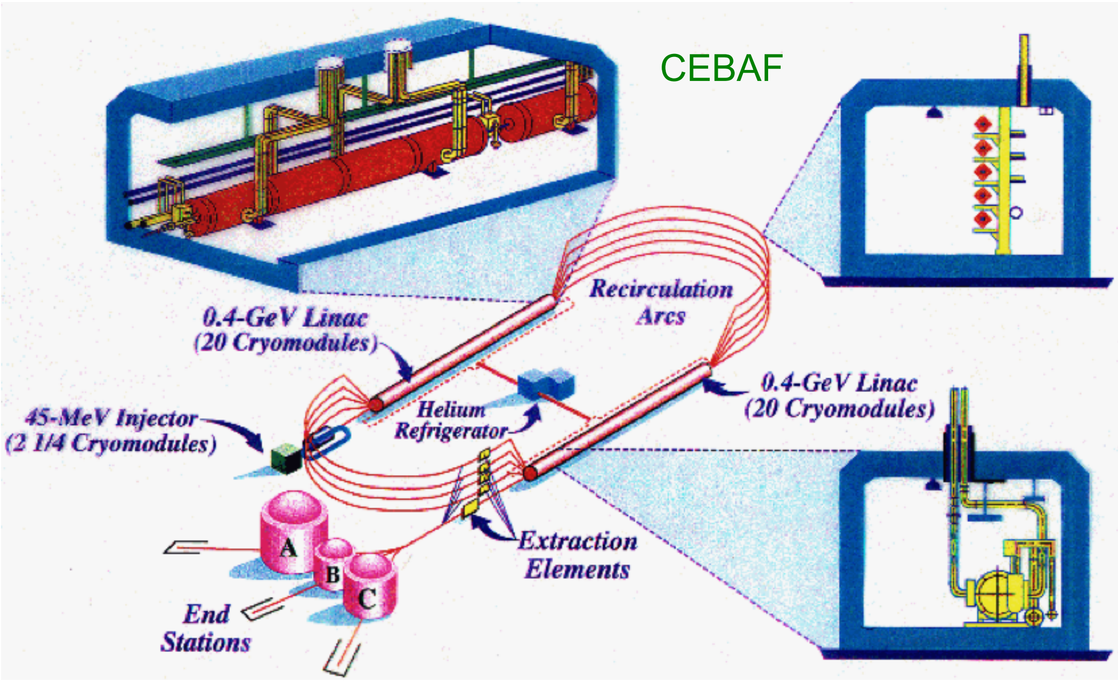


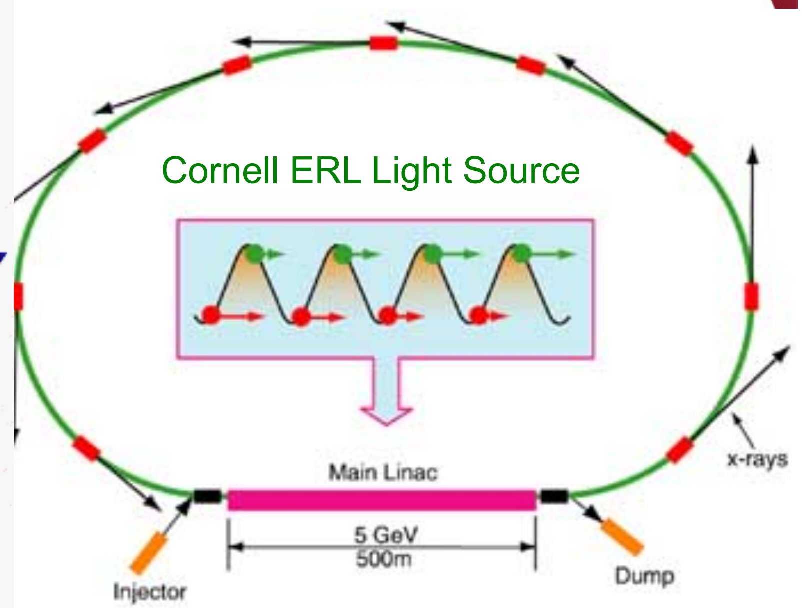
Microtrons



- What about electrons? Microtrons are like cyclotrons
  - but each revolution electrons "slip" by integer # of RF cycles
  - Trades off large # of revs for minimal RF generation cost
  - Bends must have large momentum aperture
  - Used for medical applications today (20 MeV, 1 big magnet)
  - Mainz MAMI: 855 MeV, used for nuclear physics

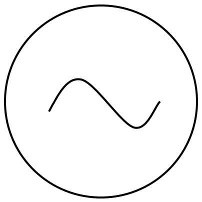
Recirculating Linacs and ERLs



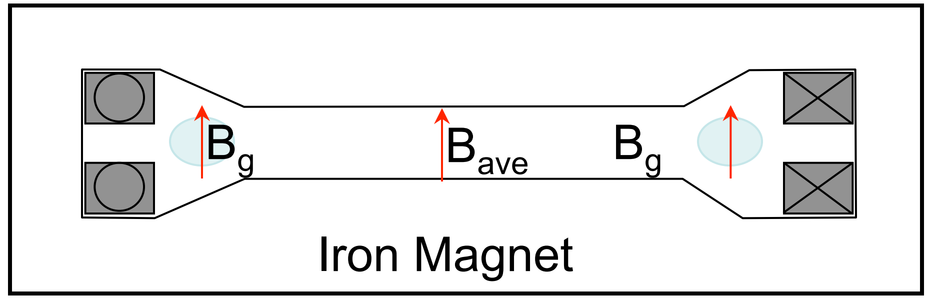


- Recirculating linacs have separate arcs, longer linacs
  - CEBAF: 4->6->12 GeV polarized electrons, 2 SRF linacs
  - Higher energy at cost of more linac, separated bends
- Energy recovery linacs recirculate exactly out of phase
  - Raise energy efficiency of linac, less beam power to dump
  - Requires high-Q SRF to recapture energy efficiently

Betatrons



$I(t)=I_0 \cos(2\pi\omega_I t)$



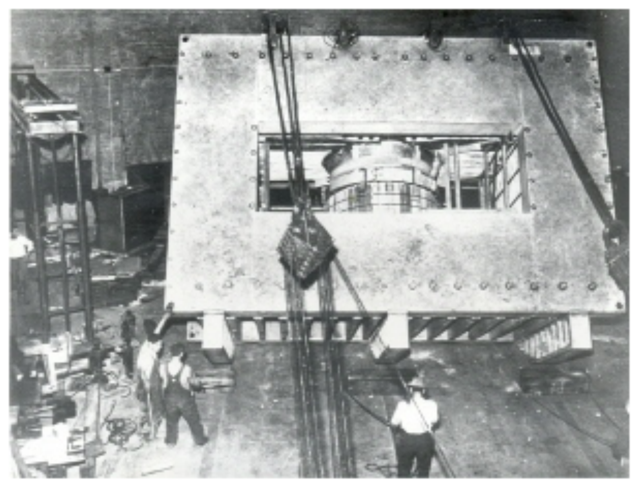
- Apply Faraday's law with time-varying current in coils
- Beam sees time-varying electric field – accelerate half the time!
- Early proofs of stability: focusing and betatron motion

Donald Kerst

UIUC 2.5 MeV

Betatron, 1940

Don't try this at home!!

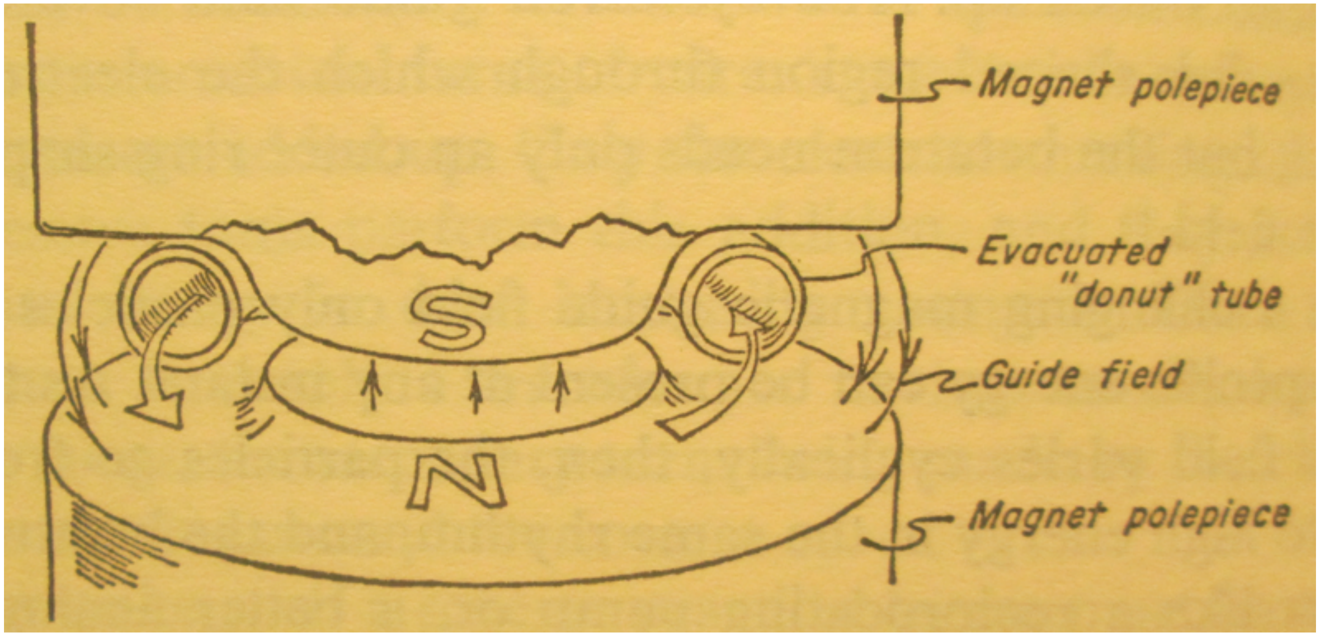


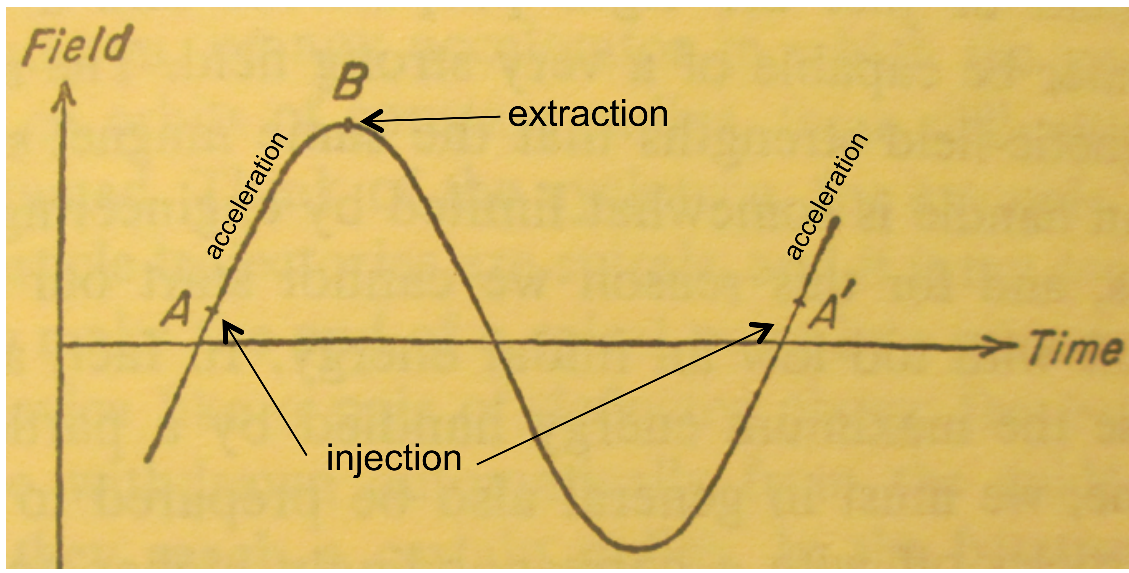
Really don't try this at home!!

UIUC 312 MeV

betatron, 1949

Betatrons



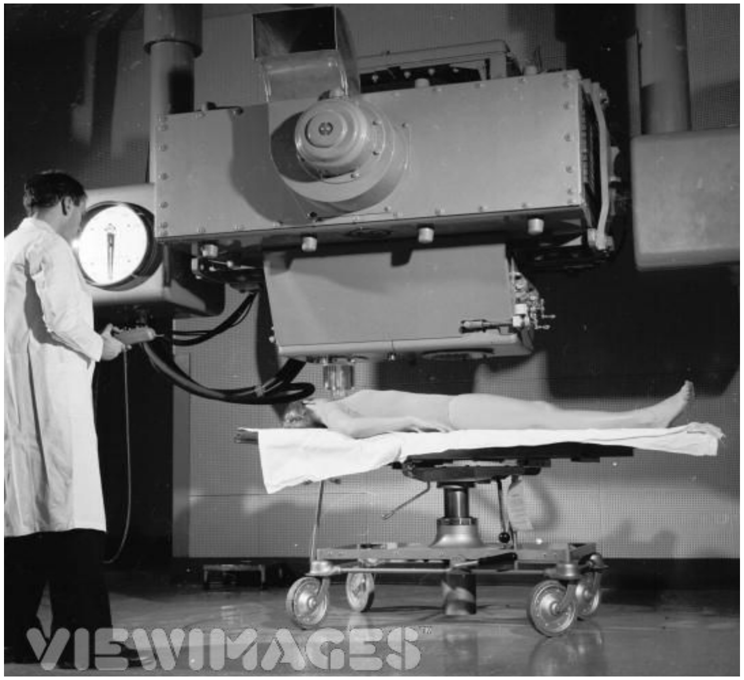


Wilson and Littauer: "Accelerators, Machines of Nuclear Physics"

Betatrons

- Betatrons produced electrons up to 300+ MeV
  - Early materials and medical research
  - Also produced medical hard X-rays and gamma rays

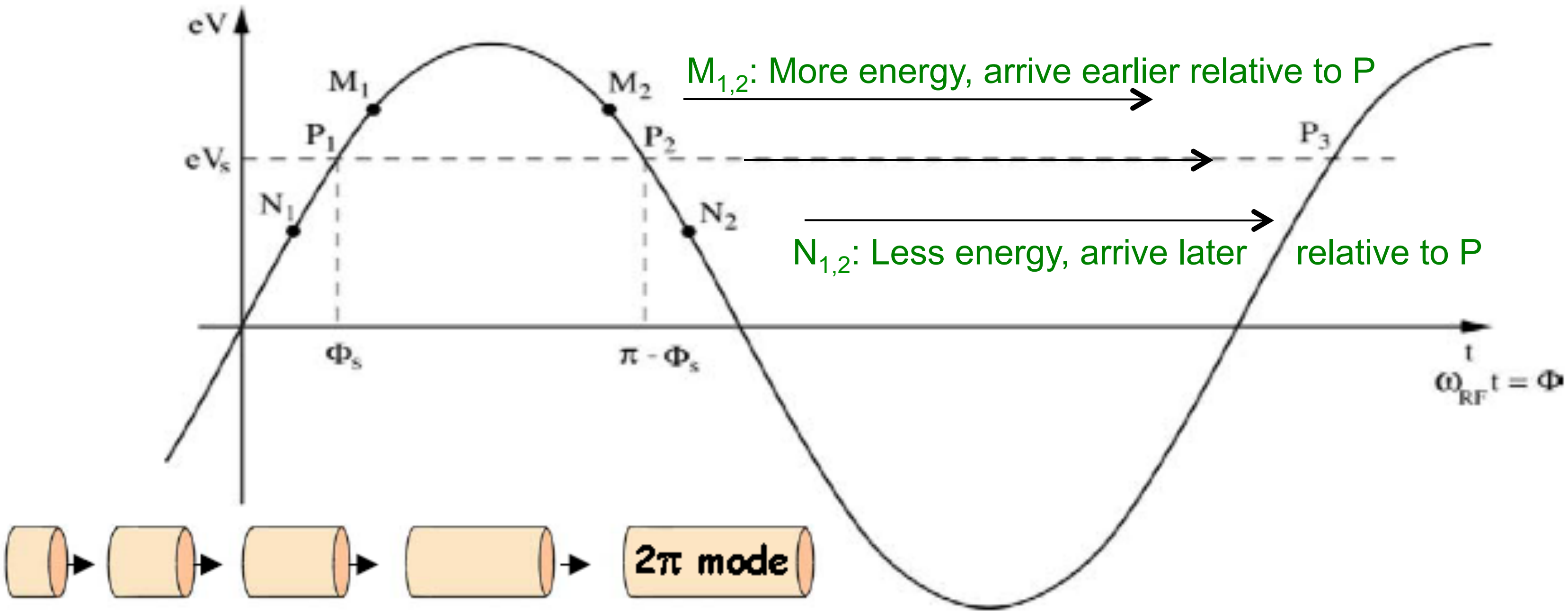
- Betatrons have their challenges
  - Linear aperture scaling
  - Large stored energy/impedance
  - Synchrotron radiation losses
  - Quarter duty cycle
  - Ramping magnetic field quality



This will only hurt a bit…

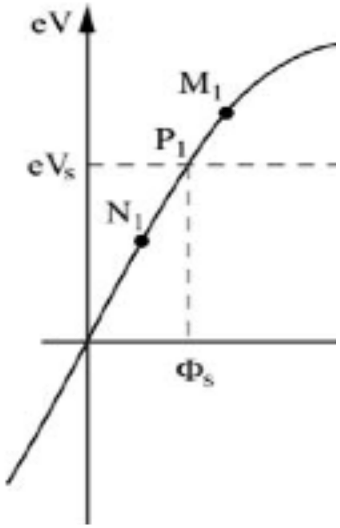
- **More on betatrons/weak focusing this afternoon**

Phase Stability



- Consider a series of accelerating gaps (or a ring with one gap)
  - **By design** there is a synchronous phase Φ_s that gains just enough energy to hit phase Φ_s in the next gap
  - $P_{1,2}$ are fixed points: they "ride the wave" exactly in phase
- If increased energy means increased velocity ("below transition")
  - M_1,N_1 will move towards P_1 (local stability) => **phase stability**
  - $M_{2,}N_2$ will move away from P_2 (local instability)

Phase Stability Implies Transverse Instability



$$\frac{\partial V}{\partial t} > 0 \quad \Rightarrow \quad \frac{\partial E_z}{\partial z} < 0$$

- For phase stability, longitudinal electric field must have a negative gradient. But then (source-free) Maxwell says

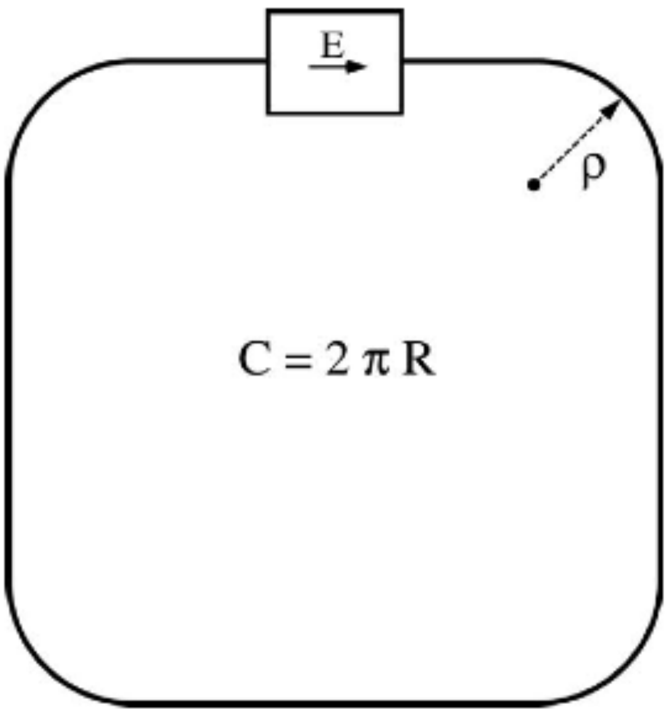
$$\vec{\nabla} \cdot \vec{E} = 0 \quad \Rightarrow \quad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \quad \Rightarrow \quad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} > 0$$

There must be some transverse defocusing/diverging force!

Any accelerator with RF phase stability (longitudinal focusing) needs transverse focusing! (solenoids, quads…)

The Synchrotron

The synchrotron is a synchronous accelerator since there is a synchronous RF phase for which the energy gain fits the increase of the magnetic field at each turn. That implies the following operating conditions:



$e\hat{V}\sin\Phi \longrightarrow$ Energy gain per turn

$\Phi = \Phi_s = cte \longrightarrow$ Synchronous particle

$\omega_{RF} = h\omega_r \longrightarrow$ RF synchronism

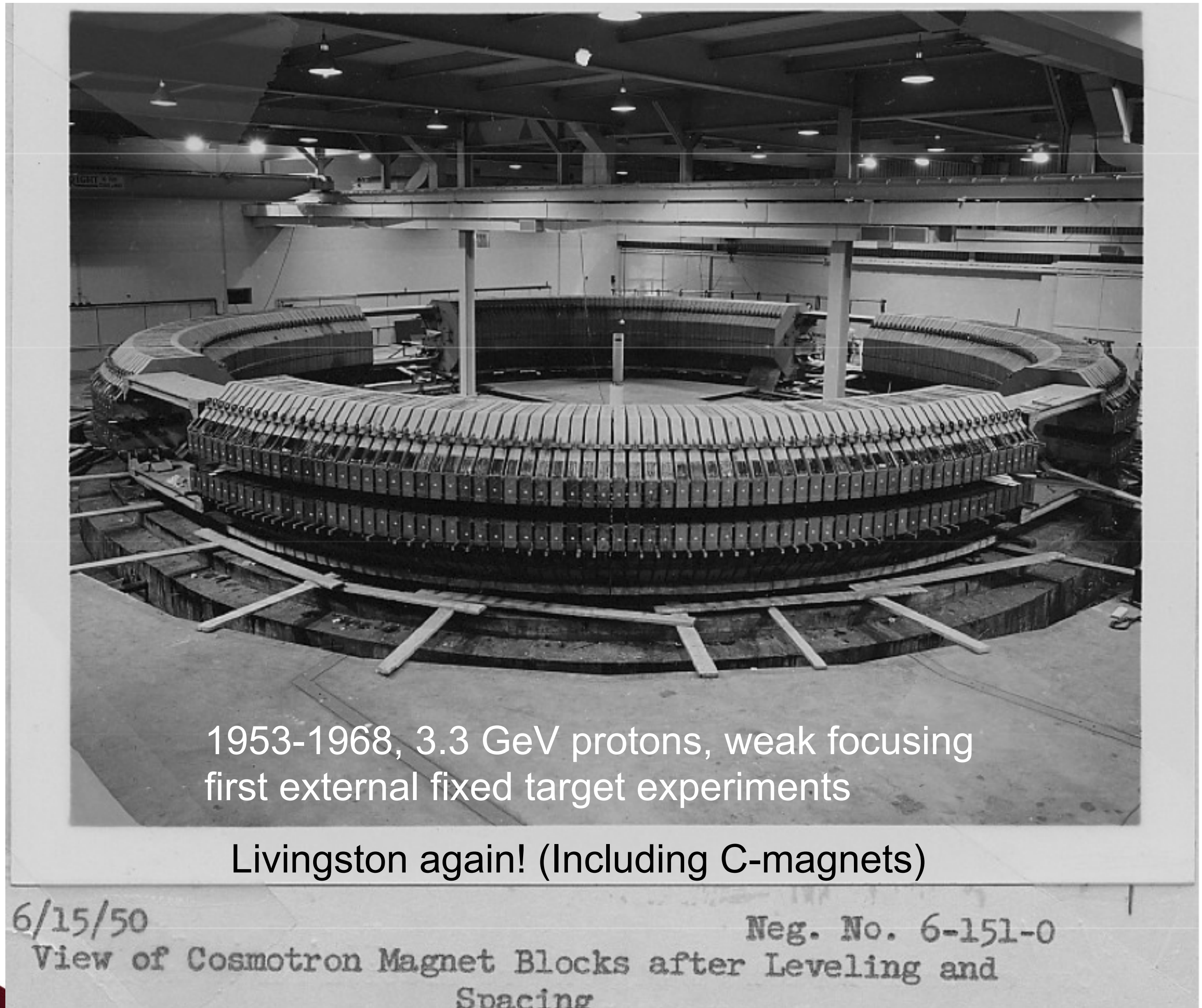
$\rho = cte \quad R = cte \longrightarrow$ Constant orbit

$B\rho = P/e \Rightarrow B \longrightarrow$ Variable magnetic field

If v = c, ω_r hence ω_{RF} remain constant (ultra-relativistic e^-)

Joel LeDuff, CERN Accel School July 2003

BNL Cosmotron



1953-1968, 3.3 GeV protons, weak focusing
first external fixed target experiments

Livingston again! (Including C-magnets)

National Academy of Sciences, Biographical Memoir of M. Stanley Livingston by Ernest D. Courant

LBL Bevatron

Ed McMillan and Ed Lofgren

- Last and largest weak-focusing proton synchrotron
- 1954, Beam aperture about 4' square!, beam energy to 6.2 GeV
- Discovered antiproton 1955, 1959 Nobel for Segre/Chamberlain

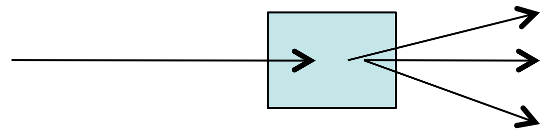
(Became Bevelac, decommissioned 1993, demolished recently)

Fixed Target Experiments

- Why did the Bevatron need 6.2 GeV protons?
  - Antiprotons are "only" 930 MeV/c^2 (times 2…)
  - Bevatron used Cu target, p+n->p+n+p+pbar
  - Mandelstam variables give:

$$\frac{E_{\rm cm}^2}{c^2} = 2\left(\frac{E_1 E_2}{c^2} + p_{\rm z1} p_{\rm z2}\right) + (m_{01}c)^2 + (m_{02}c)^2$$

- Fixed Target experiment



$$(4m_{\rm p0}c)^2 < \frac{E_{\rm cm}^2}{c^2} = 2\frac{E_1 m_{\rm p0}}{c^2} + 2(m_{\rm p0}c)^2 \quad \Rightarrow \quad E_1 > 7m_{\rm p0}c^2$$

$$E_{\rm cm} = \sqrt{2E_1(m_{02}\,c^2)}$$

  - Available CM energy scales with root of beam energy
    - Main issue: forward momentum conservation steals energy

Two Serious Problems

- These machines were getting way too big
  - Bevatron magnet was 10,000 tons
  - Apertures scale linearly with machine size, energy

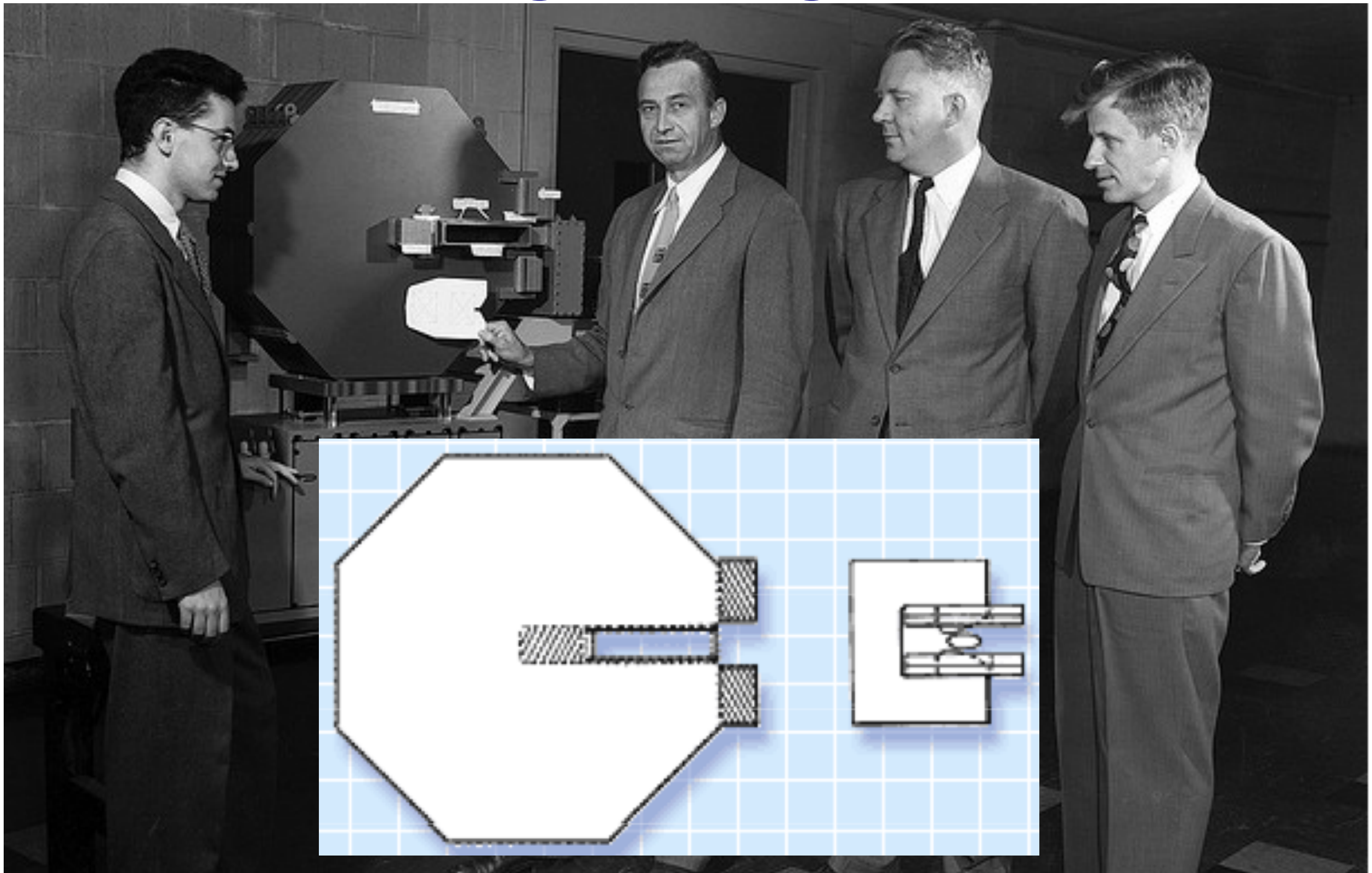
    (Length/circumference scales linearly with energy at fixed field strength too…)

- Fixed target energy scaling is painful
  - Available CM energy only scales with $\sqrt{E_{beam}}$

- Accelerator size grew with the square of desired CM energy
  - Something had to be done!!!

Strong Focusing (1952) and Colliders (1958-62ish) to the rescue!!!

Livingston *Again*?



E. Courant, M.S. Livingston, H. Snyder, and J.P. Blewett

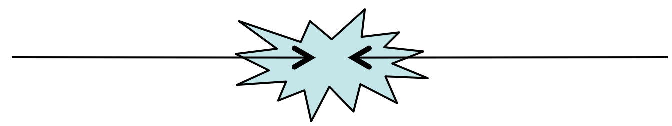
- Strong focusing (and its mathematical treatment) is really the focus (ha) of the rest of this week

Collider Experiments

- What if the Bevatron was a collider?
  - Antiprotons are "only" 930 MeV/c^2 (times 2…)
  - Two-body system (Mandelstam variables) gives (again):

$$\frac{E_{\rm cm}^2}{c^2} = 2\left(\frac{E_1 E_2}{c^2}\right) + p_{z1}p_{z2} + (m_{01}c)^2 + (m_{02}c)^2$$

- Case 2: Collider



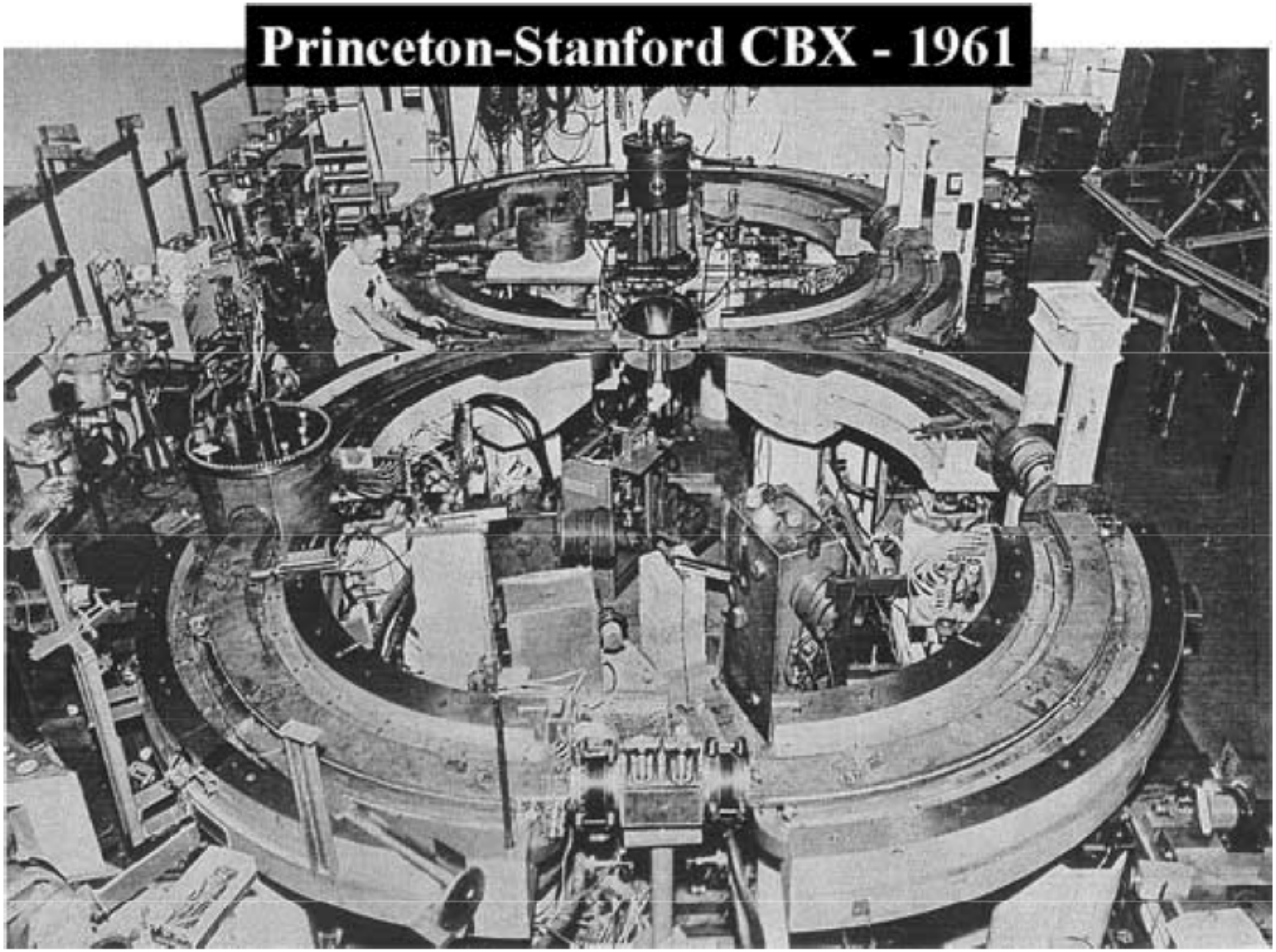
$$E_1 \gg m_{01}c^2 \qquad E_2 \gg m_{02}c^2$$

$$E_{\rm cm} = 2\sqrt{E_1 E_2} = 2E \;\; \text{if } E_1 = E_2$$

  - Linear scaling with beam energy!
  - For Bevacollidatron, e- + e+ -> p+pbar is possible!

(Although the cross section is probably pretty small)

First Electron Collider



Cambridge Electron Accelerator

THE CEA TEAM, 1959. *The group that led the Cambridge Electron Accelerator (CEA) in Cambridge, Massachusetts. The machine was later converted for colliding beam experiments, testing the technique of 'low-beta' that proved so important in storage rings. Seated from left: Thomas Collins and David Jacobus. Standing from left: Fred Barrington, CEA Director Stanley Livinston, Robert Cummings, Lee Young, John Rees, William Jones, Janez Dekkra, and Kenneth Robinson (deceased).*

SLAC Beam Line, "Colliding Beam Storage Rings", John Rees, Mar 1986

Luminosity

- **Luminosity** L is a measure of how many interactions of **cross section** σ can be created per unit time

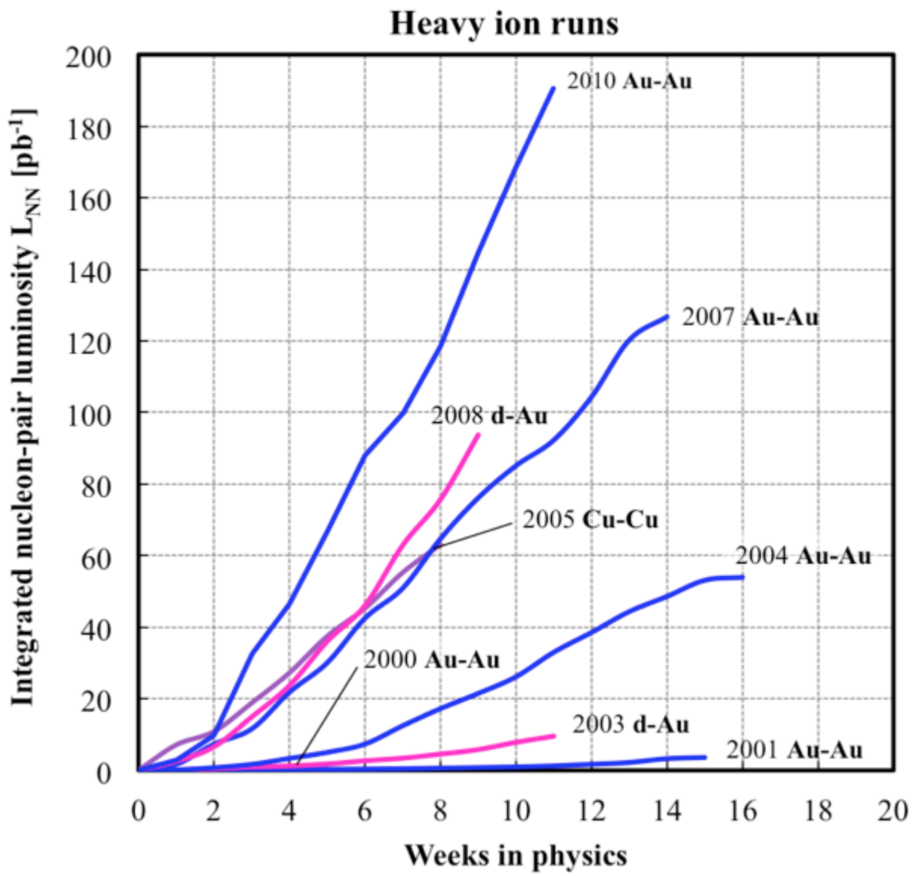
$$L\sigma = \frac{dN}{dt} \qquad N = \sigma \int L\, dt = \sigma\, L_{\text{int}}$$

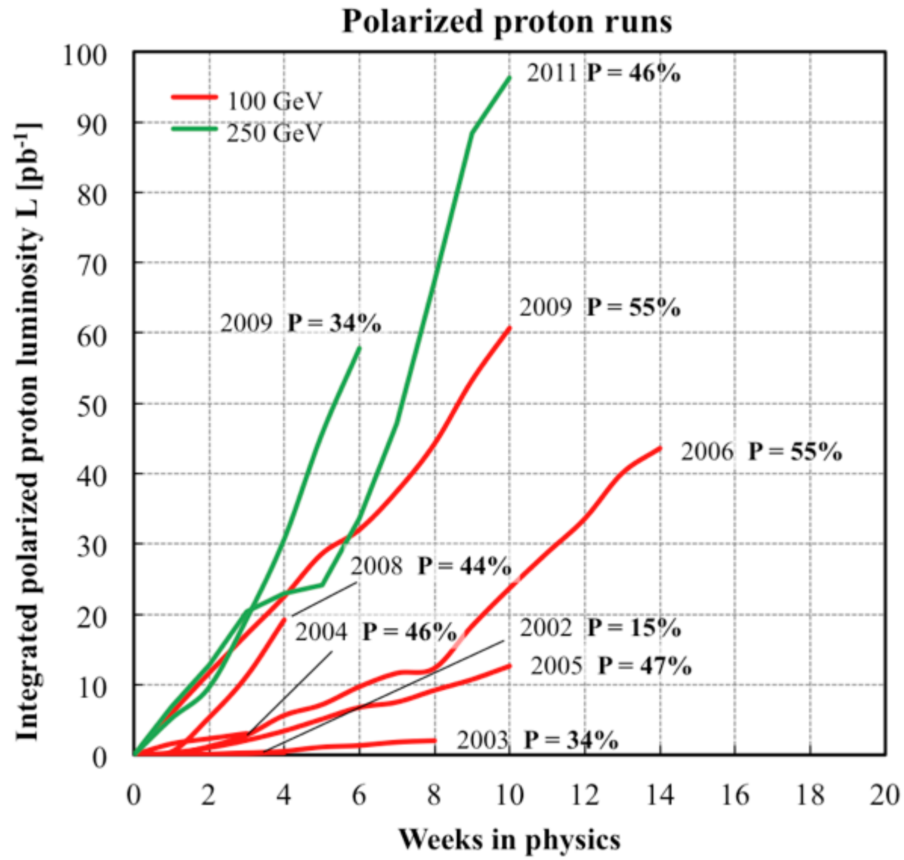
  - L_{int} is integrated luminosity, an important factor of production for colliders
  - $[L]=cm^{-2}\, s^{-1}$, $[L_{int}]=cm^{-2}$ (1 ba=10^{-24} cm; 1 $pb^{-1}=10^{36}\, cm^{-2}$)
- For equal-sized head-on Gaussian beams in a collider

$$L = \frac{f_{\text{rev}}\, h\, N_1\, N_2}{4\pi\sigma_x\sigma_y}$$

  - $\sigma_{x,y}$ are rms beam sizes, h is number of bunches
    - Colliding 100 μm 7.5e9p bunches at 100 kHz for 1 year gives about 1 pb^{-1} of integrated luminosity
    - See Appendix D of the text for more details about luminosity

Evolution of RHIC Collider Luminosities

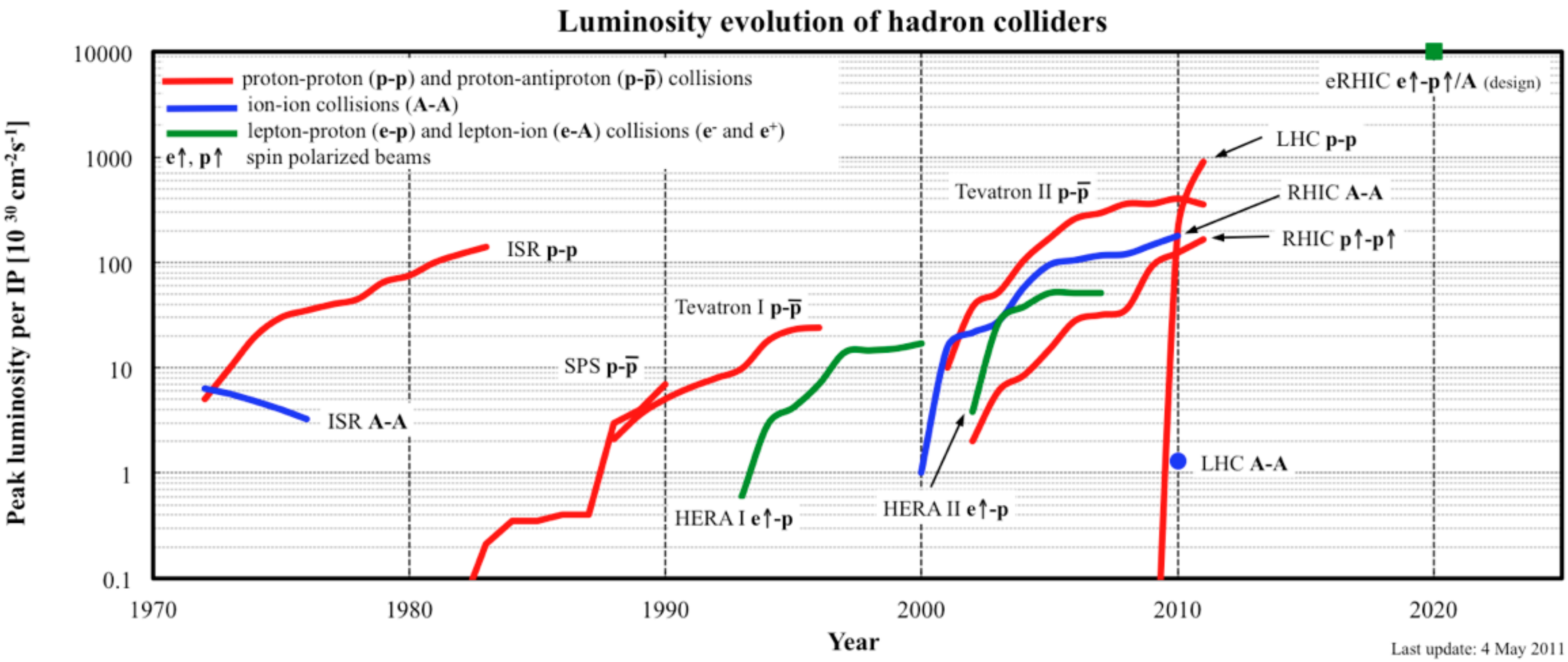




Note: The nucleon-pair luminosity is defined as $L_{NN} = A_1 A_2 L$, where L is the luminosity, and A_1 and A_2 are the number of nucleons of the ions in the two beam respectively.

W. Fischer, http://www.rhichome.bnl.gov/RHIC/Runs

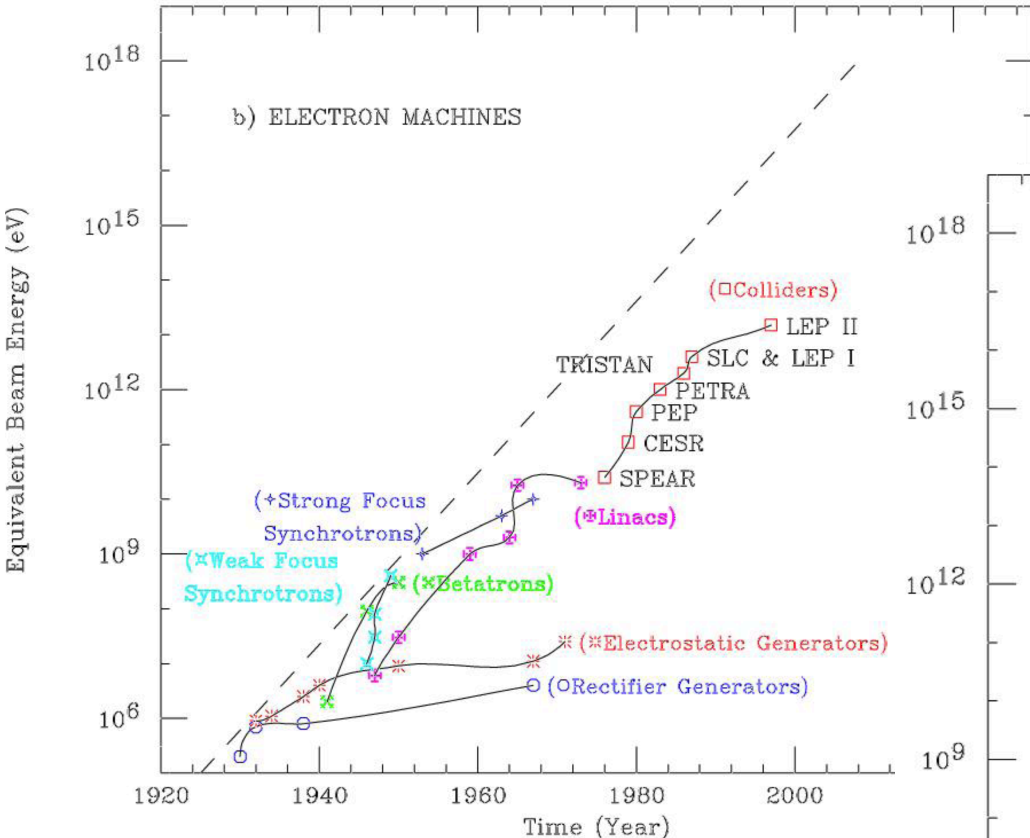
Evolution of Hadron Collider Luminosities



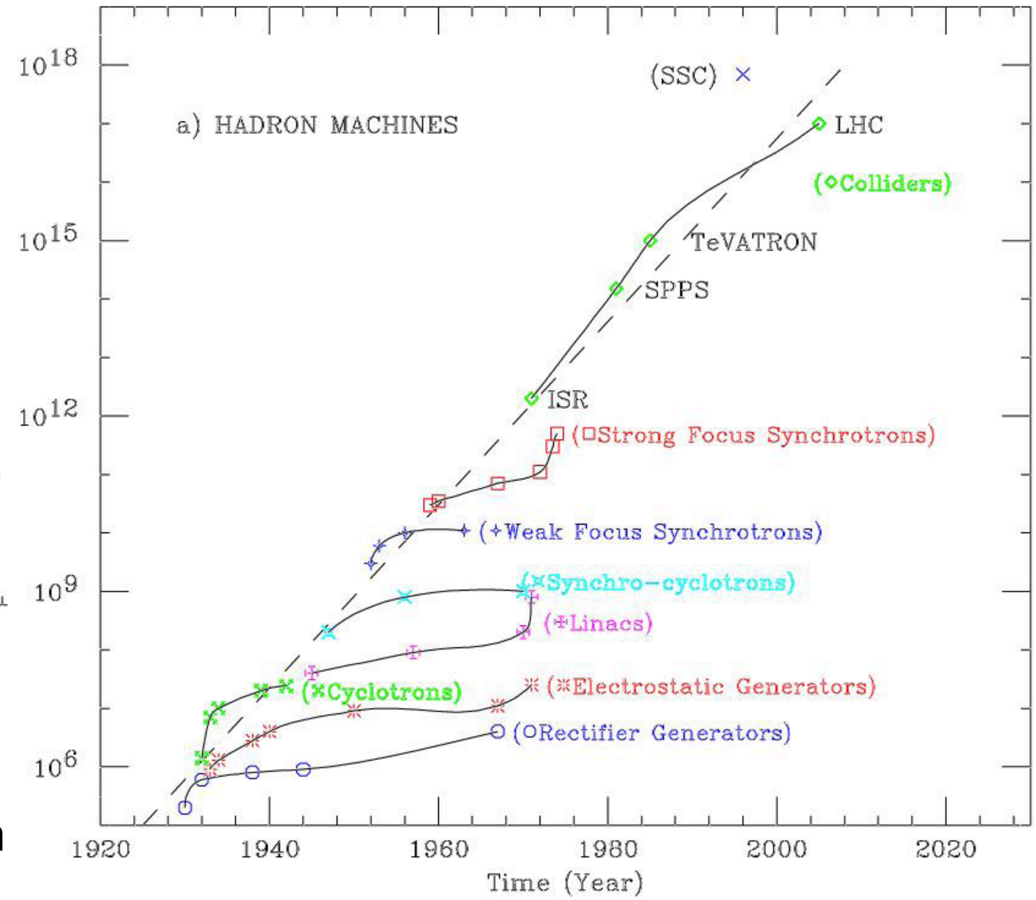
Note: For ion collisions the nucleon-pair luminosity is shown. The nucleon-pair luminosity is defined as $L_{NN} = A_1 A_2 L$, where L is the luminosity, and A_1 and A_2 are the number of nucleons of the ions in the two beam respectively. The highest energies for the machines are: ISR 31 GeV, SPS 315 GeV, Tevatron 980 GeV, HERA 920 GeV (p) 27.5 GeV (e), RHIC 250 GeV, LHC 3.5 TeV.

W. Fischer, http://www.rhichome.bnl.gov/RHIC/Runs

Livingston Plots



Livingston observed that accelerator energy was growing exponentially (in 1950!)



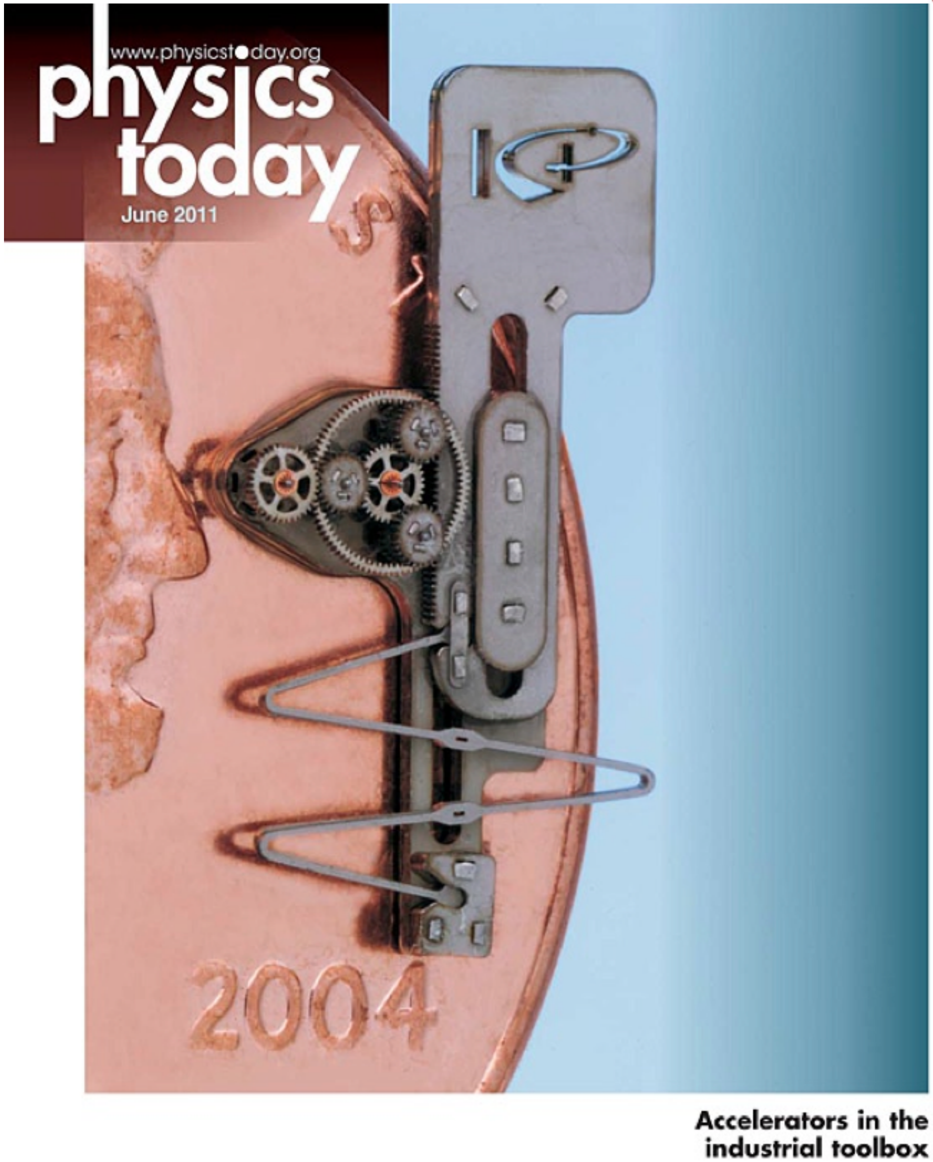
Still holds true over 60 years (!!!) later

Technologies tend to saturate then new technologies are introduced

(G. Hoffstaetter, Cornell)

Cover Of The Rolling Stone

- Accelerators make the cover of June 2011 Physics Today
  - Micromachining example from synchrotron light
- Industrial applications
  - Plenty of applications and interest across the world



========= Extra Slides =========

Lorentz Lie Group Generators I

- Lorentz transformations can be described by a Lie group where a general Lorentz transformation is

  $$A = e^{L} \qquad \det A = e^{\mathrm{Tr}\, L} = +1$$

  where L is 4x4, real, and traceless. With metric g, the matrix gL is also antisymmetric, so L has the general six-parameter form

  $$L = \begin{pmatrix} 0 & L_{01} & L_{02} & L_{03} \\ L_{01} & 0 & L_{12} & L_{13} \\ L_{02} & -L_{12} & 0 & L_{23} \\ L_{03} & -L_{13} & -L_{23} & 0 \end{pmatrix}$$

  Deep and profound connection to EM tensor $F^{\alpha\beta}$

J.D. Jackson, Classical Electrodynamics 2nd Ed, Section 11.7

Lorentz Lie Group Generators II

- A reasonable basis is provided by six generators
  - Three generate rotations in three dimensions

$$S_{1,2,3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

  - Three generate boosts in three dimensions

$$K_{1,2,3} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Lorentz Lie Group Generators III

- $(S_{1,2,3})^2$ and $(K_{1,2,3})^2$ are diagonal.
- $(\epsilon \cdot S)^3 = -\epsilon \cdot S$ and $(\epsilon \cdot K)^3 = \epsilon \cdot K$ for any unit 3-vector ϵ
- Nice commutation relations:

$$[S_i, S_j] = \epsilon_{ijk} S_k \quad [S_i, K_j] = \epsilon_{ijk} K_k \quad [K_i, K_j] = -\epsilon_{ijk} S_k$$

- We can then write the Lorentz transformation in terms of two three-vectors (6 parameters) ω, ζ as

$$L = -\omega \cdot S - \zeta \cdot K \qquad A = e^{-\omega \cdot S - \zeta \cdot K}$$

  - Electric fields correspond to boosts
  - Magnetic fields correspond to rotations
  - Deep beauty in Poincare, Lorentz, Einstein connections