USPAS Accelerator Physics 2013
Duke University

Chapter 11: Space Charge Effects
Space Charge, Beam-Beam
(Electron Cloud in separate talk from Ecloud workshop)

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Debye Length

- Comes from thermodynamic description of large systems of moving charges

$$\lambda_{\mathrm{D}} \equiv \left(\frac{\epsilon_0 k_B T}{N e^2} \right)^{1/2}$$

- **Collisional regime**: dynamics dominated by binary collisions in close encounters (e.g. Touschek)
  - Single particle scattering and single particle effects

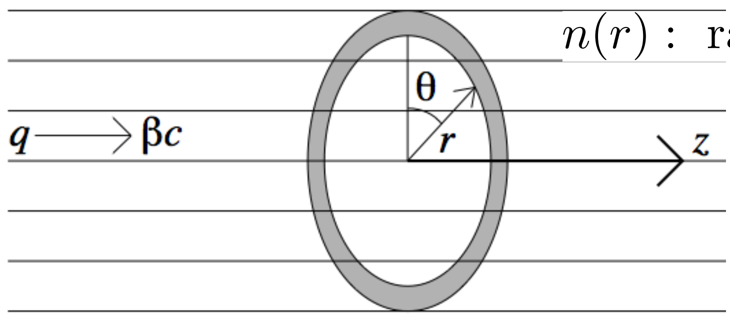
$$\sigma_{x,y,z} \ll \lambda_{\mathrm{D}}$$

- **Self-field or space charge regime**: dynamics dominated by self-field over large distances compared to average particle separation
  - Collective effects, single-component cold plasmas

$$\sigma_{x,y,z} \gg \lambda_{\mathrm{D}}$$

11.1: Transverse Space Charge Tune Shift

- In space-charge dominated synchrotrons and low-energy beams (without bunch compression), we can approximate the beam as being cylindrical and effectively infinitely long



$n(r)$: radial particle number density

  - The above figure simply establishes the coordinate system and velocity. We assume that the beam is cylindrically symmetric (for now).
  - Only radial forces: $F(r) = q(E_r - \beta c B_\theta)$
    - Electric repulsion will be stronger than magnetic attraction
    - At hyperrelativistic velocities these nearly balance (~no effect)
    - Also seen by imagining electrostatic forces only in bunch frame
    - Can compensate by shielding charges/currents (e.g. plasma)

Back to Maxwell

- Maxwell's equations in cylindrical coordinates here are

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{r}\frac{d}{dr}(rE_r) = \frac{q}{\epsilon_0}n(r) \qquad (\vec{\nabla}\times\vec{B})_z = \frac{1}{r}\frac{d}{dr}(rB_\theta) = \frac{q}{c\epsilon_0}\beta n(r)$$

Relativistic β

- We can integrate these directly

$$E_r = \frac{q}{\epsilon_0}\frac{1}{r}\int_0^r r\, n(r)\, dr \qquad B_\theta = \frac{\beta}{c}\frac{q}{\epsilon_0}\frac{1}{r}\int_0^r r\, n(r)\, dr = \frac{\beta}{c}E_r$$

- These are simple if the beam has constant density
- Also simple if we treat these beams as having a radial Gaussian distribution

$$n(r) = A\exp\left(\frac{-r^2}{2\sigma^2}\right) \qquad A = \frac{N}{2\pi l\sigma^2}$$

Bunch length l
Bunch population N

Space Charge Force

- Calculating the force from these fields

$$F(r) = q(E_r - \beta c B_\theta)$$

gives

$$F(r) = \frac{Nq^2}{2\pi\epsilon_0 l}\frac{(1-\beta^2)}{r}\left[1-\exp\left(-\frac{r^2}{2\sigma^2}\right)\right]$$

like beam-beam
but opposite sign
In B term

- This is a defocusing force that is is equal in both directions
-- how does it affect our focusing and tune?

$$\frac{d^2y}{ds^2} = \frac{1}{\beta^2c^2}\frac{d^2y}{dt^2} = \frac{1}{\beta^2c^2}\frac{F_y}{m\gamma} \qquad \frac{d^2y}{d\theta^2} = \frac{R^2F(y)}{\beta^2\gamma mc^2}$$

$$R: \text{ average radius, } s = R\theta$$

$$\frac{d^2y}{d\theta^2} + Q_\mathrm{V}^2\, y = \frac{R^2}{\beta^2\,\gamma mc^2}F(y)$$

$$r_0 \equiv q^2/(4\pi\epsilon_0\, mc^2)$$

Equation of Motion

- We get something rather unpleasant for the expansion

$$\frac{d^2y}{d\theta^2} + Q_{\mathrm{V}}^2 y = \frac{2Nr_0R^2}{l\beta^2\gamma^3}\left[\frac{1-\exp\left(-\frac{y^2}{2\sigma_{\mathrm{V}}^2}\right)}{y}\right]$$

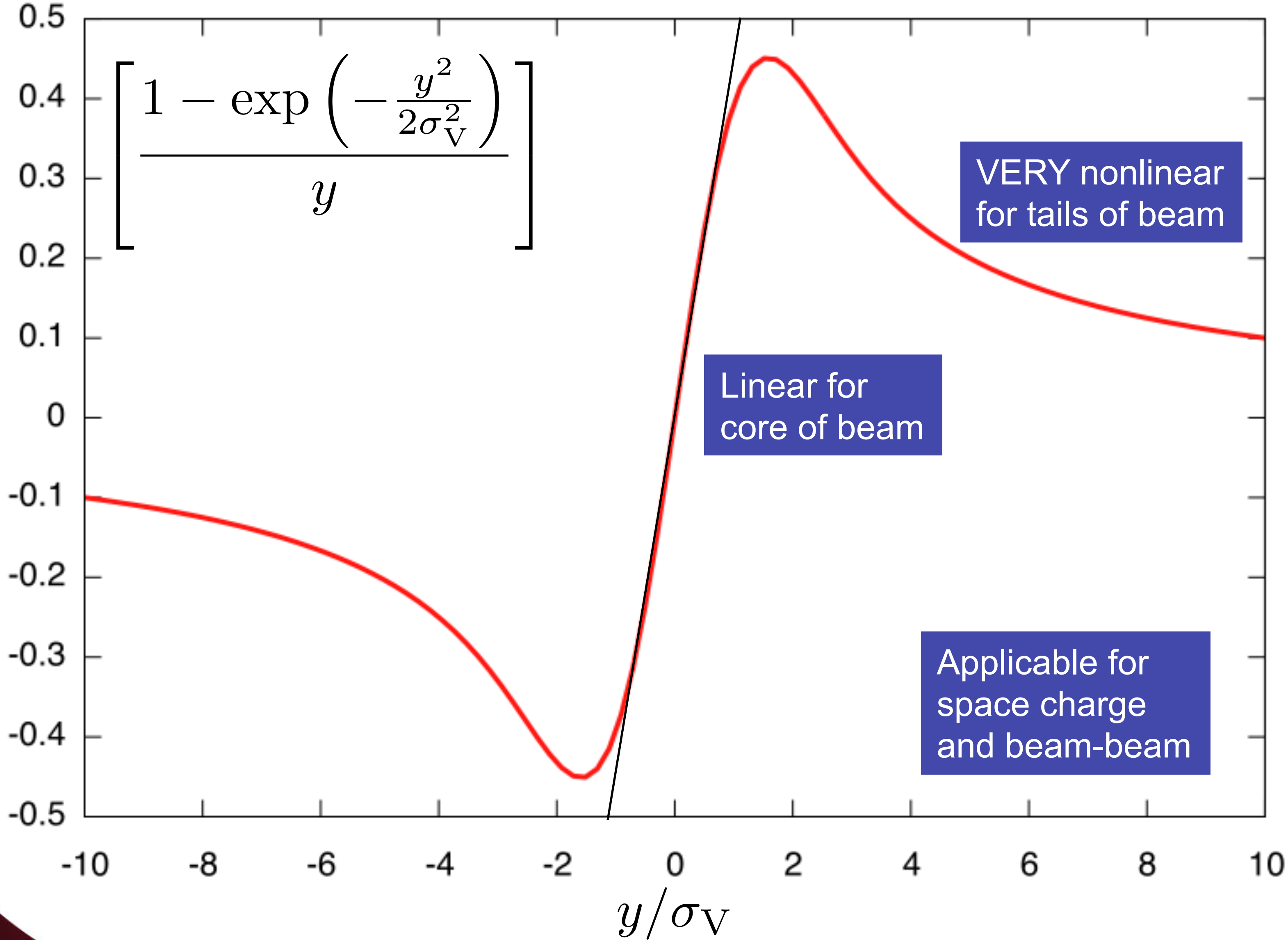
  - But we can expand the unpleasantness and ignore the higher order terms for now (which contribute nonlinearities)

$$\left[\frac{1-\exp\left(-\frac{y^2}{2\sigma_{\mathrm{V}}^2}\right)}{y}\right] = \frac{y}{2\sigma_{\mathrm{V}}^2} - \sum_{n=2}^{\infty}\frac{(-1)^n}{2^n\, n!}\frac{y^{2n-1}}{\sigma_{\mathrm{V}}^{2n}}$$

Maclaurin series
Taylor series about 0

$$\frac{d^2y}{d\theta^2} + \left(Q_{\mathrm{V}}^2 - \frac{N}{l\sigma_{\mathrm{V}}^2}\frac{r_0R^2}{\beta^2\gamma^3}\right)y = 0$$

That Messy Term



Linear Space Charge Tune Shift: Calculated

$$\frac{d^2y}{d\theta^2} + (Q_\mathrm{V} + \delta Q_\mathrm{sc})^2 y \approx 0$$

$$\boxed{\delta Q_\mathrm{sc} = -\frac{N r_0 R^2}{2l Q_\mathrm{V}\, \sigma_\mathrm{V}^2\, \beta^2\gamma^3} = -\frac{N r_0}{4\pi\, B_f\, \epsilon^\star_{\mathrm{V,rms}}\, \beta\gamma^2}}$$

- To get the last we've used a few other conversions

$$\beta_\mathrm{V} \approx R/Q_\mathrm{V} \qquad \pi\epsilon^\star_{\mathrm{V,rms}} = \pi\frac{\sigma_\mathrm{V}^2}{\beta_\mathrm{V}\beta\gamma}$$

Normalized emittance

$$\text{Bunching factor}: \quad B_f \equiv \frac{l}{2\pi R} = \frac{I_\mathrm{ave}}{I_\mathrm{peak}}$$

- δQ_sc is called the (vertical) Laslett tune shift
  - For hadron beams, this is a big effect at low energy, high N
  - Dominates high intensity boosters (FNAL, BNL, CERN)
  - Electrons escape most effects except for very low γ

Flat/Round Space Charge Tune Shift

- This tune shift is symmetric between H,V for round beams
  - e.g. most hadron beams
- For elliptical or ribbon beams one can show that the proper calculation gives

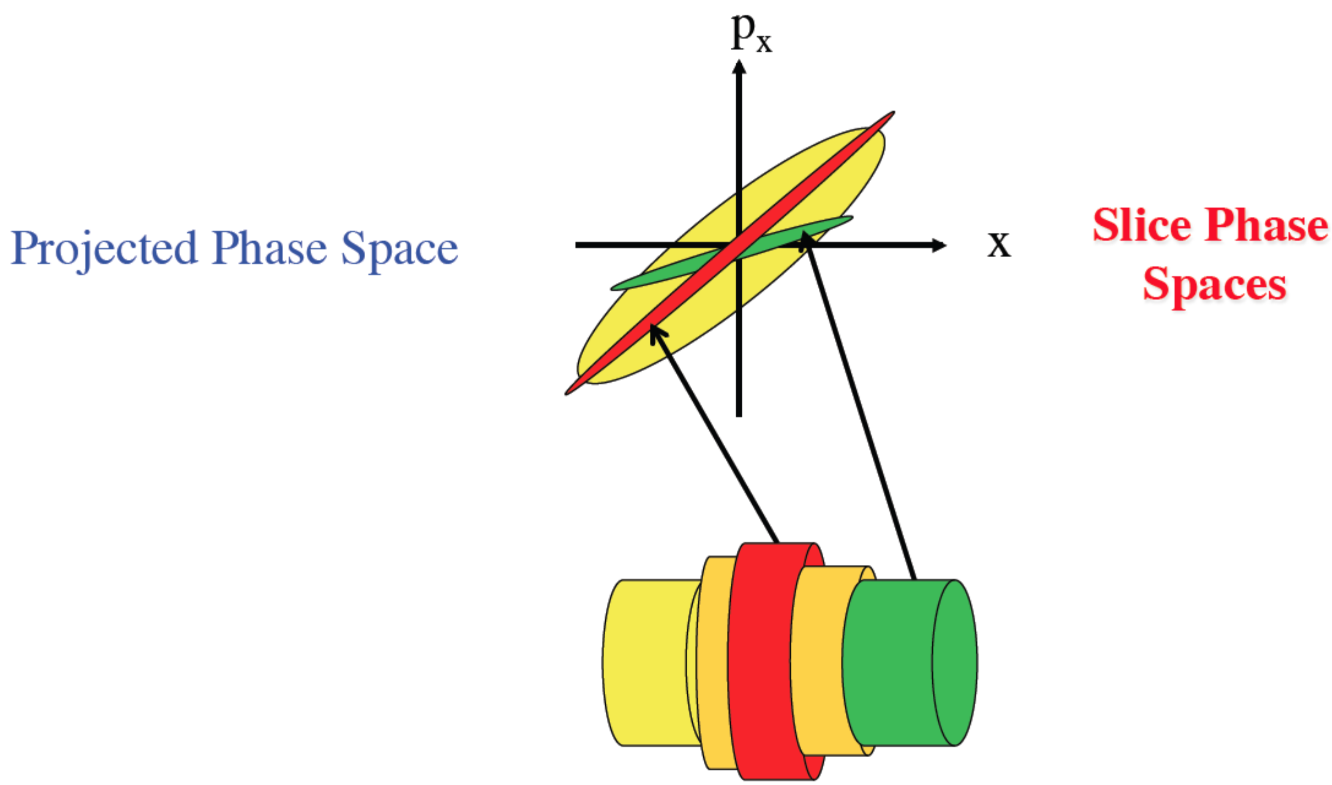
$$\boxed{\delta Q_{\mathrm{sc,V}} = -\frac{\beta_{\mathrm{V}} N r_0}{2\pi B_f \sigma_{\mathrm{V}}(\sigma_{\mathrm{H}} + \sigma_{\mathrm{V}})\beta^2\gamma^3}}$$

  - The horizontal is just given by reversing H and V
  - This is true for most synchrotron electron beams

- This tune shift is different for different parts of the beam
  - Commonly called an incoherent tune shift
  - Compare to coherent tune shift given by, e.g., quadrupoles

Head and Tail Space Charge

- Differential defocusing in head and tail of beam from space charge is a source of some (reversible) emittance growth



$$\varepsilon_{rms} = \sqrt{\sigma_x^2\sigma_{x'}^2 - \sigma_{xx'}{}^2} = \sqrt{\left(\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2\right)} \approx \left|\sin\left(\sqrt{2}k_s z\right)\right|$$

Ferrario, CAS Greece 2011

Messy Nonlinearities

$$\left[\frac{1-\exp\left(-\frac{y^2}{2\sigma_{\mathrm{V}}^2}\right)}{y}\right] = \frac{y}{2\sigma_{\mathrm{V}}^2} - \sum_{n=2}^{\infty} \frac{(-1)^n}{2^n\, n!} \frac{y^{2n-1}}{\sigma_{\mathrm{V}}^{2n}}$$

- Note that space charge also drives many nonlinearities
  - First order is quadrupole (tune shift) so higher orders are octupole (nonlinear tune shift), dodecapole, …
  - Beam size is natural scaling parameter
- How high can we run δQ_{sc}?
  - It spreads beam across resonances, so not more than 1!
  - Large amplitude particles shift away from resonance though
  - Incoherent tune spread across emittance of beam
  - Some facilities (FNAL Booster, AGS) run up to 0.7!
- Note that space charge is distributed through the accelerator
  - This makes it quite computationally expensive to model

Space Charge Compensation

- How to compensate for space charge?
- Create neutral plasma by co-propagating opposite charge beam
  - Studied at e.g. CERN PS Booster (Aiba), FNAL Booster
  - Matching beams compensate tune shift and resonances
  - Ion recombination not expected to be substantial
  - Electron columns with strong solenoidal fields (Shiltsev,FNAL, ‘07)
- Inject small mix of electronegative gases
  - Very low energy high flux ion beam (Dudnikov^2, FNAL/BINP)
- Low energy electrons: Use focusing solenoidal field in gun
  - Developed by Carlsten (LANL, ’94) for 1.3 GHz photoinjector
  - Now routinely incorporated into high-brightness photoinjector design
  - Also incorporate superconducting RF half-cell: faster acceleration
  - Ion back-bombardment problem on RF photocathodes (Pozdeyev)

11.4: Longitudinal Space Charge Defocusing

- Longitudinal space charge is most easily calculated in the center of mass frame of the beam
  - Then boost fields to lab frame with Lorentz transformation
  - Voltage for uniform beams, in the beam center of mass frame

$$V_{\rm cm} = \frac{qg}{4\pi\epsilon_0} \frac{N}{\gamma l_{\rm beam}} \qquad g \equiv \left(1 + 2\ln\frac{b}{a}\right)$$

a: beam radius
b: pipe radius

  - For a more realistic longitudinal distribution (parabolic)

$$V_{\rm cm} = \frac{qg}{4\pi\epsilon_0} \frac{3N}{2\gamma l_{\rm beam}} \left[1 - \left(\frac{2z_{cm}}{\gamma l_{\rm beam}}\right)^2\right]$$

  - Boosting back and finding the longitudinal force

$$\boxed{F_{\|} = \frac{3}{\pi} g \frac{Nq^2}{\epsilon_0 l_{\rm beam}^2 \gamma^2} z}$$

  - Modifies synchrotron frequency, important near transition
  - Creates bunch lengthening in low energy electron beams

Application of Longitudinal Space Charge

Nonlinear Longitudinal Space Charge Oscillations in Relativistic Electron Beams

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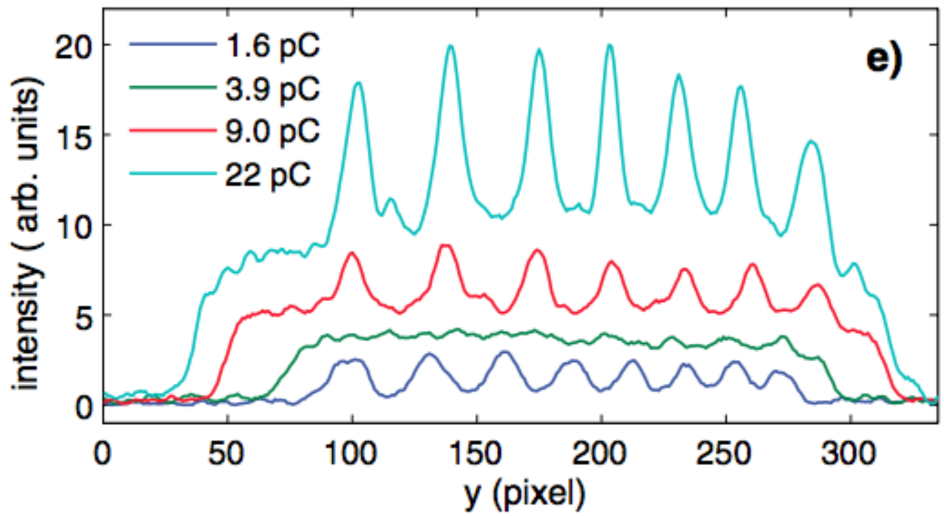
(Received 16 February 2011; published 4 May 2011)

In this Letter we study the evolution of an initial periodic modulation in the temporal profile of a relativistic electron beam under the effect of longitudinal space-charge forces. Linear theory predicts a periodic exchange of the modulation between the density and the energy profiles at the beam plasma frequency. For large enough initial modulations, wave breaking occurs after 1/2 period of plasma oscillation leading to the formation of short current spikes. We confirm this effect by direct measurements on a ps-modulated electron beam from an rf photoinjector. These results are useful for the generation of intense electron pulse trains for advanced accelerator applications.



Space-charge driven plasma oscillations modify initial modulation from SCRF gun

Musumeci, Li, and Marinelli, PRL 106, 184801 (2011)

11.3: Beam-Beam Force

- The beam-beam **force** is very similar to space charge
  - Except now use properties of colliding beam
  - Oppositely charged beams focus, same sign defocus
  - Velocities are opposite so sign of force from B term reverses

$$F(r) = q(E_r + \beta c B_\theta)$$

$$F(r) = \frac{Nq^2}{2\pi\epsilon_0 l}\frac{1+\beta^2}{r}\left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right]$$

like space charge
but opposite sign
In B term

- We also only integrate the beam-beam kick over the bunch length as a short kick, since the interaction is short
  - We can then use the same short kick approximations as before for multipoles

Beam-Beam Force (cont)

$$F(r) = \frac{Nq^2}{2\pi\epsilon_0 l}\frac{1+\beta^2}{r}\left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right]$$

- Assume particles are ultrarelativistic (β~1) and take the leading term to find linear focusing and thus a tune shift

$$F(y) \approx \frac{Nq^2}{2\pi\epsilon_0 l\sigma_{\mathrm{V}}^2} y$$

- The counterrotating bunch acts like a short lens of length $l/2$

$$\delta p_y = F(y)\delta t = F(y)(l/2\beta c) = \frac{Nq^2}{4\pi\epsilon_0\beta c\sigma_{\mathrm{V}}^2} y \left(\frac{p}{\beta\gamma mc}\right) = \frac{Nr_0 p}{\gamma\sigma_{\mathrm{V}}^2} y$$

$\beta \approx 1$

$$\Delta y' = \frac{\delta p_y}{p} = \frac{Nr_0}{\gamma\sigma_{\mathrm{V}}^2} y$$

Thin quadrupole!

Beam-Beam Force (cont)

$$\Delta y' = \frac{\delta p_y}{p} = \frac{N r_0}{\gamma \sigma_{\rm V}^2} y$$

- We can use the thin quadrupole formula we derived on Friday to derive the corresponding linear tune shift
  - Reverse sign of tune change for oppositely charged beams!

$$\delta Q = -\frac{\beta_y \Delta k}{4\pi}$$

$$\boxed{\Delta Q_{\rm bb} = -\frac{N_{\rm IP} N r_0 \beta_{\rm V}^\star}{4\pi\gamma\sigma_{\rm V}^2} = -\frac{N_{\rm IP} N r_0}{4\pi\gamma\epsilon_{\rm V,rms}}}$$

  - N_{IP} is the number of interaction points
  - Tune shift independent of beta function at collision point
  - N and emittance are properties of opposing beam
  - For elliptical beams we can similarly generalize to

$$\boxed{\Delta Q_{\rm V,bb,elliptical} = -\frac{N_{\rm IP} N r_0 \beta_{\rm V}^\star}{2\pi\gamma\sigma_{\rm V}(\sigma_{\rm H} + \sigma_{\rm V})}}$$

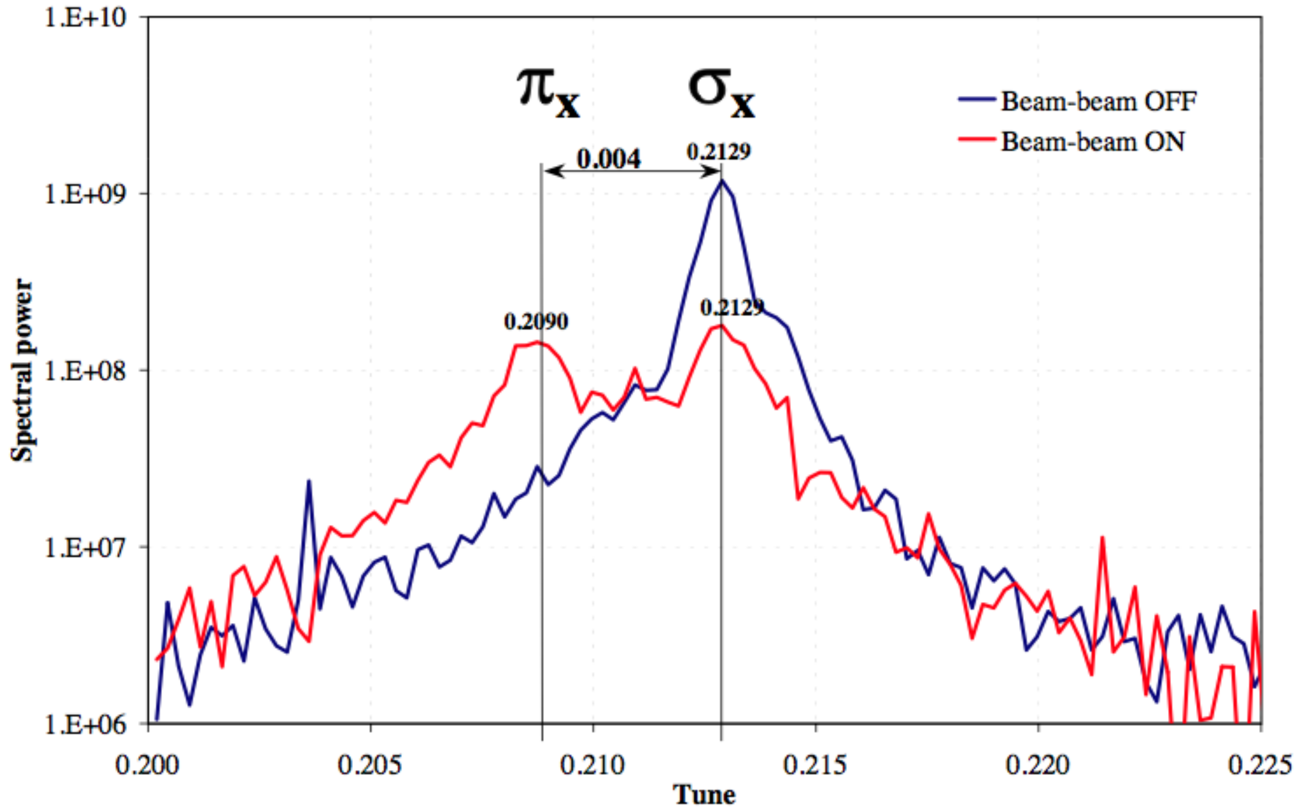
Beam-Beam Simulation

- Most beam-beam simulations are done using a **weak-strong** model
  - Assume the beam creating the kick is static or quasistatic
  - Beam-beam kick is then just a thin (very) nonlinear lens
  - Scales as particle partitioning of weak beam
  - Quick: you did this in the Java lab!

- But we can have two-beam collective effects
  - The two beams are two coupled oscillators
  - Both coherent and incoherent effects (strong nonlinearities)
  - This requires a **strong-strong** model where both distributions must evolve together consistently over time
  - Can be very computationally expensive
    - Similar to large detailed space charge calculations

Beam-Beam Coherent Effects

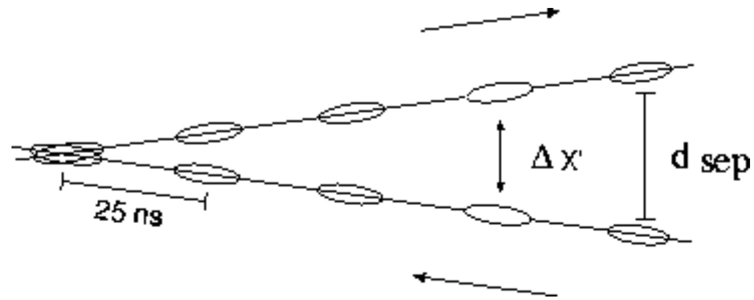
- Colliding beams are not always aligned on center
  - Offsets of beam centers can easily produce coherent coupled oscillations between the beams
  - Can give rise to π and σ oscillator modes



W. Fischer et al, "Observation of Coherent Beam-Beam Modes in RHIC", 2002

Long Range Beam-Beam

- Recall that our force is very nonlinear at large amplitudes
  - There are good reasons to separate colliding beams to moderate or large amplitudes
  - e.g. short-spaced bunch trains in long interaction regions with crossing angle (LHC with minimum bunch spacing)



  - This introduces beam-beam crossings at angles and nonlinearities from parasitic beam-beam kicks
  - Drives designs to large crossing angles for large separation
  - Luminosity optimization drives need for crab cavities

Beam Beam vs Space Charge

- Beam-beam and space charge tune shifts look similar!
  - Their nonlinear terms also look similar

$$\delta Q_{\rm sc} = -\frac{N r_0 R^2}{2l Q_{\rm V}\ \sigma_{\rm V}^2\ \beta^2\gamma^3} = -\frac{N r_0}{4\pi\ B_f\ \epsilon^\star_{\rm V,rms}\ \beta\gamma^2}$$

$$\Delta Q_{\rm V,bb} = -\frac{N_{\rm IP} N r_0 \beta^\star_{\rm V}}{4\pi\gamma\sigma^2_{\rm V}} = \frac{N_{\rm IP} N r_0}{4\pi\epsilon^\star_{\rm V,rms}\beta\gamma^2}$$

- Naively we would expect their behavior to be similar too
  - Space charge is distributed through the accelerator
    - Nearly phase-averages out many nonlinear behaviors
  - But beam-beam is localized in phase
    - Emphasizes nonlinear behaviors, little phase averaging
  - Beam-beam tune shift limits are more like 5-7x10^{-3}/IR
    - e.g. RHIC polarized proton collisions are beam-beam limited

Beam-Beam Compensation

- Similar to space charge, it's possible to compensate beam-beam behaviors with similar nonlinear magnetic fields
  - Local compensation preferred to compensate nonlinearities near source
  - Electron lens (Tevatron, Shiltsev et al)
  - Wires (RHIC, Fischer et al)

- Hollow electron beam collimation
  - Not exactly beam-beam but doesn't fit many other places
  - Requires creation of hollow electron beam at source/gun
  - Used to limit halo of high energy hadron beams
  - Surround core of beam with circular beam of electrons
  - Creates nonlinear forces that eliminate halo, do not disturb core
    - See, e.g., http://arxiv.org/abs/1202.1512 (Stancari et al)