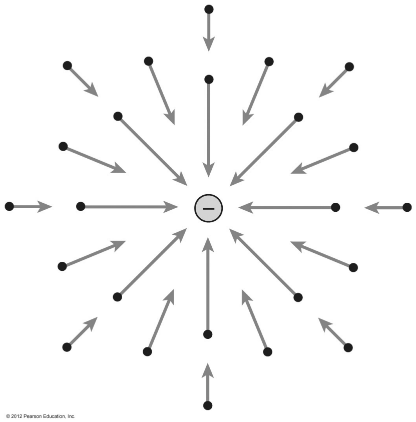

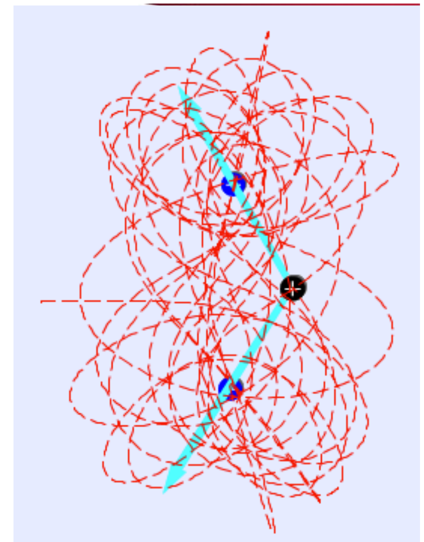

University Physics 227N/232N
Old Dominion University

More Electrostatics
20:3 Electric Field and Forces

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Wednesday, January 15 2014

Happy Birthday to Edward Teller, Gene Krupa, Dove Cameron, Skrillex, and Drew Brees!

Review: Electric Charge

- Electric charge is a fundamental property of matter.
  - Many particles, including the electron and proton, carry electric charge.
  - Charge comes in two varieties, **positive** and **negative.**
  - Most charged particles carry exactly one **elementary** charge, *e*, either positive or negative.
    - The proton carries exactly charge $+e$, the electron exactly charge $-e$.
    - The quarks, which make up protons, neutrons, and other subatomic particles, carry $\pm 1/3\ e$ or $\pm 2/3\ e$. But they're never observed in isolation.
  - The charge in a closed system is **conserved**
    - The algebraic sum of charges remains unchanged.
    - This is true even if new particles are created or destroyed.
  - The SI unit of charge is the **coulomb** (C), equal to approximately 6.25×10^{18} elementary charges.
    - **Thus *e* is approximately 1.602×10^{-19} C.**
  - Charges interact with electromagnetic radiation (light, radio, …)

Electric Charges and Atoms

- Most atoms are roughly electrically neutral
  - Same number of electrons and protons
  - Reality: many electrons are promiscuous and move among nuclei quite a lot.

- Macroscopic objects have over ~10^{20} atoms
  - If electrons and protons did not have **very nearly exactly** equal and opposite charges, large objects would experience unusual electrical forces (contrary to observation)
  - Proton and electron electric charges are equal and opposite to the best precision we have every measured them
  - Best measurement: $1.602176565(35)\times10^{-19}$ C
    - About 1 part in 46 billion!

(Most accurate physical measurement: hyperfine rubidium transition frequency: 1 part in 417 *quadrillion*)
(That's about the same relative accuracy as the thickness of a hair to the distance to the sun!)

Review: Coulomb's Law and the Electric Force

- Like charges repel, and opposite charges attract, with a force that depends on
  - The product of the two charges
  - The inverse square of the distance between them
- Mathematically, the electric force is described by **Coulomb's law** (much like the force of **gravity**):

$$\vec{F}_{12} = -\frac{G m_1 m_2}{r^2}\hat{r}$$

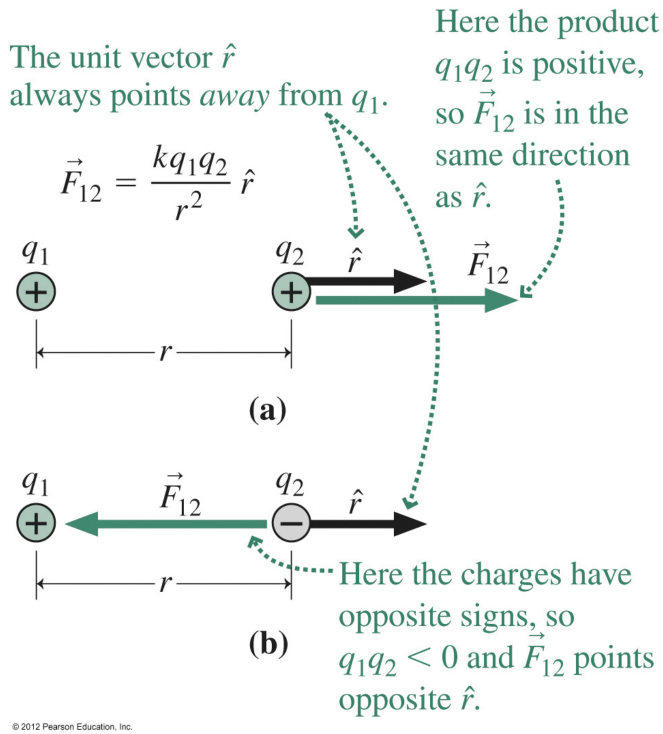
Gravity

$$\boxed{\vec{F}_{12} = \frac{k q_1 q_2}{r^2}\ \hat{r}}$$

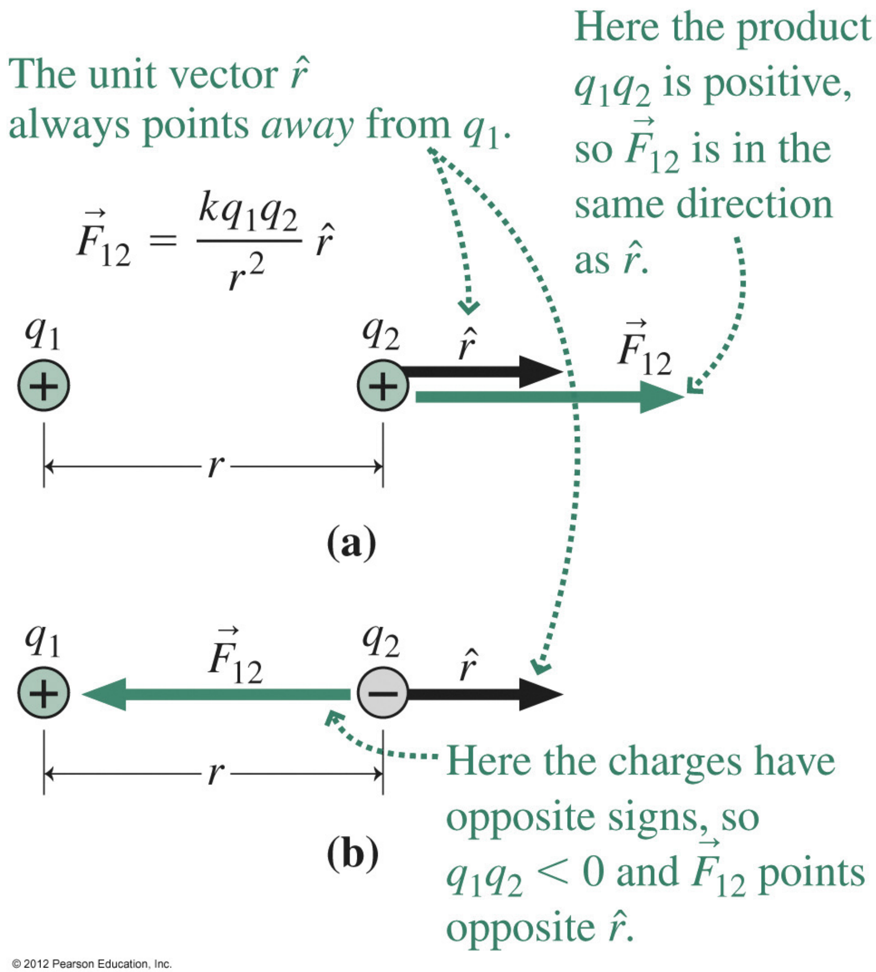
Coulomb

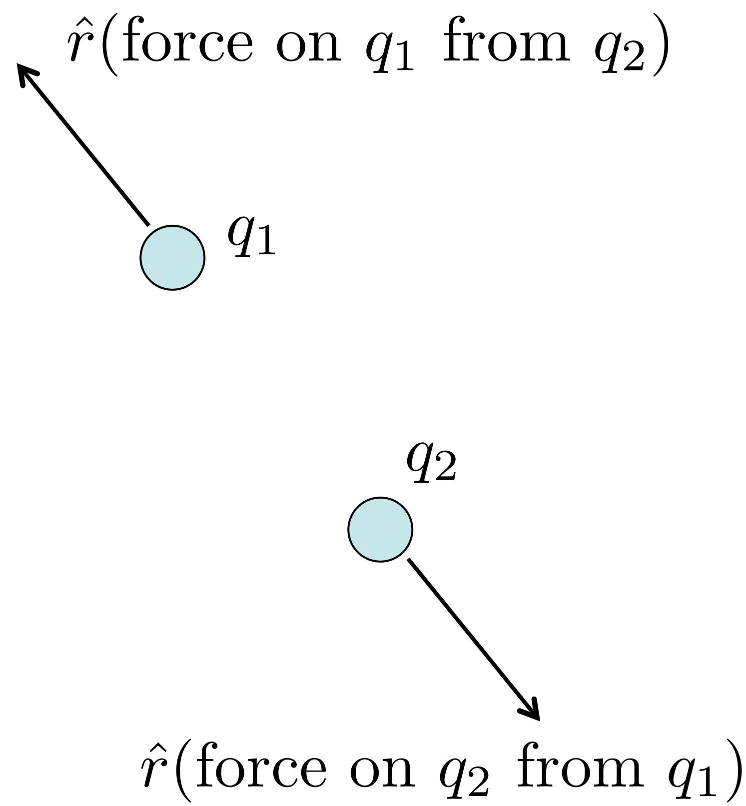
$$\boxed{k \approx 9.0 \times 10^9\ \mathrm{N\ m^2/C^2}}$$

Here $\vec{F}_{12}$ (Coulomb) is the force q_1 exerts *on* q_2 and is a unit vector pointing *from* q_1 toward q_2.



That $\hat{r}$ thing is kinda important

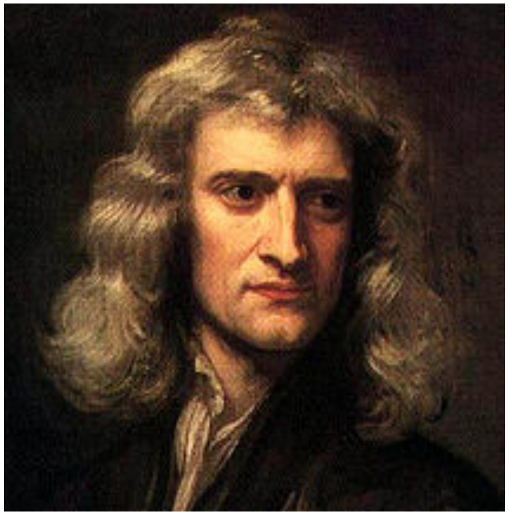


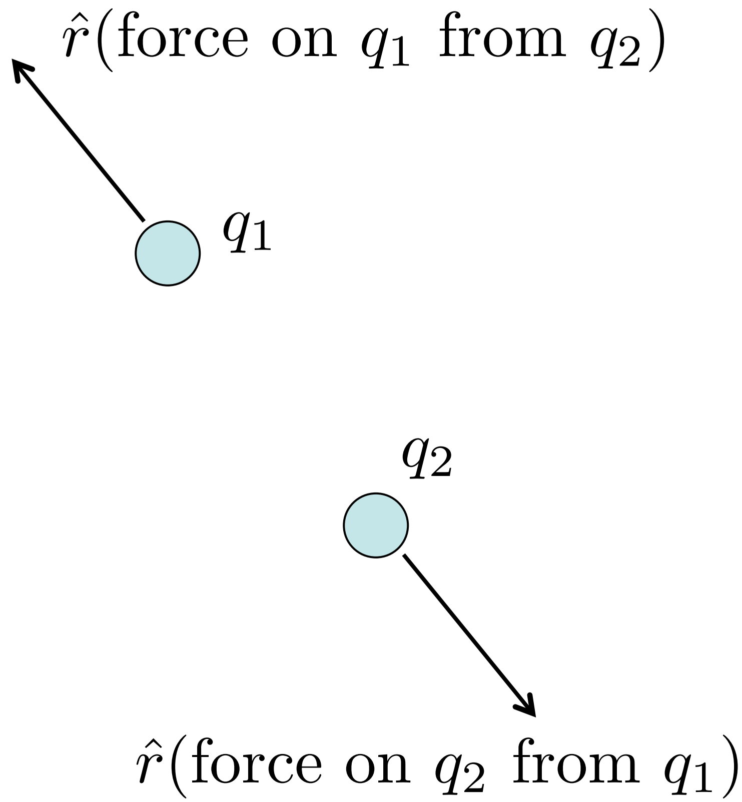


$$\boxed{\vec{F}_{12} = \frac{kq_1q_2}{r^2}\,\hat{r}}$$

Coulomb

That $\hat{r}$ thing is kinda important





Newton's Third Law

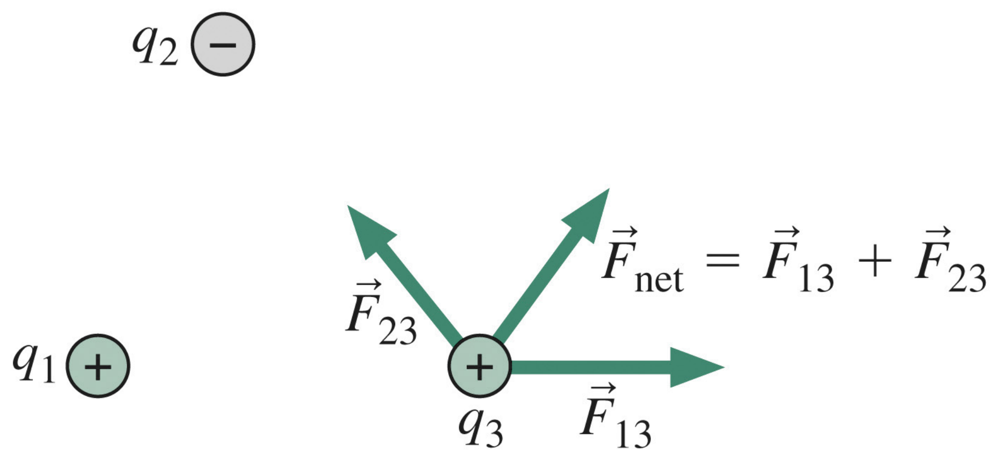
If object A exerts a force on object B, then object B exerts and **oppositely** directed force of equal magnitude on A.

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2}\,\hat{r}$$

Coulomb

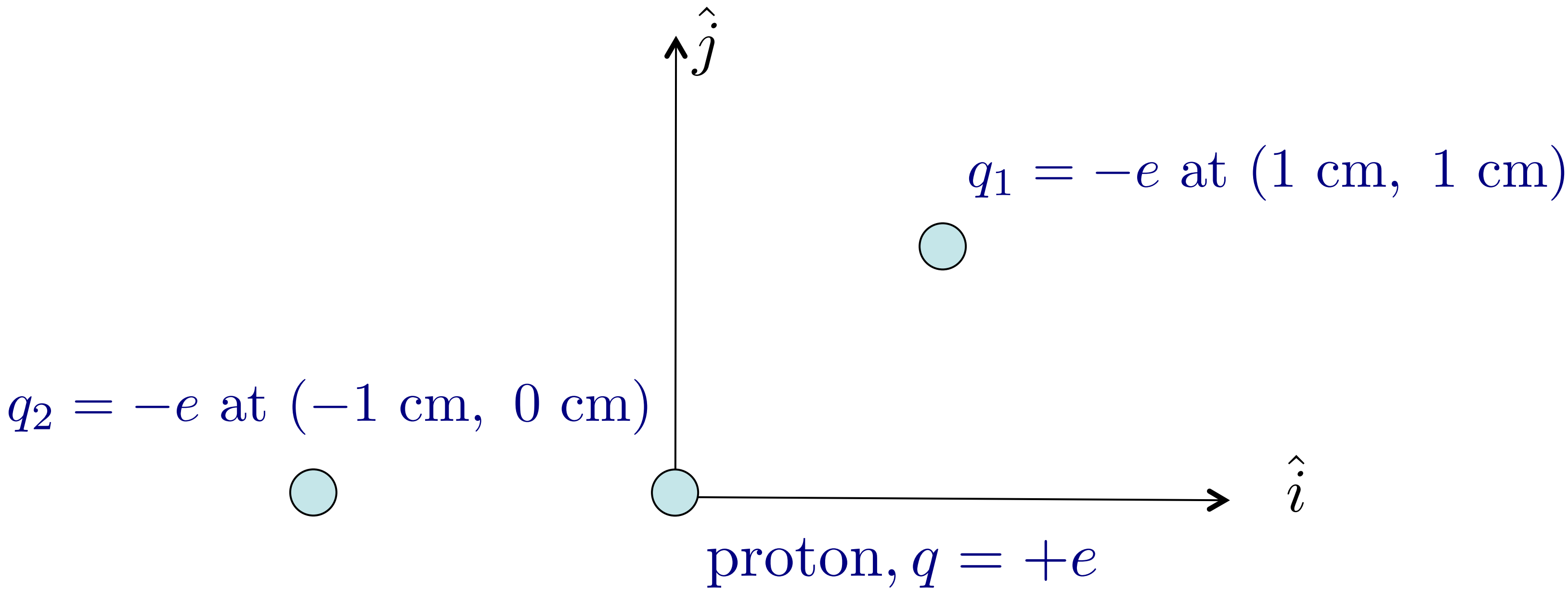
The Superposition Principle

- The electric force obeys the **superposition principle.**
  - That means the force two charges exert on a third force is just the vector sum of the forces from the two charges, each treated without regard to the other charge.
  - The superposition principle makes it mathematically straightforward to calculate the electric forces exerted by distributions of electric charge.
    - The net electric force is the sum of the individual forces.



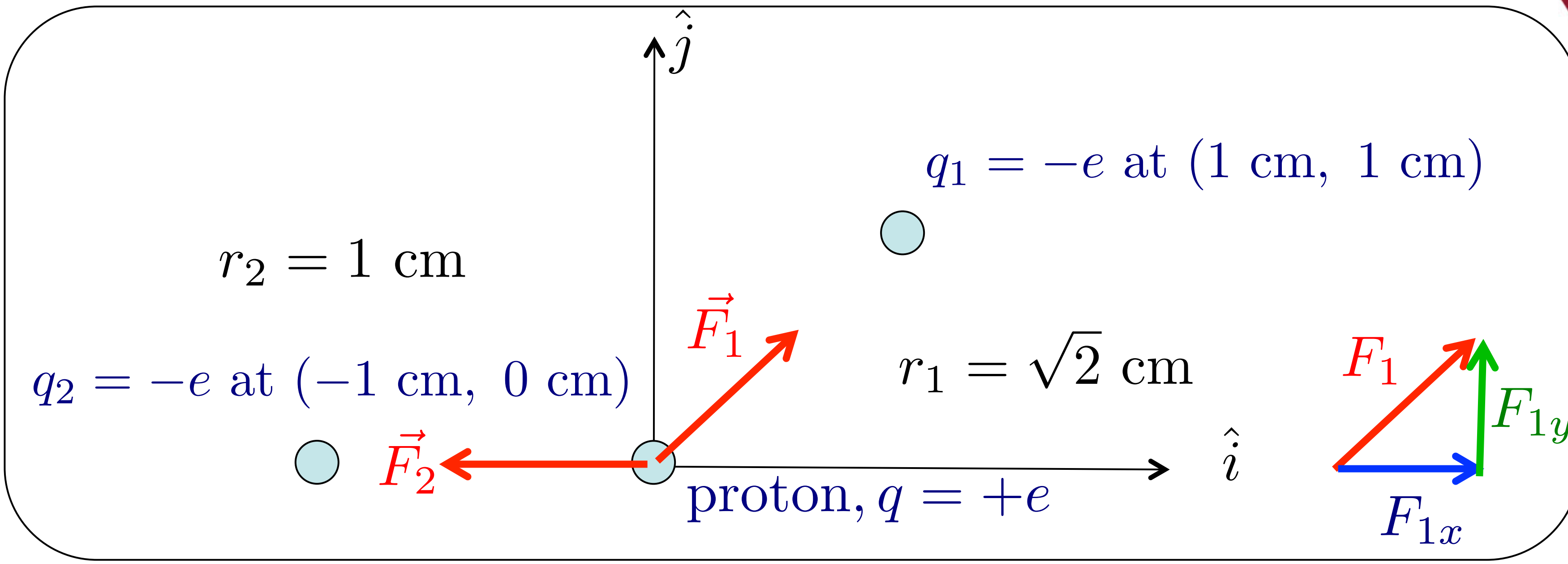
Example: Coulomb Force from Multiple Charges

- What is the force (magnitude and direction; force is a vector!) on a proton at the origin from the two other charges shown below?



Draw the picture, including vectors for forces
Calculate magnitudes of forces
Calculate and add components
Find final total force and direction; evaluate

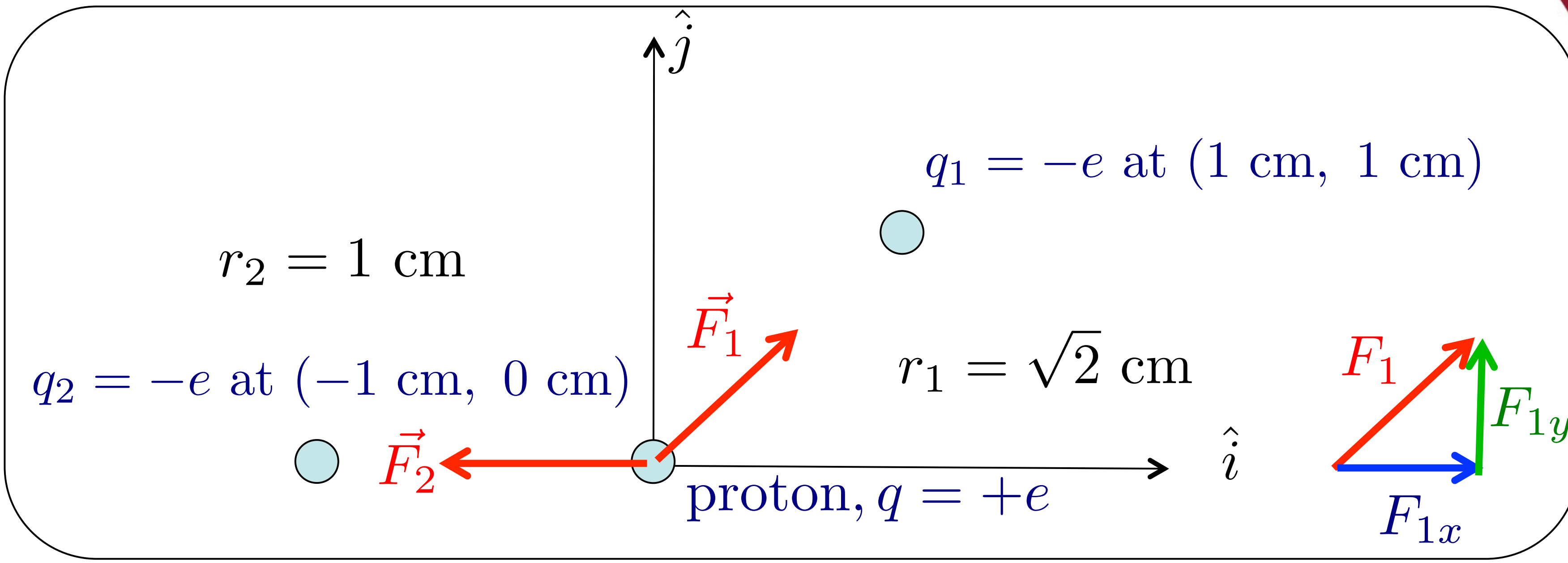
Example: Continued



$$F_{2y} = 0$$
$$F_{2x} = -F_2$$
$$F_2 = kqq_2/r_2^2$$

$$\sin(45^\circ) = F_{1y}/F_1$$
$$\cos(45^\circ) = F_{1x}/F_1$$
$$F_1 = kqq_1/r_1^2$$

Example: Continued



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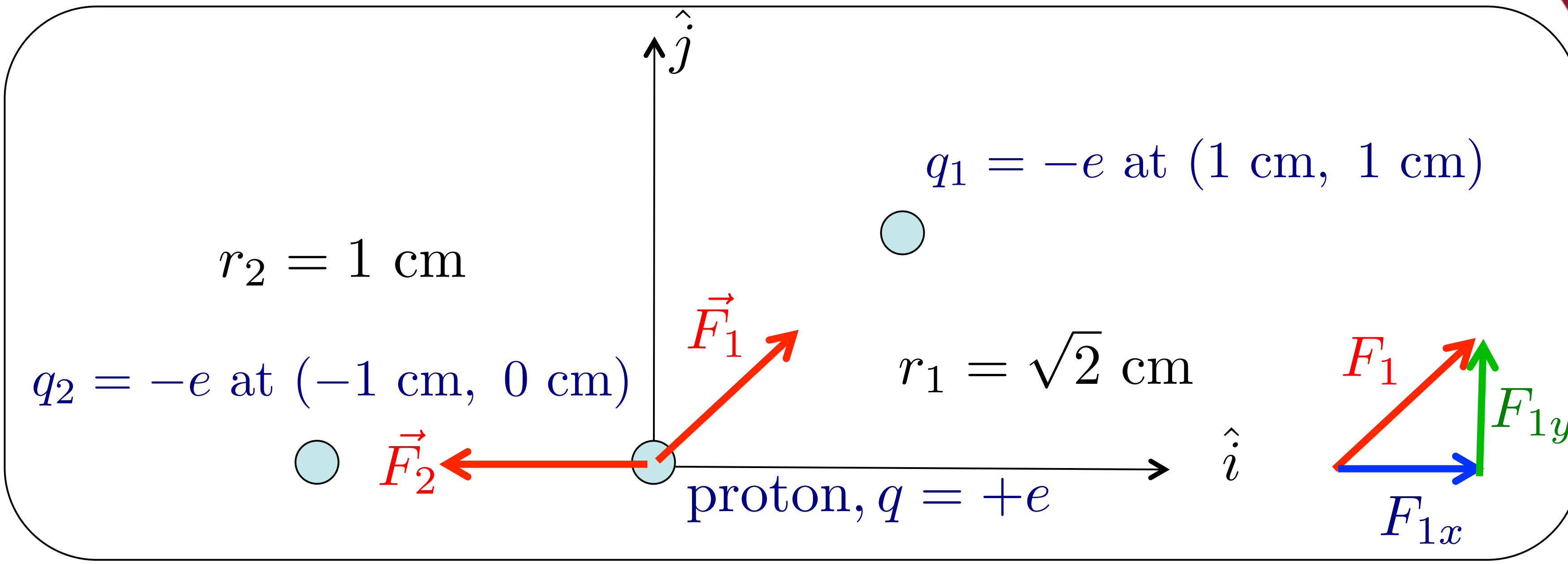
$$\cos(45^\circ) = F_{1x}/F_1$$

$$F_1 = kqq_1/r_1^2$$

$$\begin{aligned} F_{\text{tot},x} &= F_{1x} + F_{2x} \\ &= F_1\cos(45^\circ) - F_2 \\ &= kq\left(\frac{q_1}{r_1^2}\cos(45^\circ) - \frac{q_2}{r_2^2}\right) \\ &= ke^2\left(\frac{1}{r_1^2\sqrt{2}} - \frac{1}{r_2^2}\right) = \boxed{-1.5\times 10^{-24}\ \text{N} = F_{\text{tot},x}} \end{aligned}$$

(be careful!)

Example: Continued



$$F_{2y} = 0$$

$$F_{2x} = -F_2$$

$$F_2 = kqq_2/r_2^2$$

$$\begin{aligned} F_{\text{tot},y} &= F_{1y} + F_{2y} \\ &= F_1 \sin(45^\circ) + 0 \\ &= \frac{kqq_1}{r_1^2}\sin(45^\circ) \\ &= \frac{ke^2}{r_1^2\sqrt{2}} = \boxed{+8.2\times 10^{-25}\ \text{N} = F_{\text{tot},y}} \end{aligned}$$

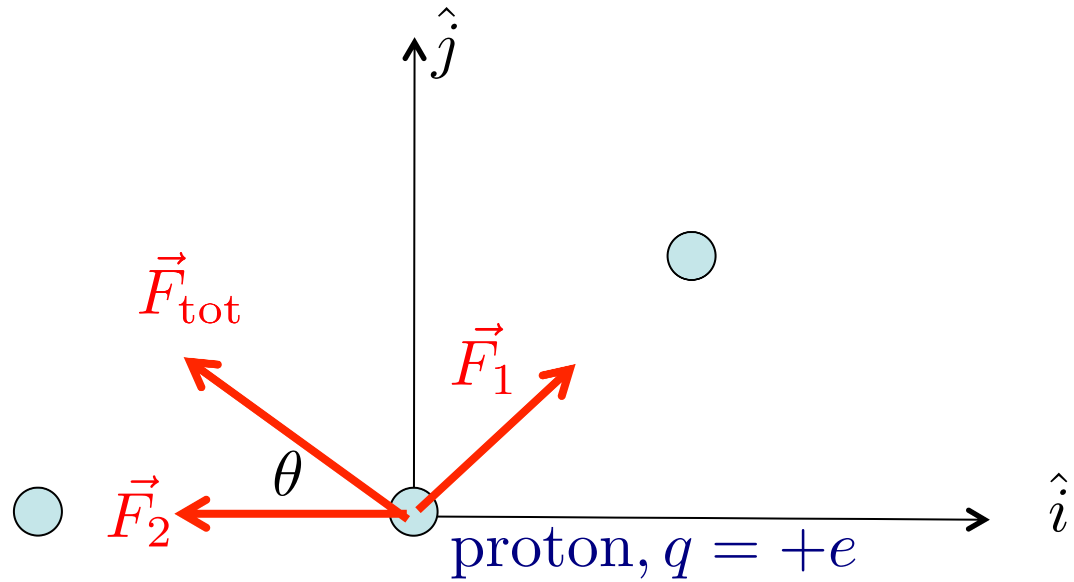
$$\sin(45^\circ) = F_{1y}/F_1$$

$$\cos(45^\circ) = F_{1x}/F_1$$

$$F_1 = kqq_1/r_1^2$$

(be careful!)

Example: Continued



$$-1.5 \times 10^{-24}\ \mathrm{N} = F_{\mathrm{tot},x}$$

$$+8.2 \times 10^{-25}\ \mathrm{N} = F_{\mathrm{tot},y}$$

$$F_{\mathrm{tot}} = \sqrt{F_{\mathrm{tot},x}^2 + F_{\mathrm{tot},y}^2}$$

$$\boxed{\theta = \tan^{-1}(F_{\mathrm{tot},y}/F_{\mathrm{tot},x}) = 29^\circ}$$

- Makes sense – more strongly attracted to closer charge!

Example: Balloon on the Ceiling

- You rub a 30g balloon on your head and stick it to the ceiling
  - Assume the balloon just barely sticks to the ceiling and the charges on the balloon (negative) are 1mm from the same number of positive charges on the ceiling
  - About how many excess electrons did you rub onto the balloon?

$$F_g = mg = (0.03\ \mathrm{kg})(9.8\ \mathrm{m/s}^2) = 0.3\ \mathrm{N} = F_{\mathrm{electrostatic}}$$

$$F_{\mathrm{electrostatic}} = \frac{kq^2}{r^2}$$

Example: Balloon on the Ceiling

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$$F_{\text{electrostatic}} = \frac{kq^2}{r^2}$$

$$q = \sqrt{\frac{F_{\text{electrostatic}} r^2}{k}} = \sqrt{\frac{(0.3 \text{ N})(10^{-3} \text{ m})^2}{9 \times 10^9 \text{ N m}^2/\text{C}^2}} = 5.8 \times 10^{-9} \text{ C}$$

$$\frac{q}{e} = \frac{5.8 \times 10^{-9} \text{ C}}{1.6 \times 10^{-19} \text{ C}} \approx \boxed{3.6 \times 10^{10} \text{ electrons}}$$

The Electric Field

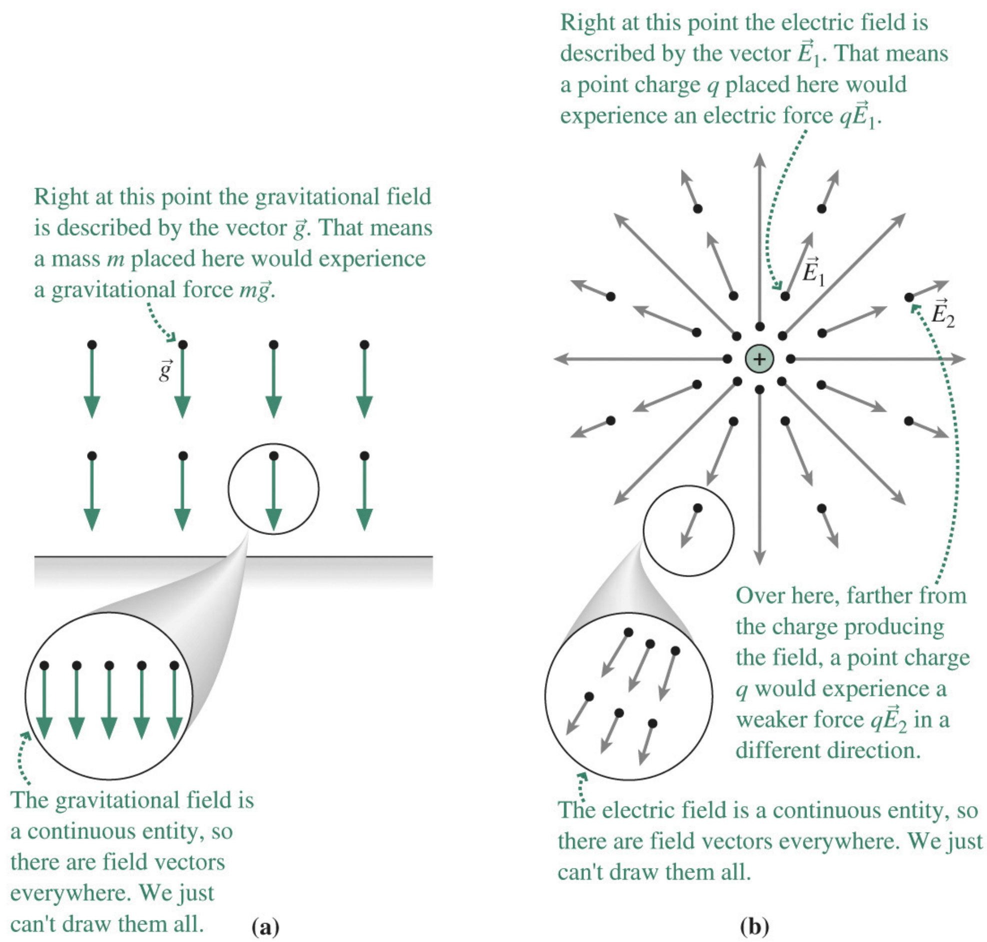
- The electric field at a point in space is the force per unit charge that a charge *q* placed at that point **would** experience:

$$\boxed{\vec{E} = \frac{\vec{F}}{q}} \qquad \frac{\mathrm{N}}{\mathrm{C}} = \frac{\mathrm{V}}{\mathrm{m}}$$

- The force on a charge *q* in an electric field $\vec{E}$ is

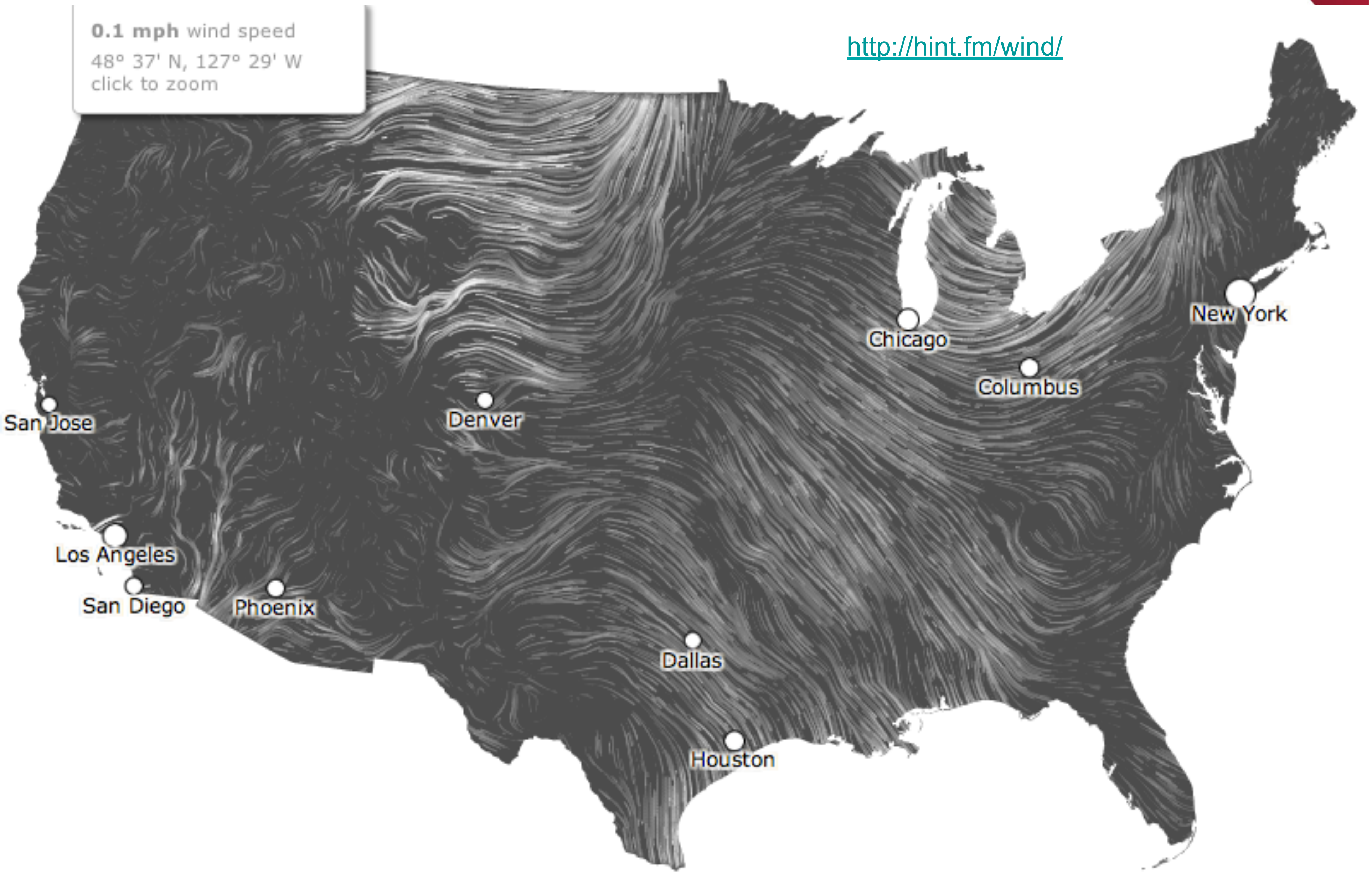
$$\boxed{\vec{F} = q\vec{E}}$$

  - Electric field is analogous to the gravitational field, which gives force per **unit** mass.



Vector Field Examples

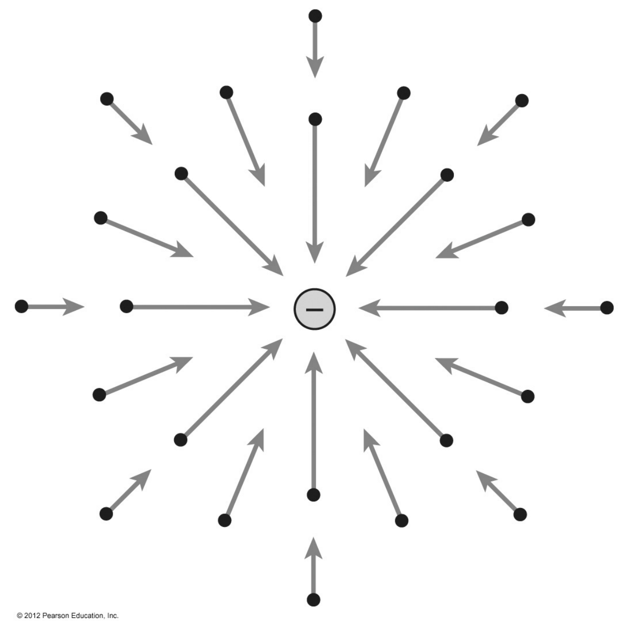
http://hint.fm/wind/



Vector Field Examples

Fields of Point Charges and Charge Distributions

- The field of a point charge is radial, **outward** for a positive charge and **inward** for a negative charge.



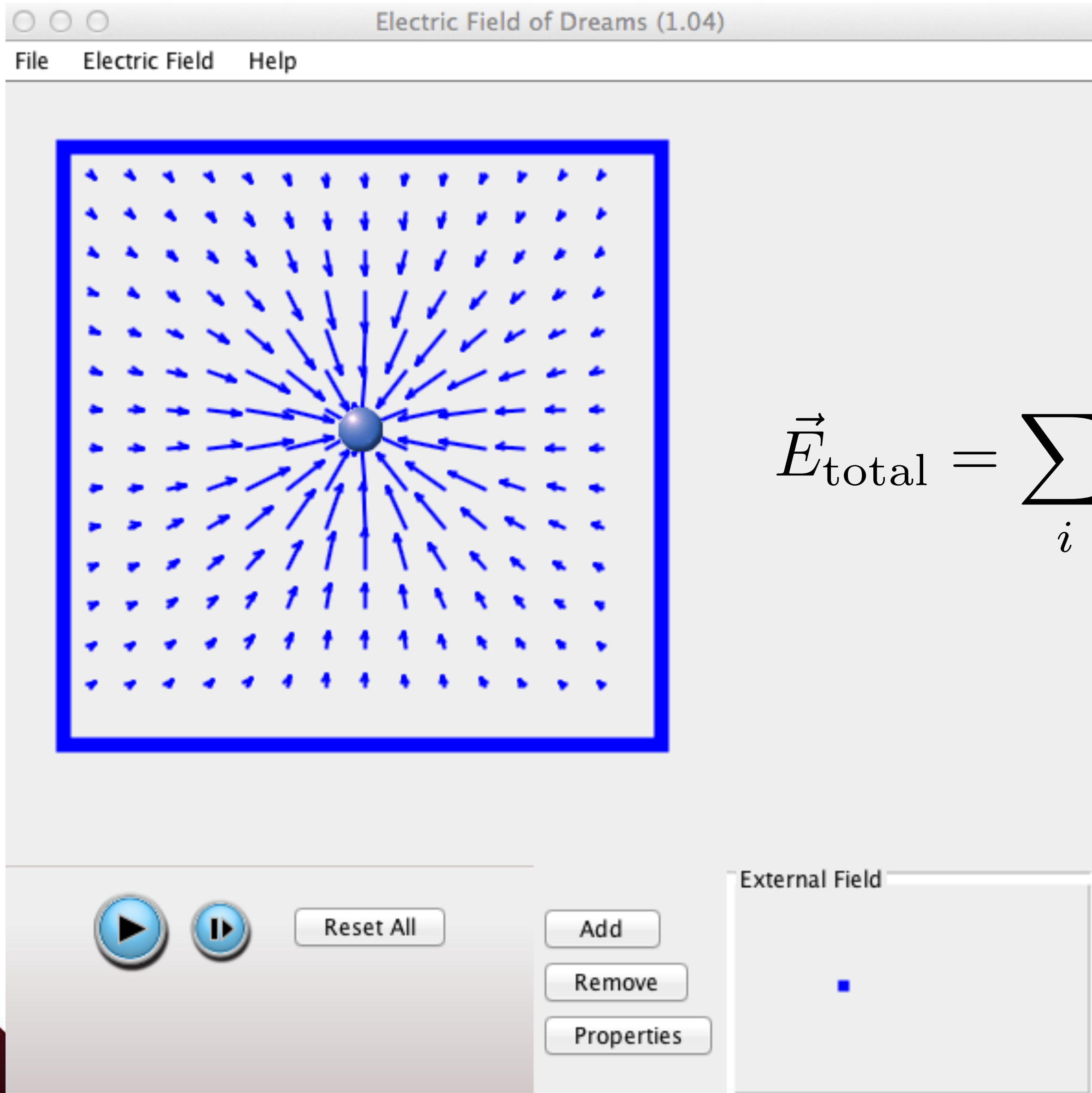
$$\vec{E}_{\text{point charge}} = \frac{kq}{r^2}\,\hat{r}$$

- The superposition principle shows that the field due to a charge distribution is the vector sum of the fields of the individual charges.

$$\vec{E}_{\text{total}} = \sum_i \vec{E}_i = \sum_i \frac{kq_i}{r_i^2}\,\hat{r}_i$$

Demo: Electric Field of Dreams (5-10 minutes)

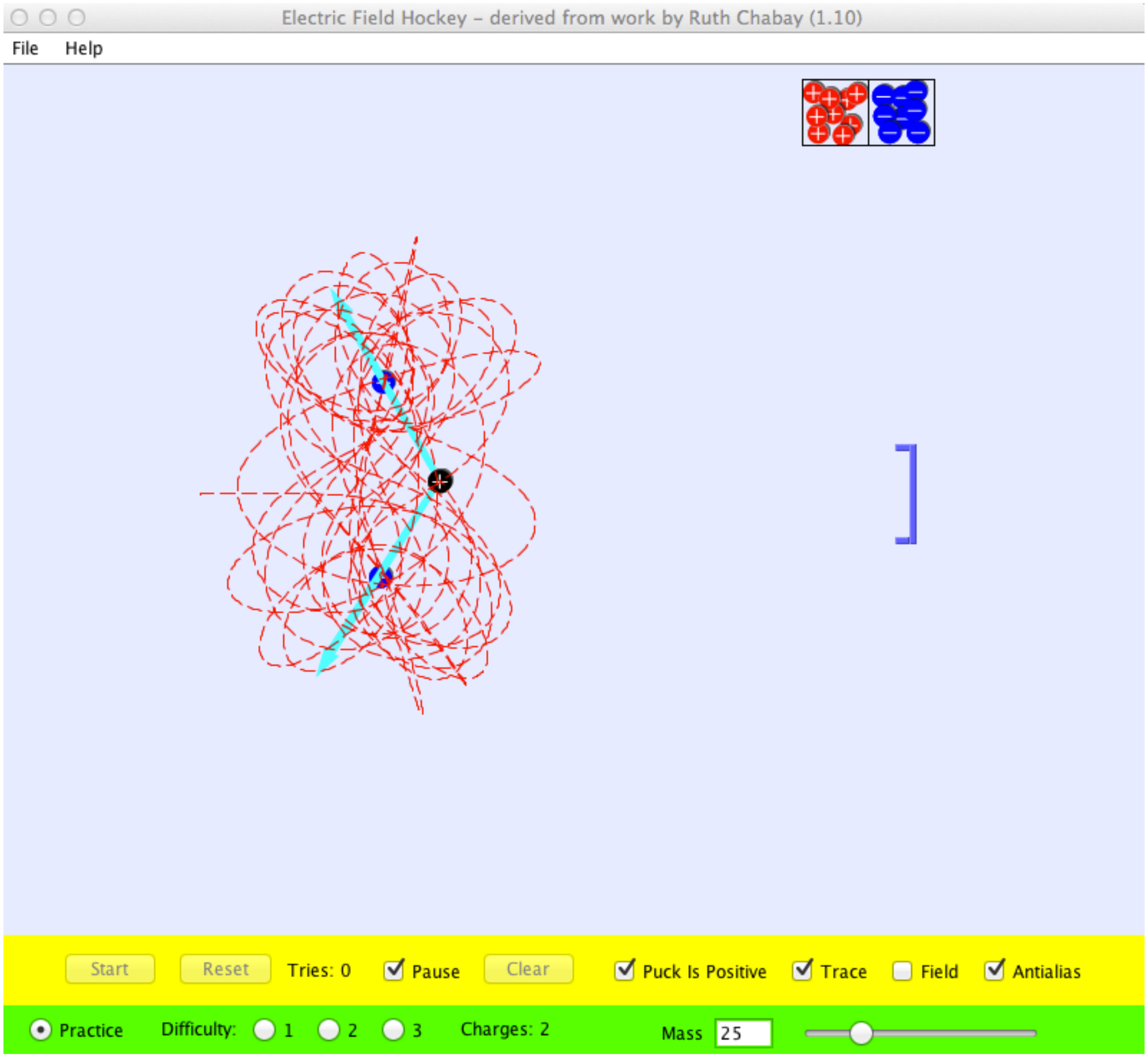
http://phet.colorado.edu/en/simulation/efield



$$\vec{E}_{\text{total}} = \sum_i \vec{E}_i = \sum_i \frac{kq_i}{r_i^2}\,\hat{r}_i$$

Demo: Electric Field Hockey

http://phet.colorado.edu/en/simulation/electric-hockey



Electric Field Hockey (10 minutes)

http://phet.colorado.edu/en/simulation/electric-hockey

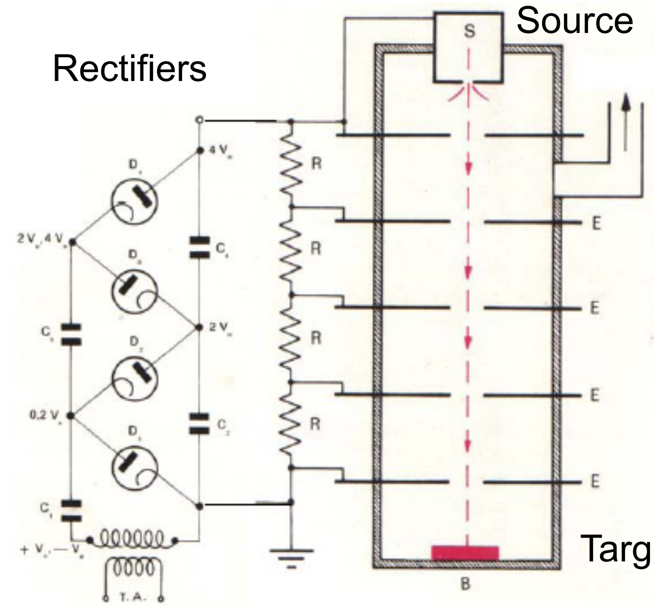
- Click “Download” or “Run Now!” to run Java simulation
- Click and drag electric charges from box in upper right
- Click “Start” to watch how the black proton moves
- Click “Field” checkbox to see net electric field
  - Remember that a positive charge moves in the direction of the electric field!
- Click “Trace” to draw the path of the proton as it moves

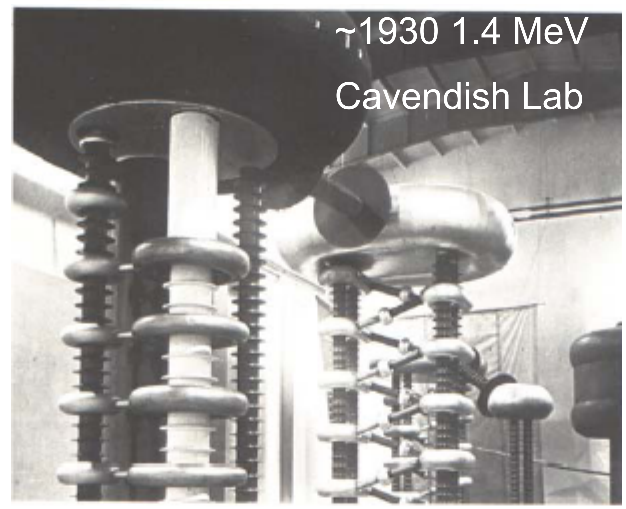
- Add more charges! Watch pretty pictures!
  - You can see how this is very complicated, nonlinear motion
  - This is **not** first semester motion with constant acceleration

- When you feel good, try Difficulties 1, 2, 3 to see if you can hit the goal on the right with your proton

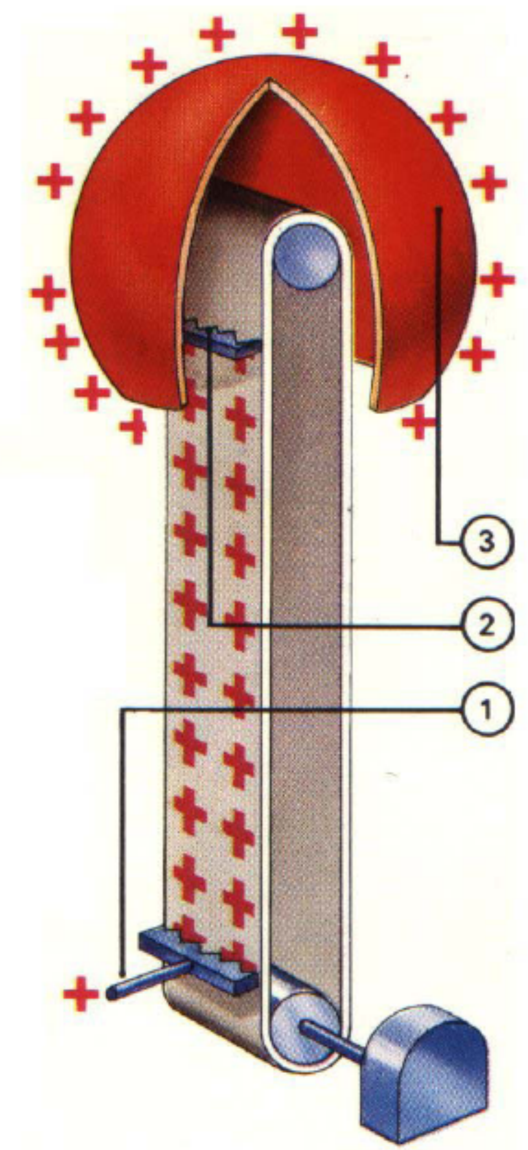
DC Accelerating Gaps: Cockcroft-Walton

- Accelerates ions through successive electrostatic voltages
  - First to get protons to >MeV
  - Continuous HV applied through intermediate electrodes
  - Rectifier-multipliers (voltage dividers)
  - Limited by HV sparking/breakdown
  - FNAL still uses a 750 kV C-W
- Also example of early ion source
  - H gas ionized with HV current
  - Provides high current DC beam





DC Accelerating Gaps: Van de Graaff



- How to increase voltage?
  - R.J. Van de Graaff: charge transport
  - Electrode (1) sprays HV charge onto insulated belt
  - Carried up to spherical Faraday cage
  - Removed by second electrode and distributed over sphere
- Limited by discharge breakdown
  - ~2MV in air
  - Up to 20+ MV in SF_6!
  - Ancestors of Pelletrons (chains)/ Laddertrons (stripes)

Van de Graaff Popularity

The Dipole: an Important Charge Distribution

- An **electric dipole** consists of two point charges of equal magnitude but opposite signs, held a short distance apart.
  - The dipole is electrically neutral, but the separation of its charges results in an electric field.
  - Many charge distributions, especially molecules, behave like electric dipoles.
  - The product of the charge and separation is the dipole moment: *p = qd.*
  - Far from the dipole, its electric field falls off as the inverse cube of the distance.