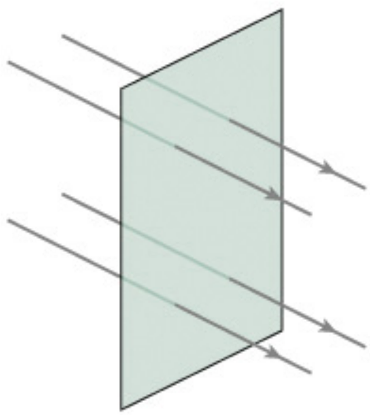
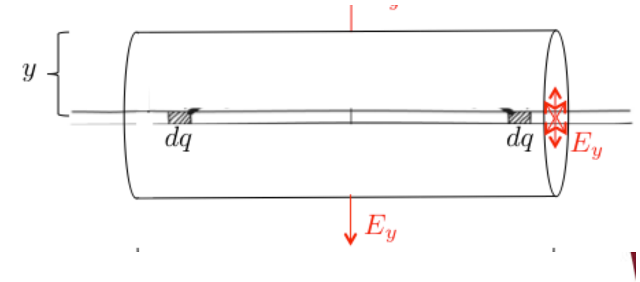




University Physics 227N/232N
Old Dominion University

Flux and Gauss's Law
Example Problems and Solutions

Dr. Todd Satogata (ODU/Jefferson Lab)
satogata@jlab.org
http://www.toddsatogata.net/2014-ODU

Wednesday, January 29 2014

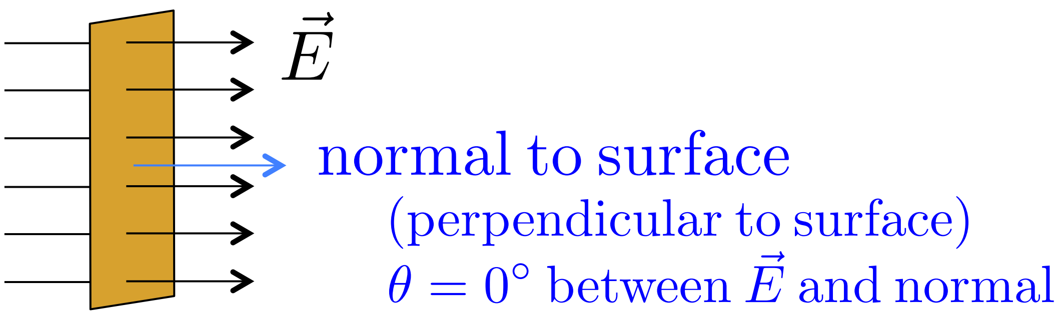
Happy Birthday to Abdus Salam, Jonny Lang, Sara Gilbert,
Paul Ryan, Oprah Winfrey, and Anton Chekhov!

Flux Problem 1

- An electric flux of $\Phi = 5\ \mathrm{N\,m^2/C}$ passes through a flat surface that is perpendicular to a constant electric field of strength $E = 3\,\mathrm{N/C}$. What is the area of the surface?

Flux Problem 1: Solution

- An electric flux of $\Phi = 5\ \mathrm{N\,m^2/C}$ passes through a flat surface that is perpendicular to a constant electric field of strength $E = 3\ \mathrm{N/C}$. What is the area of the surface?
  - Draw a picture!



  - Electric flux is defined as $\Phi = \int E\, dA\, \cos\theta$ where $\int$ is just a fancy sum.
  - Every piece of the area has the same field E passing through it
  - Every normal vector to the area has the same angle to the E field ($\theta = 0°$)
  - So every term of the sum is the same, and the flux reduces to

$$\boxed{\Phi = E\, A\, \cos\theta} = E\, A$$

$$A = \Phi/E = (5\,\mathrm{N\,m^2/C})/(3\,\mathrm{N/C}) = \boxed{1.7\,\mathrm{m^2} = A}$$

Flux Comments

- Notice that I boxed that handy equation

$$\Phi = E\ A\ \cos\theta$$

- **This holds for**
  - A constant electric field of strength E
  - A surface of area A
  - **When and only when** every point on the surface has the **same angle** to the electric field
    - Often the surface is chosen so this angle is 0 or 90 degrees to make the cosine simple to calculate
  - Usually these surfaces are chosen because of **symmetries**

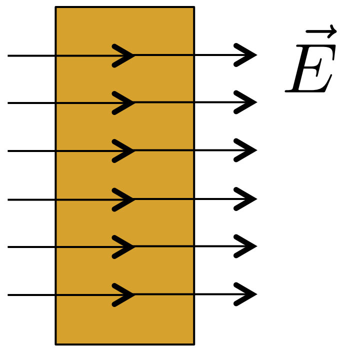
  - http://www.ic.sunysb.edu/Class/phy141md/doku.php?id=phy142:lectures:4 has a nice set of visualizations on flux, normals to surfaces, and Gauss's Law

Flux Problem 2

- The surface in Problem 1 is now turned 90 degrees so it is *parallel* to the electric field of strength $E = 3\ \mathrm{N/C}$. What is the electric flux Φ through the surface now?

Flux Problem 2: Solution

- The surface in Problem 1 is now turned 90 degrees so it is *parallel* to the electric field of strength $E = 3\,\mathrm{N/C}$. What is the electric flux Φ through the surface now?
  - Draw a picture!
    (a bit harder…)



normal to surface points

out of paper

$\theta = 90^\circ$ between $\vec{E}$ and normal

  - Every piece of the area has the same field E passing "along" it
  - Every normal vector to the area has the same angle to the E field ($\theta = 90^\circ$)
  - We can then use $\Phi = E\,A\,\cos\theta$

$$\Phi = E\,A\,\cos\theta = (3\,\mathrm{N/C})(1.7\,\mathrm{m}^2)(0) = \boxed{0\,\mathrm{N\,m^2/C} = \Phi}$$

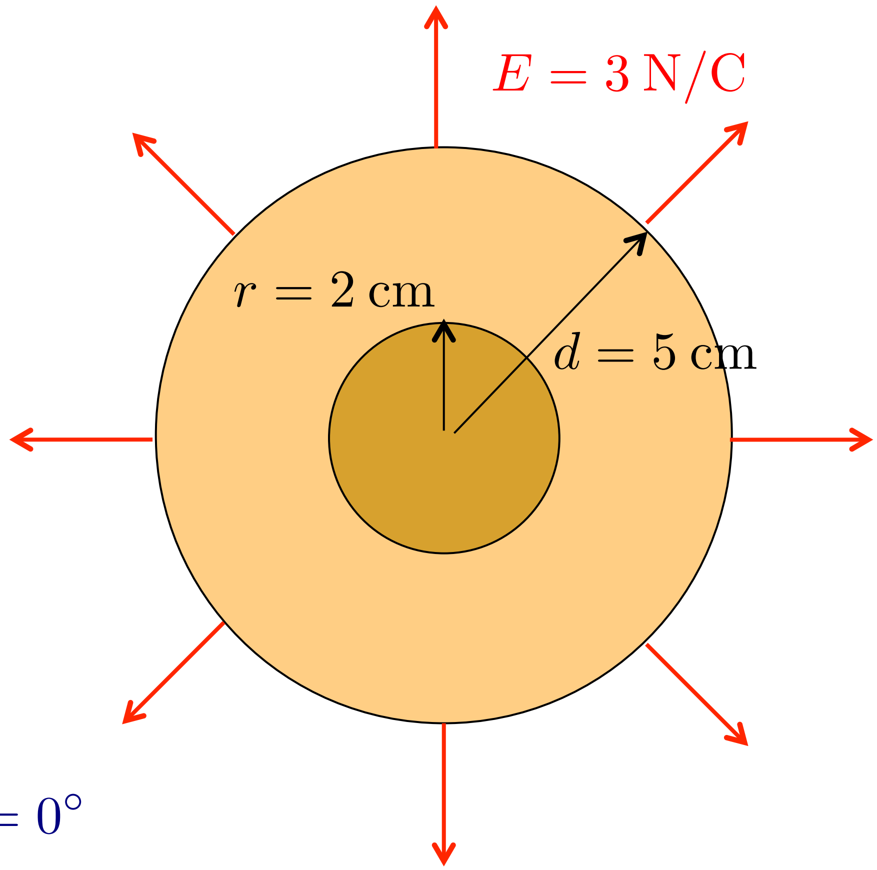
- The flux produced by a field parallel to the surface is always zero!

Flux Problem 3

- A sphere of radius $r = 2\,\mathrm{cm}$ creates an electric field of strength $E = 3\,\mathrm{N/C}$ at a distance $d = 5\,\mathrm{cm}$ from the center of the sphere. What is the electric flux through the surface of the sphere drawn at distance $d = 5\,\mathrm{cm}$?

Flux Problem 3: Solution

- A sphere of radius $r = 2\,\mathrm{cm}$ creates an electric field of strength $E = 3\,\mathrm{N/C}$ at a distance $d = 5\,\mathrm{cm}$ from the center of the sphere. What is the electric flux through the surface of the sphere drawn at distance $d = 5\,\mathrm{cm}$?
  - Draw a picture! (really spheres)
  - The E field is the same strength everywhere on the sphere
  - The E field is perpendicular to the surface of the sphere everywhere on the sphere



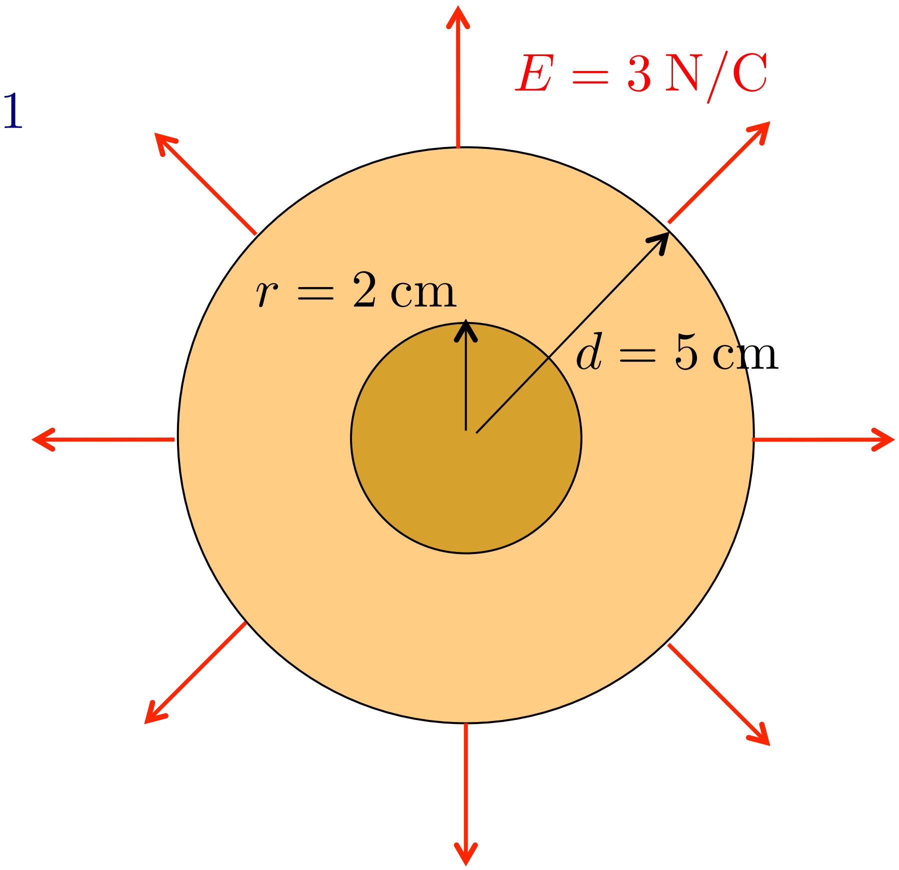
$$\Phi = E\,A\,\cos\theta \qquad \theta = 0^\circ$$

Flux Problem 3: Solution

- A sphere of radius $r = 2\,\mathrm{cm}$ creates an electric field of strength $E = 3\,\mathrm{N/C}$ at a distance $d = 5\,\mathrm{cm}$ from the center of the sphere. What is the electric flux through the surface of the sphere drawn at distance $d = 5\,\mathrm{cm}$?

$$\Phi = E\,A\,\cos\theta$$

$$\theta = 0^\circ \qquad \cos\theta = 1$$



- Surface area of sphere

$$A = 4\pi\,d^2$$

(Use d, **not** r, because the radius of the sphere that we're calculating flux for is d!)

- Putting this together

$$\begin{aligned}\Phi &= E(4\pi d^2)(1) \\ &= (3\,\mathrm{N/C})(4\pi)(0.05\,\mathrm{m})^2 \\ &= \boxed{0.1\,\mathrm{N\,m^2/C} = \Phi}\end{aligned}$$

Gauss's Law Problem 1

- A sphere of radius $r = 2\,\mathrm{cm}$ creates an electric field of strength $E = 3\,\mathrm{N/C}$ at a distance $d = 5\,\mathrm{cm}$ from the center of the sphere. What is the electric charge on the sphere?

Gauss's Law Problem 1: Solution

- A sphere of radius $r = 2\,\mathrm{cm}$ creates an electric field of strength $E = 3\,\mathrm{N/C}$ at a distance $d = 5\,\mathrm{cm}$ from the center of the sphere. What is the electric charge on the sphere?
  - Gauss's law relates the enclosed electric charge to the total flux through a surface surrounding that charge

$$\Phi = 4\pi k q_{\mathrm{enclosed}}$$

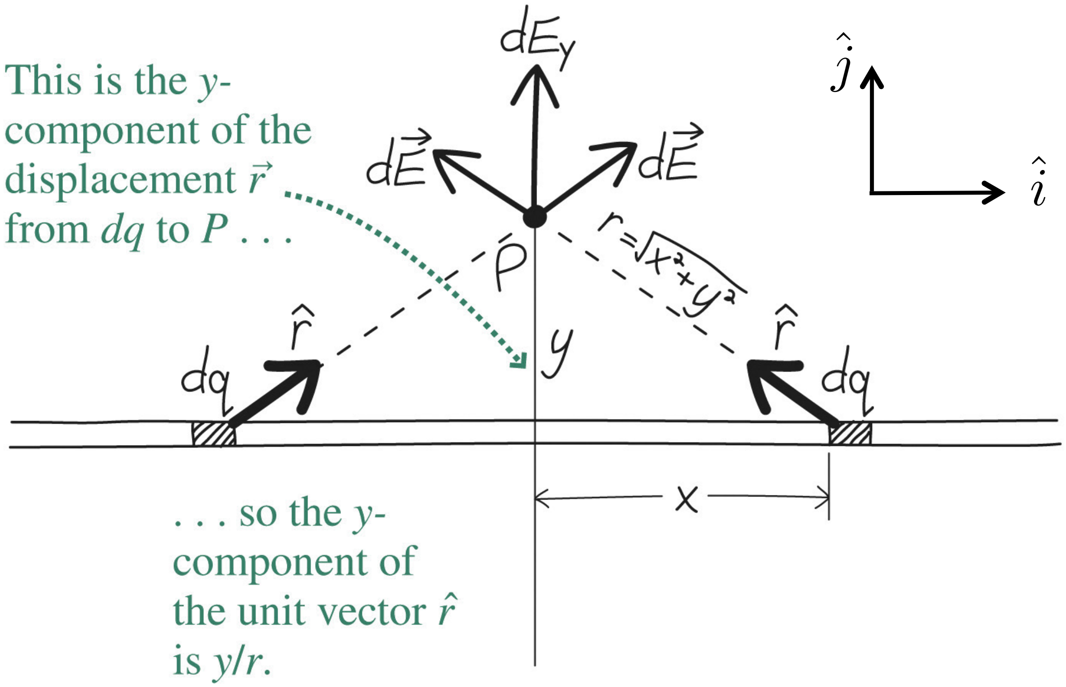
  - We calculated the flux in the last problem: $\Phi = 0.1\,\mathrm{N\,m^2/C^2}$
  - The only charge inside the sphere (Gaussian surface) is the electric charge on the sphere, so $q_{\mathrm{enclosed}} = q_{\mathrm{sphere}}$
  - The rest is just a calculation:

$$q_{\mathrm{enclosed}} = q_{\mathrm{sphere}} = \frac{\Phi}{4\pi k} = \frac{(0.1\,\mathrm{N\,m^2/C})}{4\pi(9\times 10^9\,\mathrm{N\,m^2/C^2})}$$

$$q_{\mathrm{sphere}} = 8.1\times 10^{-13}\,\mathrm{C}$$

Gauss's Law Example 1

- Let's use Gauss's Law to derive an earlier result a faster way
  - On pages 14-15 of the Jan 24 lecture, we derived the electric field from an infinitely long line of charge with charge per unit length λ



$$\vec{E} = \frac{2k\lambda}{y}\,\hat{j}$$

  - But this was painful – it involved an integral we had to look up.

Gauss's Law Example 1

- We will use Gauss's law by drawing a closed surface around some of our charge
  - In particular, we want to draw a surface where the electric field is **either perpendicular to or parallel to** the surface everywhere on the surface
  - We also want the electric field to be the **same magnitude** over large pieces of the surface
  - Then we can use the simpler calculation for flux for separate pieces of the surface that we drew, and use Gauss's Law

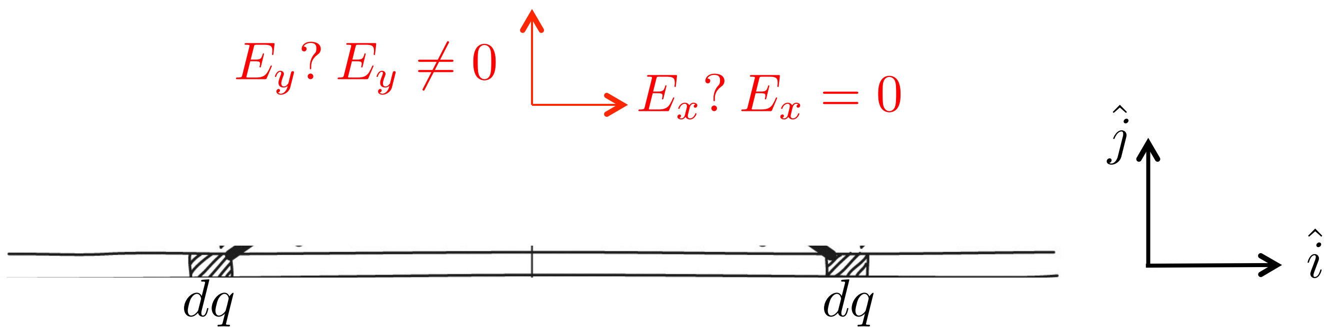
$$\Phi = E\ A\ \cos\theta$$

$$\Phi = EA \quad \text{for areas where E} \perp \text{A} \quad (\cos\theta = 1)$$

$$\Phi = 0 \quad \text{for areas where E} \parallel \text{A} \quad (\cos\theta = 0)$$

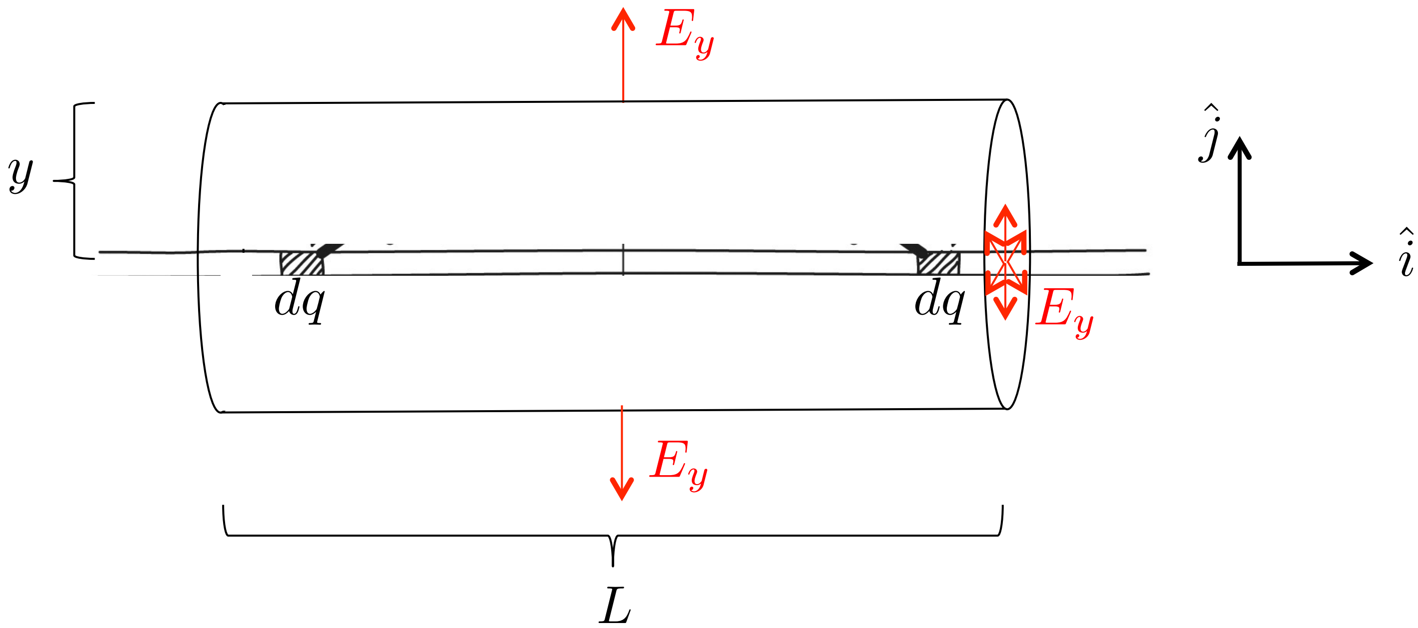
Gauss's Law Example 1

- Now let's figure out what closed surface to draw
  - Remember, we'll want pieces that are **perpendicular or parallel to the electric field**
  - So we'll need to have an idea of which direction the electric field points from this distribution of charge



  - The natural coordinate system to draw has the x direction along the line charge, and the y direction perpendicular to it.
  - In the x direction, there are always equal charges at any $\pm x$
    - So the horizontal component of the field is zero, $E_x = 0$
  - In the y direction, all charge is located in the $-y$ direction
    - So the vertical component of the field is nonzero, $E_y \neq 0$

Gauss's Law Example 1

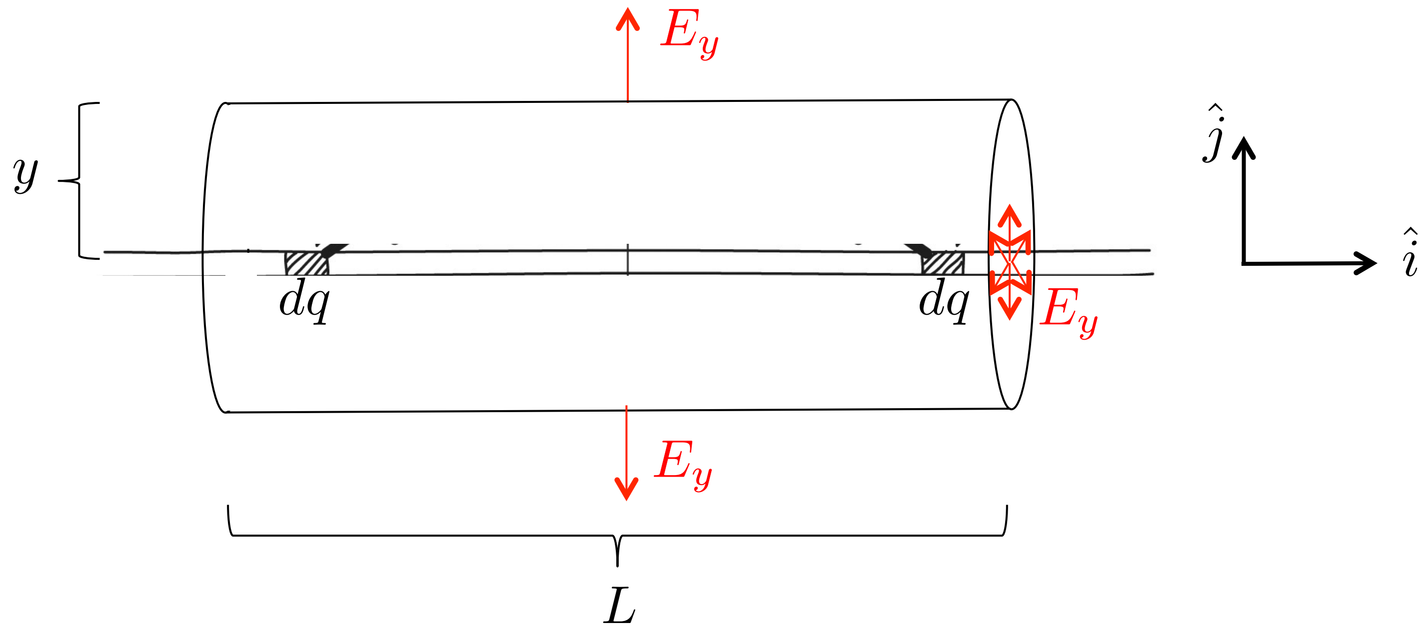


- The fields always point "out" from the line charge
  - So let's draw a closed Gaussian surface: a cylinder
  - On the ends of the cylinder, $E \parallel A$ so $\Phi = 0$
  - On the side of the cylinder, $E \perp A$ and E is constant: $\Phi = E\,A$
  - So the **total flux** through this cylinder of radius y and length L is

$$\Phi_{\text{total}} = 2\Phi_{\text{end}} + \Phi_{\text{side}} = 0 + E\,A_{\text{side}} = E(2\pi y)L$$

$(\text{cylinder side area } \mathrm{A}_{\text{side}} = (2\pi \mathrm{y})\mathrm{L})$

Gauss's Law Example 1



- We're almost there! We've calculated Φ_{total} : what is q_{enclosed}?
  - It's the charge per unit length times the cylinder length:

$$q_{\text{enclosed}} = \lambda\, L$$

  - So now Gauss's Law gives us the answer – no integrals!

$$\Phi_{\text{total}} = E\, L\, (2\pi y) = 4\pi q_{\text{enclosed}} = 4\pi k \lambda L$$

$$\Rightarrow \boxed{E = \frac{2k\lambda}{y}\hat{j}}$$

same answer as before!!