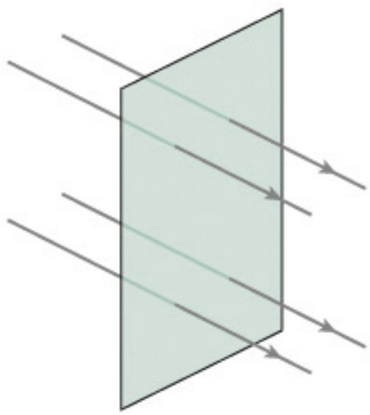

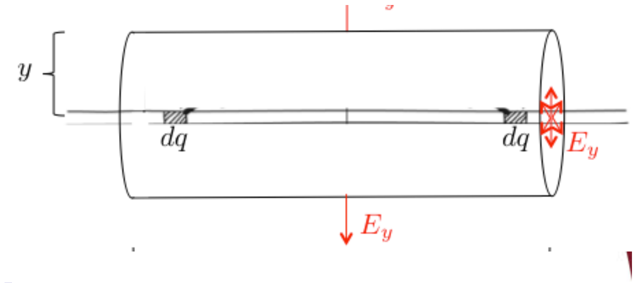

University Physics 227N/232N
Old Dominion University

Flux and Gauss's Law Review
More Example Problems and Solutions

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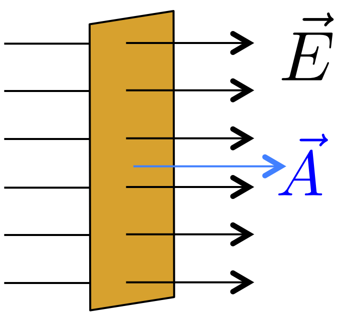
Happy Birthday to Rudolf Mossbauer, Vernon Davis, Brad Rutter,
Kerry Washington, and Jackie Robinson!

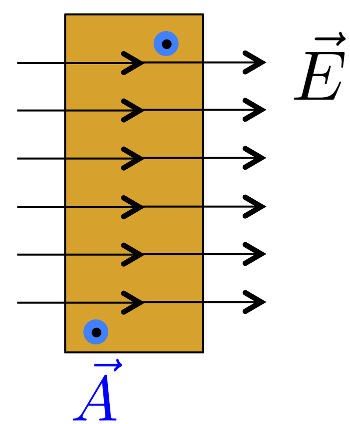
Chapter 21 Review: Flux

- **Electric Flux** of an electric field through a surface is defined as
  - … the sum of the electric field components **perpendicular** to the surface **times** the area of the surface that they intersect
  - It really is a measure of **how much electric field points through the surface**
    - Bigger surface with the same field => bigger flux
    - Bigger field with the same surface => bigger flux
  - Electric field is a **vector** and flux depends on the angle between the surface and the electric field vector at any point



$$\Phi = \vec{E} \cdot \vec{A} = E\ A\ \cos\theta$$

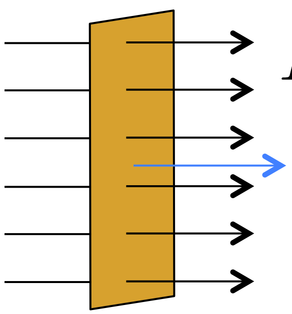
$\vec{A}$ points **perpendicular** to the surface



$\theta = 0^\circ$ $\theta = 90^\circ$

Flux Problem 1: Solution

- An electric flux of $\Phi = 5\ \mathrm{N\,m^2/C}$ passes through a flat surface that is perpendicular to a constant electric field of strength $E = 3\ \mathrm{N/C}$. What is the area of the surface?
  - Draw a picture!



  - Electric flux is defined as $\Phi = \int E\, dA\, \cos\theta$ where $\int$ is just a fancy sum.
  - Every piece of the area has the same field E passing through it
  - Every normal vector to the area has the same angle to the E field ($\theta = 0^\circ$)
  - So every term of the sum is the same, and the flux reduces to

$$\boxed{\Phi = E\,A\,\cos\theta} = E\,A$$

$$A = \Phi/E = (5\,\mathrm{N\,m^2/C})/(3\,\mathrm{N/C}) = \boxed{1.7\,\mathrm{m^2} = A}$$

Flux Comments

- Notice that I boxed that handy equation

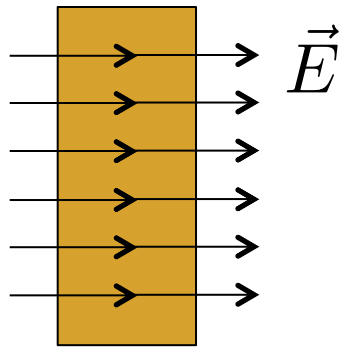
$$\boxed{\Phi = E\ A\ \cos\theta}$$

- **This holds for**
  - A constant electric field of strength E
  - A surface of area A
  - **When and only when** every point on the surface has the **same angle** to the electric field
    - Often the surface is chosen so this angle is 0 or 90 degrees to make the cosine simple to calculate
  - Usually these surfaces are chosen because of **symmetries**

  - http://www.ic.sunysb.edu/Class/phy141md/doku.php?id=phy142:lectures:4 has a nice set of visualizations on flux, normals to surfaces, and Gauss's Law

Flux Problem 2: Solution

- The surface in Problem 1 is now turned 90 degrees so it is *parallel* to the electric field of strength $E = 3\ \mathrm{N/C}$. What is the electric flux Φ through the surface now?
  - Draw a picture!
    (a bit harder…)



normal to surface points out of paper

$\theta = 90^\circ$ between $\vec{E}$ and normal

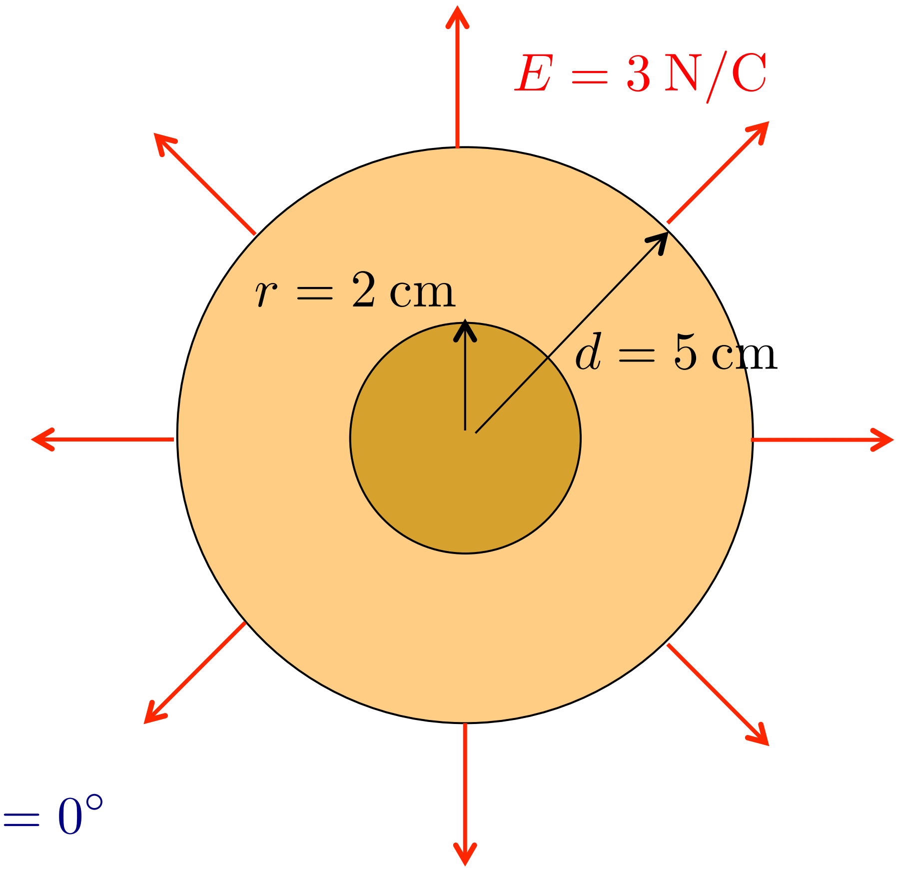
  - Every piece of the area has the same field E passing "along" it
  - Every normal vector to the area has the same angle to the E field ($\theta = 90^\circ$)
  - We can then use $\Phi = E\ A\ \cos\theta$

$$\Phi = E\ A\ \cos\theta = (3\ \mathrm{N/C})(1.7\ \mathrm{m}^2)(0) = \boxed{0\ \mathrm{N\ m^2/C} = \Phi}$$

- The flux produced by a field parallel to the surface is always zero!

Flux Problem 3: Solution

- A sphere of radius $r = 2\,\mathrm{cm}$ creates an electric field of strength $E = 3\,\mathrm{N/C}$ at a distance $d = 5\,\mathrm{cm}$ from the center of the sphere. What is the electric flux through the surface of the sphere drawn at distance $d = 5\,\mathrm{cm}$?
  - Draw a picture! (really spheres)
  - The E field is the same strength everywhere on the sphere
  - The E field is perpendicular to the surface of the sphere everywhere on the sphere



$$\Phi = E\,A\,\cos\theta \qquad \theta = 0^\circ$$

Flux Problem 3: Solution

- A sphere of radius $r = 2\,\mathrm{cm}$ creates an electric field of strength $E = 3\,\mathrm{N/C}$ at a distance $d = 5\,\mathrm{cm}$ from the center of the sphere. What is the electric flux through the surface of the sphere drawn at distance $d = 5\,\mathrm{cm}$?

$$\Phi = E\,A\,\cos\theta$$

$$\theta = 0^\circ \qquad \cos\theta = 1$$

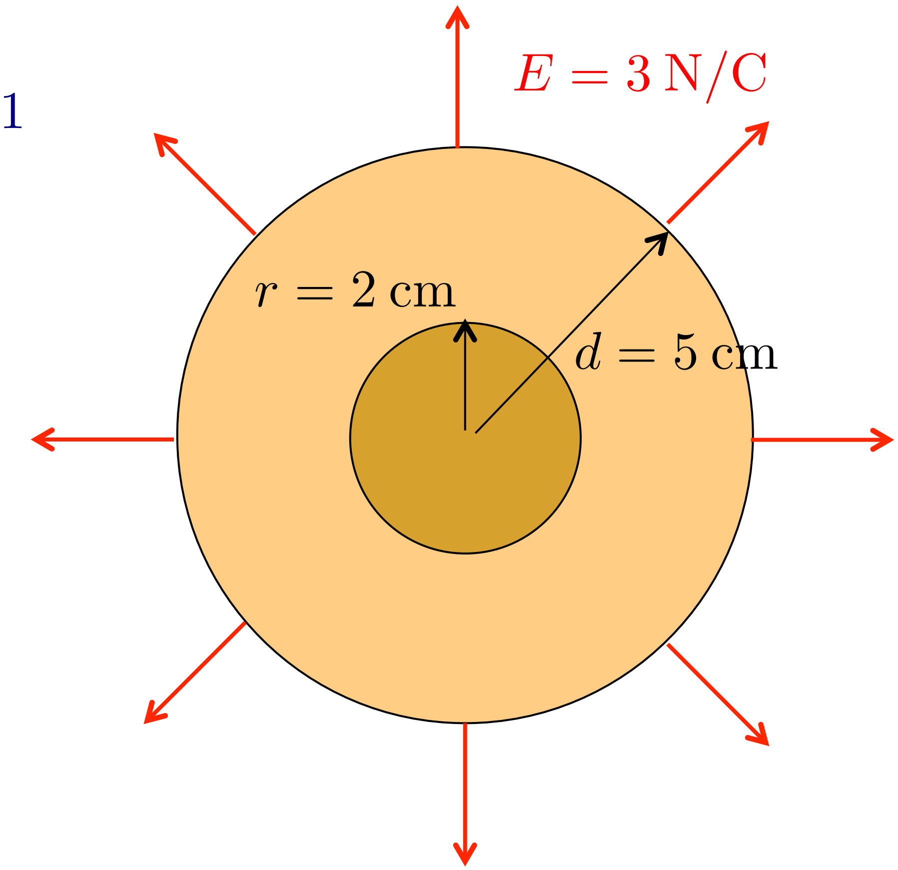
- Surface area of sphere

$$A = 4\pi\,d^2$$

(Use d, **not** r, because the radius of the sphere that we're calculating flux for is d!)

- Putting this together

$$\begin{aligned}\Phi &= E(4\pi d^2)(1)\\ &= (3\,\mathrm{N/C})(4\pi)(0.05\,\mathrm{m})^2\\ &= \boxed{0.1\,\mathrm{N\,m^2/C} = \Phi}\end{aligned}$$



Gauss's Law Problem 1: Solution

- A sphere of radius $r = 2\,\mathrm{cm}$ creates an electric field of strength $E = 3\,\mathrm{N/C}$ at a distance $d = 5\,\mathrm{cm}$ from the center of the sphere. What is the electric charge on the sphere?
  - Gauss's law relates the enclosed electric charge to the total flux through a surface surrounding that charge

$$\Phi = 4\pi k q_{\mathrm{enclosed}}$$

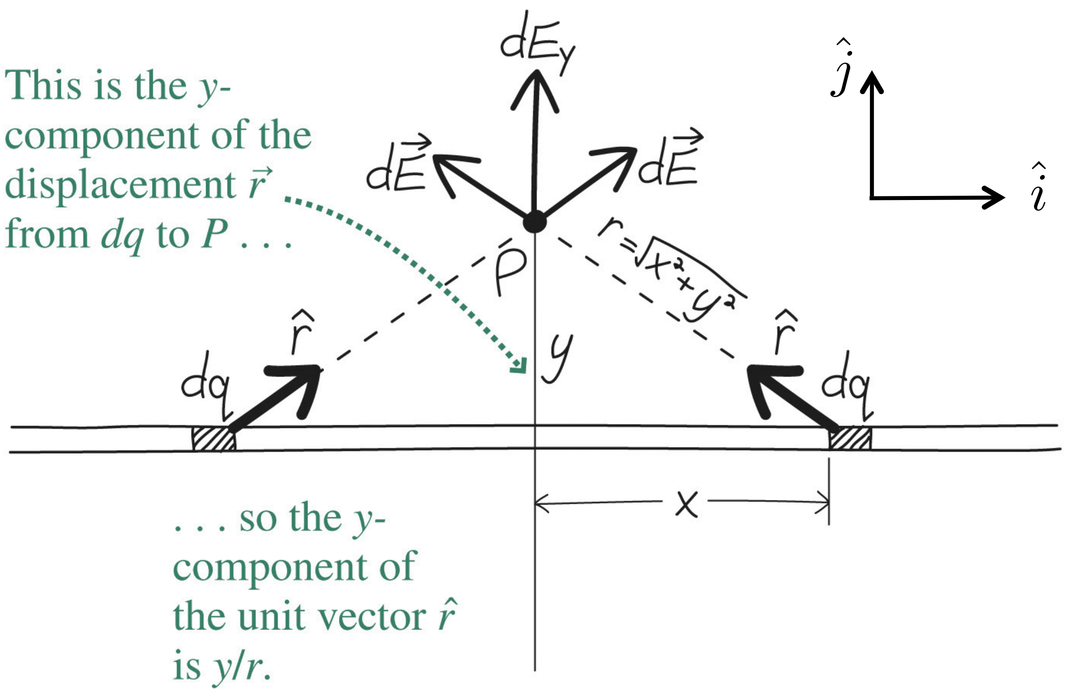
  - We calculated the flux in the last problem: $\Phi = 0.1\,\mathrm{N\,m^2/C^2}$
  - The only charge inside the sphere (Gaussian surface) is the electric charge on the sphere, so $q_{\mathrm{enclosed}} = q_{\mathrm{sphere}}$
  - The rest is just a calculation:

$$q_{\mathrm{enclosed}} = q_{\mathrm{sphere}} = \frac{\Phi}{4\pi k} = \frac{(0.1\,\mathrm{N\,m^2/C})}{4\pi(9\times 10^9\,\mathrm{N\,m^2/C^2})}$$

$$q_{\mathrm{sphere}} = 8.1 \times 10^{-13}\,\mathrm{C}$$

Gauss's Law Example 1

- Let's use Gauss's Law to derive an earlier result a faster way
  - On pages 14-15 of the Jan 24 lecture, we derived the electric field from an infinitely long line of charge with charge per unit length λ



$$\vec{E} = \frac{2k\lambda}{y}\,\hat{j}$$

  - But this was painful – it involved an integral we had to look up.

Gauss's Law Example 1

- We will use Gauss's law by drawing a closed surface around some of our charge
  - In particular, we want to draw a surface where the electric field is **either perpendicular to or parallel to** the surface everywhere on the surface
  - We also want the electric field to be the **same magnitude** over large pieces of the surface
  - Then we can use the simpler calculation for flux for separate pieces of the surface that we drew, and use Gauss's Law

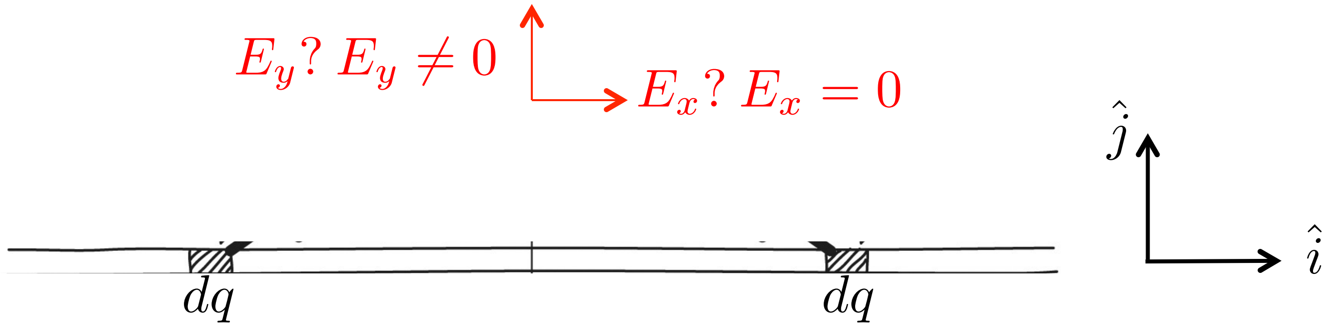
$$\Phi = E\ A\ \cos\theta$$

$$\Phi = EA \quad \text{for areas where E} \perp \text{A} \quad (\cos\theta = 1)$$

$$\Phi = 0 \quad \text{for areas where E} \parallel \text{A} \quad (\cos\theta = 0)$$

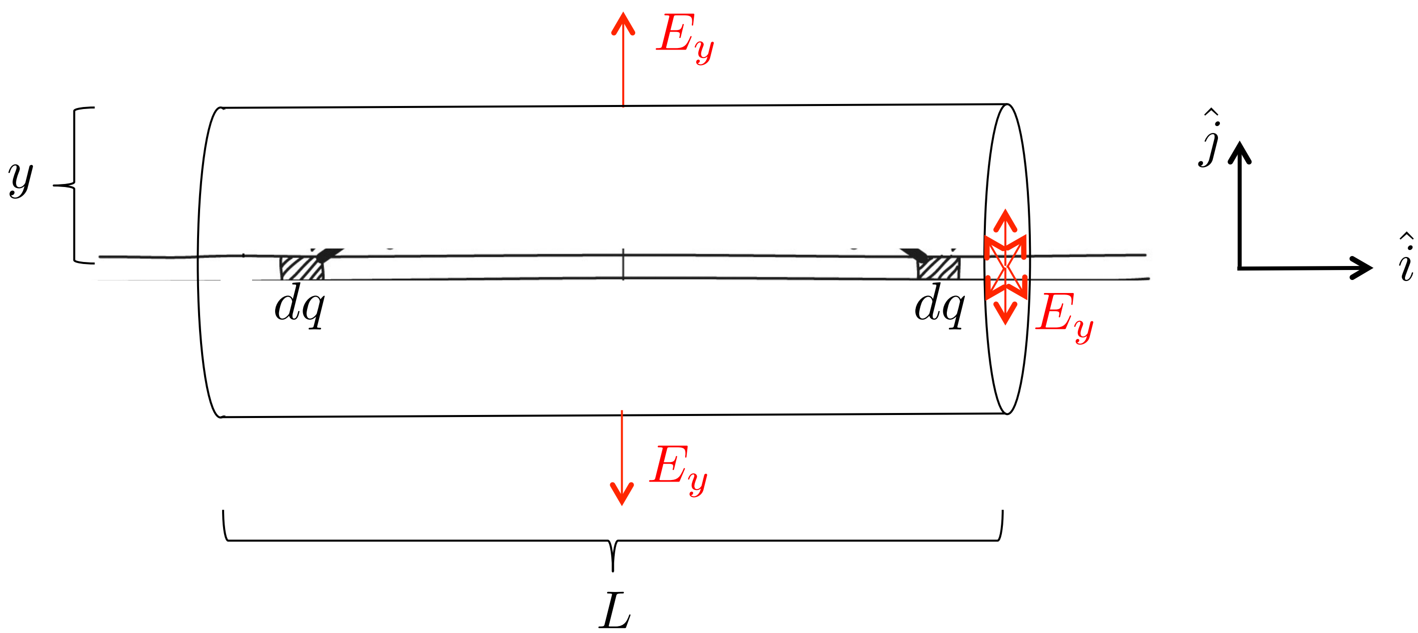
Gauss's Law Example 1

- Now let's figure out what closed surface to draw
  - Remember, we'll want pieces that are **perpendicular or parallel to the electric field**
  - So we'll need to have an idea of which direction the electric field points from this distribution of charge



  - The natural coordinate system to draw has the x direction along the line charge, and the y direction perpendicular to it.
  - In the x direction, there are always equal charges at any $\pm x$
    - So the horizontal component of the field is zero, $E_x = 0$
  - In the y direction, all charge is located in the $-y$ direction
    - So the vertical component of the field is nonzero, $E_y \neq 0$

Gauss's Law Example 1

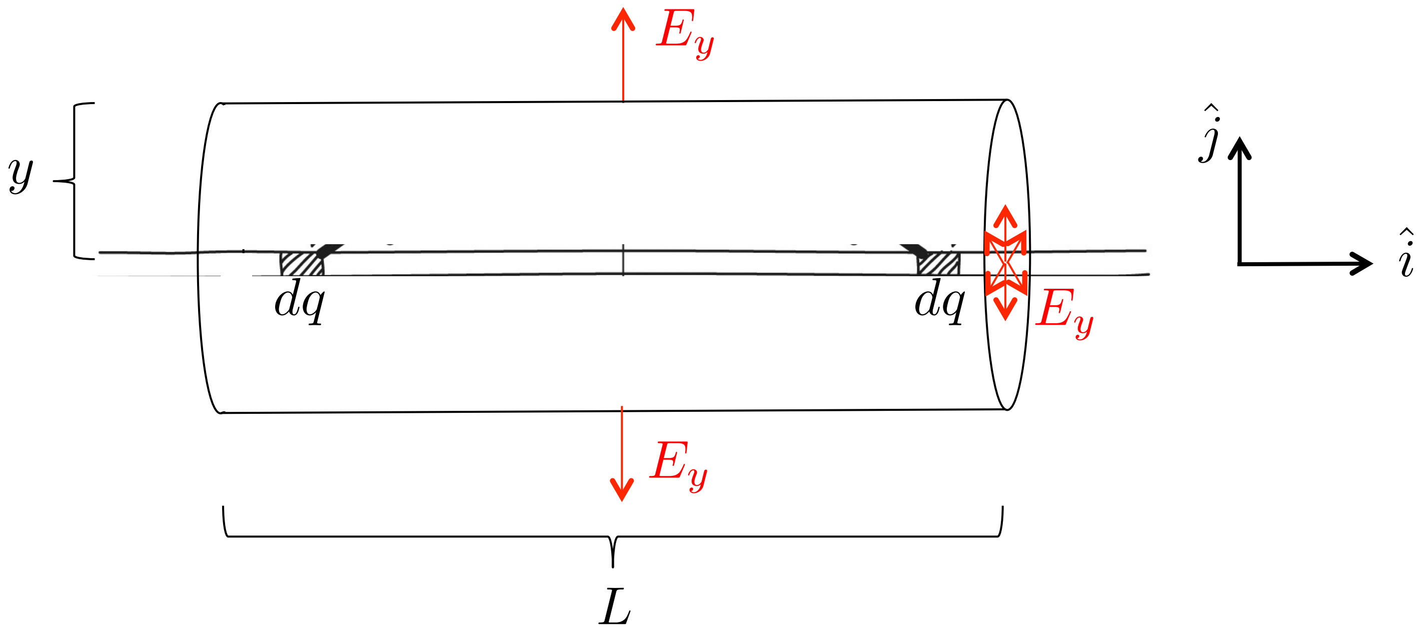


- The fields always point "out" from the line charge
  - So let's draw a closed Gaussian surface: a cylinder
  - On the ends of the cylinder, $E \parallel A$ so $\Phi = 0$
  - On the side of the cylinder, $E \perp A$ and E is constant: $\Phi = E\,A$
  - So the **total flux** through this cylinder of radius y and length L is

$$\Phi_{\text{total}} = 2\Phi_{\text{end}} + \Phi_{\text{side}} = 0 + E\,A_{\text{side}} = E(2\pi y)L$$

$$(\text{cylinder side area } \mathrm{A}_{\text{side}} = (2\pi \mathrm{y})\mathrm{L})$$

Gauss's Law Example 1



- We're almost there! We've calculated Φ_{total} : what is q_{enclosed}?
  - It's the charge per unit length times the cylinder length:

$$q_{\text{enclosed}} = \lambda\, L$$

  - So now Gauss's Law gives us the answer – no integrals!

$$\Phi_{\text{total}} = E\, L\, (2\pi y) = 4\pi q_{\text{enclosed}} = 4\pi k \lambda L$$

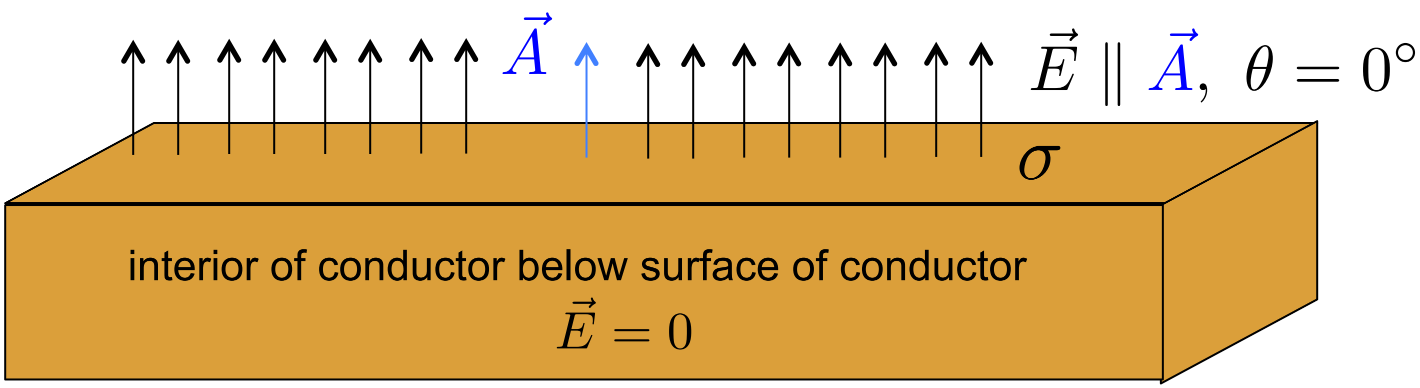
$$\Rightarrow \boxed{E = \frac{2k\lambda}{y}\hat{j}}$$

same answer as before!!

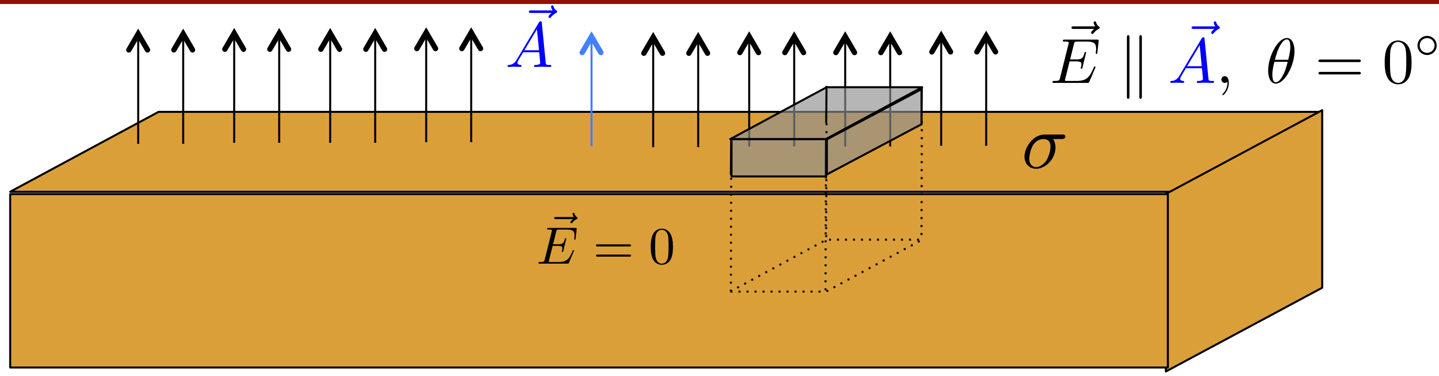
Gauss's Law and Conductors

- Remember that the **electric field on the inside of a conductor is zero**
  - Any surface that we draw for Gauss's law that's inside a conductor has zero flux, $\Phi = 0$
  - We can use this to calculate electric fields without calculus for some interesting conductors
  - This also will give us insight into electrical shielding

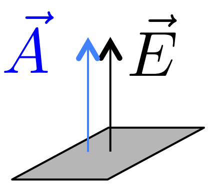
- Consider the surface of an infinitely large flat conductor that has a charge distributed evenly on it, with charge density σ



Electric Field on Flat Surface of Conductor



- E is constant and points out over the surface
- To use Gauss's law, we need a surface
  - Draw a little box with sides parallel to E
  - Flux through the box inside conductor is zero
  - Flux through the sides of the box outside of conductor is zero
  - So the total flux through the box is just flux through the **top**

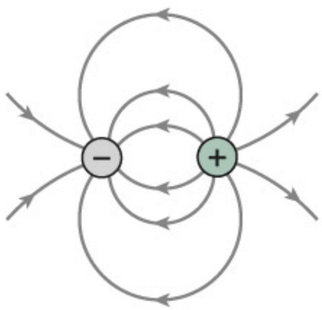
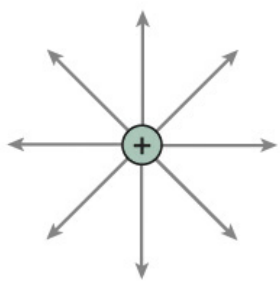


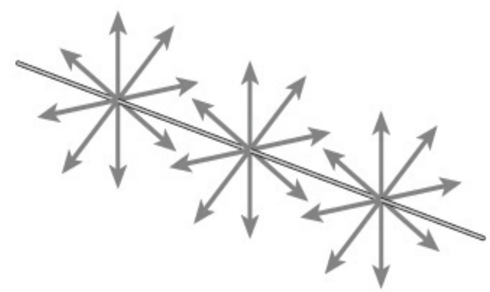
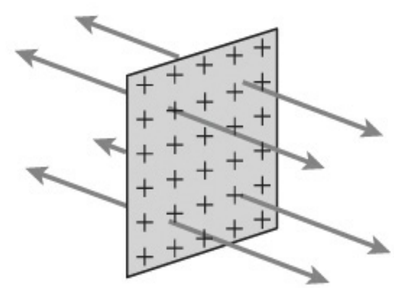
$$\Phi = \vec{E} \cdot \vec{A} = EA = 4\pi\, k\, q_{\text{enclosed}} = 4\pi\, k\, (A\sigma)$$

$$\boxed{E = 4\pi\, k\, \sigma}$$

Independent of A or distance from the (infinite flat) conductor!!

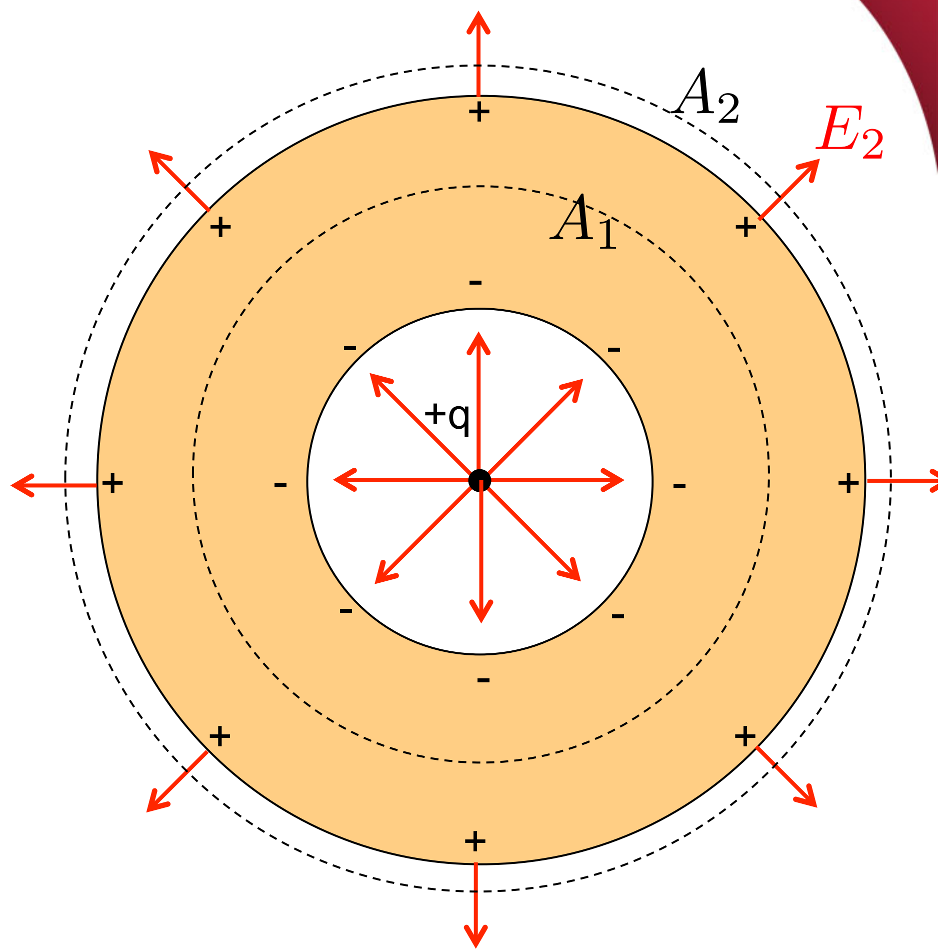
Summary of Fields (Pictures and Scaling)

Charge distribution	Field lines	Dependence of field strength on distance
Dipole		$\frac{1}{r^3}$
Point charge		$\frac{1}{r^2}$

Charge distribution	Field lines	Dependence of field strength on distance
Line charge		$\frac{1}{r}$
Plane charge		$\frac{1}{r^0}$ (constant)

Conductor Shells

- What about a shell of a neutral conductor with a hollow spot in the middle?
  - A charge q placed in the middle attracts an equal and opposite amount of charge –q to the inner surface
  - The electric field inside the conductor is still zero
  - The overall conductor is still neutral, so +q charge must be on the outer surface of the conductor



$$\Phi(A_1) = 0 \text{ (since } E = 0) \quad \Rightarrow \quad q_{\text{enclosed}}(A_1) = 0$$

$$\Phi(A_2) = 4\pi r^2 E_2 = 4\pi k(+q) \quad \Rightarrow \quad E_2 = \frac{kq}{r^2}$$

Conductor Shells

- What's the electric field inside a hollow spot in the conductor indicated here?

- What's the distribution of charges on the inner surface of that hollow spot?