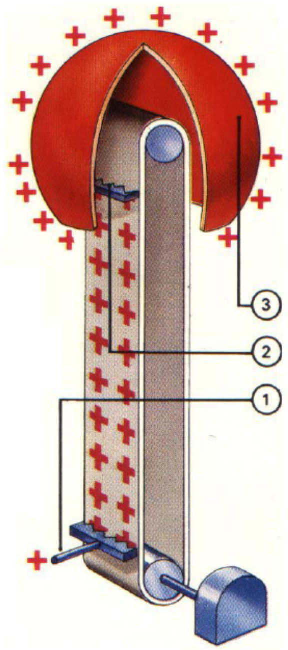
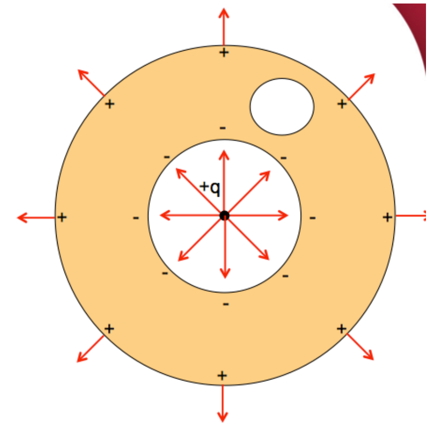




University Physics 227N/232N
Old Dominion University

Gauss's Law and Conductors
Starting Electric Potential
Exam Wed Feb 12

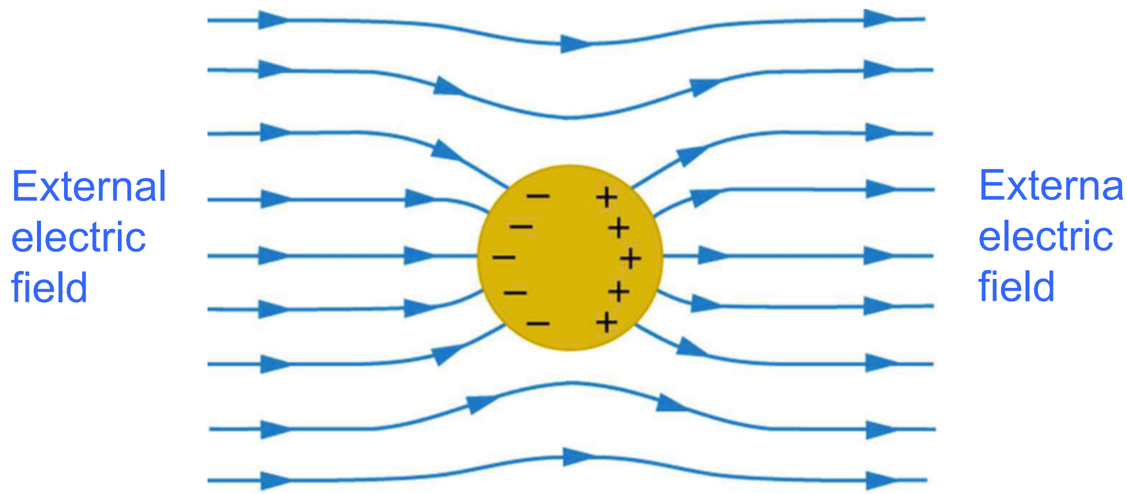
Dr. Todd Satogata (ODU/Jefferson Lab)
satogata@jlab.org
http://www.toddsatogata.net/2014-ODU

Monday, February 3 2014

Happy Birthday to Julio Jones, Isla Fisher, Felix Mendelssohn,
Norman Rockwell, and James Michner!
RIP to Philip Seymour Hoffman

Review: Conductors and Electric Fields

- Electrons move freely in conductors until **electrical forces balance**
  - The electric field **inside** a perfect conductor is **always zero**
    - Any nonzero field moves electrons until the overall electric field is zero
  - The electric field on the **surface** of a perfect conductor is always only **perpendicular** to the surface
    - Any tangential field moves electrons until the tangential field is zero



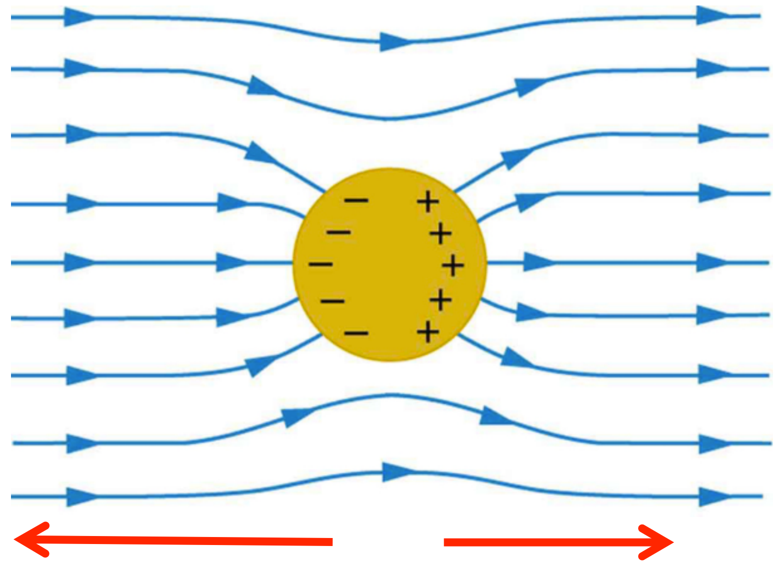
Electric field at surface of conductor perpendicular to surface

Electric field inside conductor: E=0 N/m

Review: Induced Electric Field Demo

Charged wand

Conductor is neutral

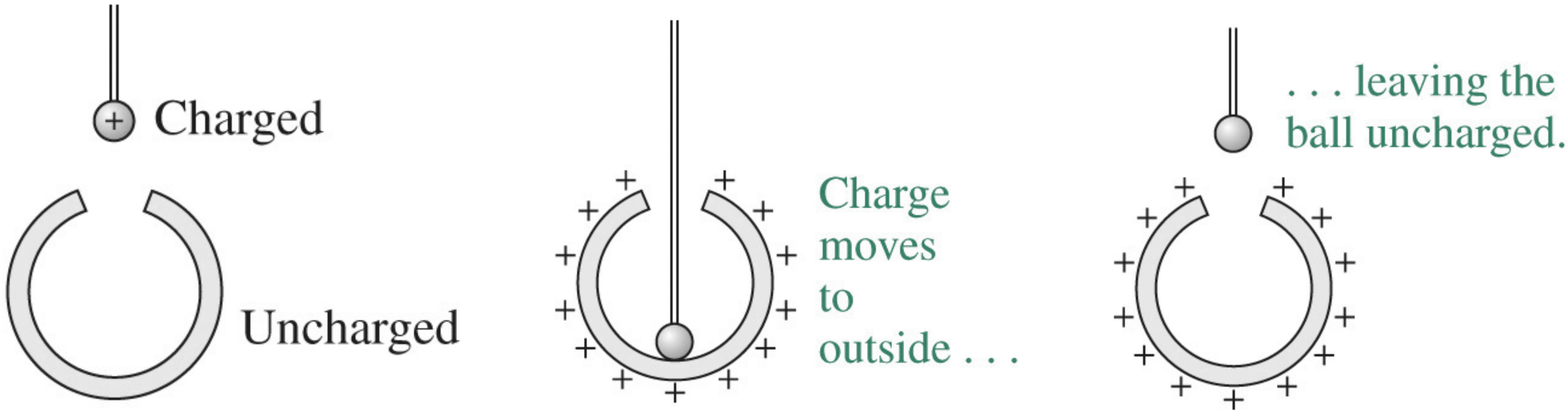


Attractive force
on - charges

Repulsive force
on + charges

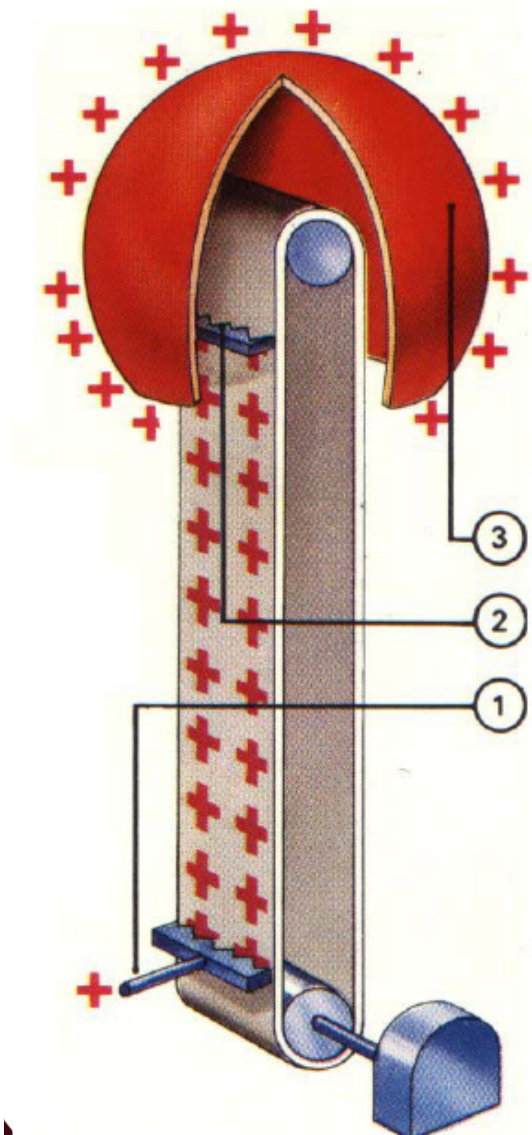
Net **attractive** force on
conductor from wand

Review: Charging A Hollow Conductor



- Electrons move freely over any connected conductive surface
  - Mutual repulsion distributes charges on "outside" of conductors
  - The inserted charged conductor emerges with zero net charge
    - Also the principle behind electrical "grounding", discharging excess electrical charge relative to the "ground"
  - This is also a very sensitive test of Gauss's law/Coulomb's law
    - And thus the inverse square power of electrostatics
      - To better than 1 part in 10^{16} !!
    - http://adsabs.harvard.edu/abs/1971PhRvL..26..721W
      - Also a limit on the photon "rest mass"

Conductors and Accelerators: Van de Graaff, SF_6

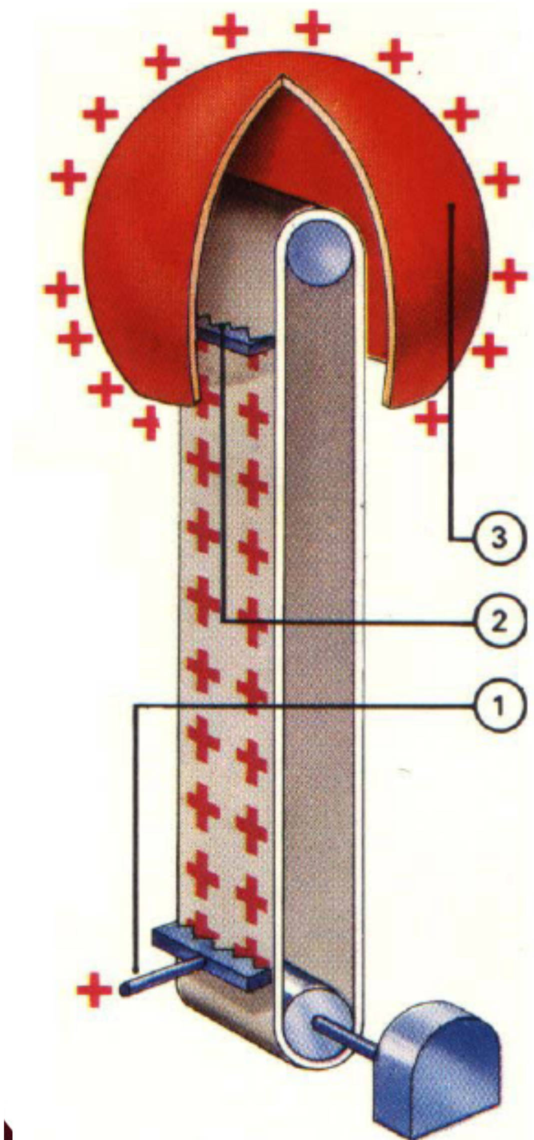


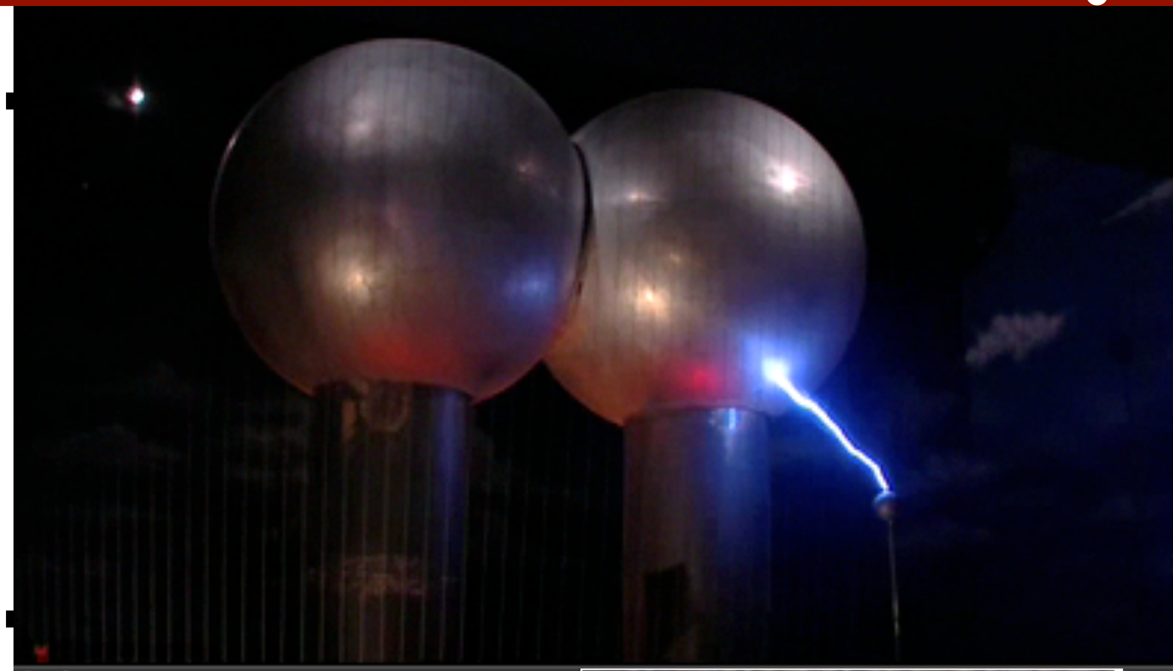
- How to increase voltage for accelerators?
  - R.J. Van de Graaff: charge transport
  - Electrode (1) sprays HV charge onto insulated belt
  - Carried up to spherical conductor
  - Removed by second electrode (2) and distributed over sphere (3)
- Limited by atmospheric electric breakdown
  - ~2 MV in air
  - 20+ MV in SF_6!

(After that, lightning bolts!)

SF_6 also happens to be the most effective greenhouse gas known
Also extremely dense: for fun, see http://www.youtube.com/watch?v=u19QfJWI1oQ

Conductors and Accelerators: Van de Graaff, SF_6





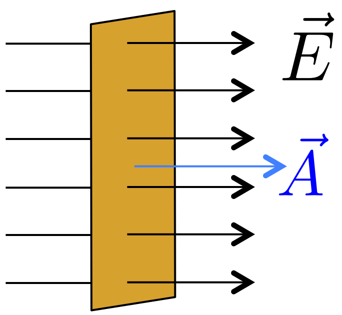
- ~2 MV in air
- 20+ MV in SF_6!

(After that, lightning bolts!)

SF_6 also happens to be the most effective greenhouse gas known
Also extremely dense: for fun, see http://www.youtube.com/watch?v=u19QfJWI1oQ

Review: Flux

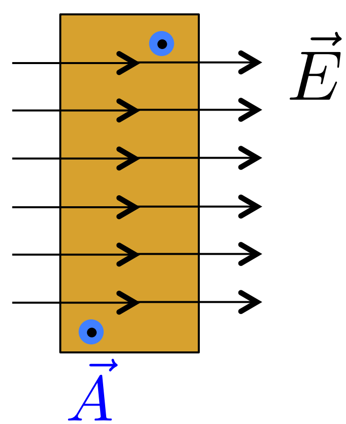
- **Electric Flux** of an electric field through a surface is defined as
  - … the sum of the electric field components **perpendicular** to the surface **times** the area of the surface that they intersect
  - It really is a measure of **how much electric field points through the surface**
    - Bigger surface with the same field => bigger flux
    - Bigger field with the same surface => bigger flux
  - Electric field is a **vector** and flux depends on the angle between the surface and the electric field vector at any point



$\theta = 0^\circ$

$$\Phi = \vec{E} \cdot \vec{A} = E\ A\ \cos\theta$$

$\vec{A}$ points **perpendicular** to the surface



$\theta = 90^\circ$

Gauss's Law Problem 1: Solution

- A sphere of radius $r = 2\,\mathrm{cm}$ creates an electric field of strength $E = 3\,\mathrm{N/C}$ at a distance $d = 5\,\mathrm{cm}$ from the center of the sphere. What is the electric charge on the sphere?
  - **Gauss's law** relates the enclosed electric charge to the total flux through a surface surrounding that charge

$$\Phi = 4\pi k q_{\mathrm{enclosed}}$$

  - We calculated the flux in an earlier problem: $\Phi = 0.1\,\mathrm{N\,m^2/C^2}$
  - The only charge inside the sphere (Gaussian surface) is the electric charge on the sphere, so $q_{\mathrm{enclosed}} = q_{\mathrm{sphere}}$
  - The rest is just a calculation:

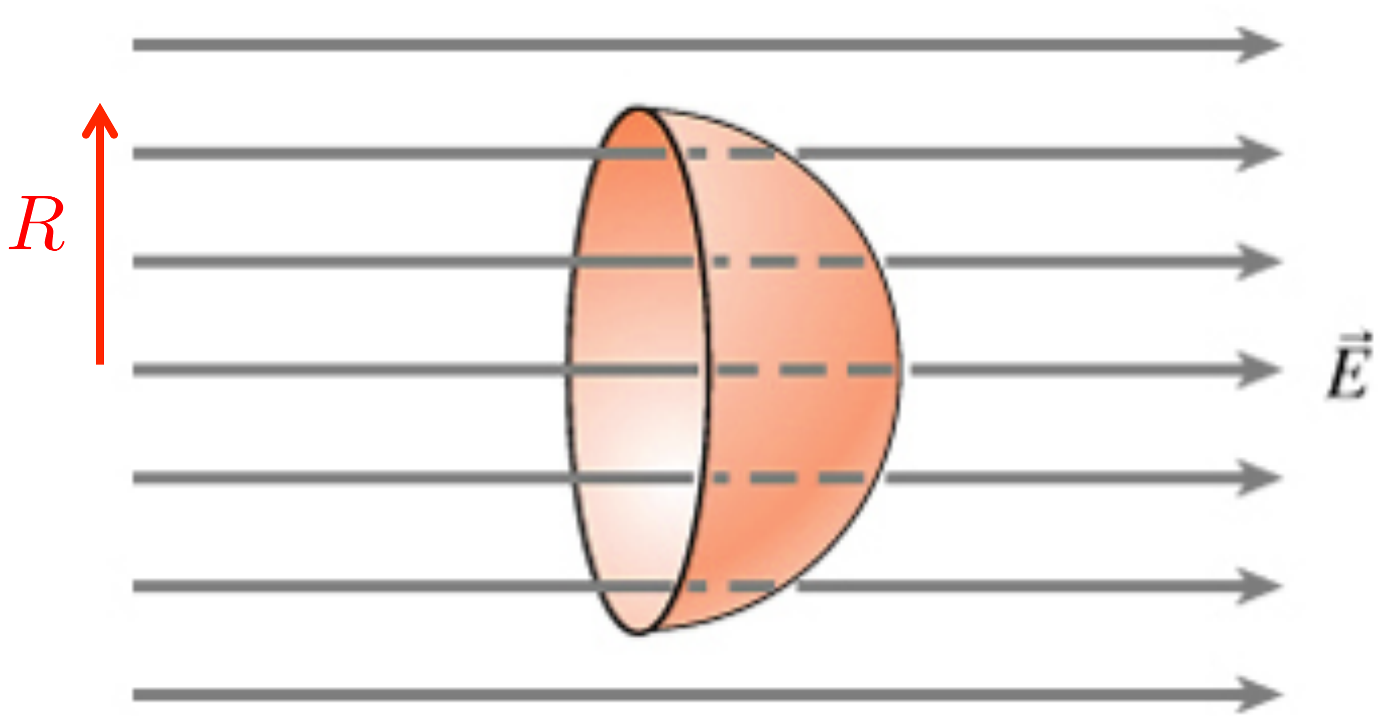
$$q_{\mathrm{enclosed}} = q_{\mathrm{sphere}} = \frac{\Phi}{4\pi k} = \frac{(0.1\,\mathrm{N\,m^2/C})}{4\pi(9\times 10^9\,\mathrm{N\,m^2/C^2})}$$

$$q_{\mathrm{sphere}} = 8.1\times 10^{-13}\,\mathrm{C}$$

Gauss's Law/Flux Ponderable (5+ minutes)

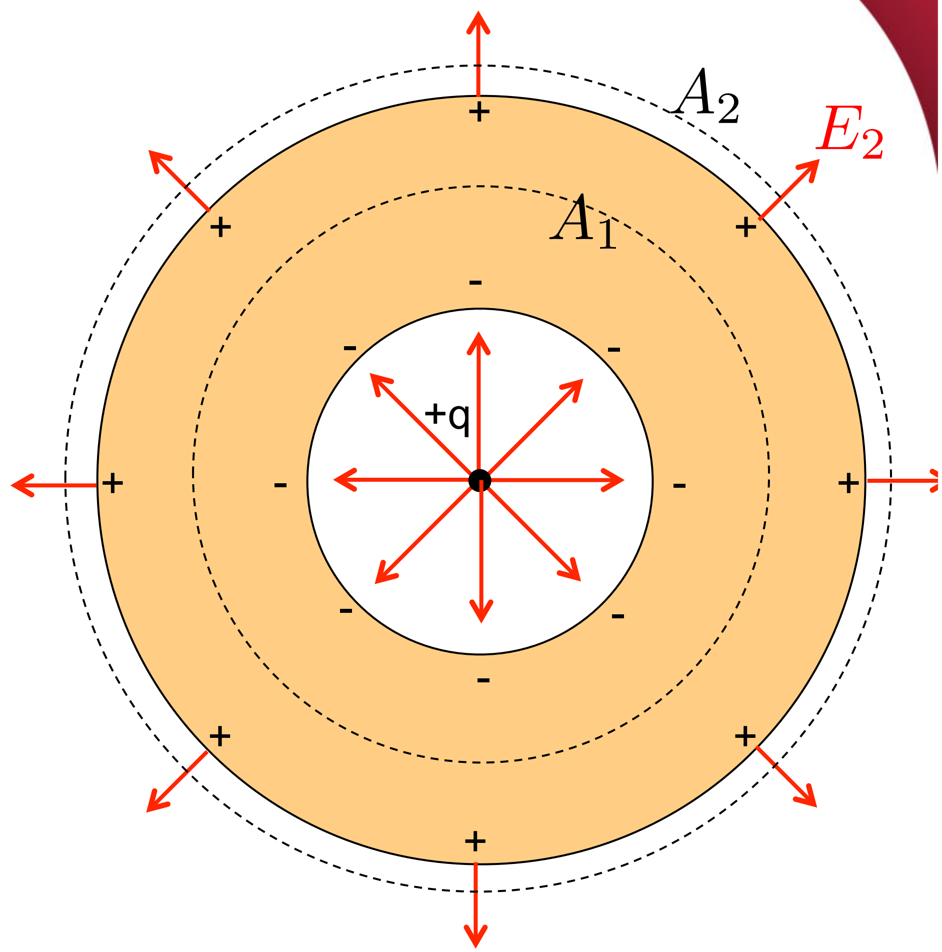
- A half-sphere of radius R is placed in a region of constant electric field $\vec{E}$ as shown below. What is the flux Φ through the surface of the half-sphere?
  - Hint: Use Gauss's Law to make the problem simpler…
  - No calculus needed

$$\Phi = 4\pi\, k\, q_{\text{enclosed}}$$



Conductor Shells $\Phi = 4\pi\, k\, q_{\text{enclosed}}$

- What about a shell of a neutral conductor with a hollow spot in the middle?
  - A charge q placed in the middle attracts an equal and opposite amount of charge –q to the inner surface
  - The electric field inside the conductor is still zero
  - The overall conductor is still neutral, so +q charge must be on the outer surface of the conductor

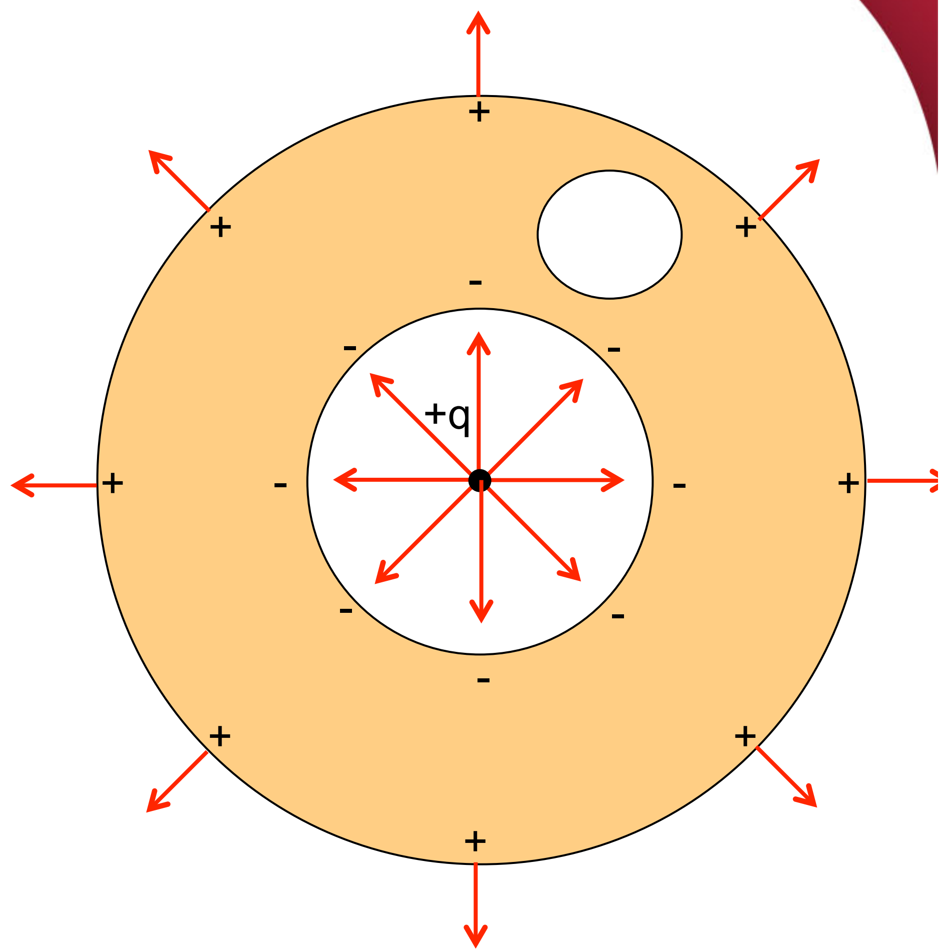


$$\Phi(A_1) = 0 \ (\text{since } E = 0) \quad \Rightarrow \quad q_{\text{enclosed}}(A_1) = 0$$

$$\Phi(A_2) = 4\pi r^2 E_2 = 4\pi k(+q) \quad \Rightarrow \quad E_2 = \frac{kq}{r^2}$$

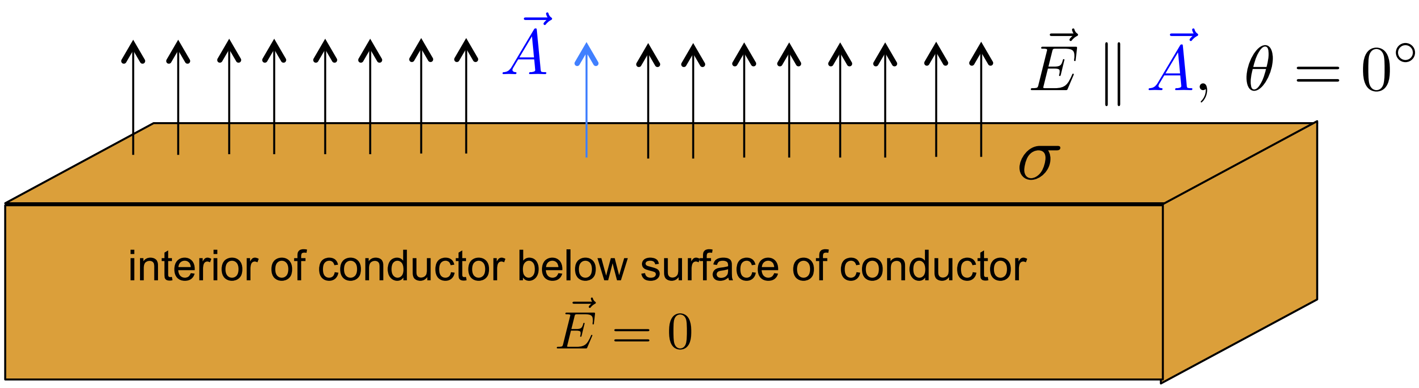
Conductor Shells $\Phi = 4\pi\, k\, q_{\text{enclosed}}$

- What's the electric field inside a hollow spot in the conductor indicated here?
- What's the distribution of charges on the inner surface of that hollow spot?

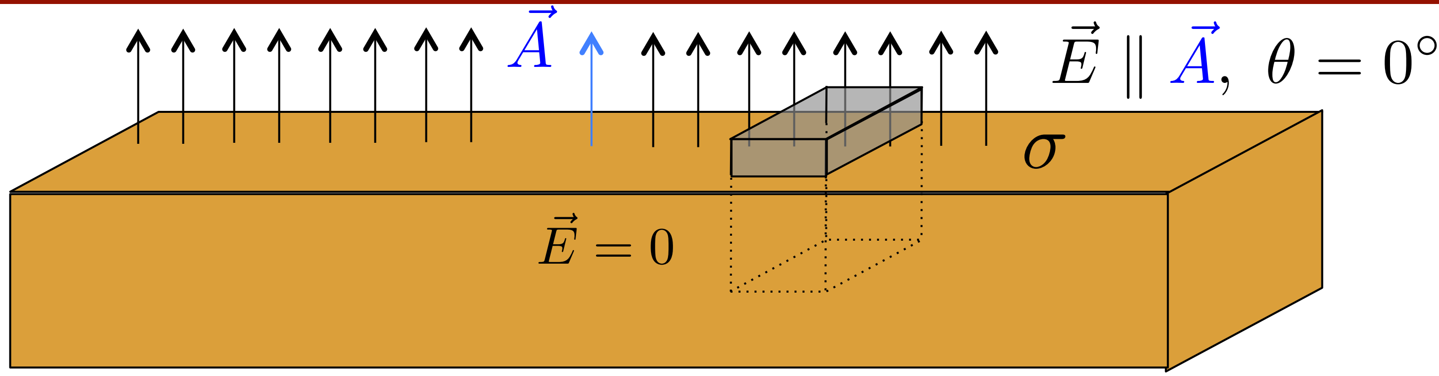


Electric Field on Flat Surface of Conductor

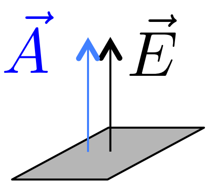
- Remember that the **electric field on the inside of a conductor is zero**
  - Any surface that we draw for Gauss's law that's **inside** a conductor has zero flux, $\Phi = 0$
  - We can use this to calculate electric fields without calculus for some interesting conductors
  - Very close to any surface, the surface basically **looks flat** and **looks like it has a constant charge distribution**
- Consider the surface of an infinitely large flat conductor that has a charge distributed evenly on it, with charge density σ



Gauss's Law on Flat Surface of Conductor



- E is constant and points out over the surface
- To use Gauss's law, we need a surface
  - Draw a little box with sides parallel to E
  - Flux through the box inside conductor is zero
  - Flux through the sides of the box outside of conductor is zero
  - So the total flux through the box is just flux through the **top**



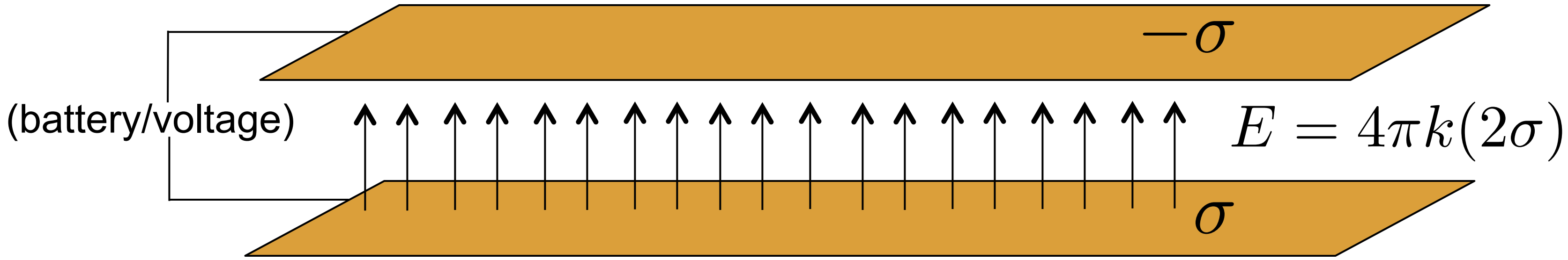
$$\Phi = \vec{E} \cdot \vec{A} = E A = 4\pi \, k \, q_{\text{enclosed}} = 4\pi \, k \, (A\sigma)$$

$$\boxed{E = 4\pi \, k \, \sigma} \quad = \sigma / \epsilon_0 \qquad \epsilon_0 = \frac{1}{4\pi k}$$

Independent of A or distance from the (infinite flat) conductor!!

Also true very close to the surface of **any** conductor

Parallel Plate Capacitors

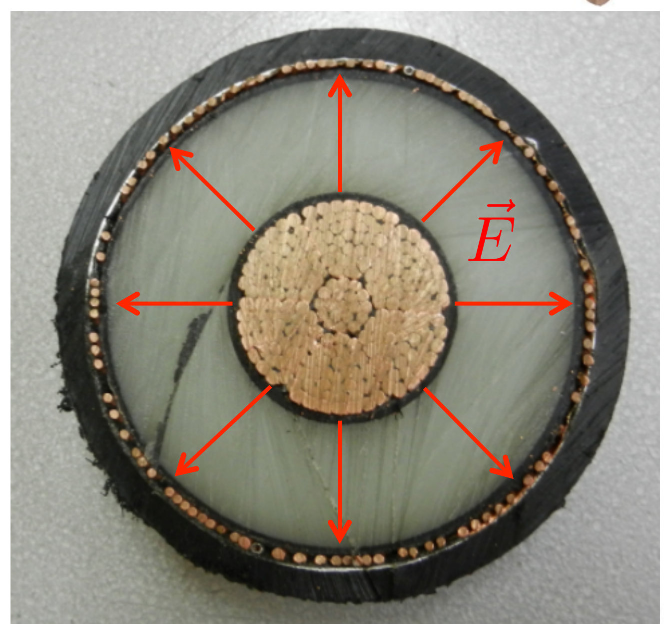


- A **capacitor** is a electrical device that stores energy in an electric field between two (or more) conductors
  - The simplest example is a **parallel plate capacitor**
  - Both conductors have equal and opposite charge density
  - No field outside conductors
  - Twice the field (sum of both plane fields) between them
  - The easiest way to create the different surface charges is with a battery
    - There is a "voltage difference" between the battery terminals
    - **What exactly is voltage?**

Coaxial Cable (homework)

- Coaxial cables are used in **many** electrical signal applications
  - Inner and outer conductors carry equal and opposite charges
- Like parallel plane capacitor
  - Electric field restricted to region between conductor surfaces
- In the region of the electric field, Gauss's Law (with a cylindrical Gaussian surface) gives

$$E = \frac{2k\lambda}{r}\hat{r}$$



Chapter 22: Electric Potential

- What exactly is **voltage**?
  - It's related to **energy** and **work** for electrical forces and fields

- Refresher
  - **Work**: a scaler (number) defined by physicists as the total force exerted over a distance
    - Like flux, it's really the component of the force in the direction of motion times the distance the object moves in that direction

$$W = \vec{F} \cdot \Delta\vec{r} = F\,\Delta r\,\cos\theta$$

  - **Energy**: a useful bookkeeping term to calculate ability to do work
    - Potential energy (springs, gravity, chemical energy, etc)
    - Kinetic energy (energy of motion of objects with mass)
    - (Friction dissipates energy in ways that make it difficult to do work)

Work Due To Electric Fields

$$W = \vec{F} \cdot \Delta\vec{r} = F\,\Delta r\,\cos\theta$$

- But an electric charges q experience a **force** $\vec{F}$ (a vector) from an electric field $\vec{E}$
  - We even have an equation for how those relate

$$\vec{F} = q\vec{E}$$

  - What is the work I have to do to move a particle against this force?

$$W = \vec{F} \cdot \Delta\vec{r} = q\vec{E} \cdot \Delta\vec{r}$$

   or adding this up over an entire path the charge moves

$$W = \int \vec{F} \cdot d\vec{r} = q \int \vec{E} \cdot d\vec{r}$$

  - This is the change in energy (potential, kinetic) of q that I put in
  - My energy change (the work I did) is **equal and opposite** to this

Electric Potential

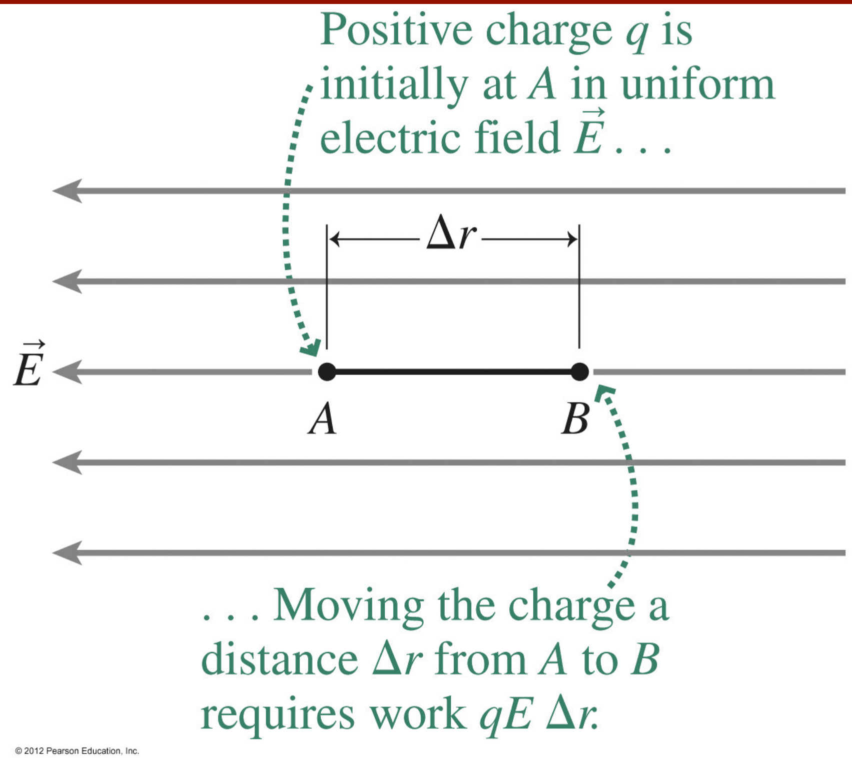
$$W_{\text{done on q}} = \int \vec{F} \cdot d\vec{r} = q \int \vec{E} \cdot d\vec{r}$$

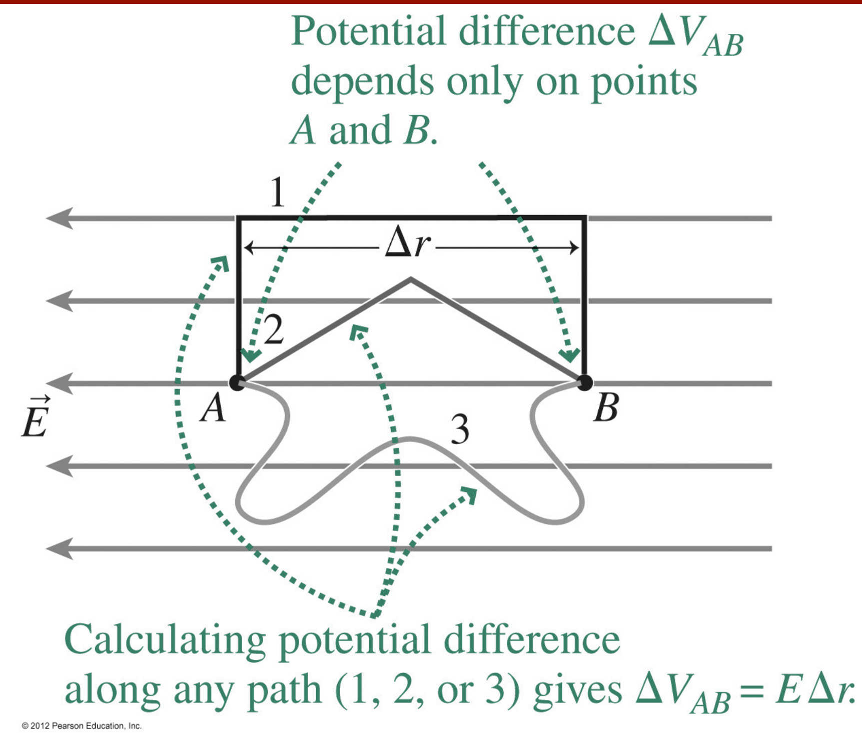
- The charge q is a test charge that I selected (so divide by it)
- The **electric potential of an electric field** between two points (A and B) is the **work done per unit charge** in moving a charge between those two points

$$\Delta V_{\text{AB}} = -\int_{\text{A}}^{\text{B}} \vec{E} \cdot d\vec{r}$$

- It turns out that for a given electric field, **this only depends on the points A and B, not the path between them**
  - There is a direct analogue to gravitational potential energy
  - (The electric field force is "conservative", i.e. frictionless)

Electric Potential of a Constant Field



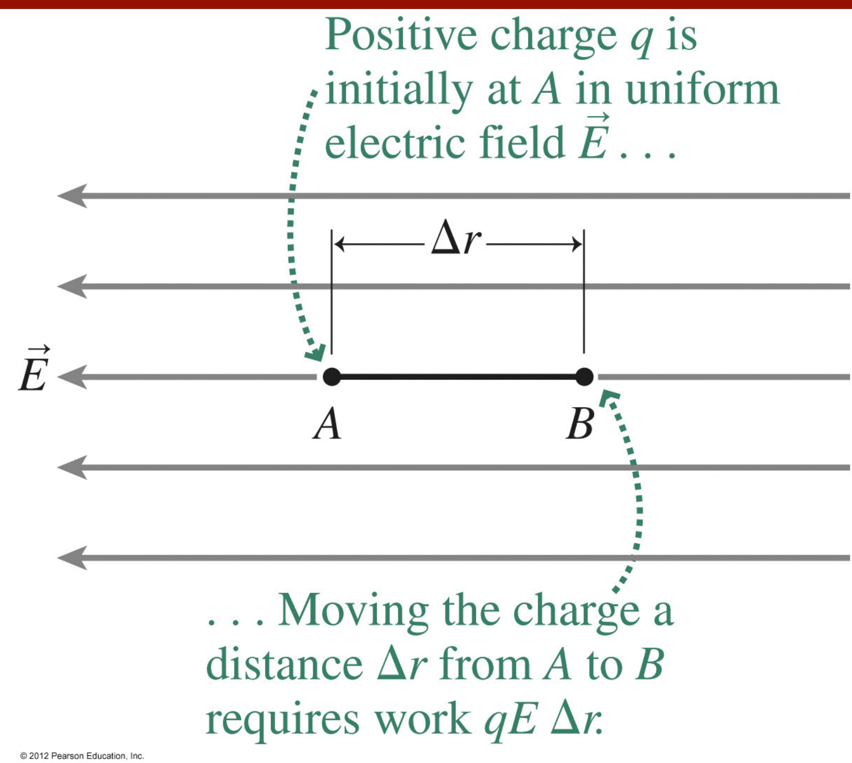


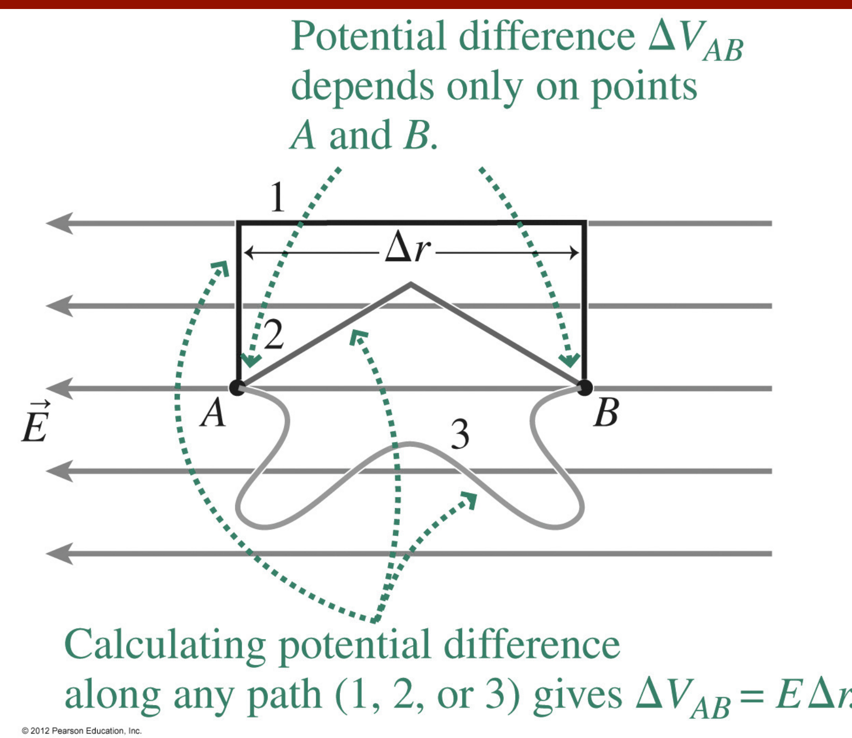
- Moving a charge along a constant electric field involves exertion of a force over distance: work (and energy)
  - Depends on the charge and which direction the particle moves

$$\Delta V_{\mathrm{AB}} = -\vec{E} \cdot \Delta\vec{r}_{\mathrm{AB}} = E\Delta r_{\mathrm{AB}} \qquad \Delta V_{\mathrm{BA}} = -\vec{E} \cdot \Delta\vec{r}_{\mathrm{BA}} = -E\Delta r_{\mathrm{BA}}$$

$$W = q\Delta V$$

Electric Potential of a Constant Field





$$\Delta V_{\rm AB} = -\vec{E}\cdot\Delta\vec{r}_{\rm AB} = E\Delta r_{\rm AB} \qquad \Delta V_{\rm BA} = -\vec{E}\cdot\Delta\vec{r}_{\rm BA} = -E\Delta r_{\rm BA}$$

- "B has a higher potential than A"
  - I usually think of this as being similar to "B is uphill from A"
  - I have to "push uphill" (do work that becomes potential energy of my charge q) to move a +q charge from A to B
  - Conversely, a +q charge will "accelerate downhill" from B to A

The Volt and the Electron Volt (eV)

- The unit of electric potential difference is the **volt** (V).
  - 1 volt is 1 joule per coulomb (1 V = 1 J/C).
  - Example: A 9-V battery supplies 9 joules of energy to every coulomb of charge that passes through an external circuit connected between its two terminals.

- The volt is *not* a unit of energy, but of energy per charge—that is, of electric potential difference.
  - A related *energy* unit is the **electron volt** (eV), defined as the energy gained by one elementary charge *e* "falling" through a potential difference of 1 volt.
  - Therefore, 1 eV = 1.6×10^{-19} J.

Table 22.2 Typical Potential Differences

Between human arm and leg due to heart's electrical activity	1 mV
Across biological cell membrane	80 mV
Between terminals of flashlight battery	1.5 V
Car battery	12 V
Electric outlet (depends on country)	100–240 V
Between long-distance electric transmission line and ground	365 kV
Between base of thunderstorm cloud and ground	100 MV

Electric Potential: Problem

- An electron starts at rest, and moves between two points with electric potential difference $\Delta V = 9\ \mathrm{V}$. What is its final velocity, and does it move from lower to higher electric potential, or higher to lower?

Electric Potential: Solution

- An electron starts at rest, and moves between two points with electric potential difference $\Delta V = 9\ \mathrm{V}$. What is its final velocity, and does it move from lower to higher electric potential, or higher to lower?
  - A + charge "goes downhill" in moving from a higher to lower electric potential
  - So the – charge electron "goes downhill" in moving from **lower** to **higher** electric potential. It accelerates through a potential difference of $\Delta V = -9\ \mathrm{V}$
  - All the energy it gains goes into kinetic energy, $\mathrm{KE} = \frac{1}{2} m_{\mathrm{e}} v^2$

$$\mathrm{KE} = \frac{1}{2} m_{\mathrm{e}} v^2 = q\Delta V = (-e)(-9\ \mathrm{V})$$

$$v = \sqrt{\frac{(2)(-1.6 \times 10^{-19}\ \mathrm{C})(-9\ \mathrm{V})}{(9.1 \times 10^{31}\ \mathrm{kg})}} = \boxed{1.8 \times 10^6\ \mathrm{m/s} = v}$$

0.6% the speed of light!

Quick Question 1

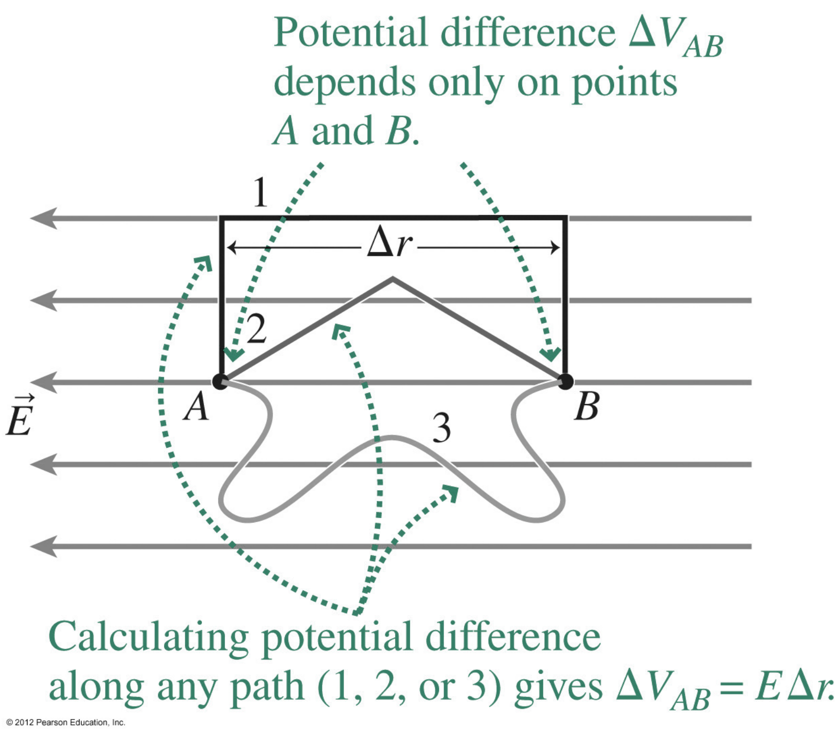
- What would happen to the potential difference between points *A* and *B* in the figure if the distance Δr were doubled?

A. ΔV would be doubled.

B. ΔV would be halved.

C. ΔV would be quadrupled

D. ΔV would be quartered.



Quick Question 2

- An alpha particle (charge 2*e*) moves through a 10-V potential difference. How much work, expressed in eV, is done on the alpha particle?

A. 5 eV
B. 10 eV
C. 20 eV
D. 40 eV