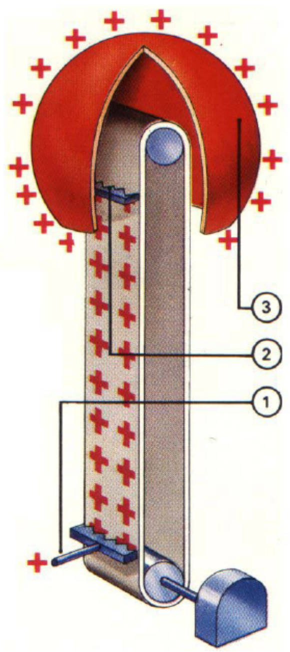

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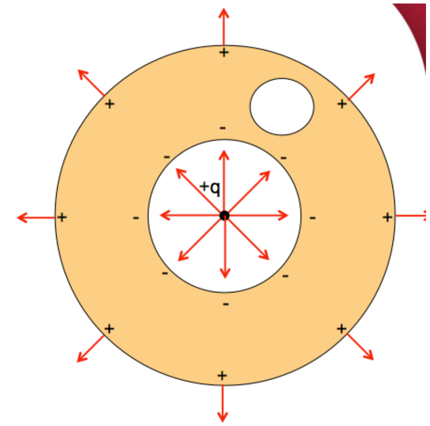

More Electric Potential
Exam Wed Feb 12
Lab Fri Feb 14

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Wednesday, February 5 2014 (Wednesday, OMG!)

Happy Birthday to Darren Criss, Laura Linney, Jennifer Jason Leigh, Christopher Guest, and Robert Hofstadter (Physics Nobel 1961)!

Review: Electric Potential

- What exactly is **voltage**?
  - It's related to **energy** and **work** for electrical forces and fields

- Refresher
  - **Work**: a scaler (number) defined by physicists as the total force exerted over a distance
    - Like flux, it's really the component of the force in the direction of motion times the distance the object moves in that direction

$$W = \vec{F} \cdot \Delta\vec{r} = F\,\Delta r\ \cos\theta$$

  - **Energy**: a useful bookkeeping term to calculate ability to do work
    - Potential energy (springs, gravity, chemical energy, etc)
    - Kinetic energy (energy of motion of objects with mass)
    - (Friction dissipates energy in ways that make it difficult to do work)

Review: Work Due To Electric Fields

$$W = \vec{F} \cdot \Delta\vec{r} = F\,\Delta r\ \cos\theta$$

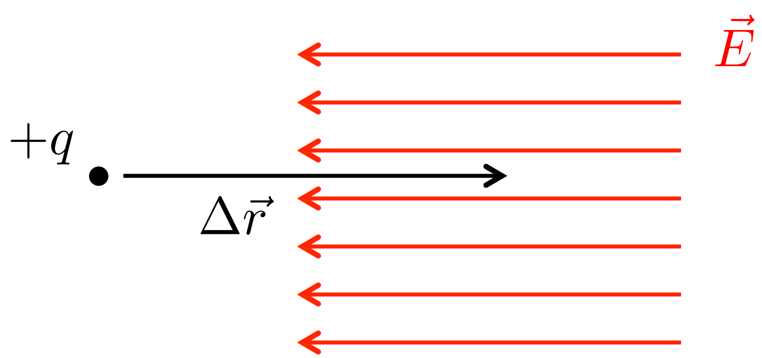
- But an electric charges q experience a **force** $\vec{F}$ (a vector) from an electric field $\vec{E}$
  - We even have an equation for how those relate

$$\vec{F} = q\vec{E}$$

  - What is the work done **by** this force on a charged particle?

$$W = \vec{F} \cdot \Delta\vec{r} = q\vec{E} \cdot \Delta\vec{r} = qE\Delta r\ \cos\theta$$

  - The change in potential energy ΔU is opposite of the work done

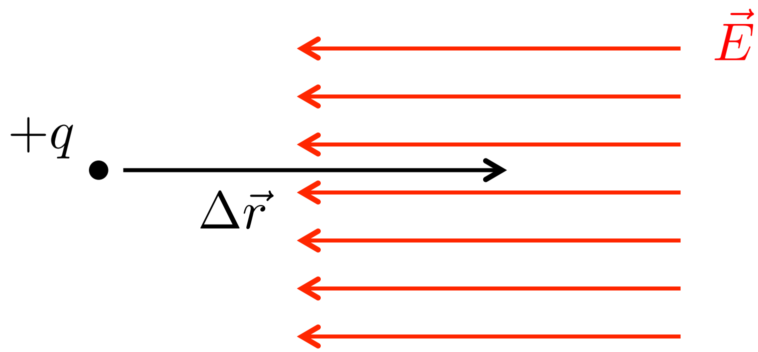


$$\theta = 180^\circ$$

$$W = -qE\Delta r$$

$$\Delta U = -W = qE\Delta r$$

Review: Work Due To Electric Fields and Potential

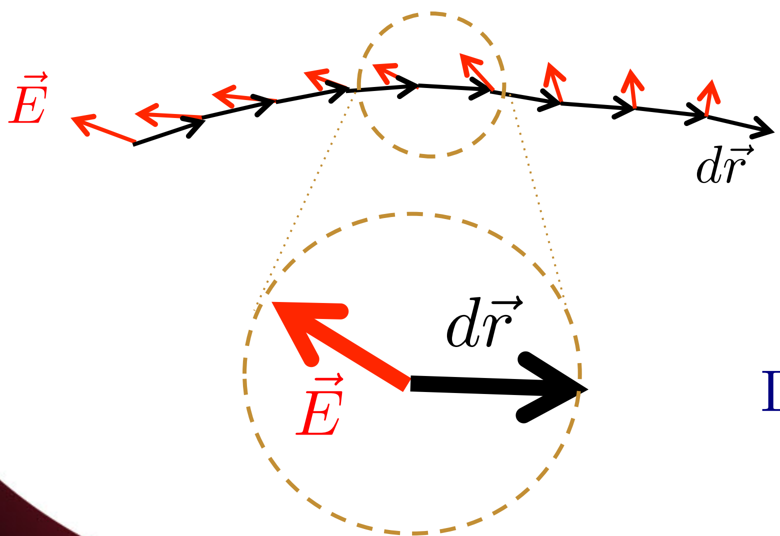


$$\text{here } \theta = 180^\circ$$

$$\text{Work} = q\vec{E}\cdot\Delta\vec{r}$$

$$\Delta U = -\text{Work} = -q\vec{E}\cdot\Delta\vec{r}$$

- We can now add these energy gains up over many pieces of a longer curve
  - Each piece has approximately constant field $\vec{E}$ and $\Delta\vec{r}$
  - Adding them all together is an integral from point A to point B



$$\Delta U_{\mathrm{AB}} = -q\int_{\mathrm{A}}^{\mathrm{B}} \vec{E}\cdot d\vec{r}$$

$$\text{Define } V = \frac{\Delta U_{\mathrm{AB}}}{q} = -\int_{\mathrm{A}}^{\mathrm{B}} \vec{E}\cdot d\vec{r}$$

Review: Electric Potential

- The **electric potential of an electric field** between two points (A and B) is the **change in potential energy** ΔU_{AB} **per unit charge** in moving a charge between those two points

$$\Delta U_{\mathrm{AB}} = q \Delta V_{\mathrm{AB}}$$

$$\Delta V_{\mathrm{AB}} = - \int_{\mathrm{A}}^{\mathrm{B}} \vec{E} \cdot d\vec{r}$$

  - Think of electric potential as “relative potential energy per unit charge”
    - **Only defined between two points**! (One may be implied!) ⚠
  - This is just like electric field is “force per unit charge” $\vec{F} = q\vec{E}$
  - (This is just like gravitational acceleration is “force per unit mass”)

Review: Electric Potential

- The **electric potential of an electric field** between two points (A and B) is the **change in potential energy** ΔU_{AB} **per unit charge** in moving a charge between those two points

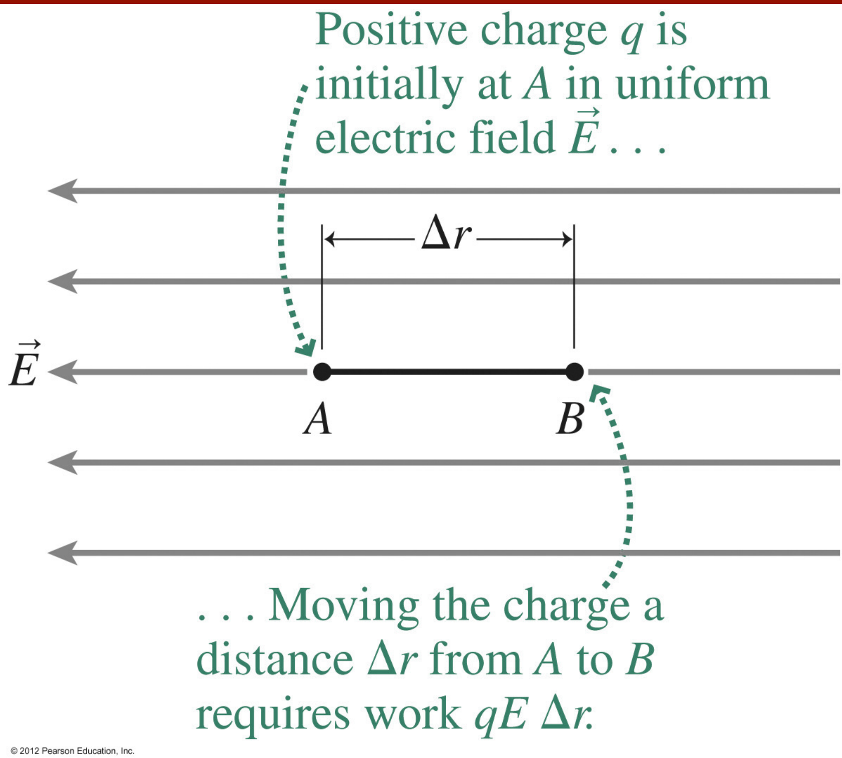
$$\Delta U_{\mathrm{AB}} = q \Delta V_{\mathrm{AB}}$$

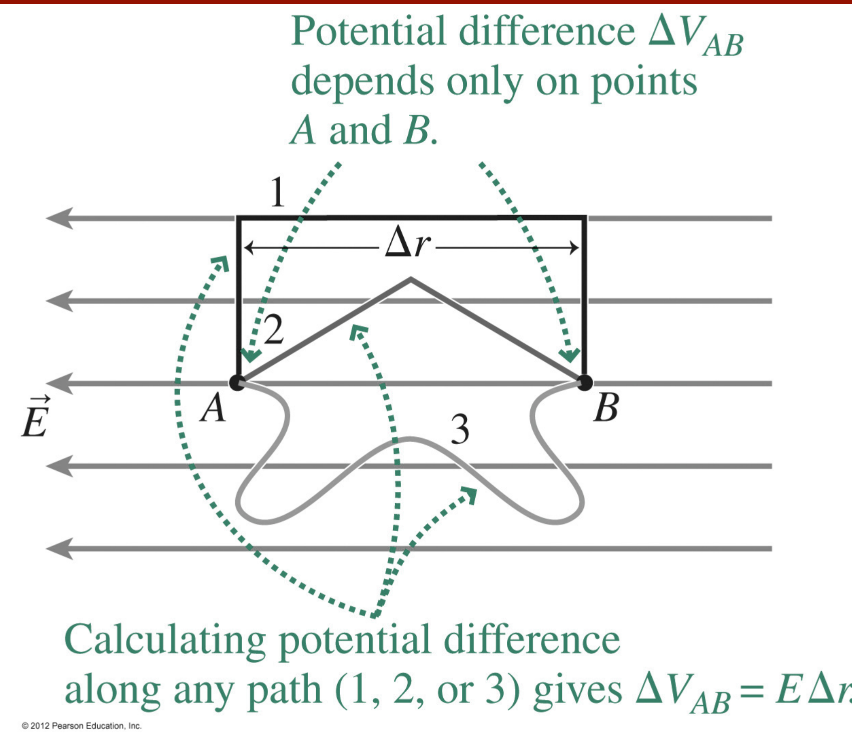
$$\Delta V_{\mathrm{AB}} = - \int_{\mathrm{A}}^{\mathrm{B}} \vec{E} \cdot d\vec{r}$$

- It turns out that for a given electric field **this only depends on the points A and B, not the path between them**
  - There is a direct analogue to gravitational potential energy
  - (The electric field force is "conservative", i.e. frictionless)

- Moving only perpendicular to an $\vec{E}$ field means that the electric potential does not change
  - The potential is the same on that path: "Equipotential"

Review: Electric Potential of a Constant Field



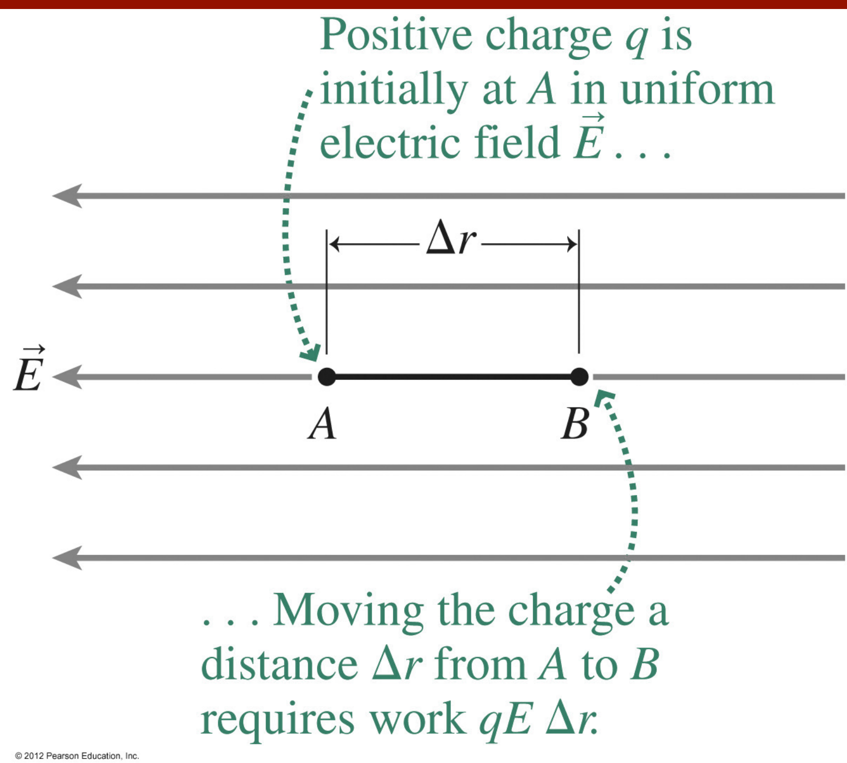


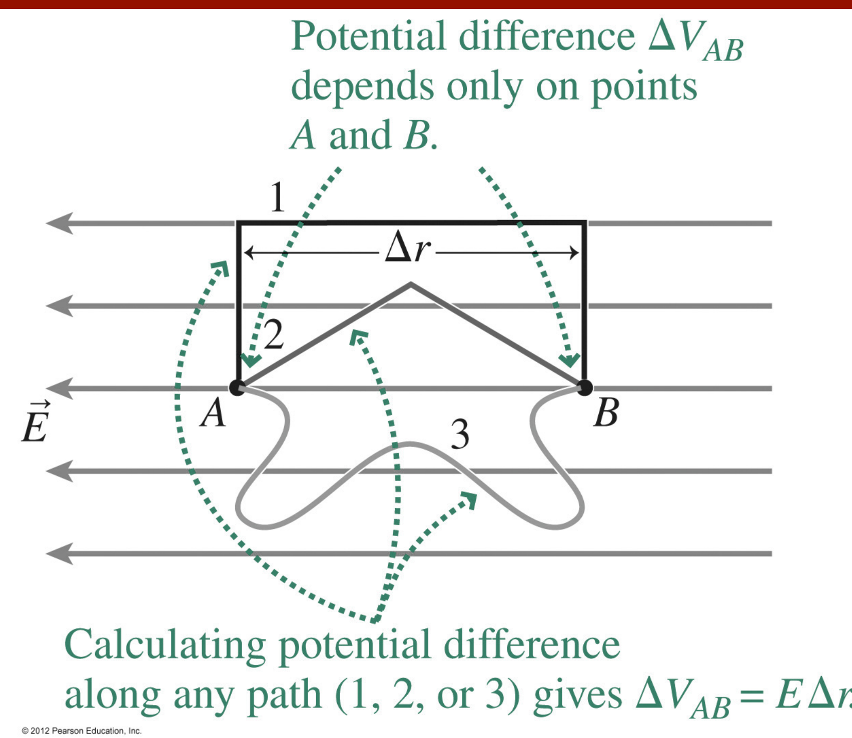
- Moving a charge along a constant electric field involves exertion of a force over distance: work (and energy)
  - Depends on the charge and which direction the particle moves

$$\Delta V_{\mathrm{AB}} = -\vec{E}\cdot\Delta\vec{r}_{\mathrm{AB}} = E\Delta r_{\mathrm{AB}} \qquad \Delta V_{\mathrm{BA}} = -\vec{E}\cdot\Delta\vec{r}_{\mathrm{BA}} = -E\Delta r_{\mathrm{BA}}$$

$$W = q\Delta V$$

Review: Electric Potential of a Constant Field





$$\Delta V_{\mathrm{AB}} = -\vec{E}\cdot\Delta\vec{r}_{\mathrm{AB}} = E\Delta r_{\mathrm{AB}} \qquad \Delta V_{\mathrm{BA}} = -\vec{E}\cdot\Delta\vec{r}_{\mathrm{BA}} = -E\Delta r_{\mathrm{BA}}$$

- “B has a higher potential than A”
  - I usually think of this as being similar to “B is uphill from A”
  - I have to “push uphill” (do work that becomes potential energy of my charge q) to move a +q charge from A to B
  - Conversely, a +q charge will “accelerate downhill” from B to A

Review: The Volt and the Electron Volt (eV)

- The unit of electric potential difference is the **volt** (V).
  - 1 volt is 1 joule per coulomb (1 V = 1 J/C).
  - Example: A 9-V battery supplies 9 joules of energy to every coulomb of charge that passes through an external circuit connected between its two terminals.

- The volt is *not* a unit of energy, but of energy per charge—that is, of electric potential difference.
  - A related *energy* unit is the **electron volt** (eV), defined as the energy gained by one elementary charge *e* "falling" through a potential difference of 1 volt.
  - Therefore, 1 eV = 1.6×10^{-19} J.

Table 22.2 Typical Potential Differences

Between human arm and leg due to heart's electrical activity	1 mV
Across biological cell membrane	80 mV
Between terminals of flashlight battery	1.5 V
Car battery	12 V
Electric outlet (depends on country)	100–240 V
Between long-distance electric transmission line and ground	365 kV
Between base of thunderstorm cloud and ground	100 MV

Electric Potential: Problem

- A 9V battery has a potential difference of $\Delta V = 9V$ between its terminals and stores about 20 kJ of energy.
  - Assuming the voltage stays constant, how much charge moves between the terminals to discharge the battery?

$$\Delta V = 9V$$

$$\Delta U_{\mathrm{AB}} = 20\,\mathrm{kJ}$$

Electric Potential: Problem

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  - Assuming the voltage stays constant, how much charge moves between the terminals to discharge the battery?

$$\Delta V = 9V$$

$$\Delta U_{\mathrm{AB}} = 20\,\mathrm{kJ}$$

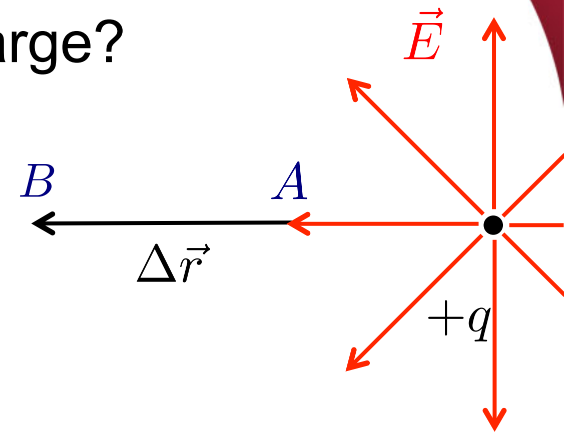
$$\Delta U_{\mathrm{AB}} = 20\,\mathrm{kJ} = q\Delta V = q(9\ \mathrm{V})$$

$$q = \frac{(20 \times 10^3\ \mathrm{J})}{(9\ \mathrm{V})} = \boxed{2 \times 10^3\ \mathrm{C} = 1.4 \times 10^{22}\ \mathrm{electrons} = q}$$

(About 1/50 of a mole of electrons!)

Electric Potential of a Point Charge

- What is the potential difference for a point charge?
  - Assume our path is radially outward
  - $\theta = 0^\circ$ so



$$\cos\theta = +1 \quad \text{and} \quad \vec{E}\cdot d\vec{r} = E\,dr$$

$$\Delta V_{\mathrm{AB}} = -\int_{\mathrm{A}}^{\mathrm{B}} \vec{E}\cdot d\vec{r} = -\int_{\mathrm{A}}^{\mathrm{B}} E\,dr$$

- For a point charge, we know the electric field: $E = \dfrac{kq}{r^2}$ Depends on r!

$$\Delta V_{\mathrm{AB}} = -kq\int_{\mathrm{A}}^{\mathrm{B}} r^{-2}\,dr = (-kq)\left(-r^{-1}\right)\Big|_{\mathrm{A}}^{\mathrm{B}}$$

An integral you probably can do!

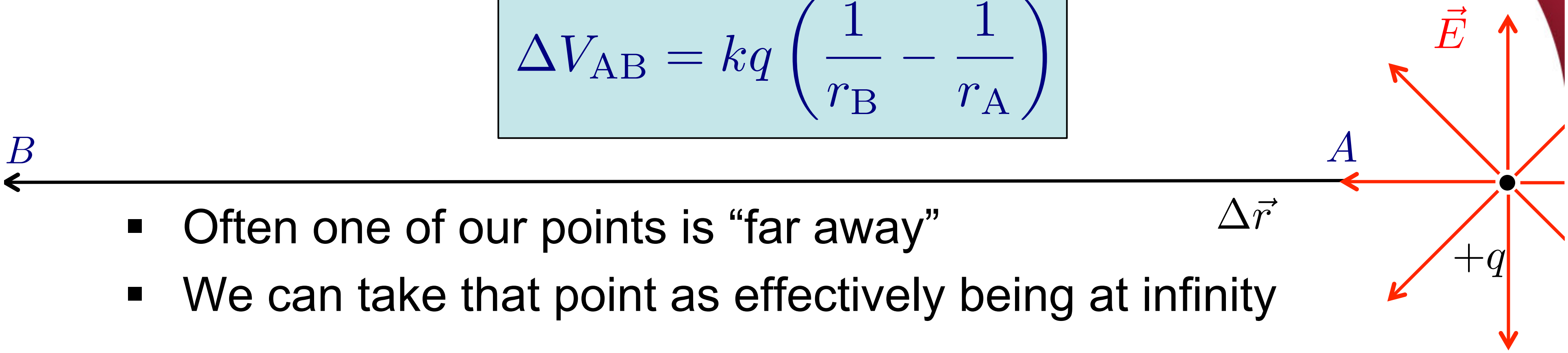
$$\Delta V_{\mathrm{AB}} = kq\left(\frac{1}{r_{\mathrm{B}}} - \frac{1}{r_{\mathrm{A}}}\right)$$

Does this make sense?

$$\Delta U_{\mathrm{AB}} = q\Delta V_{\mathrm{AB}}$$

Electric Potential of a Point Charge

$$\Delta V_{\mathrm{AB}} = kq\left(\frac{1}{r_{\mathrm{B}}} - \frac{1}{r_{\mathrm{A}}}\right)$$



- Often one of our points is “far away”
- We can take that point as effectively being at infinity

- So for potential differences with one endpoint far away
  - We can say that the point charge potential at a point *r* from a point charge *q* is

$$V_r = \frac{kq}{r}$$

Potential of a point charge

Compare to $E = \frac{kq}{r^2}$

  - We can then add up these potentials from many charges in the usual way, similar to adding up electric field from many charges
    - But this is a lot easier since **voltage is a scaler**

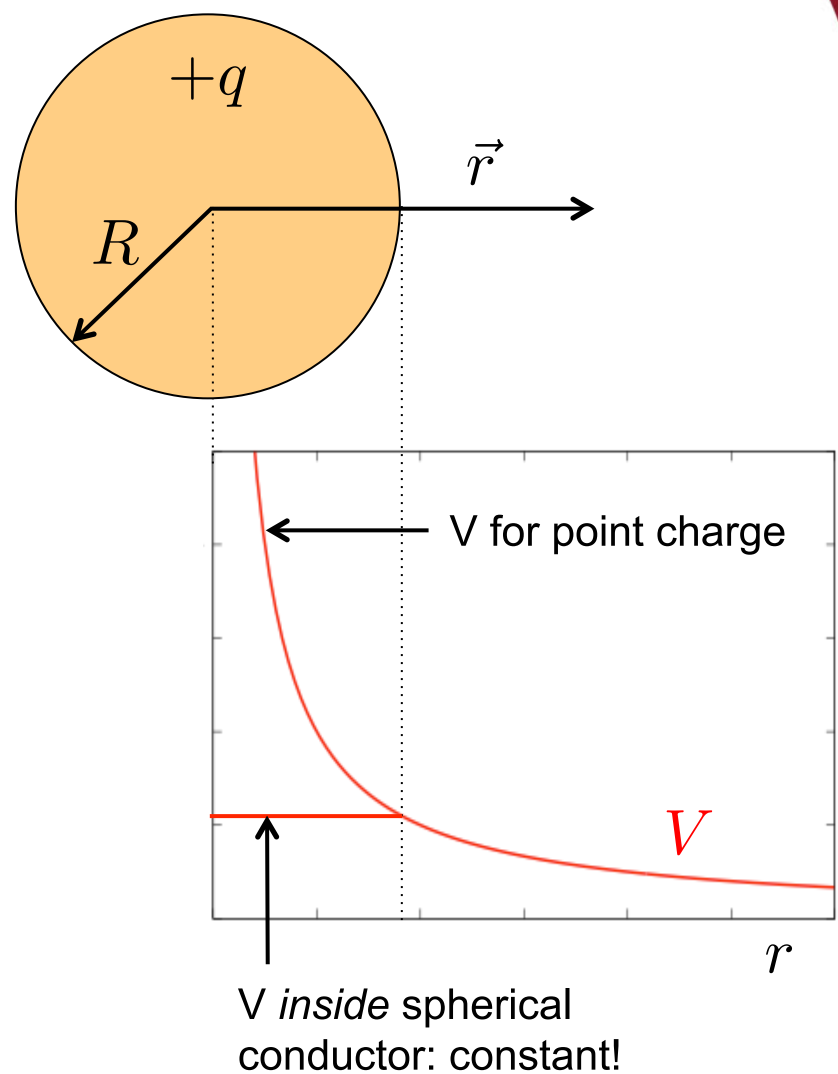
Electric Potential of a Spherical Conductor

- Remember that for a spherical conductor:
  - $E = 0$ everywhere inside
  - Electric field outside looks just like all the sphere's charge is at a point in the center (a point charge)

$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

  - So what does the **potential** V for a spherical conductor of radius *R* look like when plotted?

$$V = \frac{kq}{r} \quad \text{outside of conductor}$$



Example 22.3 from text

- Last class I linked you to a video about a Van de Graaff at the Boston Museum of Science
- This is a large spherical conductor that is loaded with excess charge until the air around it breaks down: lightning bolts!
  - If its radius is R=2.3m and the sphere is loaded with Q=640 μC of excess charge, what is its potential..

    Relative to a point at infinity?

    Relative to a point 2m away from the sphere's surface?

$$\Delta V_{\mathrm{AB}} = kq\left(\frac{1}{r_{\mathrm{B}}} - \frac{1}{r_{\mathrm{A}}}\right)$$

Example 22.3 from text

- Last class I linked you to a video about a Van de Graaff at the Boston Museum of Science
- This is a large spherical conductor that is loaded with excess charge until the air around it breaks down: lightning bolts!
  - If its radius is R=2.3m and the sphere is loaded with Q=640 μC of excess charge, what is its electric potential..

    Relative to a point at infinity?

    Relative to a point 2m away from the sphere's surface? (r_A=2.3m+2m=4.3m)

$$\Delta V_{\mathrm{AB}} = kq\left(\frac{1}{r_{\mathrm{B}}} - \frac{1}{r_{\mathrm{A}}}\right)$$

$$\Delta V(\infty) = \frac{kQ}{R} = \frac{(9\times 10^9\ \mathrm{N\,m^2/C^2})(6.4\times 10^{-4}\ \mathrm{C})}{2.3\ \mathrm{m}} = \boxed{2.5\ \mathrm{MV} = \Delta V(\infty)}$$

$$\Delta V(2\ \mathrm{m}) = kQ\left(\frac{1}{2.3\ \mathrm{m}} - \frac{1}{4.3\ \mathrm{m}}\right) = \boxed{1.1\ \mathrm{MV} = \Delta V(2\ \mathrm{m})}$$

(Calculating Field from Potential)

- Electric potential is the **integral** of electric field over a path

$$\Delta V_{\mathrm{AB}} = -\int_{\mathrm{A}}^{\mathrm{B}} \vec{E} \cdot d\vec{r}$$

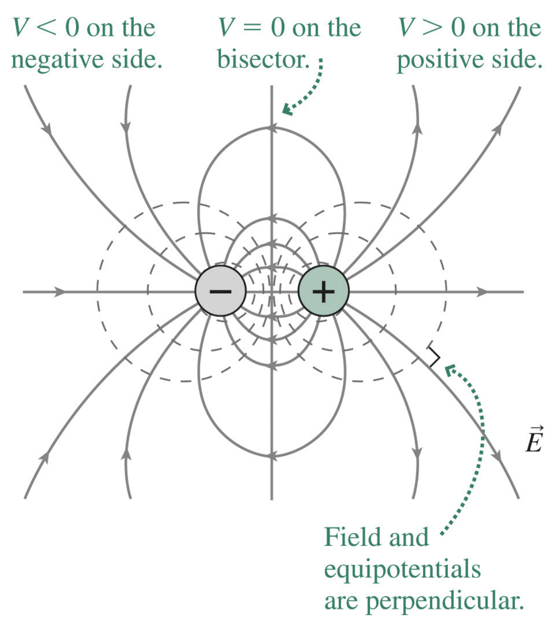
  - You have learned that integration and differentiation (derivatives) are inverses of each other
  - So we can probably calculate electric field if we are given the electric potential everywhere, $V(r)$
    - We can't just take a simple derivative though
    - We need something that gives a **vector** ($\vec{E}$) from a **scalar** (V)
    - The multidimensional derivative that does this is called the **gradient**

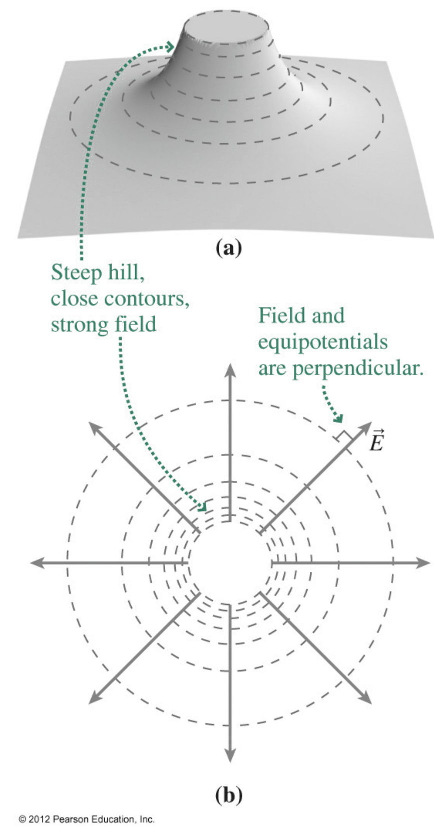
$$\vec{E} = \vec{\nabla} V \qquad \vec{\nabla} \equiv \left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz} \right)$$

$\vec{\nabla} \times \vec{A}$ is called the **curl** and returns a vector instead of a scalar

Equipotentials

- An **equipotential** is a surface on which the potential is constant.
  - In two-dimensional drawings, we represent equipotentials by curves similar to the contours of height on a map.
    - The electric field is always perpendicular to the equipotentials.
    - Equipotentials for a dipole:



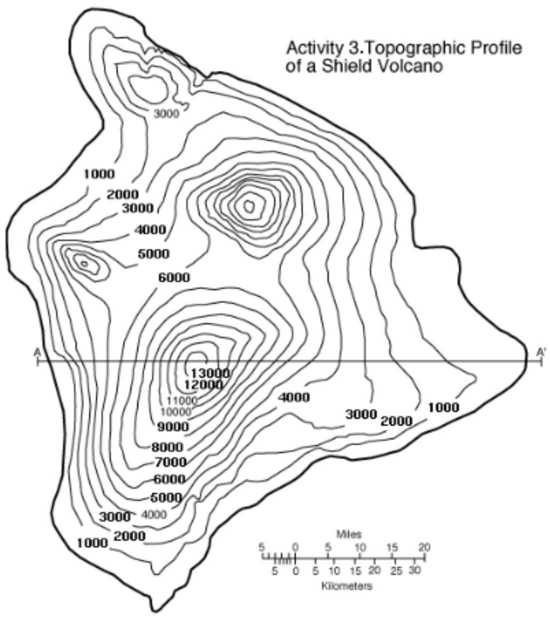


Other Examples

Isobars: lines of constant barometric pressure on weather maps

Wind is usually perpendicular to these isobars, and is strongest where the isobars are closest together.

(Similar to electric field!)



Isoclines: lines of constant elevation on geographical topographical maps.

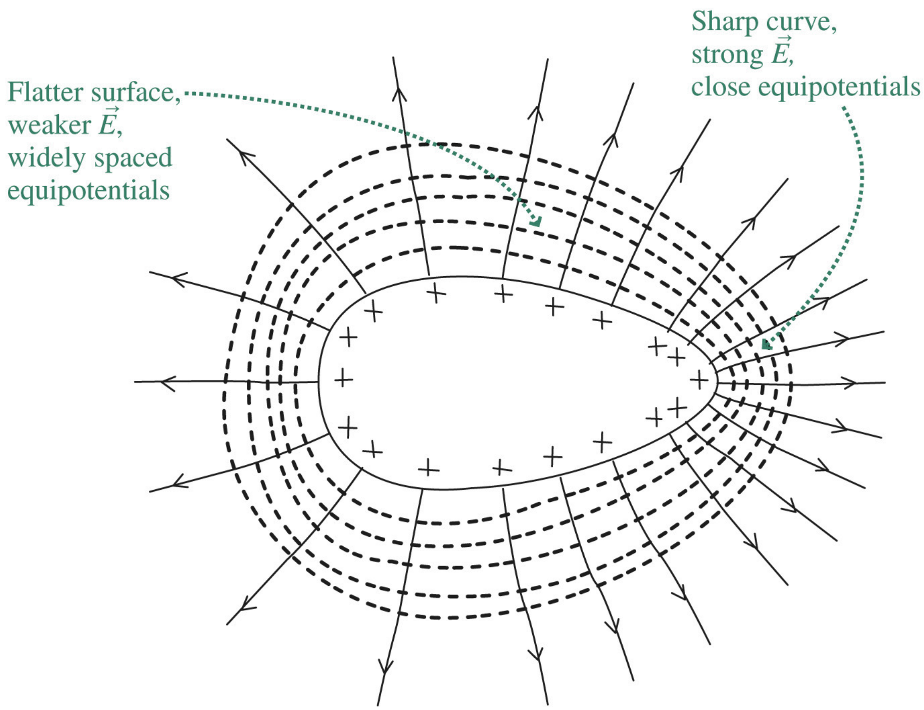
Hills are steepest where the isoclines are closest together (where elevation changes the fastest).

Moving on an isocline keeps you at the same elevation.

Charged Conductors

- $\vec{E} = 0$ inside a conductor in electrostatic equilibrium.
- On the surface, the electric field is perpendicular to the surface.
- So it takes no work to move charge inside or on the surface of a perfect conductor in electrostatic equilibrium. (makes sense: no resistance)
  - So **a conductor in electrostatic equilibrium is an equipotential.**
  - That means equipotential surfaces *near* a charged conductor roughly follow the shape of the conductor surface.

- That generally makes the equipotentials closer, and therefore the **electric field stronger** and the **charge density higher**, where the conductor curves more sharply.



Summary: Chapter 22

- **Electric potential difference** describes the work per unit charge involved in moving charge between two points in an electric field:

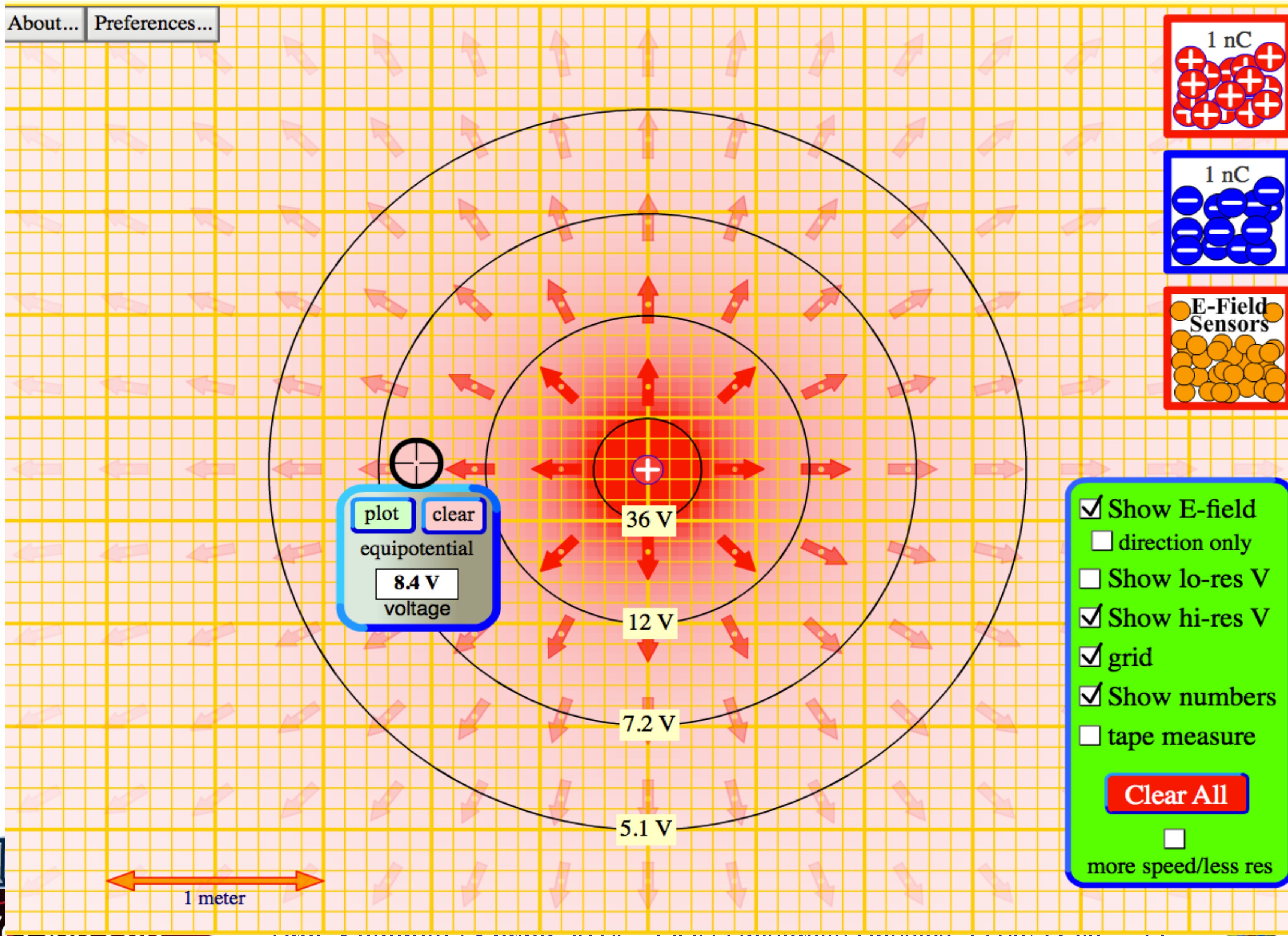
$$\Delta U_{\mathrm{AB}} = q\Delta V_{\mathrm{AB}} \qquad \Delta V_{\mathrm{AB}} = -\int_{\mathrm{A}}^{\mathrm{B}} \vec{E}\cdot d\vec{r}$$

  - The SI unit of electric potential is the volt (V), equal to 1 J/C.
  - Electric potential *always* involves two points; to say "the potential at a point" is to assume a second reference point at which the potential is defined to be zero.
  - Electric potential differences of a point charge is $V_r = \frac{kq}{r}$ where the zero of potential is taken at infinity.

- **Equipotentials** are surfaces of constant potential.
  - The electric field and the equipotential surfaces are always perpendicular.
  - Equipotentials near a charged conductor approximate the shape of the conductor.
    - A conductor in equilibrium is itself an equipotential.

Demo: Electric Potentials and Equipotentials

http://phet.colorado.edu/en/simulation/charges-and-fields



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