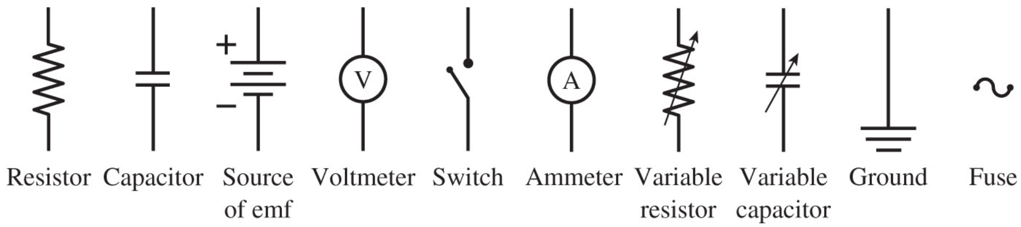
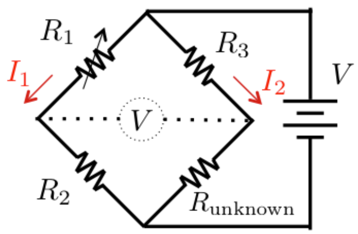


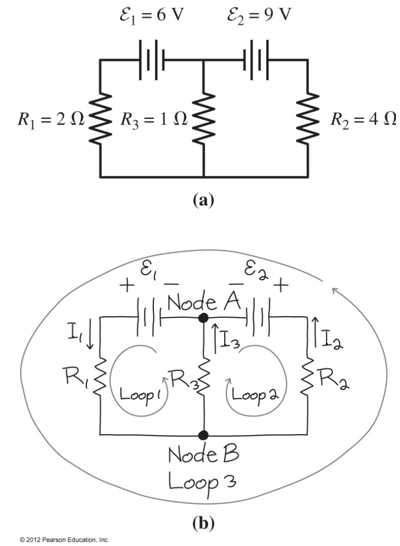


University Physics 227N/232N

Resistors, Circuits, and Kirchoff Wheatstone, AC Circuits

Lab this Friday, Feb 28

So NO QUIZ this Friday!



Dr. Todd Satogata (ODU/Jefferson Lab)

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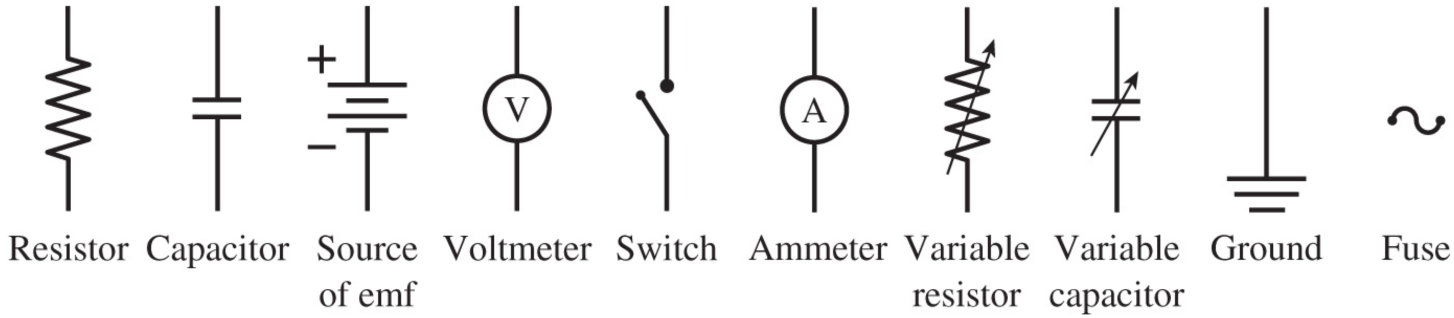
http://www.toddsatogata.net/2014-ODU

Wednesday, February 26 2014

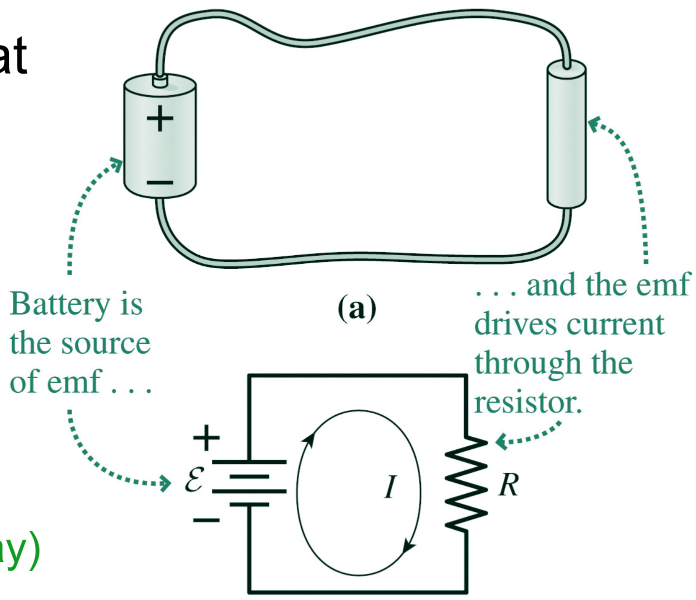
Happy Birthday to Erykah Badu, Susan Helms, Johnny Cash, Fats Domino, and Ahmed Zewail (Nobel Prize, 1999)

Review: Circuits, Symbols, and EMF

- Electric circuits are portrayed with diagrams using standard symbols, showing interconnections among their components.
  - We've seen wires, capacitors, and resistors so far



- EMF, or **electromotive force**, is a device that creates a potential difference between its sides.
  - For example, a battery or power supply
  - It supplies **electrical energy**
  - It has a + side and a – side
    - Current flows from + to – on an EMF or battery
    - (Remember that electrons flow the opposite way)



Review: Ground and Grounding

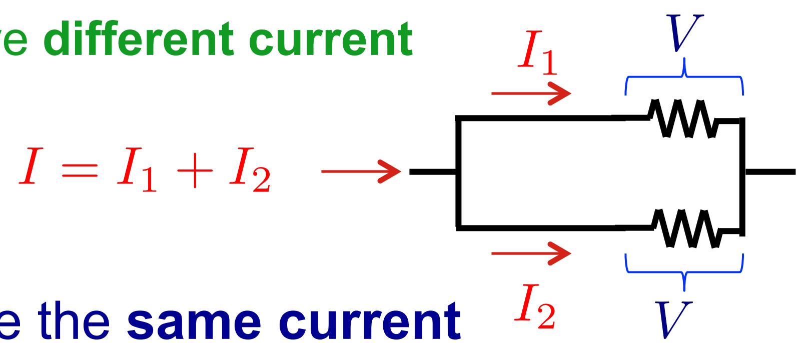
- One of the electrical symbols above was for **ground**
  - All wires connected to ground have an electric potential of zero
  - Current flows from positive potential to ground
  - Current flows from ground to negative potential
- This is the electrical equivalent of us setting electric potential to zero at infinity (or very far away)
  - "Ground wants to make stuff electrically neutral"
- Ground can be a source or sink of an effectively infinite amount of electrical charges or current
  - Think of it as a vast conductor with many charges
    - Perhaps as vast as the earth
      - Hmm, maybe that's why it's called "ground"
        - » ya think?

I wonder how small this font gets…

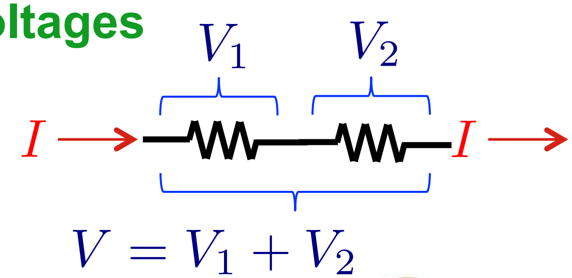
Review: Back to Resistors

- Just like capacitors, we'll have to deal with resistors in series and parallel

- Our important principles still hold true, but now for resistors!
  - Elements that are in **parallel** have the **same voltage**
    - They share the same conductors on both sides
    - But they can (and often do) have **different current**



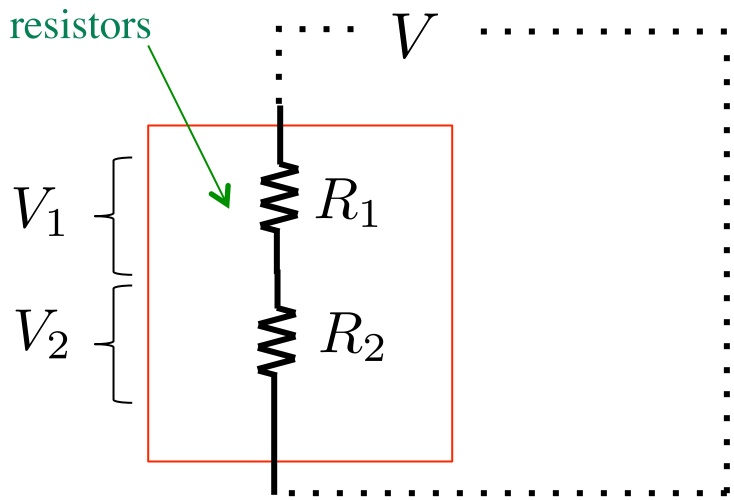
  - Elements that are in **series** have the **same current**
    - Or, for capacitors, the **same charge**
    - But they can (and often do) have **different voltages**



Review: Connecting Resistors in Series

Here the resistors have the **same current *I*** but **different voltages *V_1 and V_2***

Same current *I* in both resistors



$V = IR$

$$V_1 = IR_1$$

$$V_2 = IR_2$$

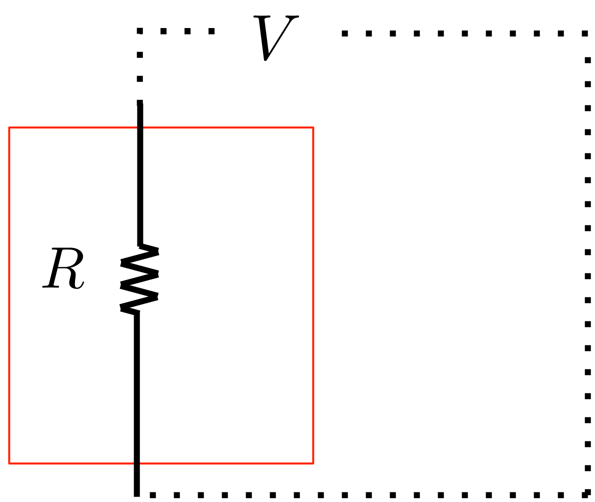
$$V = V_1 + V_2$$

$$IR = IR + 1 + IR_2$$

$$V = IR$$

$$R = R_1 + R_2$$

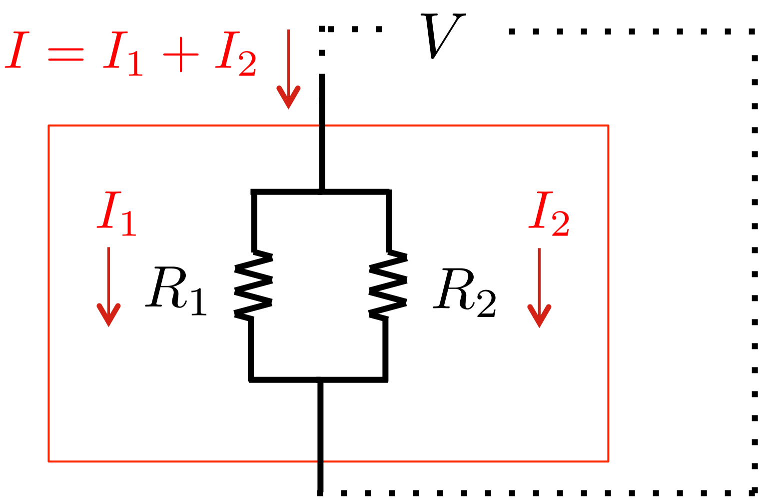
Equivalent resistance



Review: Connecting Resistors in Parallel

Here the resistors have the **same potential difference *V*** but **different currents *I***

$$V = IR$$



$$I_1 = \frac{V}{R_1}$$

$$I_2 = \frac{V}{R_2}$$

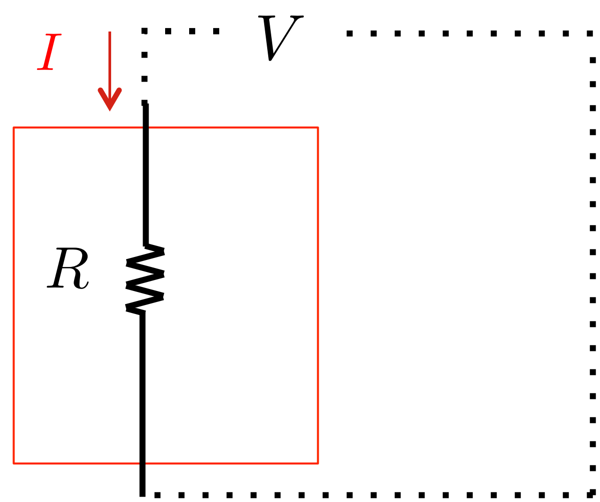
$$I = I_1 + I_2$$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$I = \frac{V}{R}$$

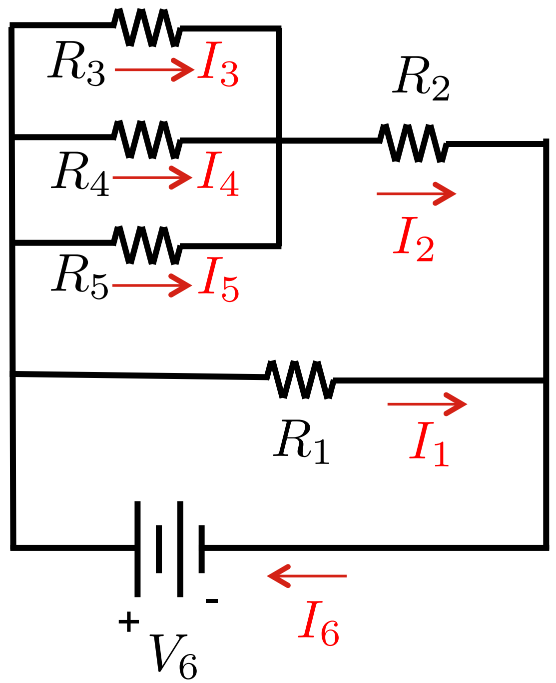
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Equivalent resistance



DC Circuits with Series and Parallel Components

- Strategy for a new electric circuit problem
  - Label all elements (*R, V, C*)
    - Number everything separately
  - Draw and label current arrows
    - Don't fret over their directions (V - to +)
    - The math to calculate them works regardless of which way you draw them
    - Negative answer means that actual current flows in the direction opposite your arrow
  - Look for places where you know two out of three of
    - DC Resistor: *I, V, R* (for V=IR)
    - DC Capacitor: Q, C, V (for Q=CV)
  - Use parallel/series rules when necessary to figure out equivalent *Q* or *R*
    - Junctions: current in equals current out
    - Wires are neutral equipotentials
      - Circle known elements as you calculate them



Voltage goes DOWN in direction of resistor V=IR ("voltage drop")

$$0 - V_6 = -I_1 R_1$$
$$V_6 = I_1 R_1$$

DC Circuits with Series and Parallel Components

- Strategy for a new electric circuit problem
  - Label all elements (*R, V, C*)
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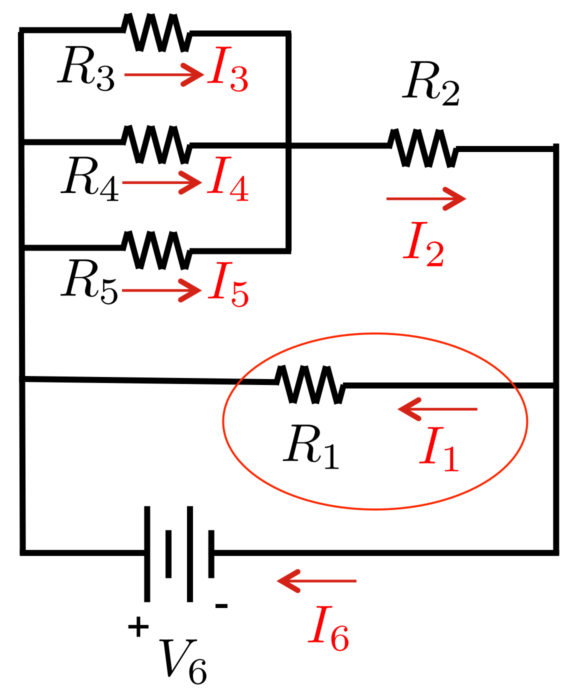

Voltage goes DOWN in direction of resistor V=IR ("voltage drop")

$$0 - V_6 = I_1 R_1$$
$$V_6 = -I_1 R_1$$

Trying It Out

- Let's try it out:

  $V_6 = 15\ \mathrm{V}$ All resistors have $R = 3\ \Omega$

  - Look for places where you know two out of three of
    - DC Resistor: *I, V, R* (for V=IR)
  - Use parallel/series rules when necessary to figure out equivalent *R*
    - Junctions: current in equals current out
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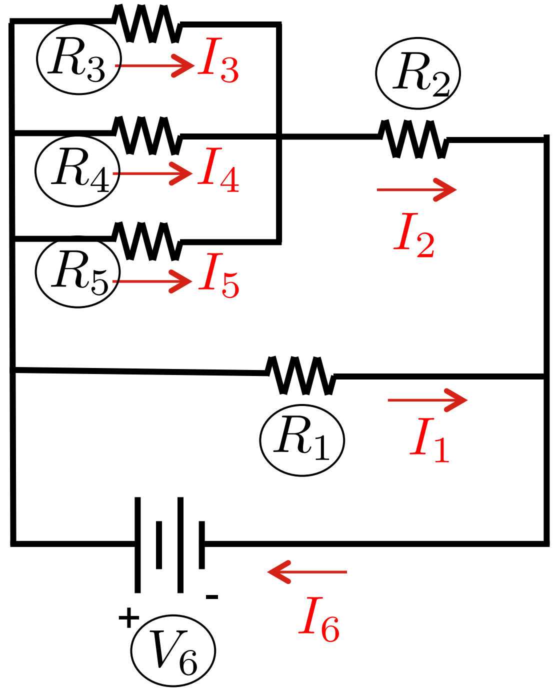


Voltage goes DOWN in direction of resistor *V=IR* ("voltage drop")

$$V_6 = I_1 R_1 \quad I_1 = V_6/R_1 = 5\ \mathrm{A}$$

Need to reduce $R_{3,4,5}$ in parallel

$$\frac{1}{R_{3,4,5}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \quad \Rightarrow \quad R_{3,4,5} = 1\ \Omega$$

And R_2 in series with $R_{3,4,5}$

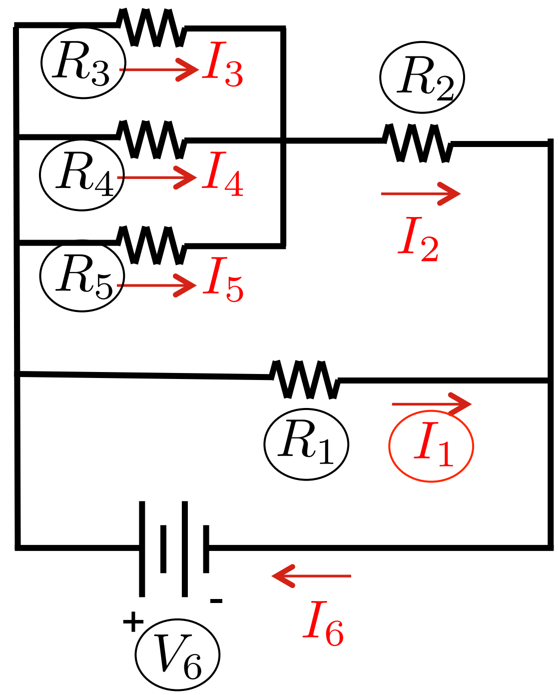
$$R_{2,3,4,5} = R_{3,4,5} + R_2 = 4\ \Omega$$

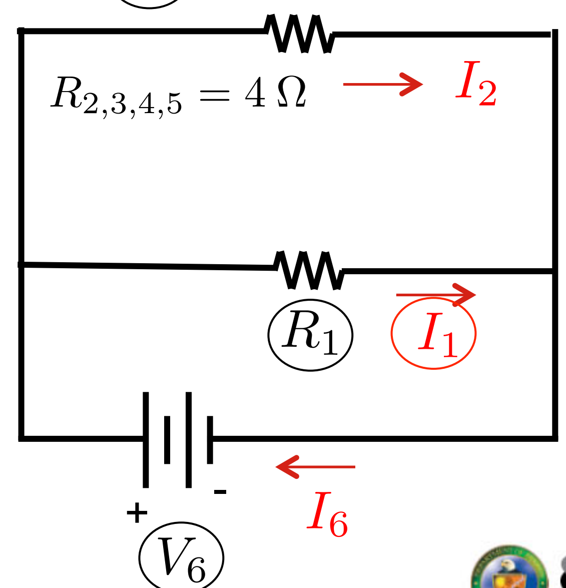
Trying It Out 2

- Let's try it out:

  $V_6 = 15\text{ V}$ $\qquad$ All resistors have $R = 3\,\Omega$

  - Look for places where you know two out of three of
    - DC Resistor: *I, V, R* (for V=IR)
  - Use parallel/series rules when necessary to figure out equivalent *R*
    - Junctions: current in equals current out
    - Wires are neutral equipotentials
      - Circle known elements as you calculate them





Voltage goes DOWN in direction of resistor *V=IR* ("voltage drop") $\qquad V_6 = I_2 R_{2,3,4,5}$

$$I_2 = V_6/R_{2,3,4,5} = 3.75\text{ A}$$

Current in equals current out $\qquad I_6 = I_1 + I_2 = 8.75\text{ A}$

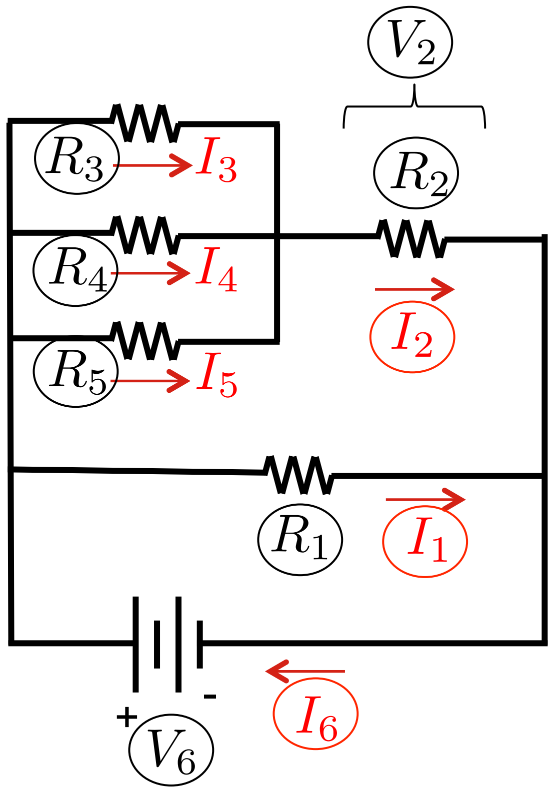
V=IR for R_2 $\qquad V_2 = I_2 R_2 = 11.25\text{ V}$

Trying It Out 3

- Let's try it out:

  $V_6 = 15\ \mathrm{V}$ All resistors have $R = 3\ \Omega$

  - Look for places where you know two out of three of
    - DC Resistor: *I, V, R* (for V=IR)
  - Use parallel/series rules when necessary to figure out equivalent *R*
    - Junctions: current in equals current out
    - Wires are neutral equipotentials
      - Circle known elements as you calculate them



Wires are equipotentials $V_3 = V_4 = V_5 = 15\ \mathrm{V} - 11.25\ \mathrm{V} = 3.75\ \mathrm{V}$

V=IR for $R_{3,4,5}$

$$V_3 = I_3 R_3 \qquad I_3 = V_3/R_3 = 1.25\ \mathrm{A}$$

$$V_4 = I_4 R_4 \qquad I_4 = V_4/R_4 = 1.25\ \mathrm{A}$$

$$V_5 = I_5 R_5 \qquad I_5 = V_5/R_5 = 1.25\ \mathrm{A}$$

Done!!

This is all painfully artful – Enter Kirchhoff

- This process involves lots of decisions
  - Which rule do I use when?
  - How do I see which rule to use when?
- It can also get very tedious
  - The bookkeeping can be quite error-prone

- Fortunately we can break everything down into two rules
  - These are called Kirchhoff's rules
  - We can use these to write down a set of equations for a given DC electrical circuit
  - We still have to solve equations but now the entire approach is more systematic

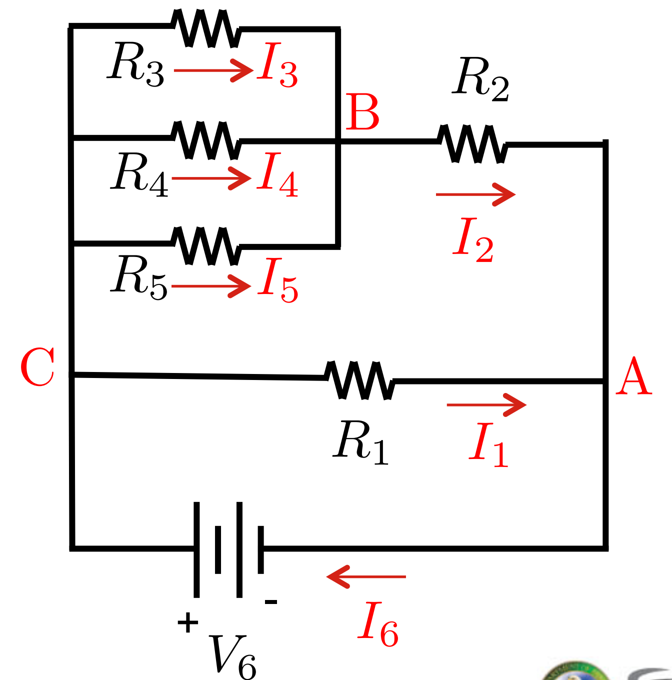
Kirchhoff's Node Rule

- You already know this one
  - At any point in a circuit, the **sum of all current in is equal to the sum of all current out**
  - This is really saying that nothing in the circuit builds up a net electrical charge
    - Capacitors build up equal and opposite charges, stay net neutral
  - Write these for all **but one** of the junction points (or "nodes")
  - For example

$$\text{Point A}: I_6 = I_1 + I_2$$

$$\text{Point B}: I_2 = I_3 + I_4 + I_5$$

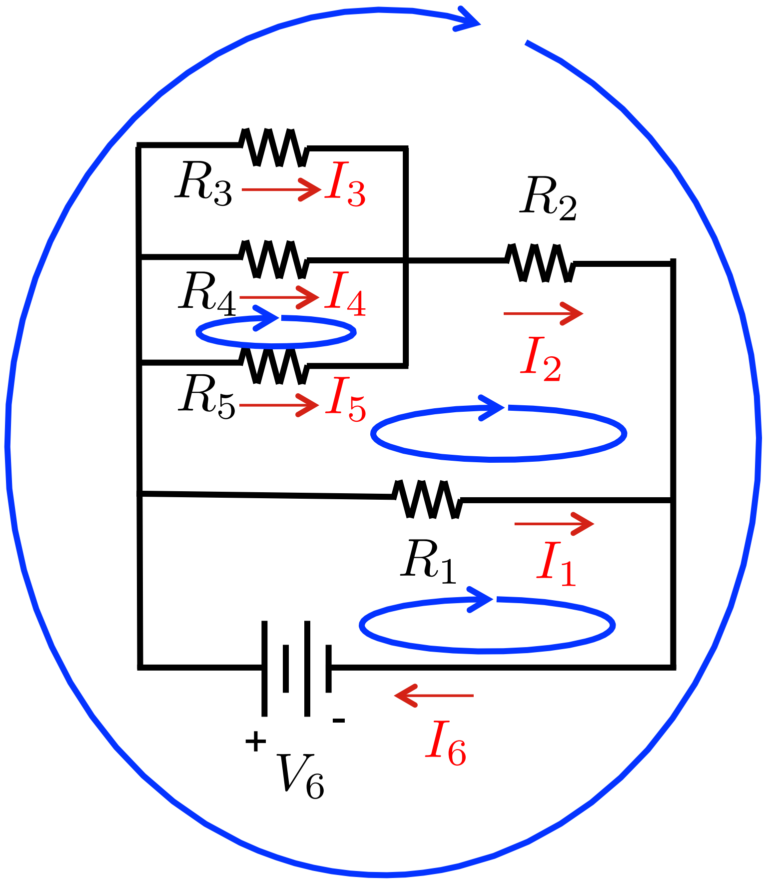
$$\text{Point C}: I_1 + I_3 + I_4 + I_5 = I_6$$



  - Note that the only two of these equations are independent

Kirchhoff's Loop Rule

- The sum of voltages around every loop in the circuit is equal to zero
  - This is really saying that energy is conserved
  - Directions matter!!
    - V in direction of loop is positive
    - IR in direction of loop is **negative**
  - Write these for as many leftover unknowns as you have
    - Here we have 6 total unknowns
    - 3-1 = 2 node equations
    - So we need 4 loops for 4 equations



$$\text{Large loop}: V_6 - I_3R_3 - I_2R_2 = 0$$

$$\text{Top loop}: R_5I_5 - R_4I_4 = 0$$

$$\text{Middle loop}: R_1I_1 - R_5I_5 - R_2I_2 = 0$$

$$\text{Bottom loop}: V_6 - R_1I_1 = 0$$

Does This Really Work?

$V_6 = 15\ \mathrm{V}$ All resistors have $R = 3\,\Omega$

$$\text{Point A}: I_6 = I_1 + I_2 \qquad I_6 = I_1 + I_2$$

$$\text{Point B}: I_2 = I_3 + I_4 + I_5 \qquad I_2 = I_3 + I_4 + I_5$$

$$\text{Large loop}: V_6 - I_3 R_3 - I_2 R_2 = 0 \qquad 15\ \mathrm{V} = (3\,\Omega)(I_2 + I_3) \qquad I_2 + I_3 = 5\ \mathrm{A}$$

$$\text{Top loop}: R_5 I_5 - R_4 I_4 = 0 \qquad I_5 = I_4$$

$$\text{Middle loop}: R_1 I_1 - R_5 I_5 - R_2 I_2 = 0 \qquad I_1 = I_5 + I_2$$

$$\text{Bottom loop}: V_6 - R_1 I_1 = 0 \qquad 15\ \mathrm{V} - (3\,\Omega) I_1 = 0 \qquad I_1 = 5\ \mathrm{A}$$

Does This Really Work? Yep…

$V_6 = 15\ \mathrm{V}$ All resistors have $R = 3\ \Omega$

$$\text{Point A}: I_6 = I_1 + I_2 \qquad I_6 = I_1 + I_2$$

$$\text{Point B}: I_2 = I_3 + I_4 + I_5 \qquad I_2 = I_3 + I_4 + I_5$$

$$\text{Large loop}: V_6 - I_3R_3 - I_2R_2 = 0 \qquad 15\ \mathrm{V} = (3\ \Omega)(I_2 + I_3) \qquad I_2 + I_3 = 5\ \mathrm{A}$$

$$\text{Top loop}: R_5I_5 - R_4I_4 = 0 \qquad I_5 = I_4$$

$$\text{Middle loop}: R_1I_1 - R_5I_5 - R_2I_2 = 0 \qquad I_1 = I_5 + I_2$$

$$\text{Bottom loop}: V_6 - R_1I_1 = 0 \qquad 15\ \mathrm{V} - (3\ \Omega)I_1 = 0 \qquad \boxed{I_1 = 5\ \mathrm{A}}$$

Top oval: $I_2 = (5\ \mathrm{A} - I_2) + 2I_5 \qquad 2I_2 = (5\ \mathrm{A}) + 2I_5$

Bottom oval: $5\ \mathrm{A} = I_5 + I_2 \qquad I_5 = 5\ \mathrm{A} - I_2$

$$2I_2 = (5\ \mathrm{A}) + 2(5\ \mathrm{A} - I_2) = 15\ \mathrm{A} - 2I_2$$

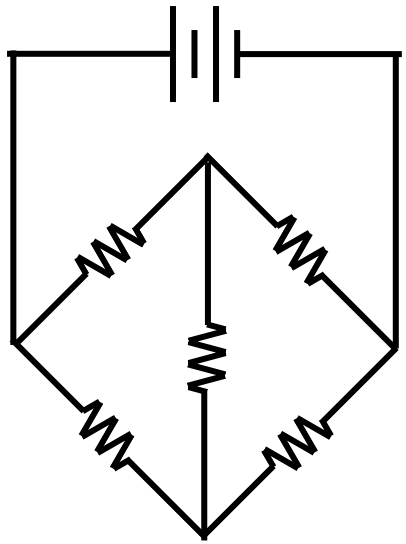
$$4I_2 = 15\ \mathrm{A} \quad \Rightarrow \quad \boxed{I_2 = 3.75\ \mathrm{A}}$$

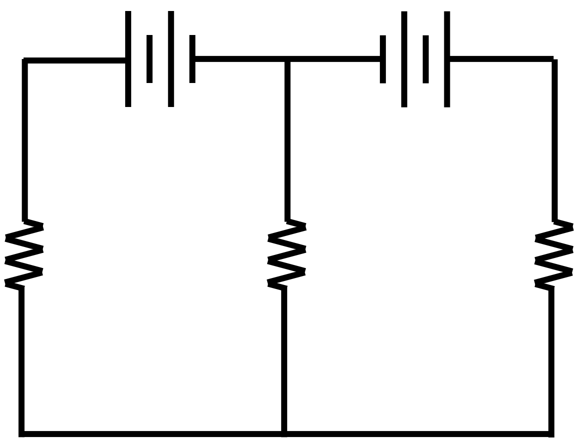
$$I_3 = (5 - 3.75)\ \mathrm{A} = \boxed{1.25\ \mathrm{A} = I_3} \qquad I_5 = 5\ \mathrm{A} - I_2 = \boxed{1.25\ \mathrm{A} = I_5}$$

(Linear Algebra makes this much more straightforward…)

Why So Complicated?

- The point is that Kirchhoff's rules will work for (nearly) ANY circuit
  - A **systematic** way of writing down solvable circuit equations
  - This includes circuits that have multiple EMFs
  - This includes circuits that aren't obviously parallel or series
  - This even includes circuits with capacitors and time-dependent currents and voltages!



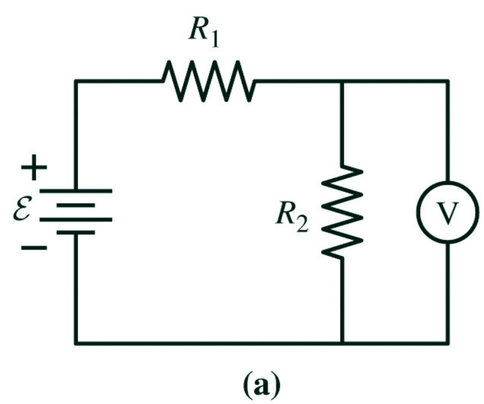


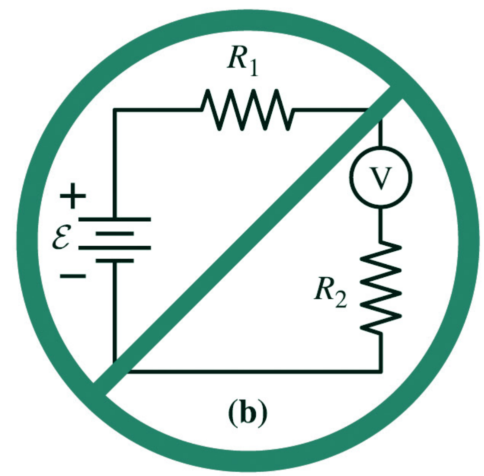
- Kirchhoff can break down though
  - Circuits with large self-impedance, or magnetic stored energy
    - We'll get to that after spring break

Electrical Measurements: Voltage Difference

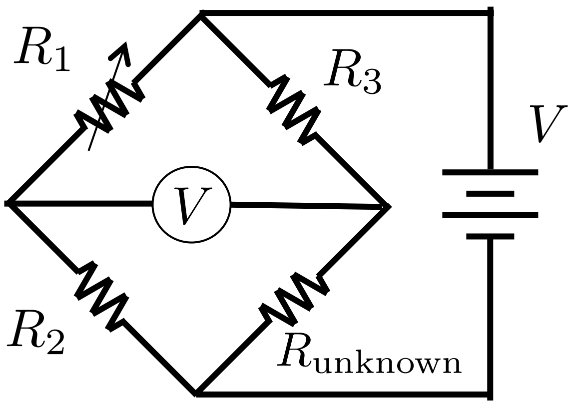
- A **voltmeter** measures potential difference between two points (the two terminals of the voltmeter)
  - Connect the voltmeter in **parallel** with the component you're measuring
  - It will draw a tiny bit of current to do the measurement
  - Ideally, a voltmeter has much larger resistance than the object being measured (digital voltmeters are order 10 MΩ)
    - So it should not draw much current and change the voltage that you're measuring

- Correct and incorrect ways to use a voltmeter to measure voltage across R_2





The Wheatstone Bridge



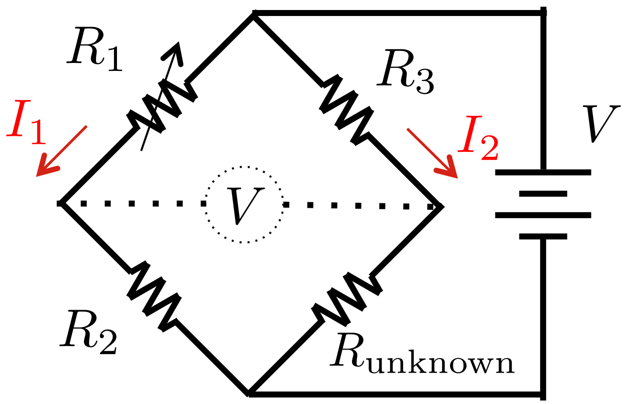
If voltmeter reads zero

$$R_{\text{unknown}} = \frac{R_2 R_3}{R_1}$$

- Voltmeters can be very sensitive
  - They really convert the small current to a deflection of a coil
- A zero voltage measurement can be used to measure an unknown resistance
  - Adjust variable resistor R_1
  - Use Kirchhoff's laws to calculate currents and voltage
  - When voltage is equal on voltmeter (or no current flows", the bridge is "balanced"
- This type of circuit is a **standard** part of many electrical systems

See http://en.wikipedia.org/wiki/Wheatstone_bridge

Deriving the Wheatstone Bridge Equation



If the voltmeter reads zero, there is no current on the central wire so we can behave as if it isn't there.

The voltage across each of the top resistors must be the same since they have the same voltages at the top (same conductor) and bottom (no voltmeter difference)

$$I_1 R_1 = I_2 R_3$$

The same goes for the bottom two resistors

$$I_1 R_2 = I_2 R_{\mathrm{unknown}}$$

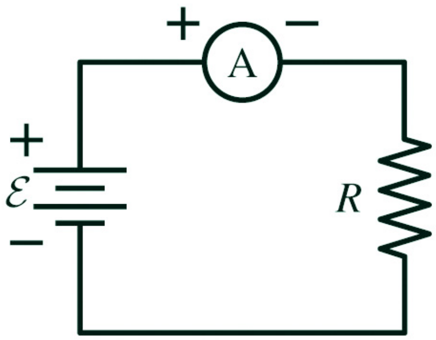
Taking a ratio of these equations gives

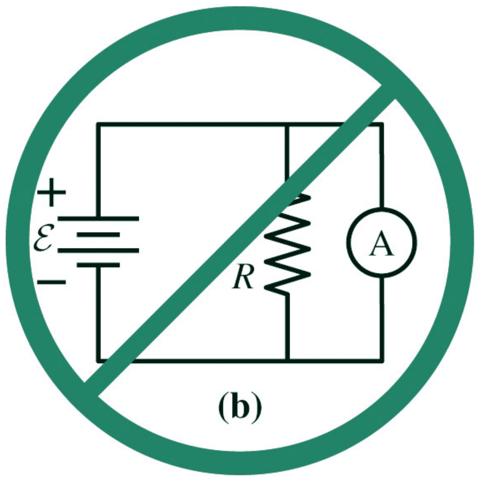
$$\frac{R_1}{R_2} = \frac{R_3}{R_{\mathrm{unknown}}}$$

$$R_{\mathrm{unknown}} = \frac{R_2 R_3}{R_1}$$

Electrical Measurements: Current Flow

- An **ammeter** measures current flowing through itself
  - Current must flow through it to be measured, so it must be connected in **series** with the device you want to measure
  - An ammeter should have very **low resistance** compared to the objects that it's measuring
    - Don't want it to "resist" the current flowing and change the current
  - Correct and incorrect ways to connect an ammeter to measure current flowing through the resistor *R*

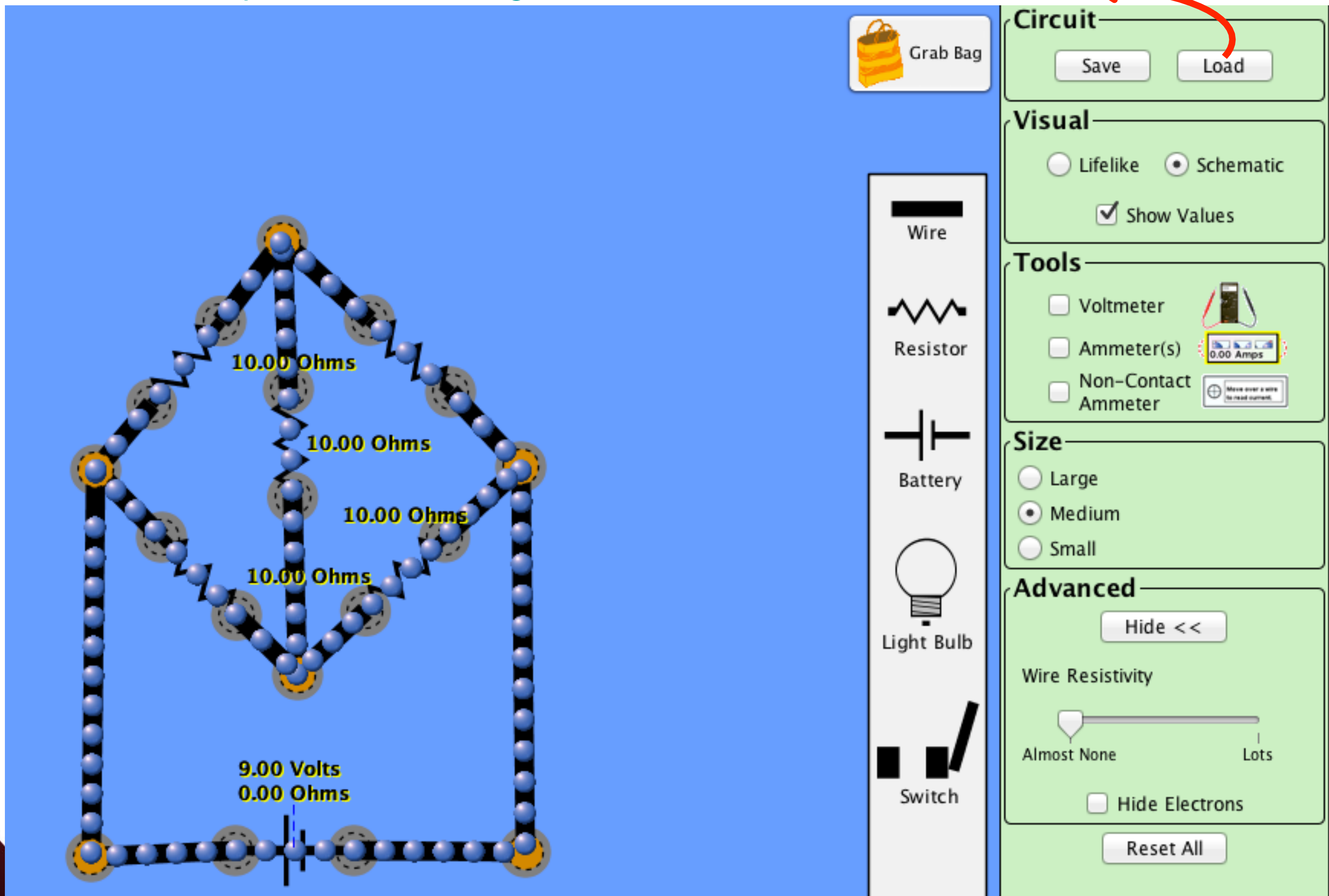




It's Time for Java App Wednesday™

http://phet.colorado.edu/en/simulation/circuit-construction-kit-dc

http://www.toddsatogata.net/2014-ODU/wheatstone.xml

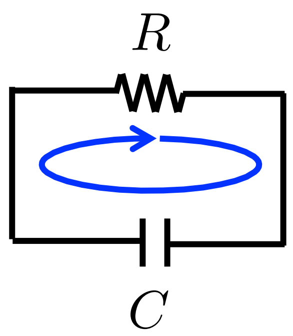


Capacitors in Circuits

- Remember that Kirchhoff's Laws work for capacitors in circuits too
  - Capacitors introduce time-dependent behavior to circuits.
  - The voltage across a capacitor is proportional to the charge on the capacitor.
  - The charge can't change instantaneously, because that would require an infinite current to move a finite amount of charge onto the capacitor in zero time.
  - Therefore, **the voltage across a capacitor cannot change instantaneously.**

- We can calculate how a basic RC circuit changes over time
  - RC circuits are the simple harmonic oscillators of the electronics world
    - The capacitor is the "spring"
    - The resistance is the "friction" (or dissipative inertia)

The RC Circuit: Time Constant and Exponential Decay



Kirchhoff's Node rule:
No nodes!

Kirchhoff's Loop rule:
One loop!

$$C \equiv \frac{Q}{V}$$

So for capacitor

$$V = \frac{Q}{C}$$

$$IR + \frac{Q}{C} = 0$$

$$I \equiv \frac{dQ}{dt}$$ Definition of current

$$\frac{dQ}{dt}R + \frac{Q}{C} = 0$$

$$\frac{dQ}{Q} = -\frac{dt}{RC}$$

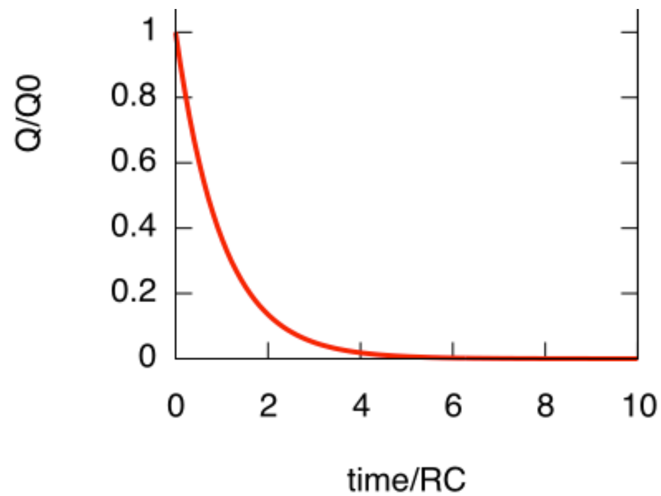
Integrate!

$$\ln(Q) = -\frac{t}{RC} + \text{constant}$$

$$Q(t) = Q_0\, e^{-t/RC}$$

$$V(t) = \frac{Q}{C} = \frac{Q_0}{C}\, e^{-t/RC}$$

Exponential decay!



RC has dimensions of **time** and is known as the **RC time constant** of the circuit

One use: high-pass and low-pass AC filters