University Physics 227N/232N

~~Lab: Ohm's Law and (maybe) DC RC Circuits~~
Or not: Watch Todd Improvise ☺
Gauss's Law and Ampere's Law

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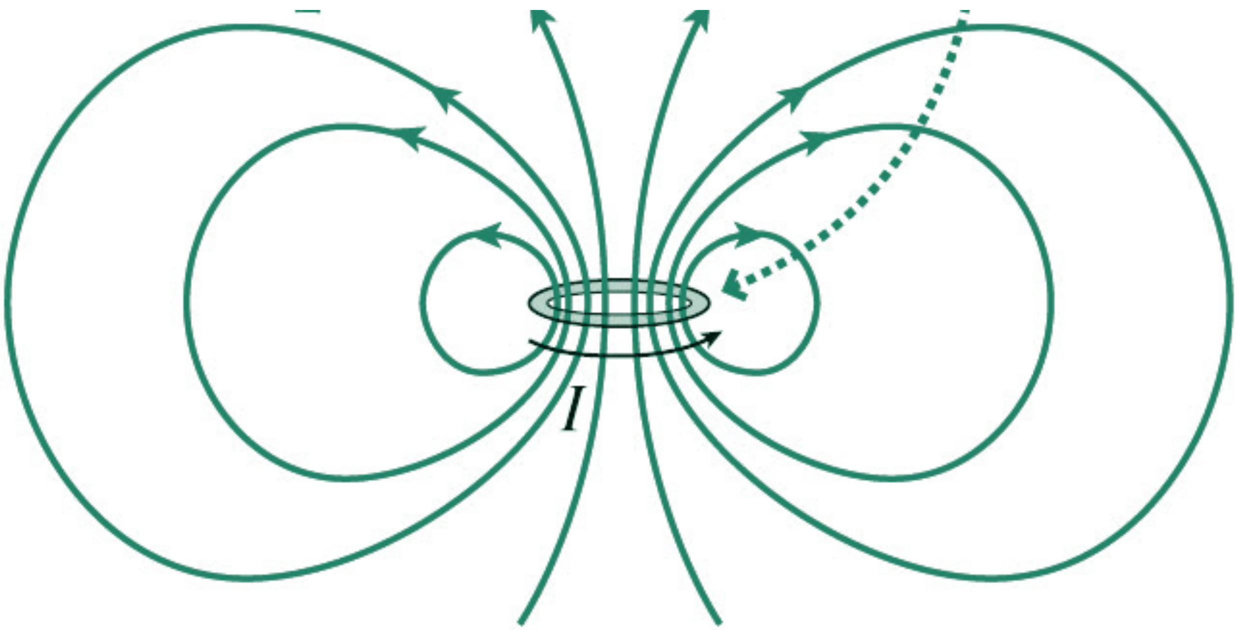
Happy Birthday to Adrian Peterson, Rosie O'Donnell, Matthew Broderick, Joseph Fourier, and Walter Gilbert (1980 Nobel)

Current Loops Again

- Last class we talked about the magnetic fields created by current loops and made a few observations
  - Magnetic field lines are always closed loops
    - they don't start or end anywhere: there are no magnetic "charges"
  - Recall: Gauss's Law for electric fields
    - Related electric flux through a closed surface to the total amount of charge inside
    - We can write a similar equation for magnetism but simpler: the total charge enclosed is always zero!



Gauss's Law for Magnetism

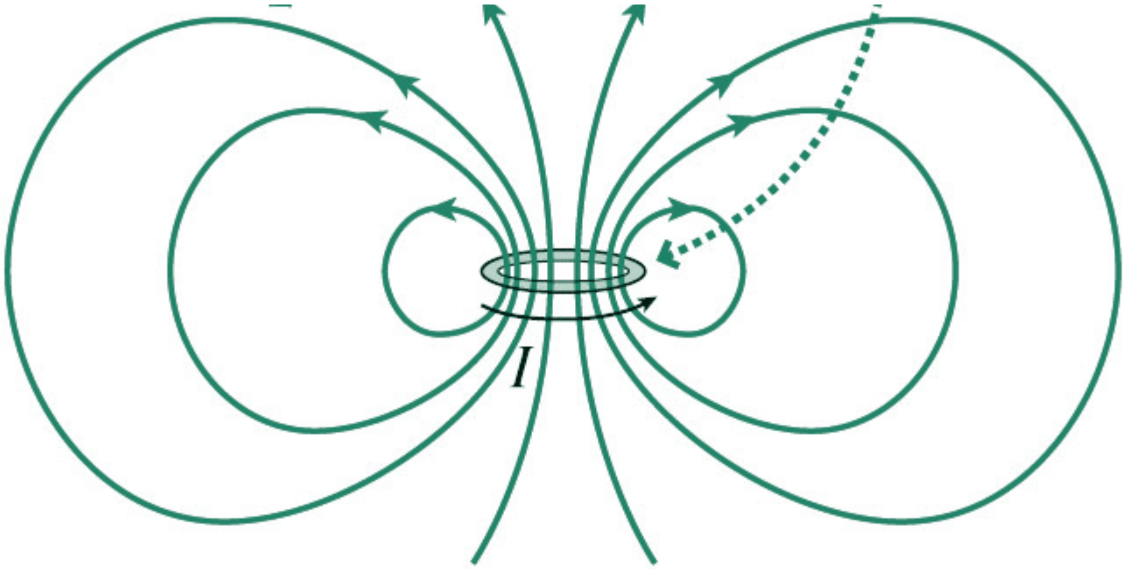
- For electric fields we had Gauss's Law:

$$\Phi_{\text{electric}} = \oint \vec{E} \cdot d\vec{A} = 4\pi k q_{\text{enclosed}}$$

- For magnetic fields this becomes

$$\Phi_{\text{magnetic}} = \oint \vec{B} \cdot d\vec{A} = 0$$

  - The net magnetic flux through **any** closed surface is **always** zero

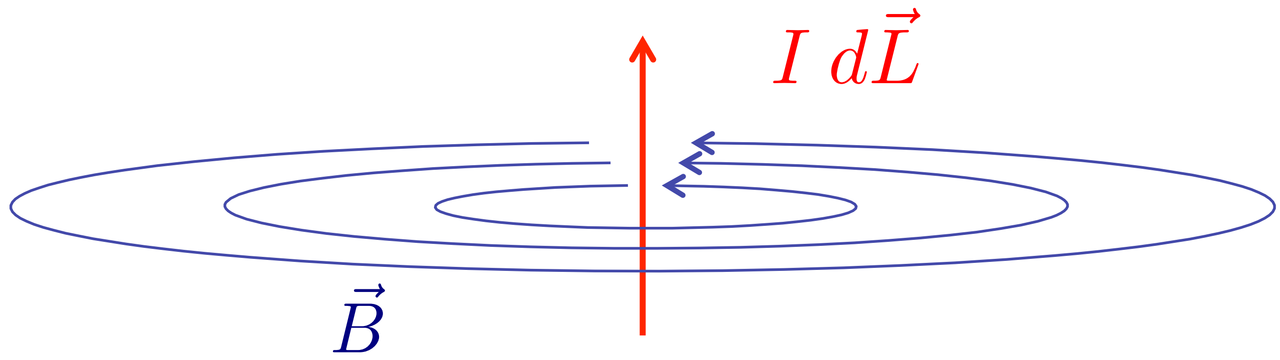


Gauss's Law: Applications?

- Magnetic fields we calculated with Biot-Savart are kinda complicated

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{L} \times \hat{r}}{r^2} \qquad \mu_0 \equiv 4\pi \times 10^{-7}\ \mathrm{T-m/A}$$

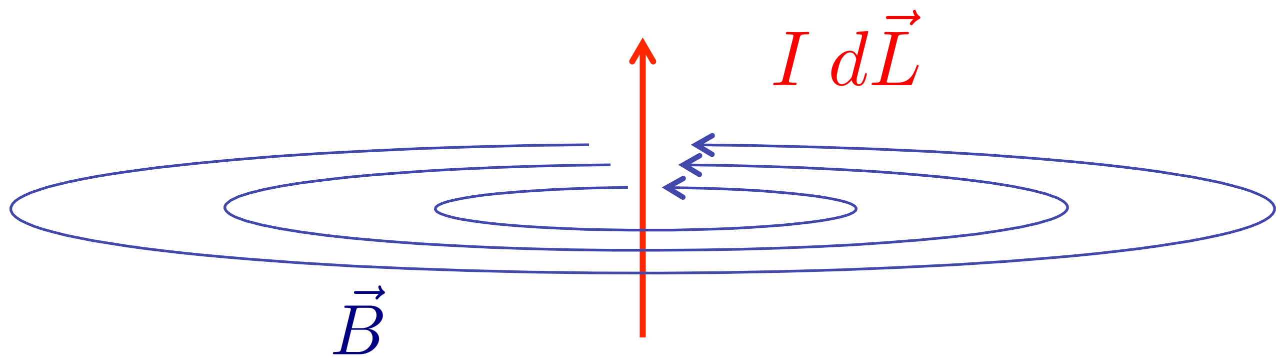
  - The current loop magnetic field is only easily calculated on the axis of the magnetic field
  - The magnetic field from an infinite straight line of current is (right-hand) circles going around the current



  - Here calculating magnetic flux is either boring (cylindrical Gaussian surface) or hard (other surfaces)
  - Gauss's Law for magnetism is more useful as a **concept**

Infinite Line Current and Ampere's Law

- So Gauss's Law isn't very useful in helping us calculate magnetic fields from symmetry
  - But let's go back to the infinite line of current and its magnetic field calculated from Biot-Savart and notice something



$$B = \frac{\mu_0 I}{2\pi r} \quad (r \text{ is distance from line, direction is right hand around } I)$$

$$(2\pi r)B = \mu_0 I$$

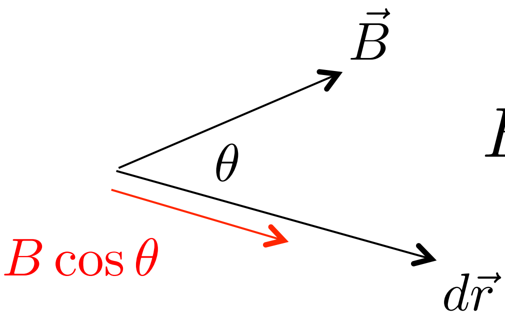
  - The magnetic field lines are just circles -- and $2\pi r$ is just the circumference of a circle of radius r.
  - Maybe summing up (integrating) B over the circumference of the circle is related to the total current "enclosed" by that circle

Ampere's Law

- Indeed, this was discovered to be true and is known as **Ampere's Law**:

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{\text{enclosed}}$$

  - I_{enclosed} is the current going **through** the closed 'Amperian" loop
  - This really only applies for steady (constant) currents
    - Changing currents create a complicated mix of B and E fields
  - Remember the **vector dot product** is a scaler:
    - really just taking a "component in the direction of" the other vector



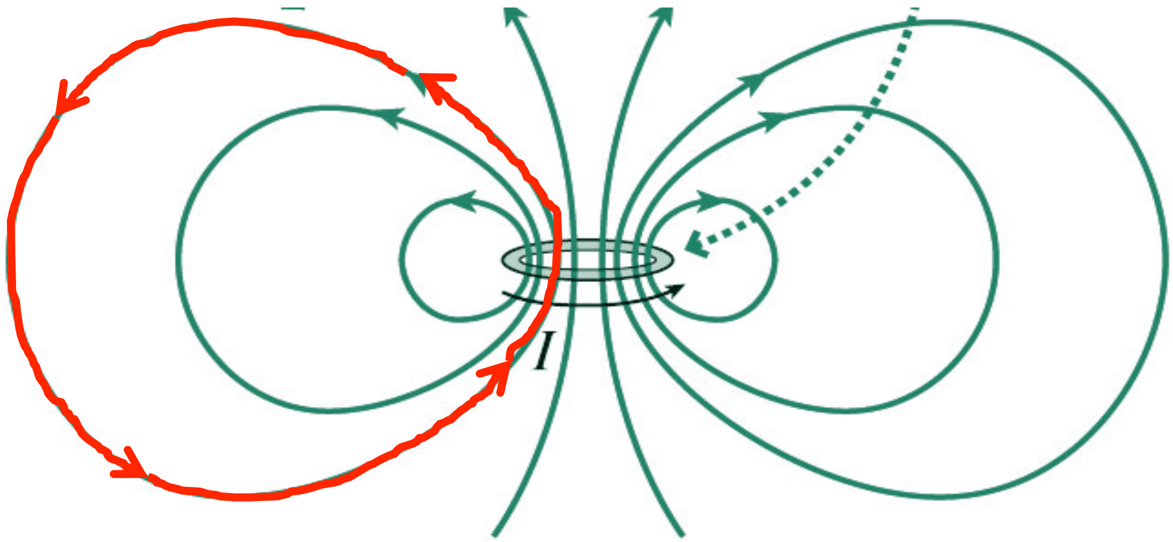
$$\vec{B} \cdot d\vec{r} = B\, dr\, \cos\theta$$

$$\vec{B} \cdot d\vec{r} = B\, dr \text{ when } \vec{B} \text{ and } d\vec{r} \text{ are parallel}$$

$$\vec{B} \cdot d\vec{r} = 0 \text{ when } \vec{B} \text{ and } d\vec{r} \text{ are perpendicular}$$

Applying Ampere's Law in General Cases

- Ampere's law is always true but it's usually hard to apply
- Example: Magnetic field from a current loop (but generally true)
  - Draw our Amperian loop along a closed magnetic field line
  - Then $\vec{B}$ and $d\vec{r}$ are parallel everywhere on the closed path
  - $\vec{B}$ also has constant magnitude everywhere on the closed path

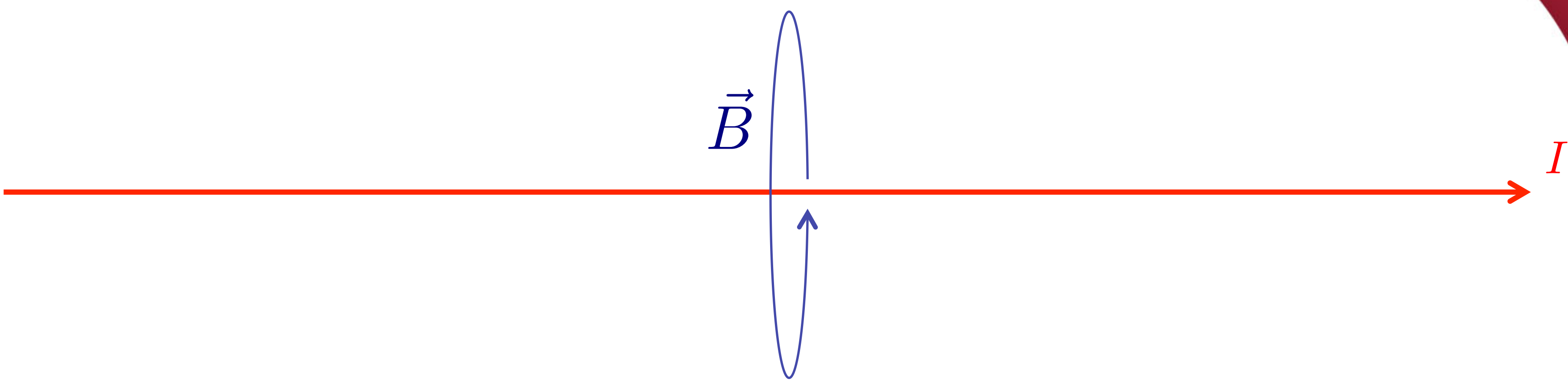


  - So if the field line has length $L_{\text{field line}}$, Ampere's Law gives

$$\oint \vec{B} \cdot d\vec{r} = BL_{\text{field line}} = \mu_0 I \quad \Rightarrow \quad \boxed{B = \frac{\mu_0 I}{L_{\text{field line}}}}$$

- But the length of that field line is usually quite hard to calculate!

Applying Ampere's Law: Infinite Line Current



- There is one case where it's easy to calculate the length of the field lines: the line of infinite current
  - Field lines go in (right-hand) circles around the current
  - Rotational symmetry: B is same everywhere on the circle
  - Then Ampere's Law gives

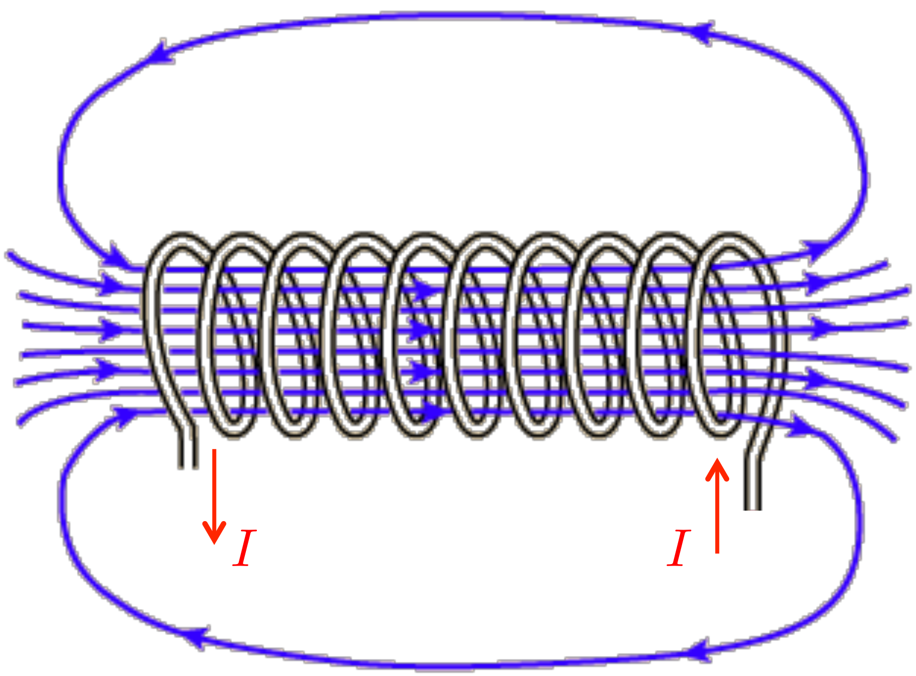
$$\oint \vec{B} \cdot d\vec{r} = BL_{\text{field line}} = \mu_0 I_{\text{enclosed}}$$

$$L_{\text{field line}} = 2\pi r$$

$$B = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r}$$

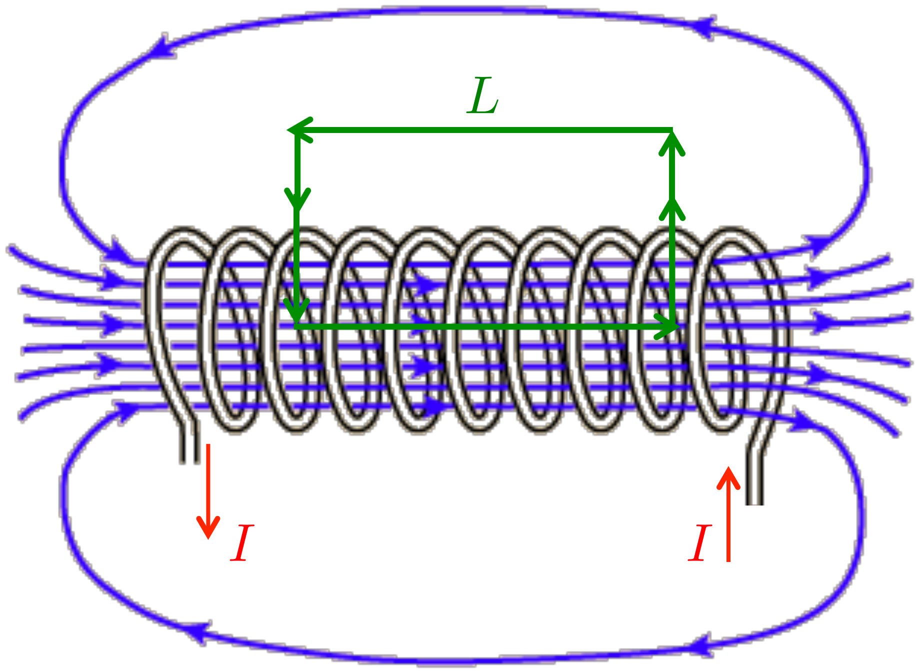
Ampere's Law: Solenoid

- Instead of a current loop, we can make many circular loops of wire with the same radius and evenly spaced
  - This is called a **solenoid**
  - A very long solenoid has a nearly **constant magnetic field** in the center of the loops
  - Outside of the loops the path is long and the field is quite small



B field direction follows right hand rule around current loops

Ampere's Law: Solenoid



Coiling of loops:
n "turns" per unit length

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{\text{enclosed}} = \mu_0 (nLI)$$

- Draw a rectangular Amperian loop with three parts
  - Outside horizontal path: $B \approx 0$ so $\int \vec{B} \cdot d\vec{r} = 0$
  - Up/down sides: $\vec{B} \perp d\vec{r}$ so $\vec{B} \cdot d\vec{r} = 0$ and $\int \vec{B} \cdot d\vec{r} = 0$
  - Inside: $\vec{B} \parallel d\vec{r}, B \approx \text{constant}$ so $\int \vec{B} \cdot d\vec{r} = BL$

$$B = \mu_0 n I$$