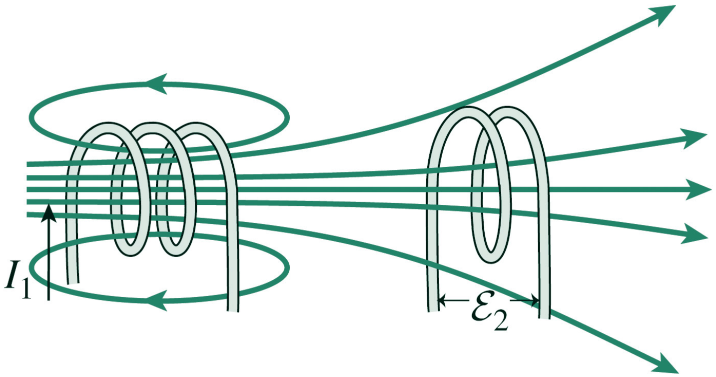

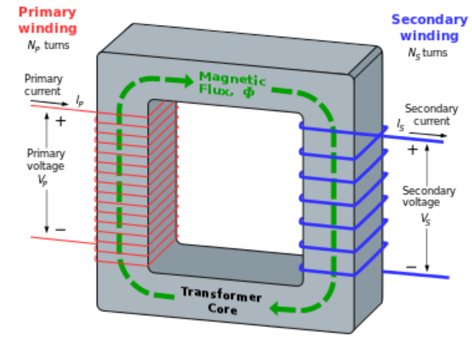

University Physics 227N/232N

Ch 27: Inductors, towards Ch 28: AC Circuits

Quiz and Homework Due This Week

Exam Next Wednesday! (April 9)

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Wednesday, April 2 2014

Happy Birthday to Michael Fassbender, Marvin Gaye, Carl Kasell,
Sir Alec Guinness, Hans Christian Andersen, and Casanova!

Some Comments About Units

- Units in the electricity/magnetism section may seem confusing
  - That's because they are!
  - We're relating more concepts than motion and kinematics that you learned in the first semester.
  - We use **metric mks** or **SI** units in this class: if you use these units in your calculations, they should "just work"
  - I'll also include some units and conversions on the cheat sheet

Quantity	Unit
Electric charge	Coulomb [C]
Magnetic field	Tesla [T] (10^{-4} T = 1 Gauss)
Electric potential	Volt [V]
Electric current	Ampere or Amp [A] = 1 C/s
Magnetic Flux	Weber [Wb] = T m^2 = V s
Inductance (to be covered today)	Henry [H]

- Conversions are easiest to figure out through physical formulas

$$\vec{F} = q\vec{v} \times \vec{B} \quad \Rightarrow \quad 1\,\mathrm{N} = (1\,\mathrm{kg\,m/s^2}) = (1\,\mathrm{C})(1\,\mathrm{m/s})(1\,\mathrm{T}) \quad \Rightarrow 1\,\mathrm{T} = 1\,\mathrm{kg/(C\,s)}$$

Review: Faraday's Law

- **Faraday's law** describes induction by relating the EMF induced in a circuit to the rate of change of magnetic flux through the circuit:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad \left(\mathrm{V} = \frac{\mathrm{T\,m^2}}{\mathrm{s}}\right)$$

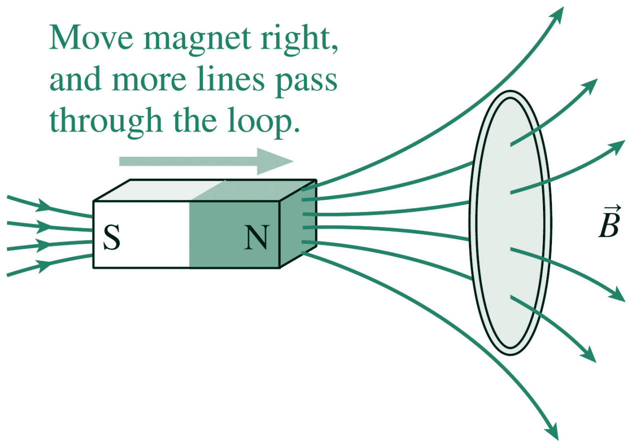
where the **magnetic flux** is given by

$$\Phi_{\mathrm{B}} = \int \vec{B} \cdot d\vec{A} \qquad \text{Weber, } \mathrm{Wb} \equiv \mathrm{T\,m^2}$$

- With a flat area and uniform field, this becomes

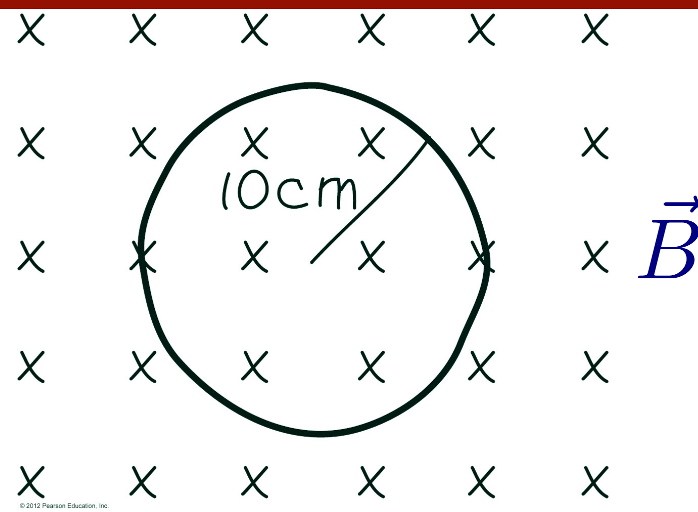
$$\Phi_B = B\,A\,\cos\theta$$

- The flux can change by changing the field *B*, the area *A*, or the orientation θ.



Moving a magnet near a wire loop increases the flux through the loop. The result is an induced EMF given by Faraday's law. The induced EMF drives an induced current in the loop.

Review: Example



- A wire loop of radius *r* = 10 cm is in the plane of the screen, perpendicular to a magnetic field $\vec{B}$ = 1.1 T pointing into the screen.
  - What is the flux through the loop?

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = BA\cos\theta = BA$$

$$\Phi_B = (1.1\ \mathrm{T})(\pi(0.1\ \mathrm{m})^2) = \boxed{0.034\ \mathrm{Wb} = \Phi_\mathrm{B}}$$

(definition of flux)

  - If the magnetic field smoothly changes to a zero value over 0.1 s, what is the induced EMF in the loop?

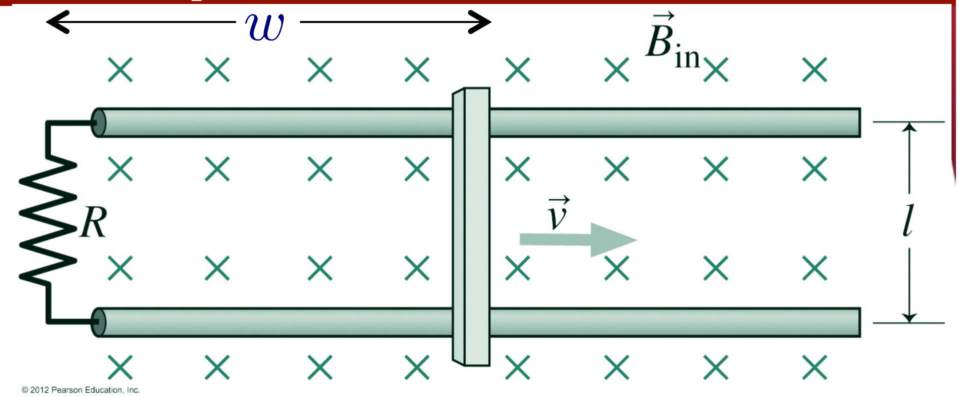
$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{(0.034\ \mathrm{Wb})}{0.1\ \mathrm{s}} = \boxed{-0.34\ \mathrm{V} = \mathcal{E}}$$

(Faraday's Law)

  - Which direction does the induced current flow in the loop?
    - Creates a flux that opposes the flux change: **clockwise current**

Review: Example

- A closed circuit is created by a conductor of length *l*=20 cm sliding along a track to the right with velocity *v* = 5.9 m/s, through a magnetic field *B* = 0.85 T pointing into the screen.



Area $= A = wl$

  - What is the change in flux though the closed loop per unit time?

$$\Phi_B = BA \quad \Rightarrow \quad \frac{d\Phi_{\mathrm{B}}}{dt} = B\frac{dA}{dt} = Bl\frac{dw}{dt} = Blv = \frac{d\Phi_{\mathrm{B}}}{dt}$$

$$\frac{d\Phi_{\mathrm{B}}}{dt} = (0.85\ \mathrm{T})(0.2\ \mathrm{m})(5.9\ \mathrm{m/s}) = 1.0\ \mathrm{V}$$

  - If the resistor has resistance *R* = 10 Ω, what is the magnitude and direction of the induced current?

$$V = \mathcal{E} = IR \quad \Rightarrow \quad I = \frac{\mathcal{E}}{R} = \frac{1.0\ \mathrm{V}}{10\ \Omega} = \boxed{0.1\ \mathrm{A} = I}$$

(Ohm's Law)

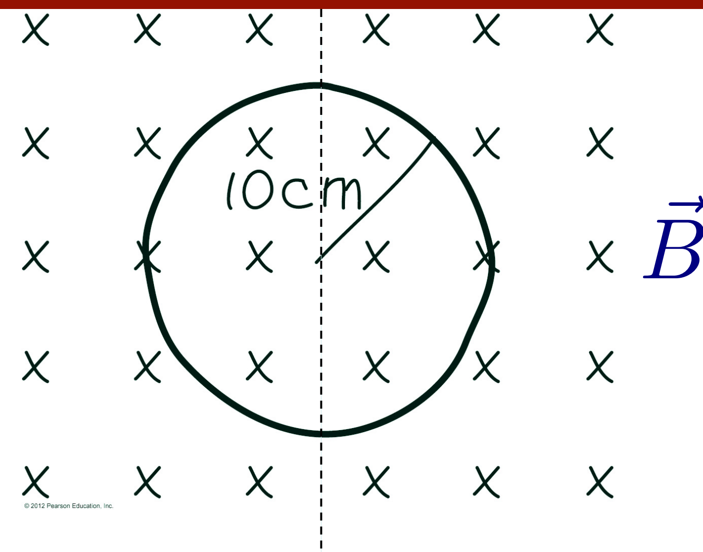
Creates a flux that opposes flux change: **counterclockwise current**

Review: Example

- A wire loop of radius r = 10 cm is in the plane of the screen, perpendicular to a magnetic field $\vec{B}$ = 1.1 T pointing into the screen.
  - What is the flux through the loop?

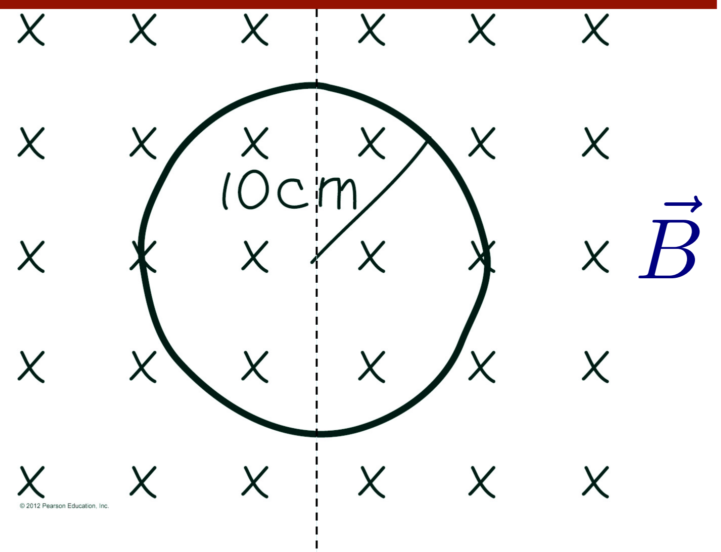
$$\Phi_B = \int \vec{B} \cdot d\vec{A} = BA\cos\theta = BA$$

$$\Phi_B = (1.1\,\mathrm{T})(\pi(0.1\,\mathrm{m})^2) = \boxed{0.034\,\mathrm{Wb} = \Phi_\mathrm{B}}$$



  - If the loop rotates around the dotted axis with frequency f = 1Hz, what is the induced EMF in the loop?

Review: Example



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$$\Phi_B = \int \vec{B} \cdot d\vec{A} = BA\cos\theta = BA$$

$$\Phi_B = (1.1\,\mathrm{T})(\pi(0.1\,\mathrm{m})^2) = \boxed{0.034\,\mathrm{Wb} = \Phi_\mathrm{B}}$$

  - If the loop rotates around the dotted axis with frequency *f* = 1Hz, what is the induced EMF in the loop?

$$\Phi_B = BA\cos\theta = BA\cos(2\pi f t)$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = BA(2\pi f)\sin(2\pi f t) = (1.1\,\mathrm{T})(\pi(0.1\,\mathrm{m})^2)(2\pi(1\,\mathrm{Hz}))\sin(2\pi f t)$$

$$\boxed{\mathcal{E} = (0.22\,\mathrm{V})\sin(2\pi f t)}$$

An AC (alternating current) generator

Induced Electric Fields

- The induced emf in a circuit subject to changing magnetic flux actually results from an **induced electric field.**
  - Induced electric fields result from changing magnetic flux.
    - This is described by the **full form of Faraday's law**, one of the four fundamental laws of electromagnetism:

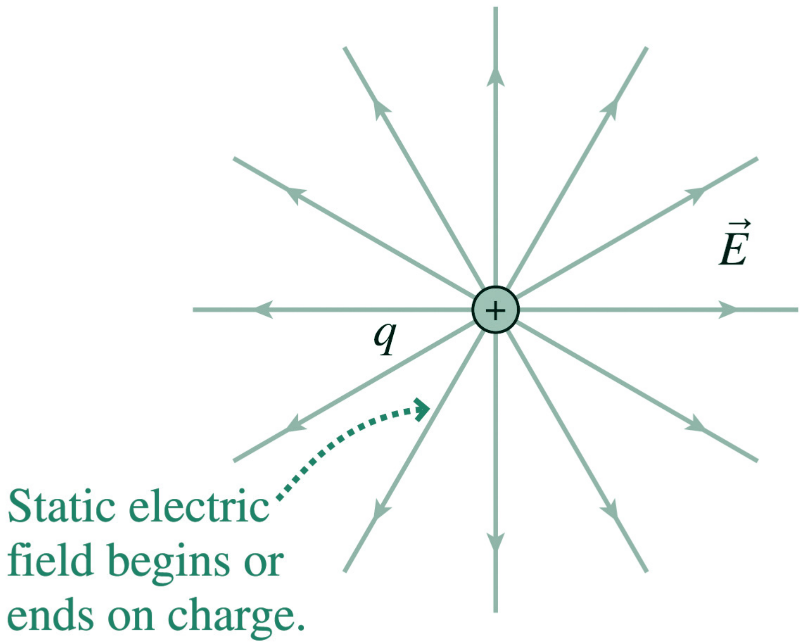
$$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_{\mathrm{B}}}{dt}$$

where the integral is taken around any closed loop, and where the flux is through any area bounded by the loop.

  - The equation states that **a changing magnetic field produces an electric field.**
    - Thus not only charges but also changing magnetic fields are sources of electric field.
    - Unlike the electric field of a static charge distribution, the induced electric field is *not conservative.*

Static and Induced Electric Fields

- Static electric fields begin and end on charges.
- Induced electric fields generally form closed loops.
  - Similar to magnetic fields created by currents.



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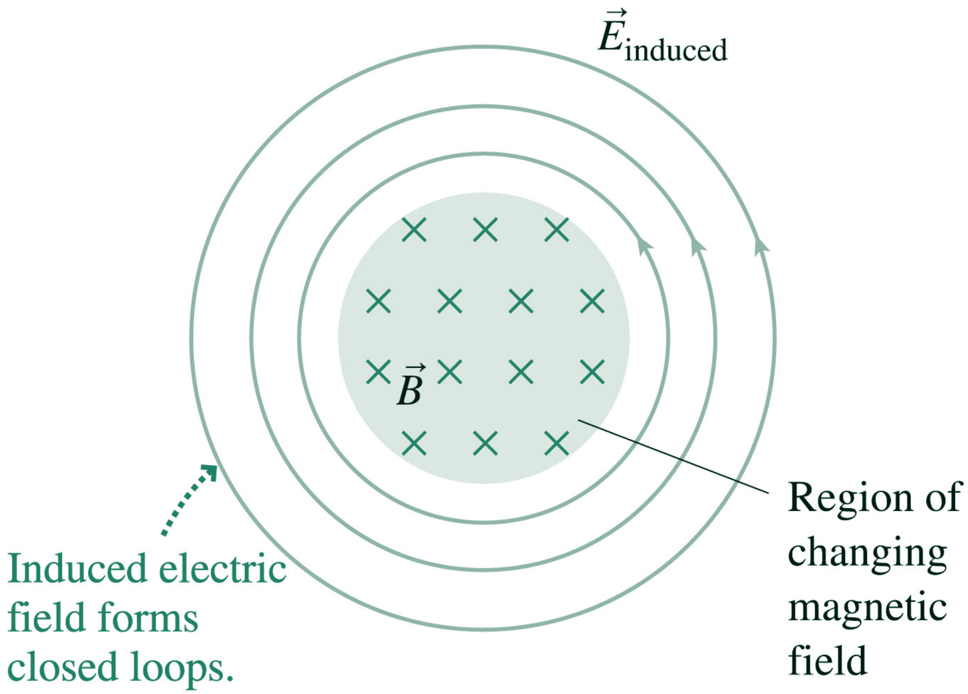
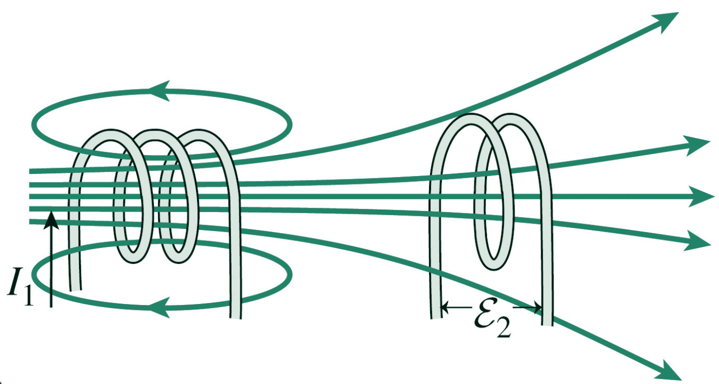


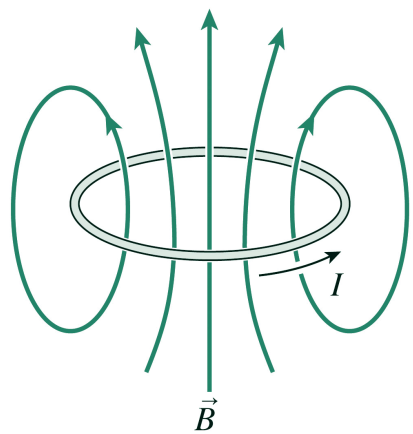
(b)

Review: Inductance

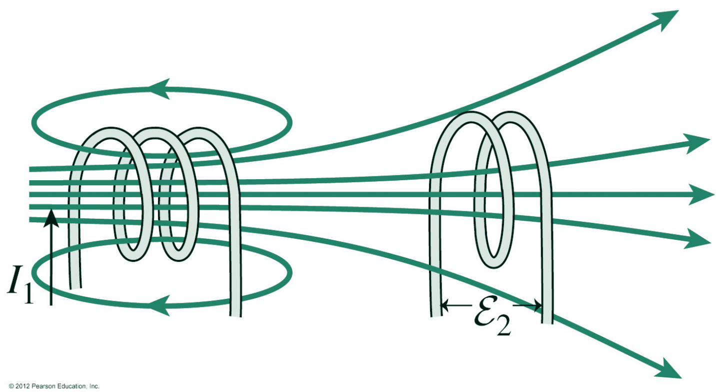
- **Mutual inductance** occurs when a changing current in one circuit results, via changing magnetic flux, in an induced emf and thus a current in an adjacent circuit.
  - Mutual inductance occurs because some of the magnetic field produced by one circuit passes through the other circuit's area, producing a changing flux – two current loops affect each other.

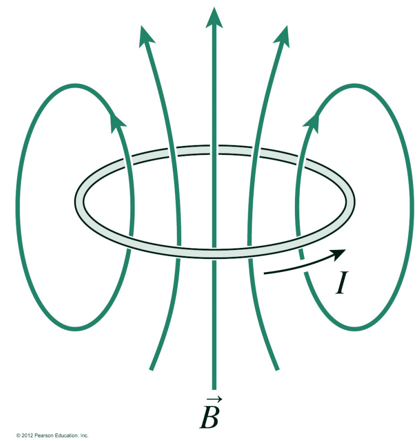
- **Self-inductance** occurs when a changing current in a circuit results in an induced emf that opposes the change in the circuit itself.
  – Self-inductance occurs because some of the magnetic flux produced in a circuit passes through that *same* circuit.





Electric and Magnetic Fields are Intertwined

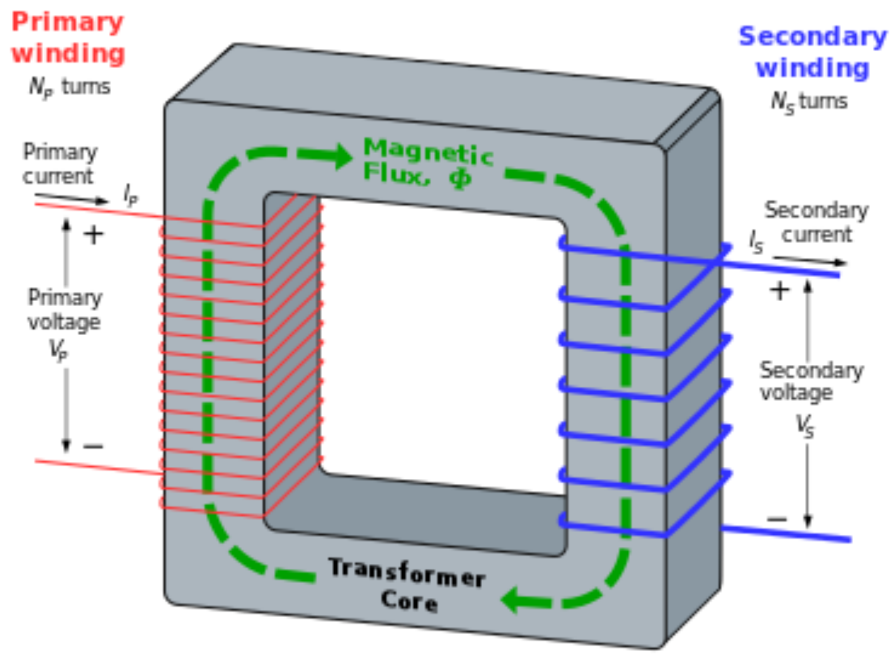




- Current is charge moving around due to an applied electric field
  - (definition of current and force on charges from electric fields)
- Current (moving electric charges) is a source of magnetic fields and magnetic flux
  - Biot-Savart and Ampere's Laws
- Changing magnetic flux is a source of electric field/EMF
  - Faraday's Law
- Changing electric fields is a source of more magnetic field
  - Lenz's law: This magnetic field "fights" the original changing flux

Mutual Inductance: Transformers

- An electrical transformer transmits electrical energy through mutual inductance
  - A purely AC device
    - Requires AC current to produce changing magnetic flux and induced current
    - AC frequency is same on both sides
  - Primary and secondary solenoid windings, usually around same core
  - Note opposite directions of windings!
  - Used to "step up" or "step down" AC voltage
    - Also used for AC circuit isolation



Flux is approximately the same on both sides

$$V_{\rm P} = -N_{\rm P} \frac{d\Phi_B}{dt} \qquad V_{\rm S} = -N_{\rm P} \frac{d\Phi_B}{dt}$$

$$\text{Transformer}: \quad \frac{V_P}{N_P} = \frac{V_S}{N_S}$$

Example: Transformer

- You are traveling to Europe and want to use your 120V device with a European 240V outlet. Neglecting for a moment the frequency difference (50 Hz European vs 60 Hz US AC frequency), you want to construct a homemade transformer to use temporarily.
  - If you wrap N_P=50 turns of wire on the European side of your transformer, how many turns N_S should you wrap on the American side?
    - 25
    - 50
    - 75
    - 100

$$\text{Transformer}: \quad \frac{V_P}{N_P} = \frac{V_S}{N_S}$$

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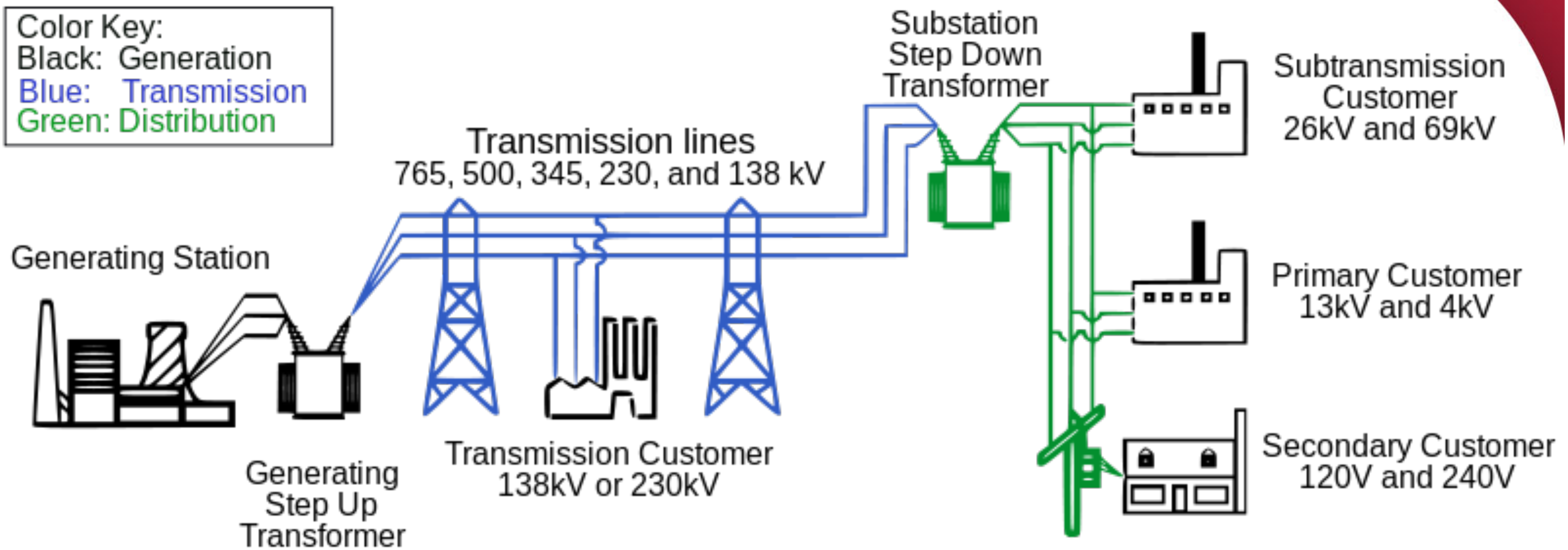
    - **25**
    - 50
    - 75
    - 100

$$\text{Transformer}: \quad \frac{V_P}{N_P} = \frac{V_S}{N_S}$$

$$N_S = \left(\frac{V_S}{V_P}\right) N_P = \left(\frac{120\text{ V}}{240\text{ V}}\right)(50\text{ turns}) = \boxed{25\text{ turns} = N_S}$$

- Don't try this at home!

Transformers and Electrical Power Distribution



- Transformers are crucial to electric power distribution
  - High-voltage AC wires lose less power over long distribution distances
  - Lower voltage is needed for use and electrical safety in homes
  - A transformer/diode combination can be used to produce DC voltage: a **rectifier**

Self-inductance

- The self-inductance *L* of a circuit is defined as the ratio of the magnetic flux through the circuit to the current in the circuit:

$$\boxed{L \equiv \Phi_{\mathrm{B}}/I}$$

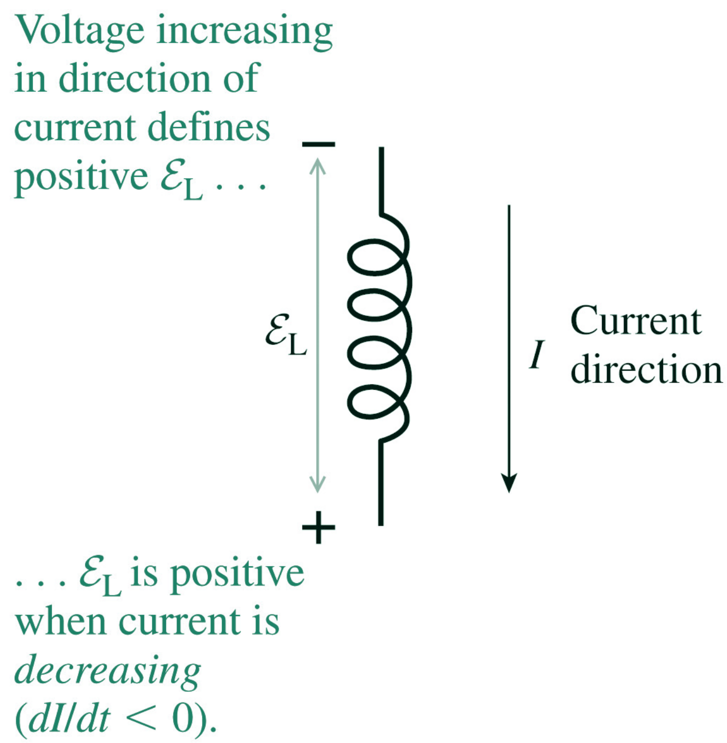
  - In differential form:

$$\frac{d\Phi_{\mathrm{B}}}{dt} = L\frac{dI}{dt}$$

  - The SI units of *L* are **Henry** or T-m^2/A
  - By Faraday's law, the EMF or voltage across an inductor is

$$\boxed{\mathcal{E}_{\mathrm{L}} = -L\frac{dI}{dt}}$$

  - The minus sign shows that the direction of the inductor EMF is such as to *oppose* the change in the inductor current.



Self-Inductance of a Solenoid

- Consider a long solenoid of cross-sectional area *A*, length *l*, with *n* turns per unit length.
  - The magnetic field inside the solenoid is

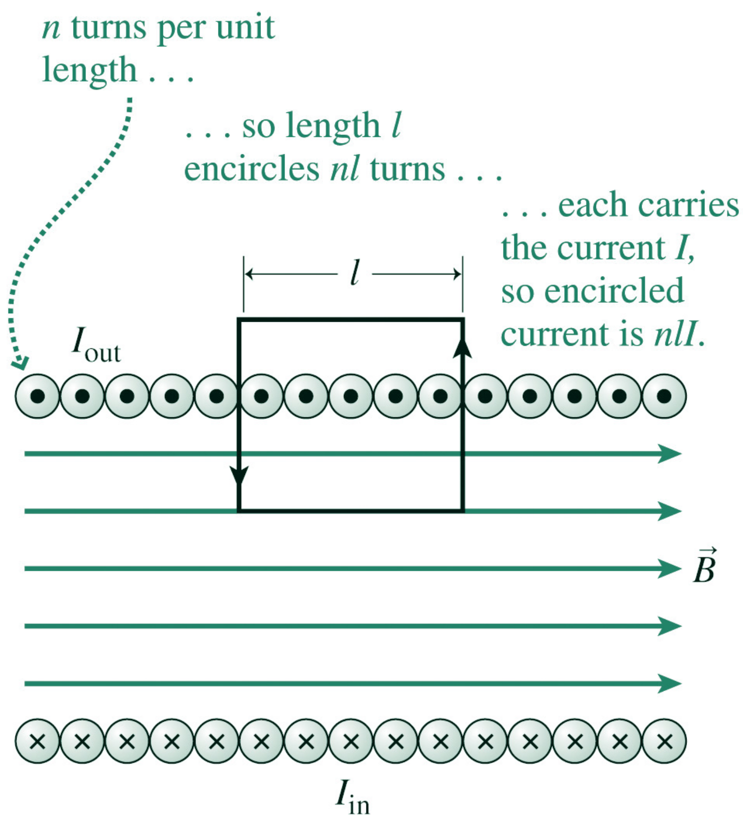
$$B = \mu_0 n I$$

  - With *n* turns per unit length, the solenoid contains a total of *nl* turns, so the flux through all the turns is

$$\Phi_B = nlBA = nl(\mu_0 n I)A = \mu_0 n^2 I A l$$

  - The self-inductance of the solenoid is

$$\boxed{L = \frac{\Phi_B}{I} = \mu_0 n^2 A l}$$



Note that this only depends on the construction/geometric details of the solenoid – similar to capacitance for a capacitor.

(Magnetic Energy)

- As current builds up in an inductor, the inductor absorbs energy from the circuit. That energy is stored in the inductor's magnetic field.
  - The rate at which the inductor stores energy is

$$P = LI\frac{dI}{dt}$$

  - For an inductor, the stored energy is

$$U_{\mathrm{B}} = \int P dt = \int_0^I (LI) dI = \frac{1}{2} LI^2$$

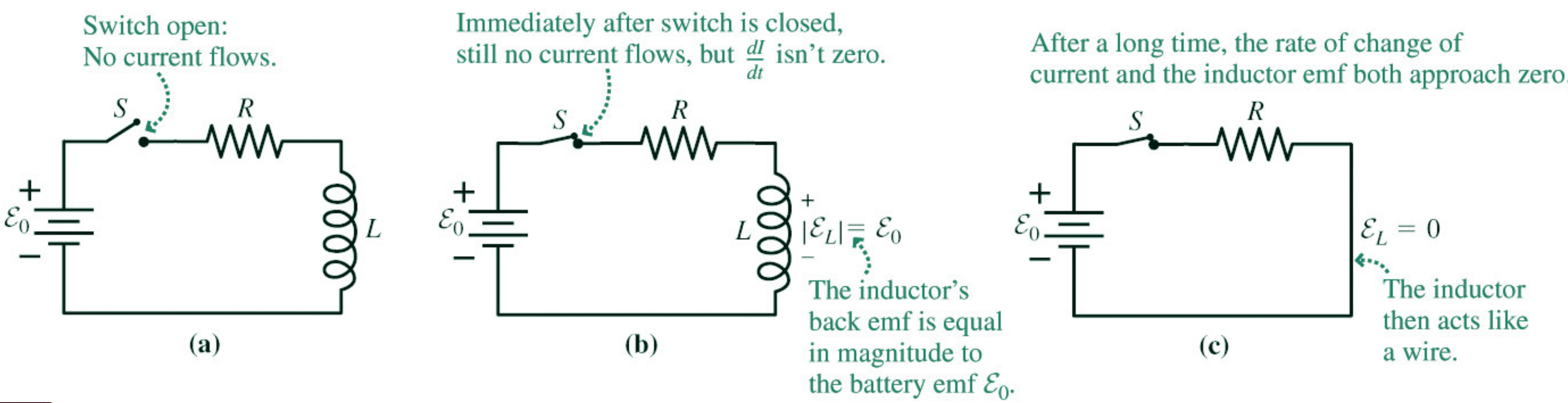
  - Considering the uniform magnetic field inside a solenoid shows that the **magnetic energy density** is

$$u_{\mathrm{B}} = \frac{B^2}{2\mu_0}$$

  - This is a universal expression: wherever there's a magnetic field, there is energy with density $B^2/2\mu_0$.

Connecting Magnetism and Circuits

- The equation $\mathcal{E}_{\mathrm{L}} = -L\dfrac{dI}{dt}$ shows that **the current through an inductor can't change instantaneously.**
  - Otherwise an impossible infinite emf would develop.
  - Rapid changes in current result in large, possibly dangerous emfs.
  - So buildup of current in an *RL* circuit occurs gradually.

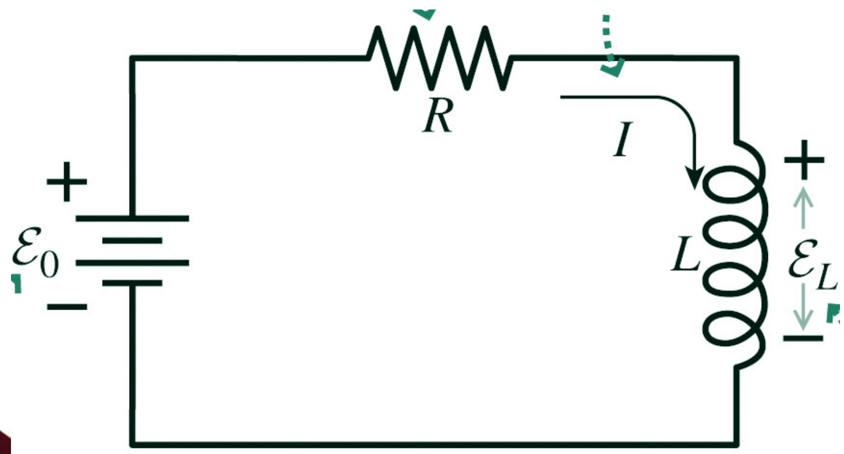


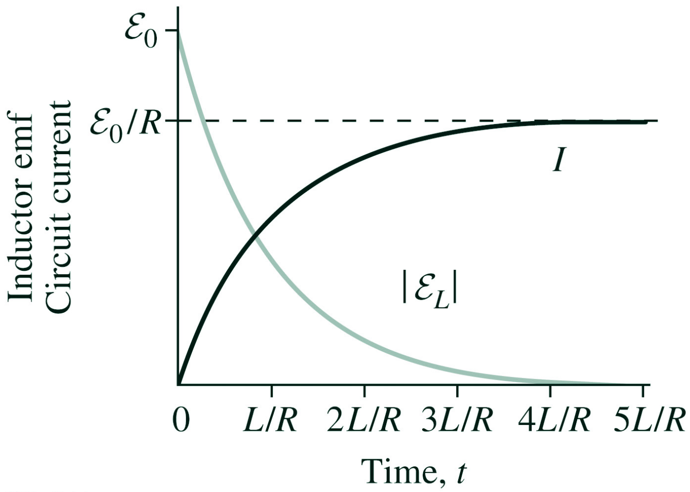
The Inductive Time Constant

- The loop rule for *RL* circuit gives $\mathcal{E}_0 - IR + \mathcal{E}_\mathrm{L} = 0$
- With $dI/dt = -\mathcal{E}_L/L$ we can solve for current to find

$$I(t) = \frac{\mathcal{E}_0}{R}\left(1 - e^{-Rt/L}\right)$$

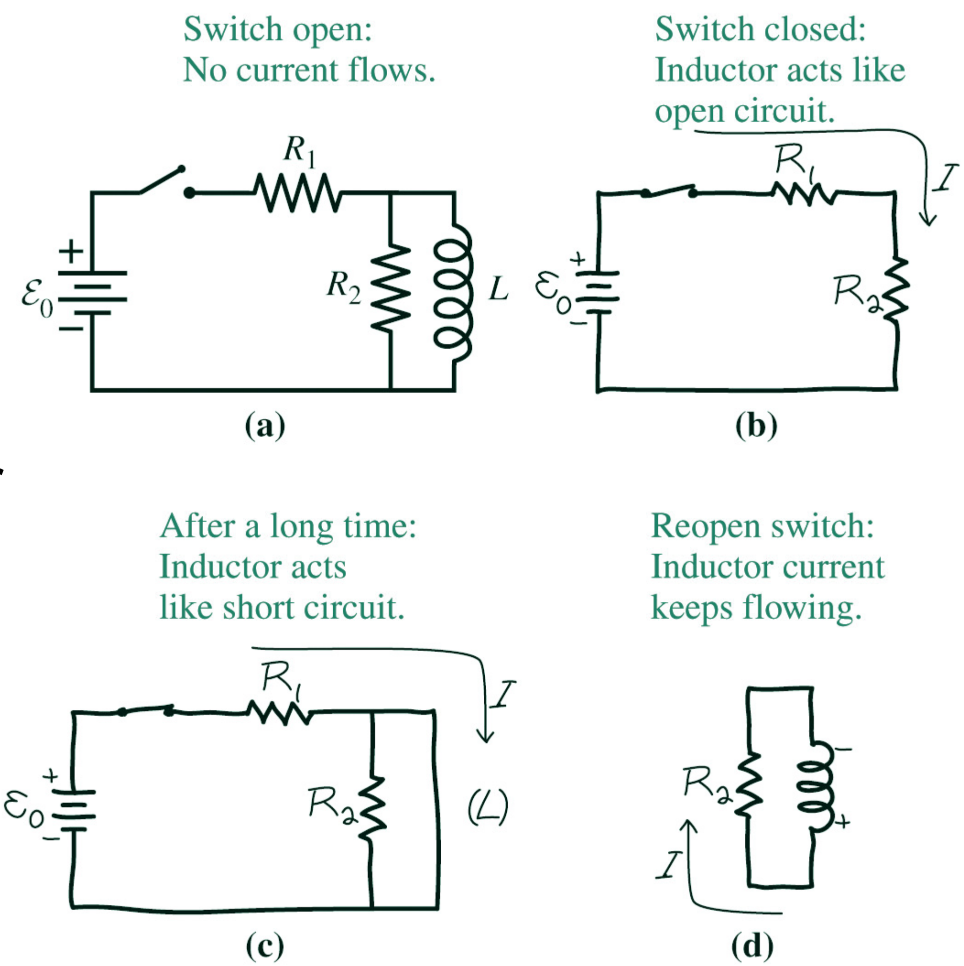
- The inductor current starts at zero and builds up with time constant *L*/*R.*
  - As the current increases, its rate of change decreases.
  - The inductor emf therefore decays exponentially to zero.
    - This decay has the same time constant *L*/*R*.





Short- and Long-Term Behavior of Inductors

- Since current can't change instantaneously, an inductor in an *RL* circuit with no current through it acts instantaneously like an open circuit.
  - If there's current flowing, it keeps flowing momentarily despite changes in the circuit.
- After a long time, the inductor current stops changing.
  - Therefore, the inductor emf is zero.
  - So the inductor acts like an ordinary wire.

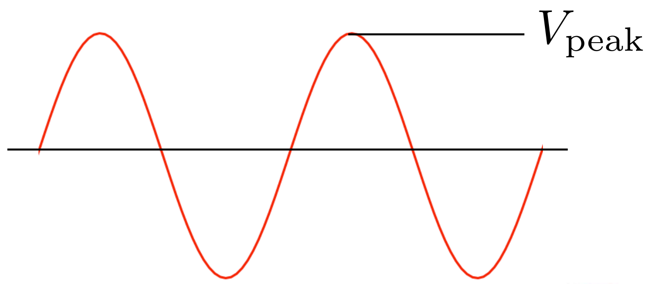


AC Circuits

- We've seen two types of time-dependent electrical circuits and one time of time-independent electrical circuit
  - For resistor circuits, V=IR and voltage and current weren't changing over time
  - For capacitor and inductor circuits, we saw exponential decay behavior in current and voltage with time constants (in the lab and previous slides)

- It's even more useful to look at what happens when we change voltage and/or current in a circuit in a periodic way
  - The basic building blocks of periodic motion: trig functions
  - So we'll have to review some concepts from circular and simple harmonic motion from last semester
  - Start with

$$V(t) = V_{\text{peak}} \sin(2\pi f t)$$



Describing Alternating Current and Voltage

- Sinusoidally varying AC current and voltage are characterized by frequency, amplitude, and phase.
  - In most everyday situations frequency *f* is given in hertz (Hz), or cycles per second.
    - It's often more convenient to use angular frequency: $\omega = 2\pi f$.
  - The alternating voltage and current can be written as

$$V = V_\mathrm{p} \sin(\omega t + \phi_V),\ I = I_\mathrm{p} \sin(\omega t + \phi_I)$$

  - Amplitude can be given as the peak value, or as the **root-mean-square** (rms) value.
    - For a sinusoidally varying quantity, the rms value is the peak value divided by $\sqrt{2}$:

$$V_\mathrm{rms} = \frac{V_\mathrm{p}}{\sqrt{2}},\quad I_\mathrm{rms} = \frac{I_\mathrm{p}}{\sqrt{2}}$$