

# USPAS Accelerator Physics 2015

## Old Dominion University

### Chapter 4: Magnets and Magnet Technology

Todd Satogata (Jefferson Lab) / [satogata@jlab.org](mailto:satogata@jlab.org)  
Vasiliy Morozov (Jefferson Lab) / [morozov@jlab.org](mailto:morozov@jlab.org)  
Alex Castilla (ODU) / [acast020@odu.edu](mailto:acast020@odu.edu)  
<http://www.toddsatogata.net/2015-USPAS>

# Overview

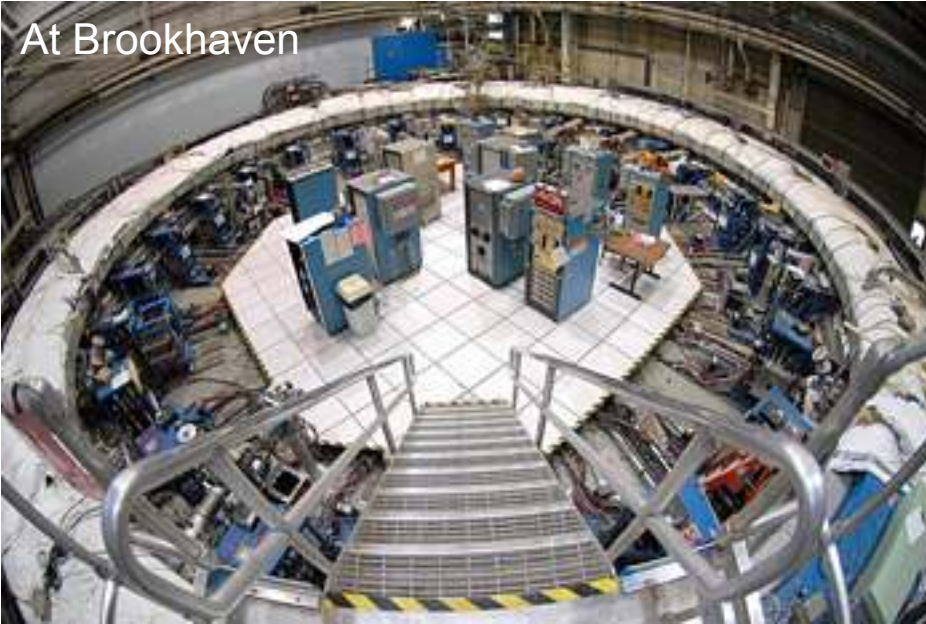
- Section 4.1: Back to Maxwell
  - Parameterizing fields in accelerator magnets
  - Symmetries, comments about magnet construction
- Sections 4.2-3: Relating currents and fields
  - Equipotentials and contours, dipoles and quadrupoles
  - Thin magnet kicks and that ubiquitous rigidity
  - Complications: hysteresis, end fields
- Section 4.4: More details about dipoles
  - Sector and rectangular bends; edge focusing
- Extras: Superconducting magnets
  - RHIC, LHC, the future
- Section 4.5: Ideal Solenoid (homework!)

## Other References

- Magnet design and a construction is a specialized field all its own
  - Electric, Magnetic, Electromagnetic modeling
    - 2D, 3D, static vs dynamic
  - Materials science
    - Conductors, superconductors, ferrites, superferrites
  - Measurements and mapping
    - e.g. g-2 experiment: 1 PPM field uniformity, 14m SC dipole
- Entire USPAS courses have been given on just superconducting magnet design
  - <http://www.bnl.gov/magnets/staff/gupta/scmag-course/>  
(Ramesh Gupta and Animesh Jain, BNL)

# g-2 magnet

At Brookhaven



- Magnet moved from Brookhaven to Fermilab in 2013
  - 17 tons, 44m circumference, 18 cm gap
  - 35 days, over 3200 miles
  - <http://muon-g-2.fnal.gov/bigmove/>



At Fermilab

On the move



# EM/Maxwell Review I

- Recall our relativistic Lorentz force

$$\frac{d(\gamma m \vec{v})}{dt} = q \left( \vec{E} + \vec{v} \times \vec{B} \right)$$

- For large  $\gamma$  common in accelerators, magnetic fields are much more effective for changing particle momenta
- Can mostly separate E (RF, septa) and B (DC magnets)
  - Some exceptions, e.g. plasma wakefields, betatrons, RFQ
- Easiest/simplest: magnets with constant B field
  - Constant-strength optics
    - Most varying B field accelerator magnets change field so slowly that E fields are negligible
    - Consistent with our assumptions for “standard canonical coordinates”, p 49 Conte and MacKay and yesterday

## EM/Maxwell Review II

- Maxwell's Equations for  $\vec{B}$ ,  $\vec{H}$  and magnetization  $\vec{M}$  are

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \vec{j} \quad \vec{H} \equiv \vec{B}/\mu - \vec{M}$$

- A magnetic vector potential  $\vec{A}$  exists

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad \text{since } \vec{\nabla} \cdot \vec{\nabla} \times \vec{A} = 0$$

- Transverse 2D ( $B_z=H_z=0$ ), paraxial approx ( $p_{x,y} \ll p_0$ )
- Away from magnet coils ( $\vec{j} = 0$ ,  $\vec{M} = 0$ )
  - Simple homogeneous differential equations for fields

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \quad \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} = 0$$

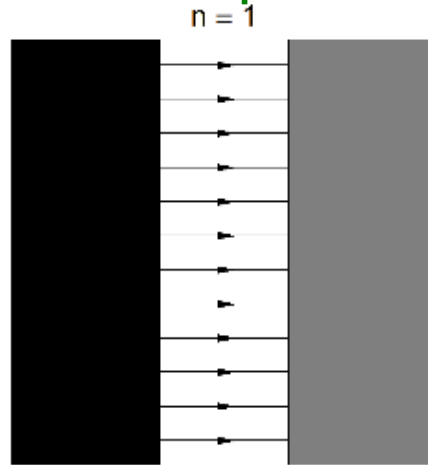
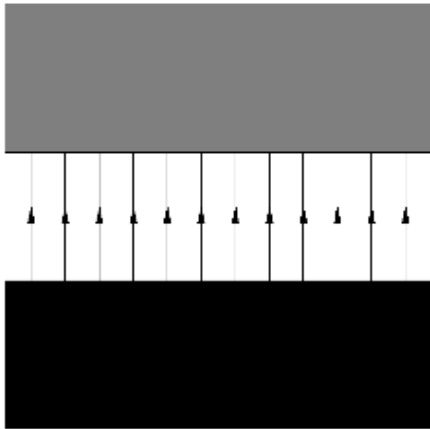
# Parameterizing Solutions

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \qquad \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} = 0$$

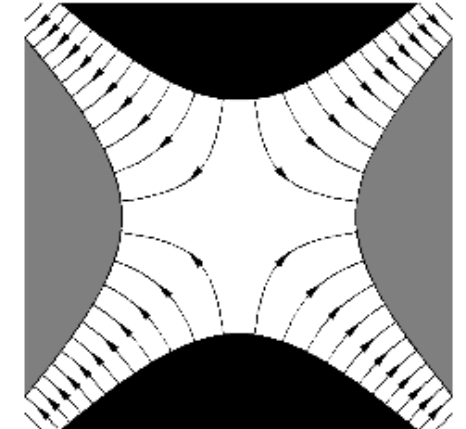
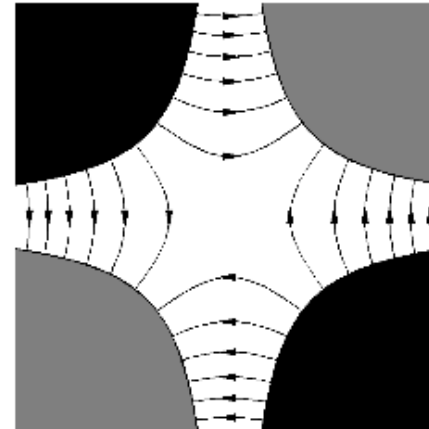
- What are solutions to these equations?
  - **Constant field:**  $\vec{B} = B_x x^0 \hat{x} + B_y y^0 \hat{y}$ 
    - Dipole fields, usually either only  $B_x$  or  $B_y$
    - 360 degree ( $2\pi$ ) rotational “symmetry”
  - **First order field:**  $\vec{B} = (B_{xx}x + B_{xy}y)\hat{x} + (B_{yx}x + B_{yy}y)\hat{y}$ 
    - Maxwell gives  $B_n = B_{xx} = -B_{yy}$  and  $B_s = B_{xy} = B_{yx}$
$$\vec{B} = B_n(x\hat{x} - y\hat{y}) + B_s(x\hat{y} + y\hat{x})$$
    - Quadrupole fields, either normal  $B_n$  or skew  $B_s$
    - 180 degree ( $\pi$ ) rotational symmetry
    - 90 degree rotation interchanges normal/skew
  - Higher order...

# Visualizing Fields I

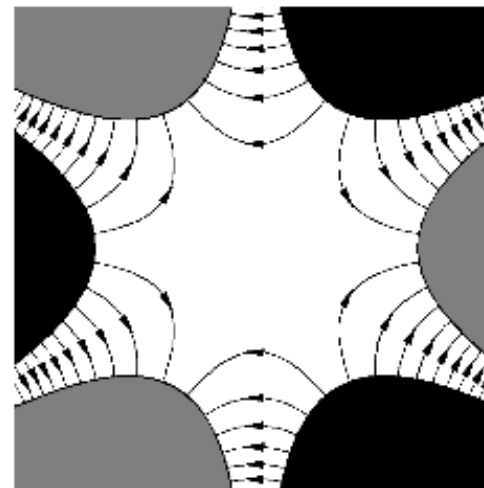
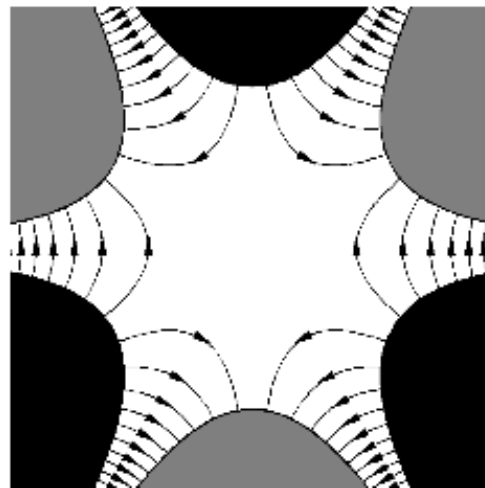
Dipole and “skew” dipole  
 $n = 1$



Quad and skew quad  
 $n = 2$



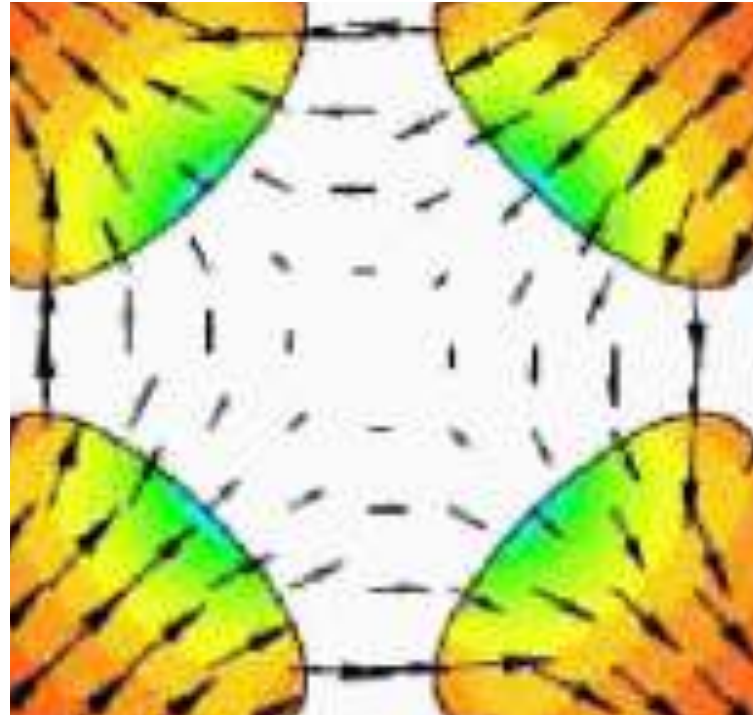
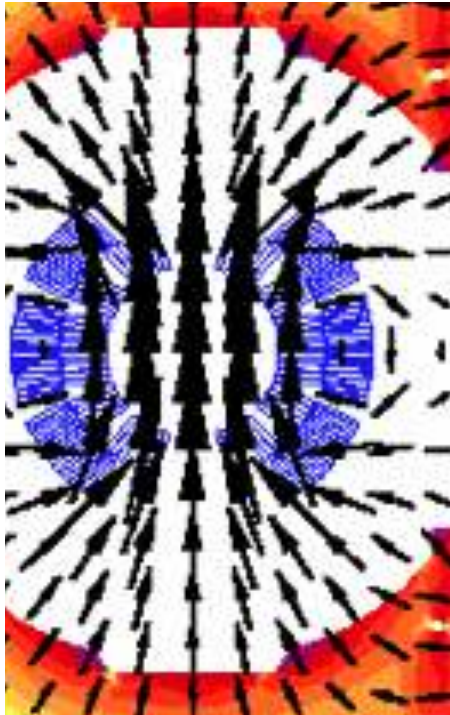
Sextupole and skew sextupole  
 $n = 3$





## Visualizing Dipole and Quadrupole Fields II

LHC  
dipole  
field



LEP  
quadrupole  
field

- LHC dipole:  $B_y$  gives horizontal bending
- LEP quadrupole:  $B_y$  on x axis,  $B_x$  on y axis
  - Horizontal focusing=vertical defocusing or vice-versa
  - No coupling between horizontal/vertical motion
    - Note the nice “harmonic” field symmetries

# General Multipole Field Expansions

- Rotational symmetries, cylindrical coordinates
  - Power series in radius  $r$  with angular harmonics in  $\theta$

$$x = r \cos \theta \quad y = r \sin \theta$$

$$B_y = B_0 \sum_{n=0}^{\infty} \left(\frac{r}{a}\right)^n (b_n \cos n\theta - a_n \sin n\theta)$$

$$B_x = B_0 \sum_{n=0}^{\infty} \left(\frac{r}{a}\right)^n (a_n \cos n\theta + b_n \sin n\theta)$$

- Need “reference radius”  $a$  (to get units right)
- $(b_n, a_n)$  are called (normal, skew) **multipole coefficients**
- We can also write this succinctly using de Moivre as

$$B_x - iB_y = B_0 \sum_{n=0}^{\infty} (a_n - ib_n) \left(\frac{x + iy}{a}\right)^n$$

## But Do These Equations Solve Maxwell?

- Yes ☺ Convert Maxwell's eqns to cylindrical coords

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \qquad \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} = 0$$

$$\frac{\partial(\rho B_\rho)}{\partial \rho} + \frac{\partial B_\theta}{\partial \theta} = 0 \qquad \frac{\partial(\rho B_\theta)}{\partial \rho} - \frac{\partial B_\rho}{\partial \theta} = 0$$

- Aligning  $r$  along the  $x$ -axis it's easy enough to see

$$\frac{\partial}{\partial x} \Rightarrow \frac{\partial}{\partial r} \qquad \frac{\partial}{\partial y} \Rightarrow \frac{1}{r} \frac{\partial}{\partial \theta}$$

- In general it's (much, much) more tedious but it works

$$\frac{\partial r}{\partial x} = \frac{1}{\cos \theta}, \quad \frac{\partial \theta}{\partial x} = \frac{-1}{r \sin \theta}, \quad \frac{\partial r}{\partial y} = \frac{1}{\sin \theta}, \quad \frac{\partial \theta}{\partial y} = \frac{1}{r \cos \theta}$$

$$\frac{\partial B_x}{\partial x} = \frac{\partial B_x}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial B_x}{\partial \theta} \frac{\partial \theta}{\partial x} \dots$$

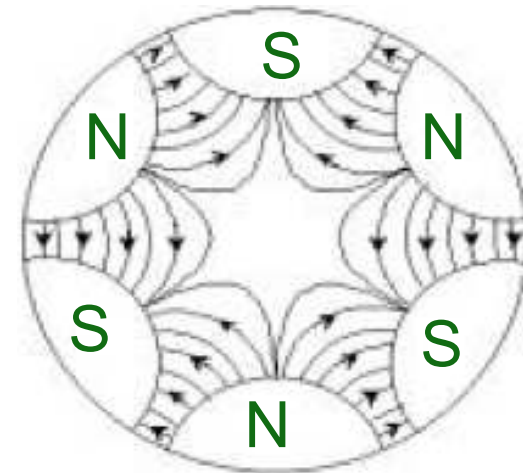
# Multipoles

$(b,a)_n$  “unit” is  $10^{-4}$  (natural scale)     $(b,a)_n$  (US) =  $(b,a)_{n+1}$

coefficient	multipole	field	notes
$b_0$	normal dipole	$B_y = B_0 b_0$	horz. bending
$a_0$	skew dipole	$B_x = B_0 a_0$	vert. bending
$b_1$	normal quadrupole	$B_x = B_0 \left(\frac{r}{a}\right) b_1 \sin \theta = B_0 \left(\frac{y}{a}\right) b_1$ $B_y = B_0 \left(\frac{r}{a}\right) b_1 \cos \theta = B_0 \left(\frac{x}{a}\right) b_1$	focusing defocusing
$a_1$	skew quadrupole	$B_x = B_0 \left(\frac{r}{a}\right) a_1 \cos \theta = B_0 \left(\frac{x}{a}\right) a_1$ $B_y = -B_0 \left(\frac{r}{a}\right) a_1 \sin \theta = -B_0 \left(\frac{y}{a}\right) a_1$	coupling
$b_2$	normal sextupole	$B_x = B_0 \left(\frac{r}{a}\right)^2 b_2 \sin(2\theta)$ $B_y = B_0 \left(\frac{r}{a}\right)^2 b_1 \cos(2\theta)$	nonlinear!



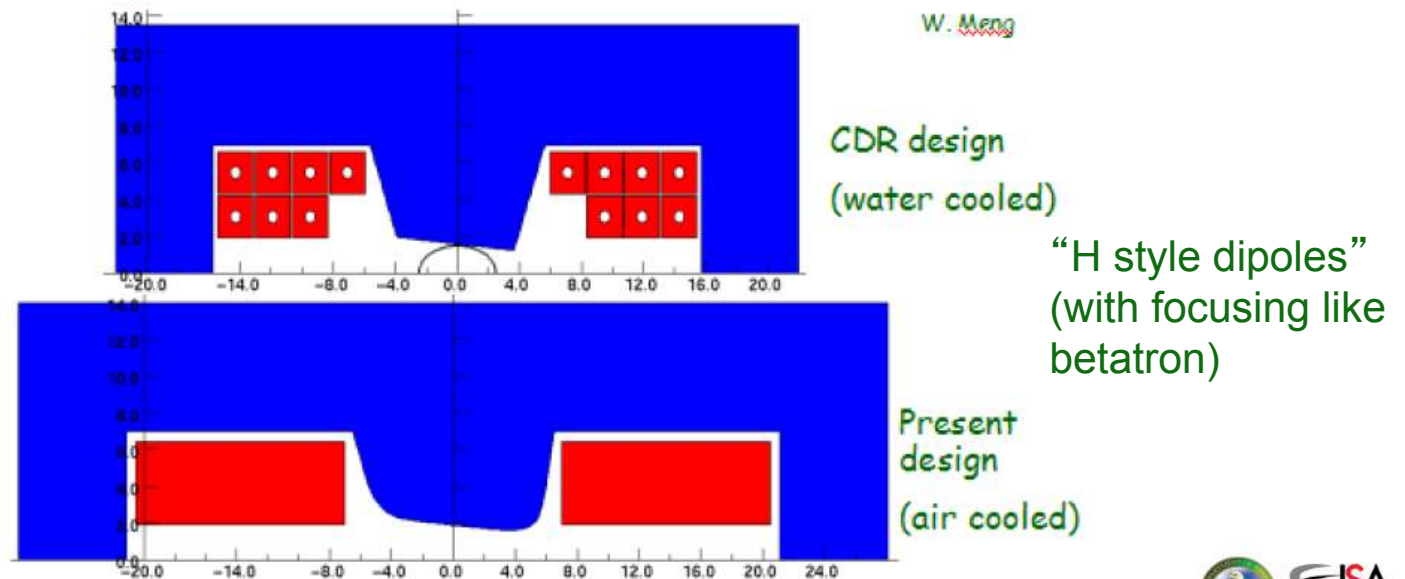
Eletra magnets



# Multipole Symmetries

- Dipole has  $2\pi$  rotation symmetry (or  $\pi$  upon current reversal)
- Quad has  $\pi$  rotation symmetry (or  $\pi/2$  upon current reversal)
- k-pole has  $2\pi/k$  rotation symmetry upon current reversal
- We try to enforce symmetries in design/construction
  - Limits permissible magnet errors
  - Higher order fields that obey main field symmetry are called allowed multipoles

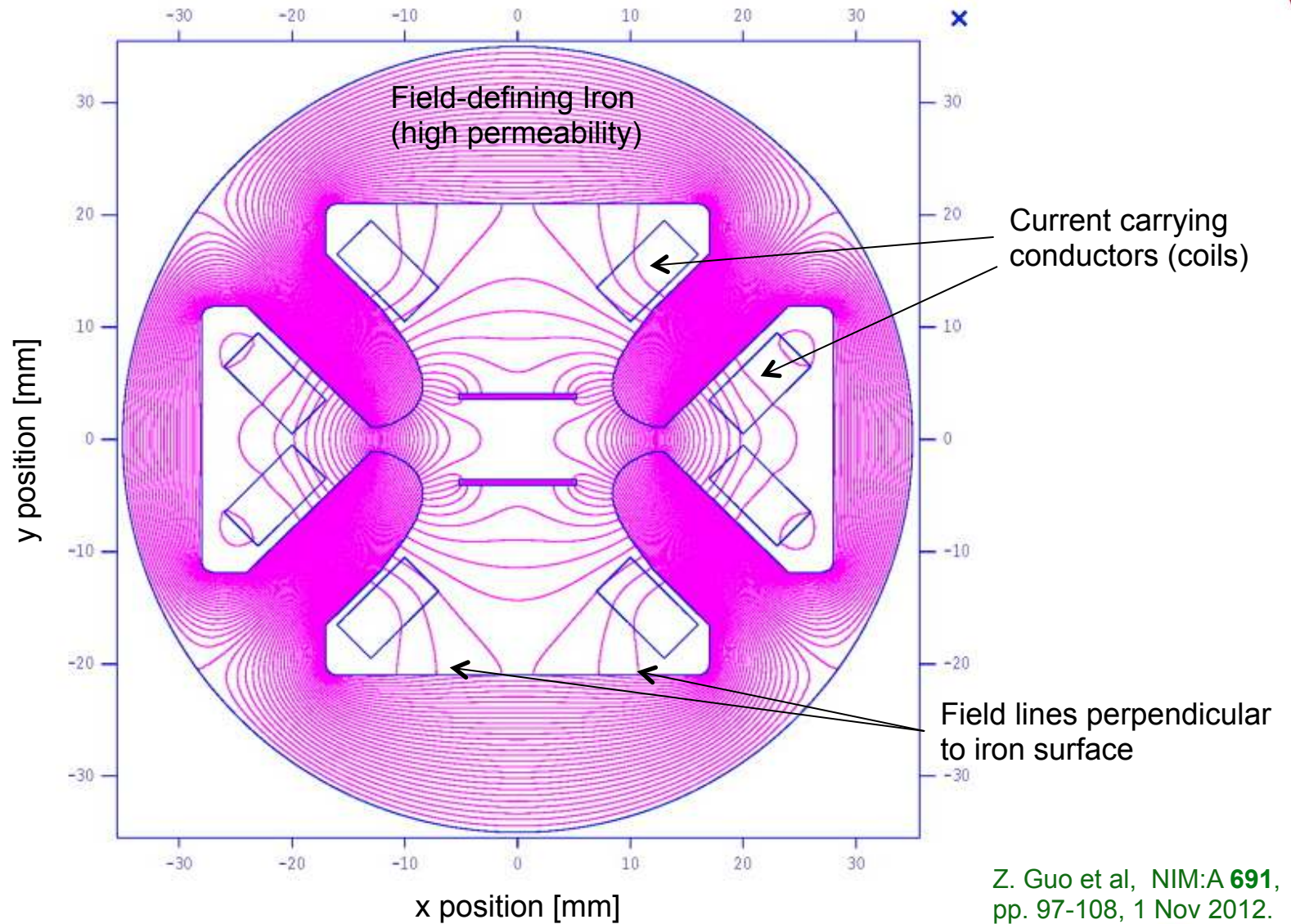
RCMS half-dipole laminations  
(W. Meng, BNL)



## Multipole Symmetries II

- So a dipole ( $n=0$ , 2 poles) has allowed multipoles:
  - Sextupole ( $n=2$ , 6 poles), Decapole ( $n=4$ , 10 poles)...
- A quadrupole ( $n=1$ , 4 poles) has allowed multipoles:
  - Dodecapole ( $n=5$ , 12 poles), Twenty-pole ( $n=9$ , 20 poles)...
- General allowed multipoles:  $(2k+1)(n+1)-1$ 
  - Or, more conceptually, (3,5,7,...) times number of poles
- Other multipoles are forbidden by symmetries
  - Smaller than allowed multipoles, but no magnets are perfect
    - Large measured forbidden multipoles mean fabrication or fundamental design problems!
- Better magnet pole face quality with punched laminations
- Dynamics are usually dominated by lower-order multipoles

## 4.2: Equipotentials and Contours



Z. Guo et al, NIM:A **691**, pp. 97-108, 1 Nov 2012.

## 4.2: Equipotentials and Contours

- Let's get around to designing some magnets
  - Conductors on outside, field on inside
  - Use high-permeability iron to shape fields: **iron-dominated**
    - Pole faces are very nearly equipotentials,  $\perp$  B,H field
    - We work with a magnetostatic *scalar* potential  $\Psi$
    - B, H field lines are  $\perp$  to equipotential lines of  $\Psi$

$$\vec{H} = -\vec{\nabla}\Psi$$

$$\Psi = \sum_{n=0}^{\infty} \frac{a}{n+1} \left(\frac{r}{a}\right)^{n+1} [F_n \cos((n+1)\theta) + G_n \sin((n+1)\theta)]$$

$$\text{where } G_n \equiv B_0 b_n / \mu_0, F_n \equiv B_0 a_n / \mu_0$$

This comes from integrating our B field expansion.  
Let's look at normal multipoles  $G_n$  and pole faces...



## Equipotentials and Contours II

- For general  $G_n$  normal multipoles (i.e. for  $b_n$ )

$$\Psi(\text{equipotential for } b_n) \propto r^{n+1} \sin[(n+1)\theta] = \text{constant}$$

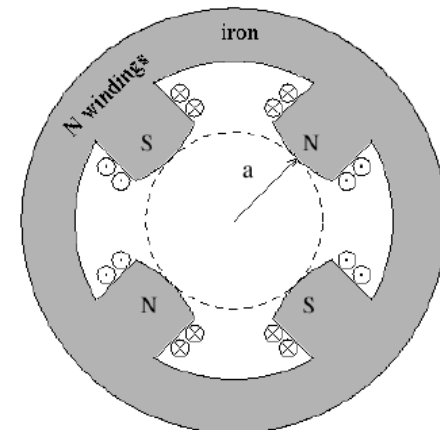
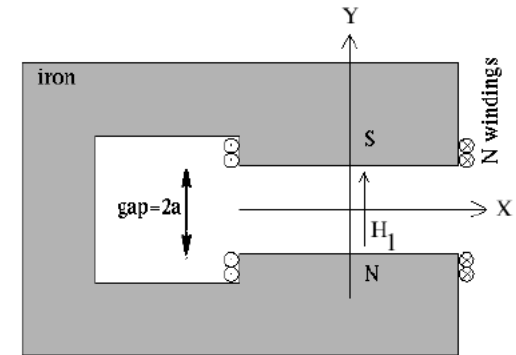
- Dipole ( $n=0$ ):  $\Psi(\text{dipole}) \propto r \sin \theta = y$ 
  - Normal dipole pole faces are  $y=\text{constant}$

- Quadrupole ( $n=1$ ):

$$\Psi(\text{quadrupole}) \propto r^2 \sin(2\theta) = 2xy$$

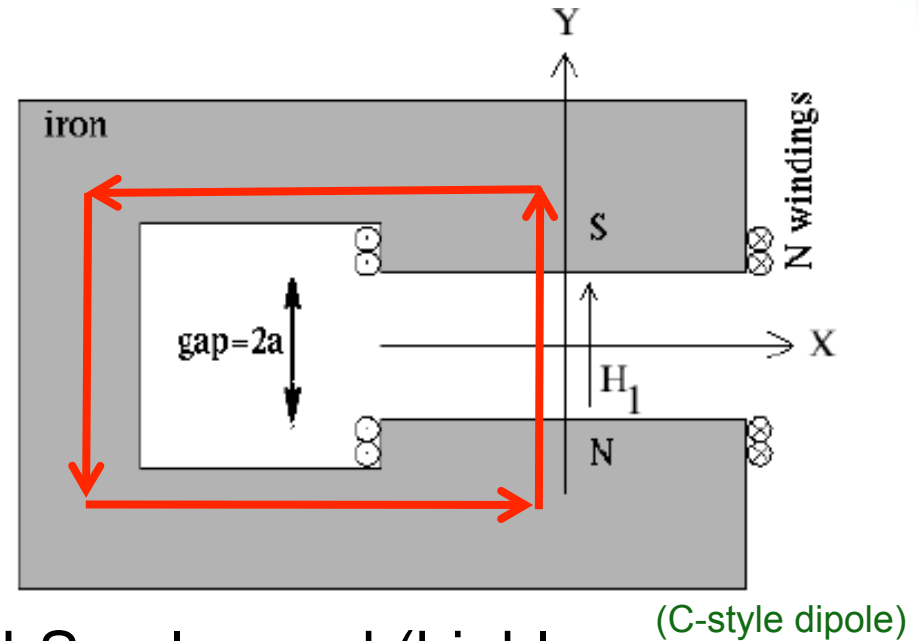
- Normal quadrupole pole faces are  $xy=\text{constant}$  (hyperbolic)

- So what conductors and currents are needed to generate these fields?



# Dipole Field/Current

- Use Ampere's law to calculate field in gap
  - N "turns" of conductor around each pole
  - Each turn of conductor carries current I



- Field integral is through N-S poles and (highly permeable) iron (including return path)

$$2NI = \oint \vec{H} \cdot d\vec{l} = 2aH \Rightarrow H = \frac{NI}{a}, \quad B = \frac{\mu_0 NI}{a}$$

- NI is in "Amp-turns",  $\mu_0 \sim 1.257 \text{ cm-G/A}$ 

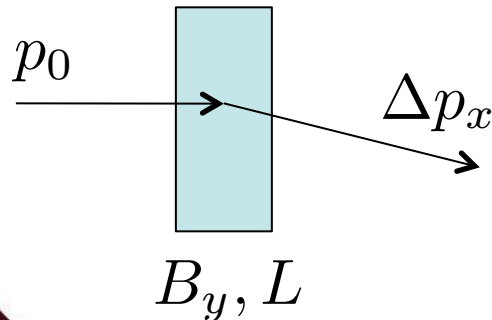
$$\Delta x' = \frac{BL}{(B\rho)}$$
  - So  $a=2\text{cm}$ ,  $B=600\text{G}$  requires  $NI \sim 955 \text{ Amp-turns}$

## Wait, What's That $\Delta x'$ Equation?

$$\Delta x' = \frac{BL}{(B\rho)} \quad \leftarrow \text{Field, length: Properties of magnet}$$

$$\Delta x' = \frac{BL}{(B\rho)} \quad \leftarrow \text{Rigidity: property of beam (really } p/q\text{!)}$$

- This is the angular transverse kick from a thin hard-edge dipole, like a dipole corrector
  - Really a change in  $p_x$  but paraxial approximation applies
  - The  $B$  in  $(B\rho)$  is not necessarily the main dipole  $B$
  - The  $\rho$  in  $(B\rho)$  is not necessarily the ring circumference/ $2\pi$
  - And neither is related to this particular dipole kick!



$$F_x = \frac{\Delta p_x}{\Delta t} = q(\beta c)B_y \quad \Delta t = L/(\beta c)$$

$$\Delta p_x = qLB_y$$

$$\Delta x' \approx \frac{\Delta p_x}{p} = \frac{q}{p}LB_y = \frac{B_y L}{(B\rho)}$$

# Quadrupole Field/Current

- Use Ampere's law again
  - Easiest to do with magnetic potential  $\Psi$ , encloses  $2NI$

$$\Psi(a, \theta) = \frac{a B_0 b_1}{2 \mu_0} \sin(2\theta)$$

$$2NI = \oint \vec{H} \cdot d\vec{l} = \Psi(a, \pi/4) - \Psi(a, -\pi/4) = \frac{a B_0 b_1}{\mu_0}$$

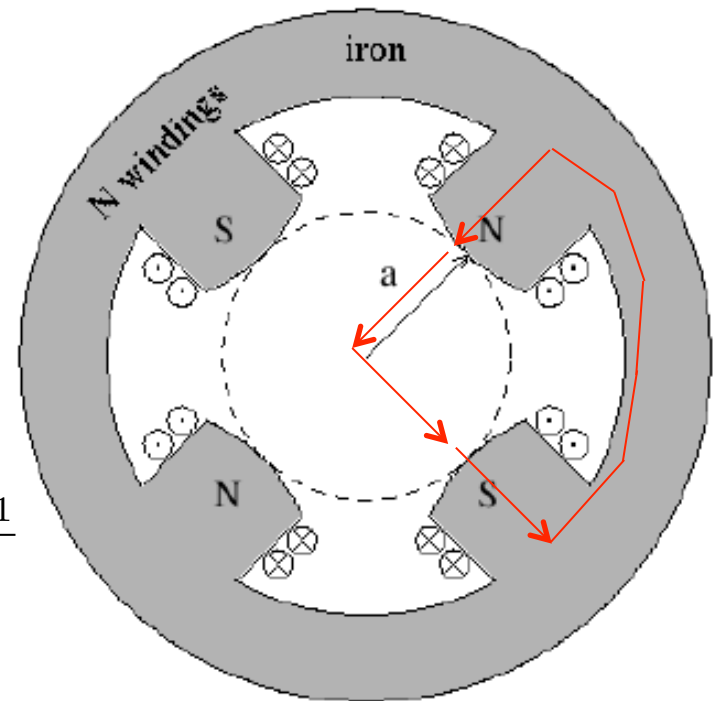
$$\Psi = NI \sin(2\theta) = \frac{2NI}{a^2} xy$$

$$\vec{H} = \nabla \Psi = \frac{2NI}{a^2} (y\hat{x} + x\hat{y}) \quad \vec{B} = \frac{2\mu_0 NI}{a^2} (y\hat{x} + x\hat{y})$$

- Quadrupole strengths are expressed as transverse gradients

$$B' \equiv \left. \frac{\partial B_y}{\partial x} \right|_{y=0} = \frac{\partial B_x}{\partial y} = \frac{2\mu_0 NI}{a^2} \quad \Delta x' = \frac{B' L}{(B\rho)} x$$

(NB: Be careful, ' has different meaning in B', B'', B''' ...)



# Quadrupole Transport Matrix

- Paraxial equations of motion for constant quadrupole field

$$\frac{d^2x}{ds^2} + kx = 0 \quad \frac{d^2y}{ds^2} - ky = 0 \quad s \equiv \beta ct$$

$$k \equiv \frac{B'}{(B\rho)} = \frac{2\mu_0 NI}{a^2} \left( \frac{q}{p} \right)$$

- Integrating over a magnet of length L gives (exactly)

Focusing  
Quadrupole

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos(L\sqrt{k}) & \frac{1}{\sqrt{k}} \sin(L\sqrt{k}) \\ -\sqrt{k} \sin(L\sqrt{k}) & \cos(L\sqrt{k}) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} = \mathbf{M}_F \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

Defocusing  
Quadrupole

$$\begin{pmatrix} y \\ y' \end{pmatrix} = \begin{pmatrix} \cosh(L\sqrt{k}) & \frac{1}{\sqrt{k}} \sinh(L\sqrt{k}) \\ \sqrt{k} \sinh(L\sqrt{k}) & \cosh(L\sqrt{k}) \end{pmatrix} \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix} = \mathbf{M}_D \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}$$

# Thin Quadrupole Transport Matrix

Focusing Quadrupole  $\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos(L\sqrt{k}) & \frac{1}{\sqrt{k}} \sin(L\sqrt{k}) \\ -\sqrt{k} \sin(L\sqrt{k}) & \cos(L\sqrt{k}) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} = \mathbf{M}_F \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$

Defocusing Quadrupole  $\begin{pmatrix} y \\ y' \end{pmatrix} = \begin{pmatrix} \cosh(L\sqrt{k}) & \frac{1}{\sqrt{k}} \sinh(L\sqrt{k}) \\ \sqrt{k} \sinh(L\sqrt{k}) & \cosh(L\sqrt{k}) \end{pmatrix} \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix} = \mathbf{M}_D \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}$

- Quadrupoles are often “thin”
  - Focal length is much longer than magnet length
- Then we can use the thin-lens approximation  $\sqrt{k}L \ll 1$

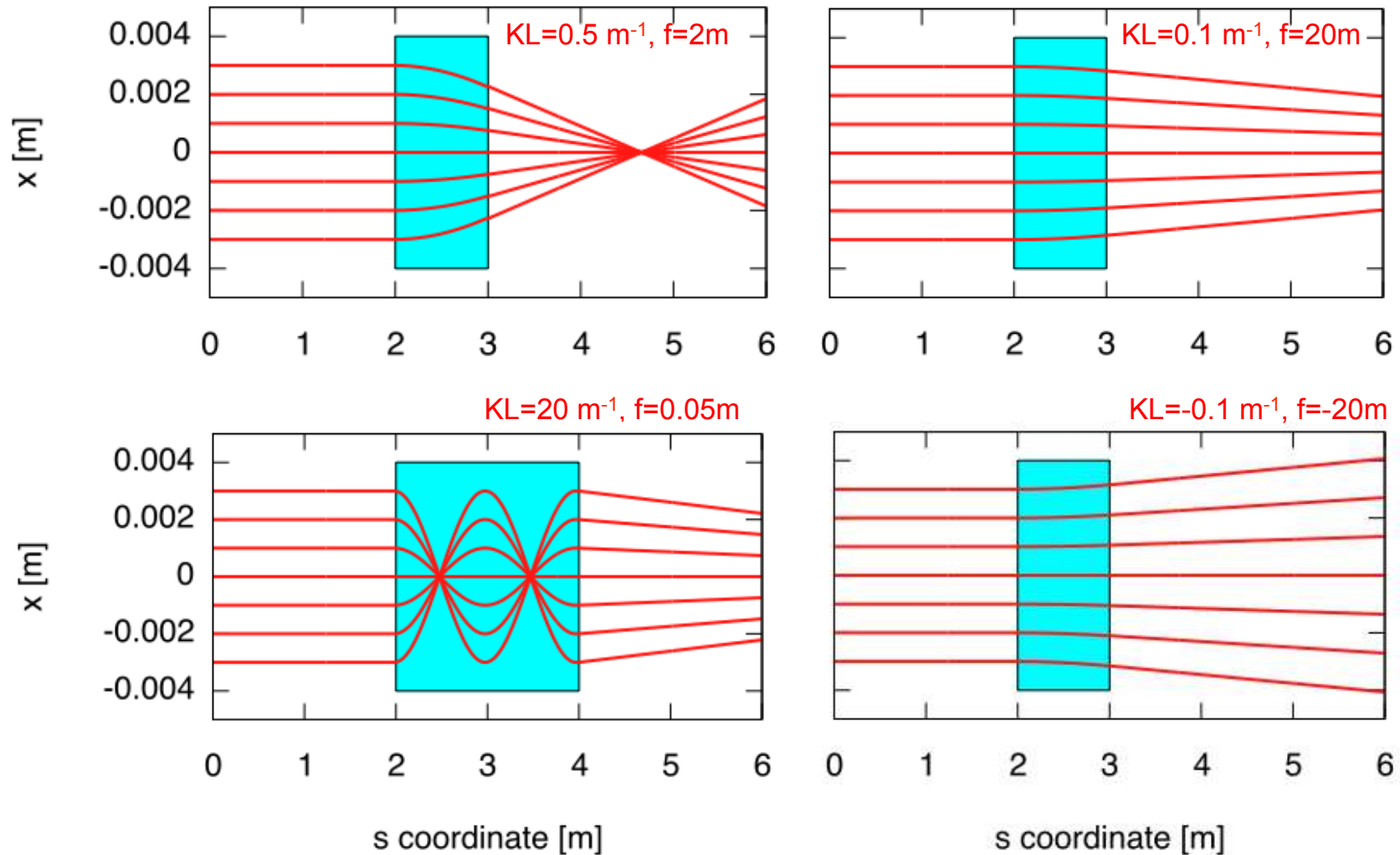
Thin quadrupole approximation

$$\mathbf{M}_{F,D} = \begin{pmatrix} 1 & 0 \\ \mp kL & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \mp \frac{1}{f} & 1 \end{pmatrix}$$

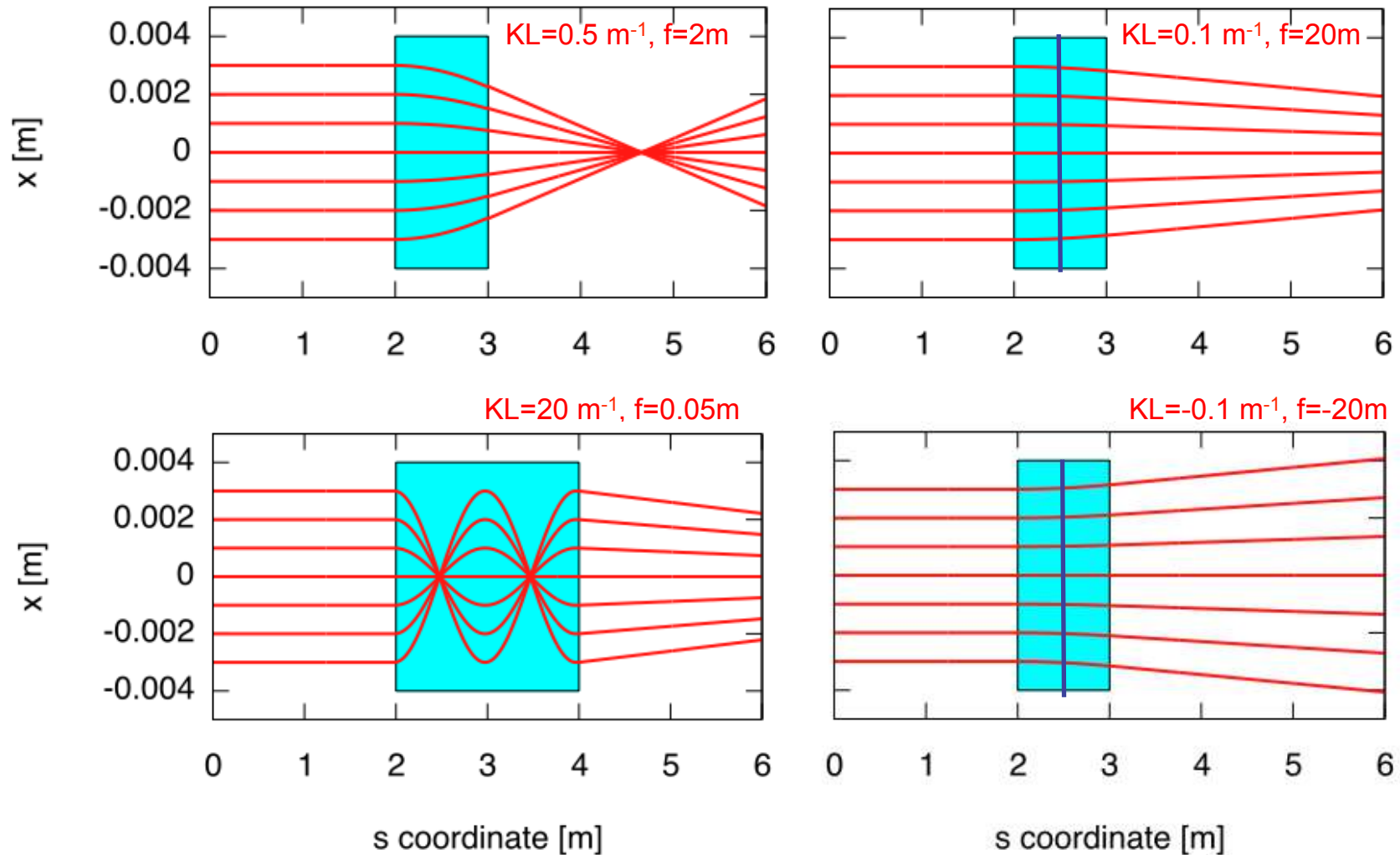
where  $f=1/(kL)$  is the quadrupole focal length

$$\Delta x' = \frac{B' L}{(B\rho)} x$$

# Picturing Drift and Quadrupole Motion



# Picturing Drift and Quadrupole Motion



Thin Quadrupole Approximations



## Higher Orders

- We can follow the full expansion for 2(n+1)-pole:

$$\Psi_n = NI \left(\frac{r}{a}\right)^{n+1} \sin((n+1)\theta)$$

$$H_x = (n+1) \frac{NI}{a} \left(\frac{r}{a}\right)^n \sin n\theta \quad H_y = (n+1) \frac{NI}{a} \left(\frac{r}{a}\right)^n \cos n\theta$$

- For the sextupole (n=2) we find the nonlinear field as

$$\vec{B} = \frac{3\mu_0 NI}{a^3} [2xy\hat{x} + (x^2 + y^2)\hat{y}]$$

- Now define a strength as an n<sup>th</sup> derivative

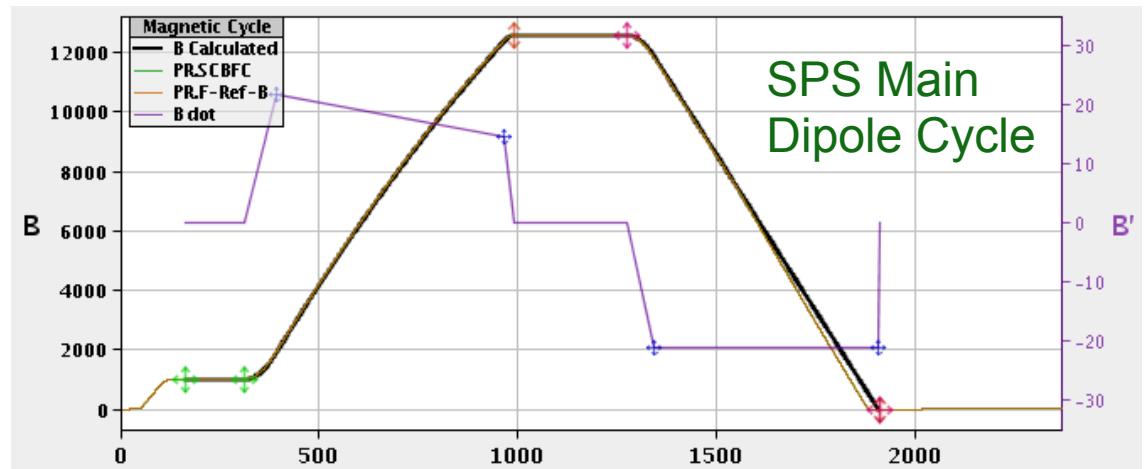
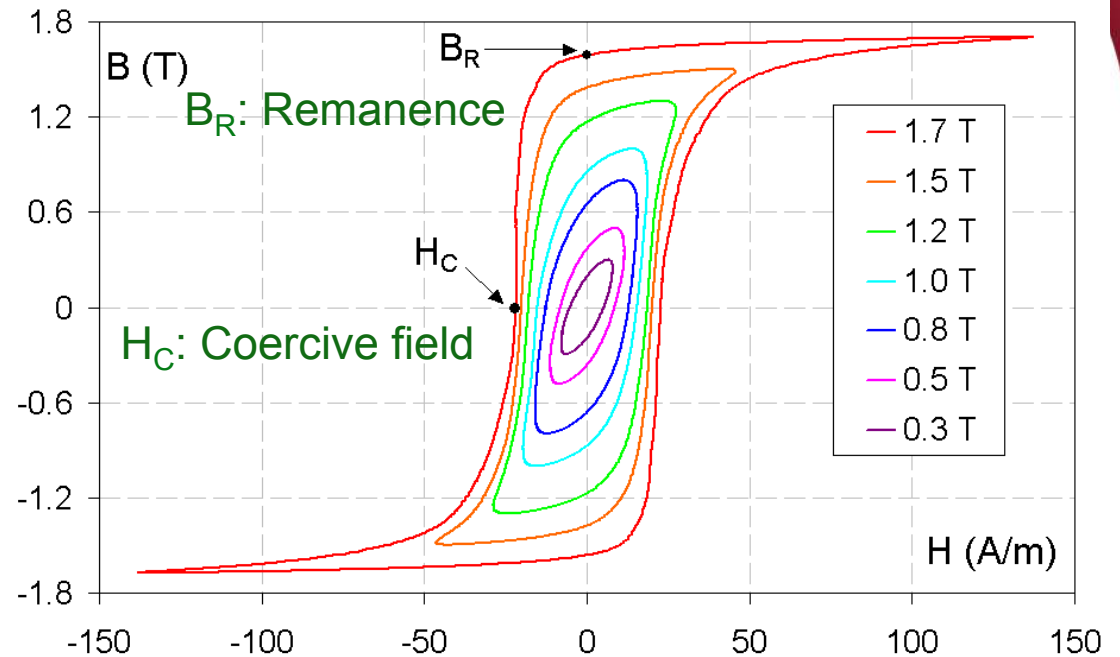
$$B'' \equiv \frac{\partial^2 B_y}{\partial x^2} \Big|_{y=0} = \frac{6\mu_0 NI}{a^3} \quad \Delta x' = \frac{1}{2} \frac{B'' L}{(B\rho)} (x^2 + y^2)$$

(NB: Be careful, ' has different meaning in B', B'', B''' ...)

# Hysteresis

- Magnets with variable current carry “memory”  
Hysteresis is quite important in iron-dominated magnets
- Usually try to run magnets “on hysteresis”

e.g. always on one side of hysteresis loop  
Large spread at large field (1.7 T): saturation  
Degaussing



# End Fields

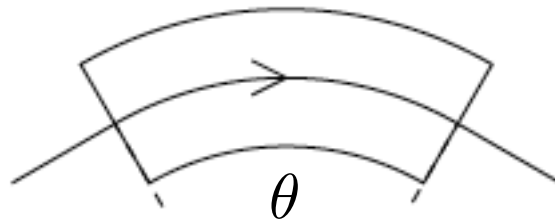
- Magnets are not infinitely long: ends are important!
  - Conductors: where coils usually come in and turn around
  - Longitudinal symmetries break down
  - Sharp corners on iron are first areas to saturate
  - Usually a concern over distances of  $\pm 1-2$  times magnet gap
    - A big deal for short, large-aperture magnets; ends dominate!
- Solution: simulate... a lot
  - Test prototypes too
  - Quadratic end chamfer eases sextupoles from ends (first allowed harmonic of dipole)
- More on dipole end focusing...



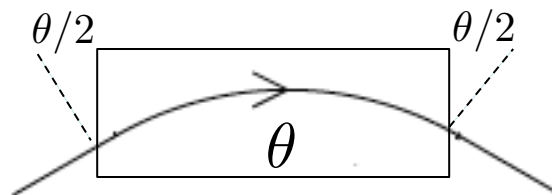
PEFP prototype magnet (Korea)  
9 cm gap, 1.4m long

## 4.4: Dipoles, Sector and Rectangular Bends

- Sector bend (sbend)
  - Beam design entry/exit angles are  $\perp$  to end faces



- Simpler to conceptualize, but harder to build
- Rectangular bend (rbend)
  - Beam design entry/exit angles are half of bend angle



- Easier to build, but must include effects of edge focusing

# Sector Bend Transport Matrix

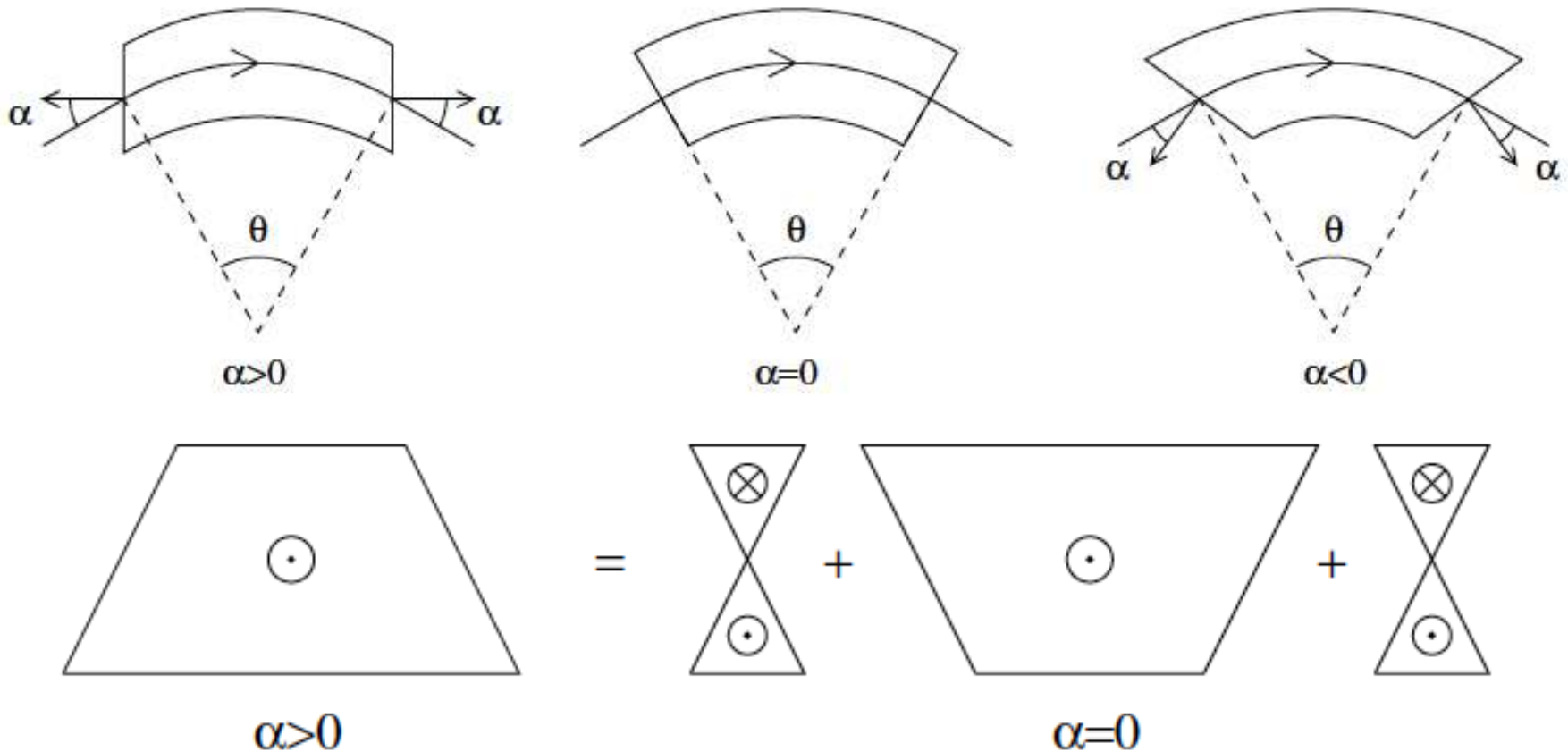


- You did this earlier (eqn 3.109 of text)

$$M_{\text{sector dipole}} = \begin{pmatrix} \cos \theta & \rho \sin \theta & 0 & 0 & 0 & \rho(1 - \cos \theta) \\ -\frac{1}{\rho} \sin \theta & \cos \theta & 0 & 0 & \sin \theta & \\ 0 & 0 & 1 & \rho\theta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\sin \theta & -\rho(1 - \cos(\theta)) & 0 & 0 & 1 & -\rho(\theta - \sin \theta) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- Has all the “right” behaviors
  - But what about rectangular bends?

# Dipole End Angles



- We treat general case of symmetric dipole end angles
  - Superposition: looks like wedges on end of sector dipole
  - Rectangular bends are a special case

## Kick from a Thin Wedge

- The edge focusing calculation requires the kick from a thin wedge

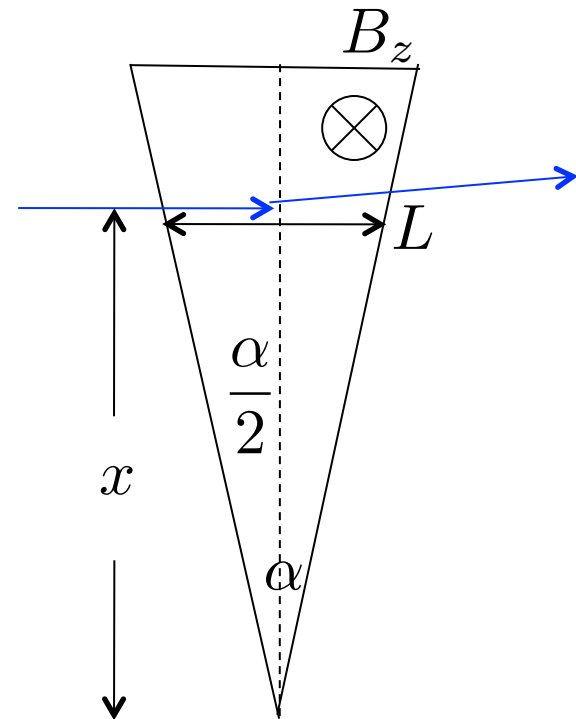
$$\Delta x' = \frac{B_z L}{(B\rho)}$$

What is L? (distance in wedge)

$$\tan\left(\frac{\alpha}{2}\right) = \frac{L/2}{x}$$

$$L = 2x \tan\left(\frac{\alpha}{2}\right) \approx x \tan \alpha$$

$$\text{So } \Delta x' = \frac{B_z \tan \alpha}{(B\rho)} x = \frac{\tan \alpha}{\rho} x$$



Here  $\rho$  is the curvature for a particle of this momentum!!

## Dipole Matrix with Ends

- The matrix of a dipole with thick ends is then

$$M_{\text{sector dipole}}(x, x', \delta) = \begin{pmatrix} \cos \theta & \rho \sin \theta & \rho(1 - \cos \theta) \\ -\frac{1}{\rho} \sin \theta & \cos \theta & \sin \theta \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_{\text{end lens}} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{\tan \alpha}{\rho} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

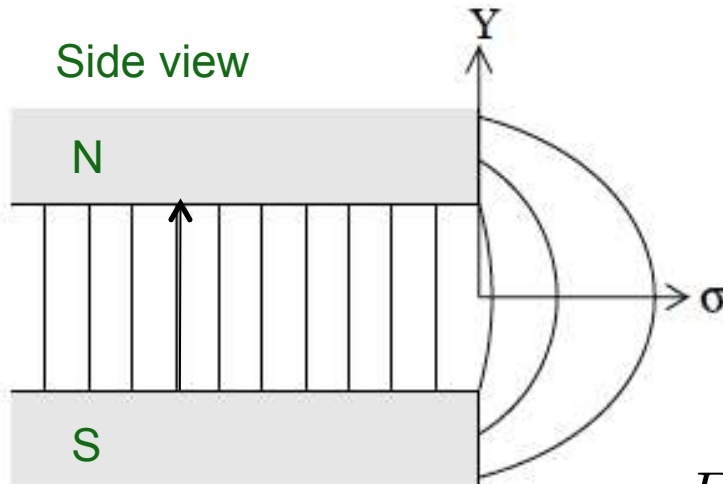
$$M_{\text{edge-focused dipole}} = M_{\text{end lens}} M_{\text{sector dipole}} M_{\text{end lens}}$$

$$M_{\text{edge-focused dipole}} = \begin{pmatrix} \frac{\cos(\theta - \alpha)}{\cos \alpha} & \rho \sin \theta & \rho(1 - \cos \theta) \\ -\frac{\sin(\theta - 2\alpha)}{\rho \cos^2 \alpha} & \frac{\cos(\theta - \alpha)}{\cos \alpha} & \frac{\sin(\theta - \alpha) + \sin \alpha}{\cos \alpha} \\ 0 & 0 & 1 \end{pmatrix}$$

- Rectangular bend is special case where  $\alpha = \theta/2$

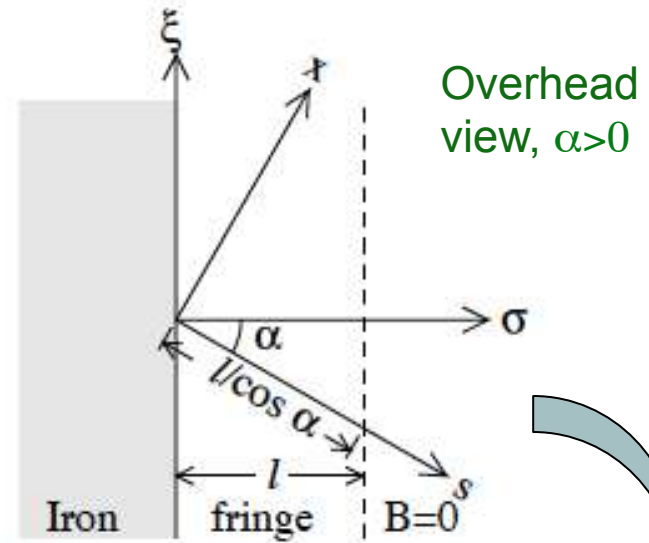


# End Field Example (from book)



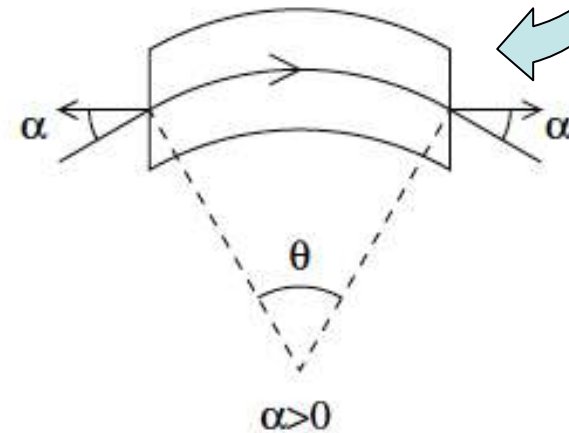
a)

$$\Delta y' = \frac{B_x l_{\text{fringe}}}{(B\rho)}$$

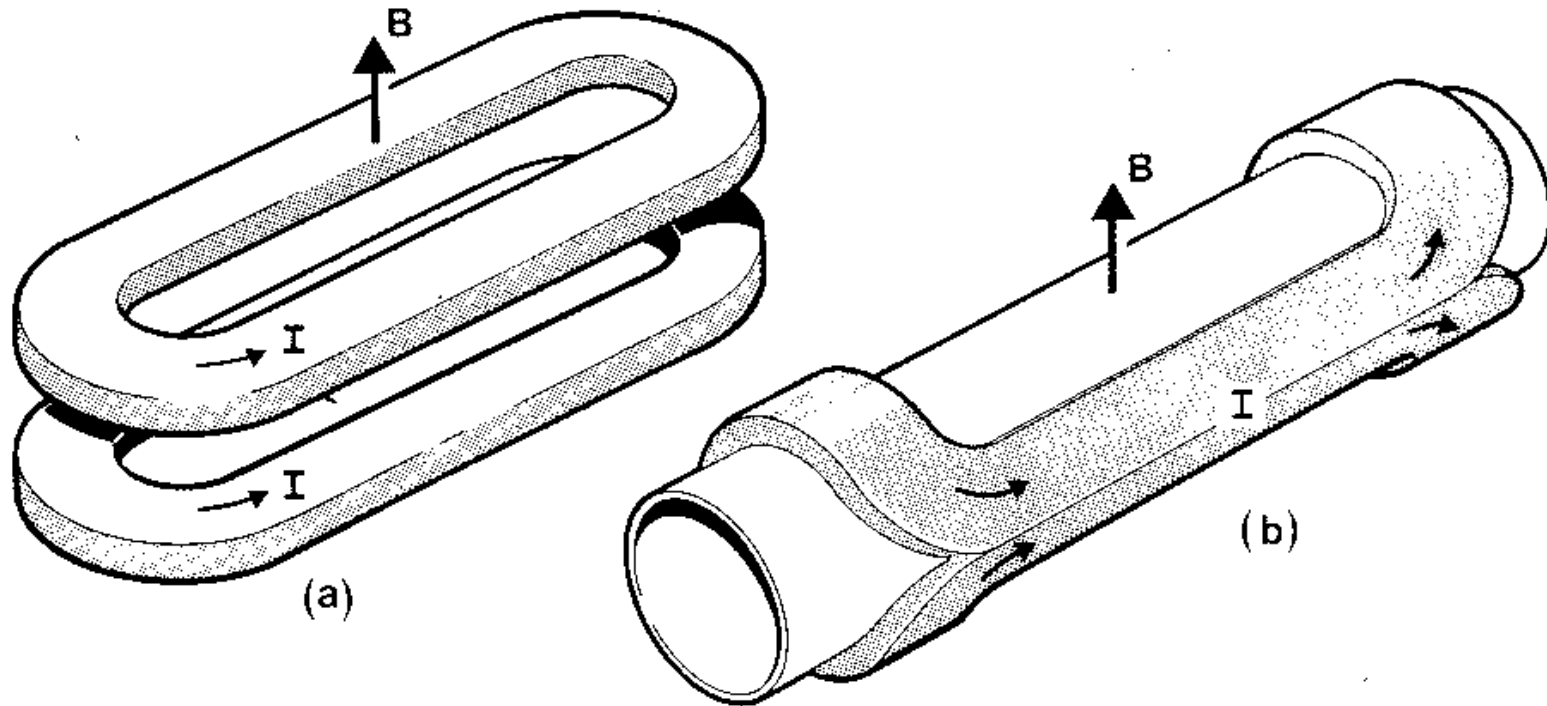


b)

- p. 85 of text
- Field lines go from  $-y$  to  $+y$  for a positively charged particle
  - $B_x < 0$  for  $y > 0$ ;  $B_x > 0$  for  $y < 0$ 
    - Net focusing!
  - Field goes like  $\sin(\alpha)$ 
    - get  $\cos(\alpha)$  from integral length

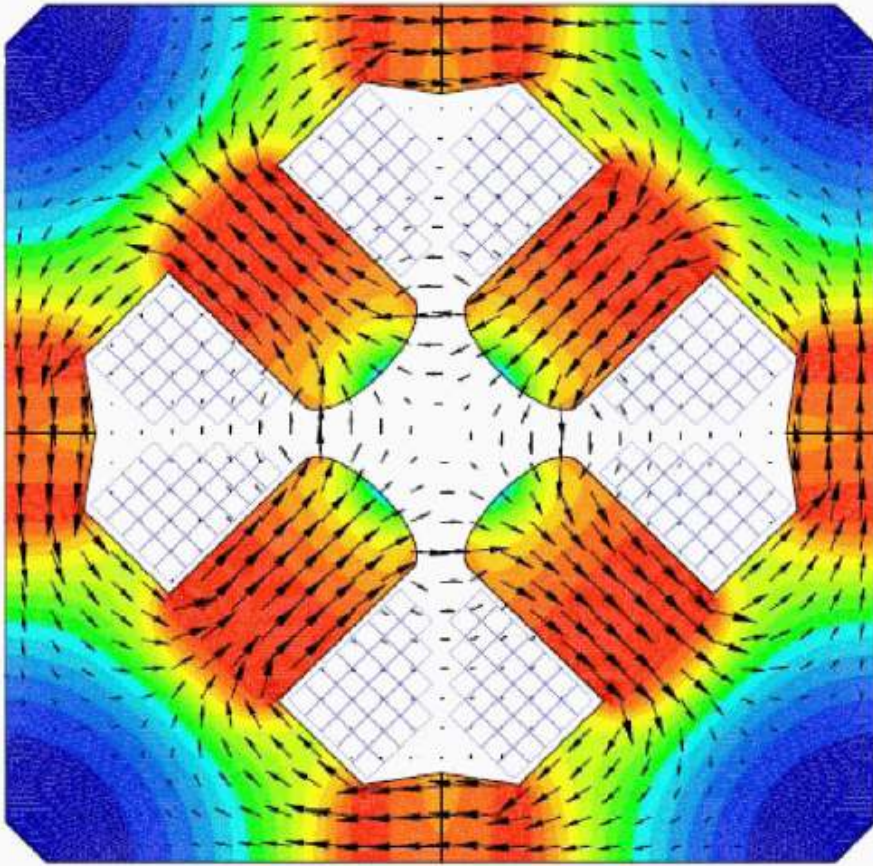


## Other Familiar Dipoles

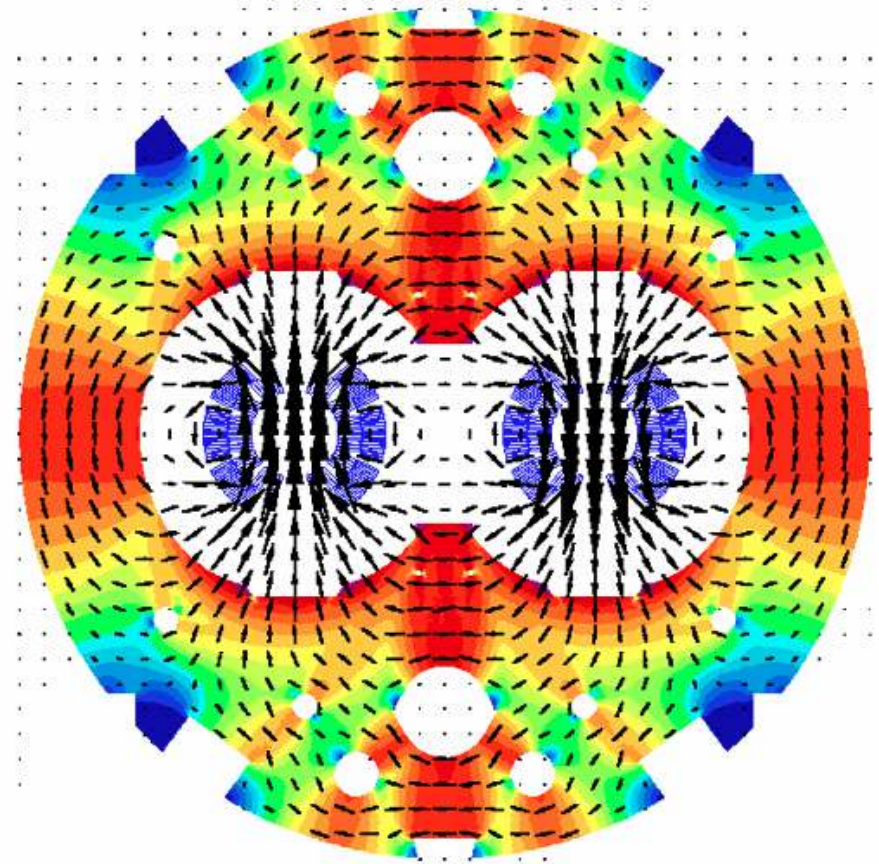


- Weaker, cheaper dipoles can be made by conforming coils to a beam-pipe (no iron)
- Relatively inexpensive, but not very precise
  - Field quality on the order of percent

# Normal vs Superconducting Magnets



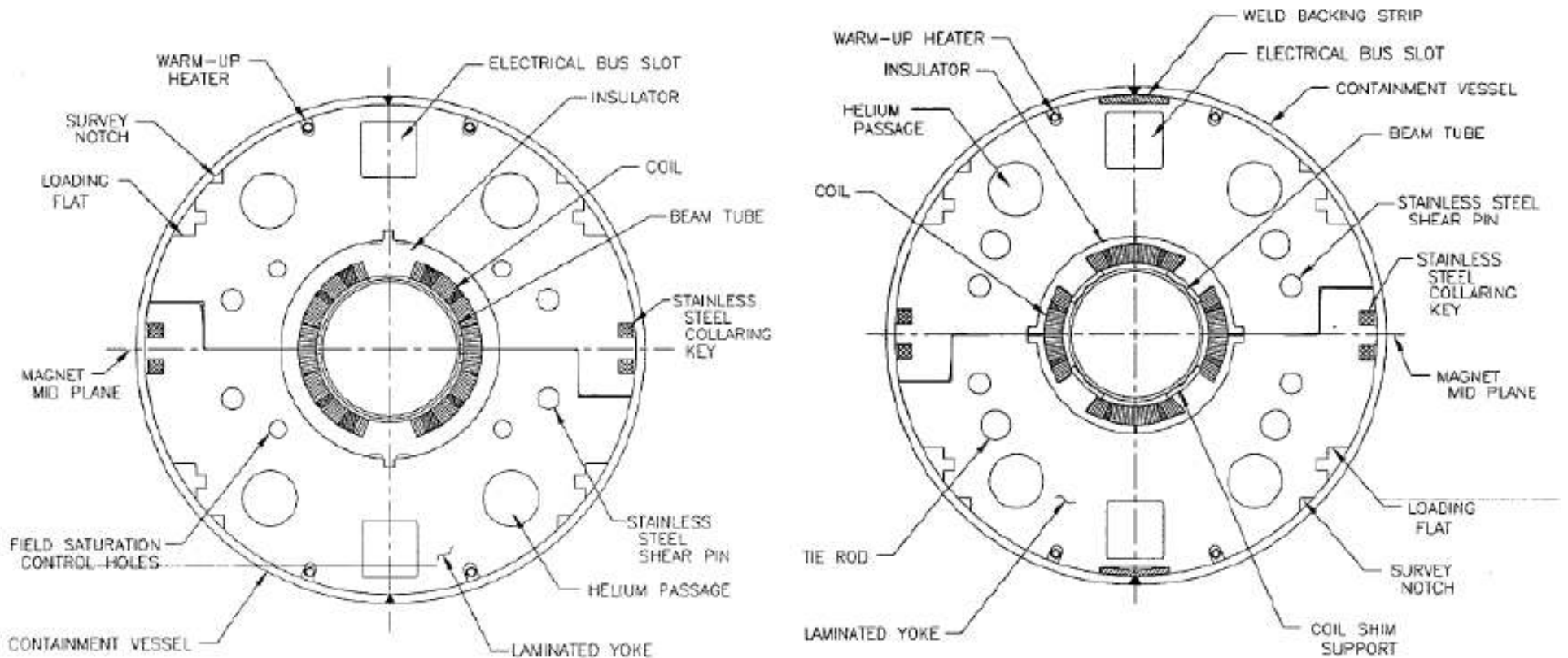
LEP quadrupole magnet (NC)



LHC dipole magnets (SC)

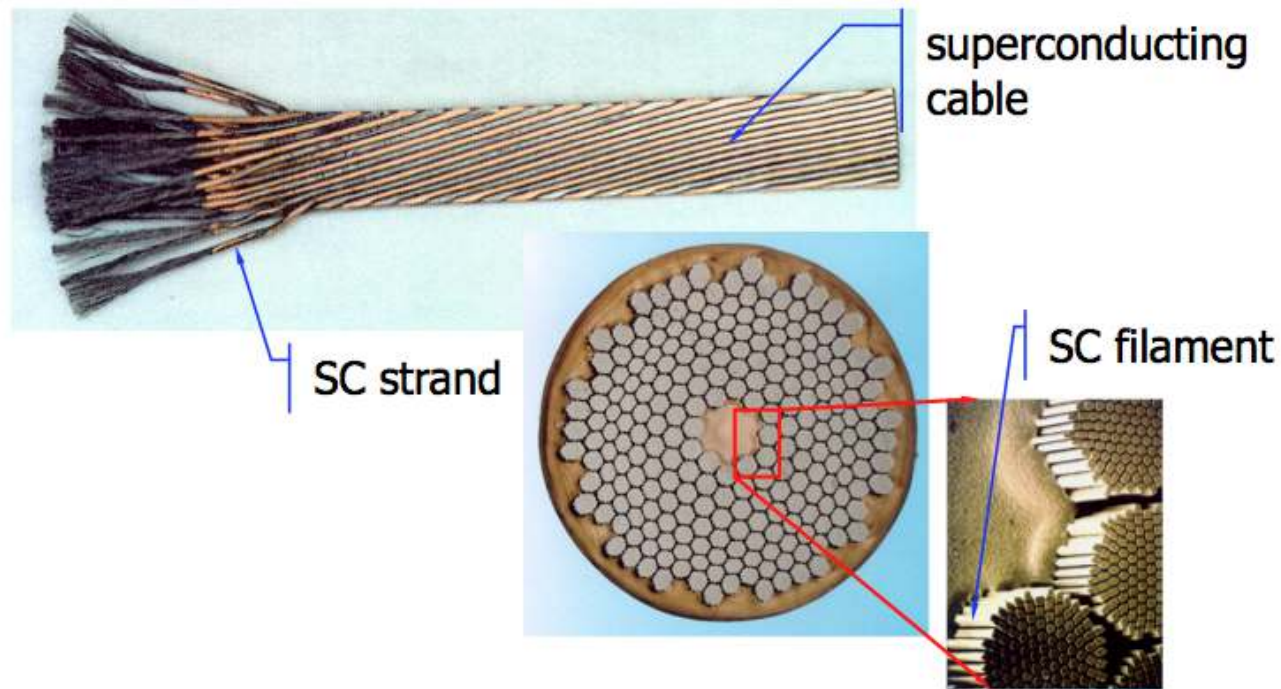
- Note high field strengths (red) where flux lines are densely packed together

# RHIC Dipole/Quadrupole Cross Sections



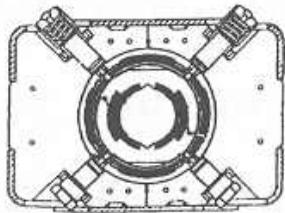
RHIC  $\cos(\theta)$ -style superconducting magnets and yokes  
 NbTi in Cu stabilizer, iron yokes, saturation holes  
 Full field design strength is up to 20 MPa (3 kpsi)  
 4.5 K, 3.45 Tesla

# Rutherford Cable

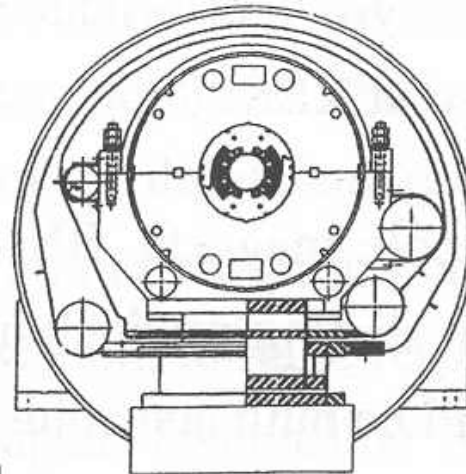


- Superconducting cables: NbTi in Cu matrix
  - Single 5  $\mu\text{m}$  filament at 6T carries  $\sim 50$  mA of current
  - Strand has 5-10k filaments, or carries 250-500 A
  - Magnet currents are often 5-10 kA: 10-40 strands in cable
    - Balance of stresses, compactable to stable high density

# Superconducting Dipole Magnet Comparison

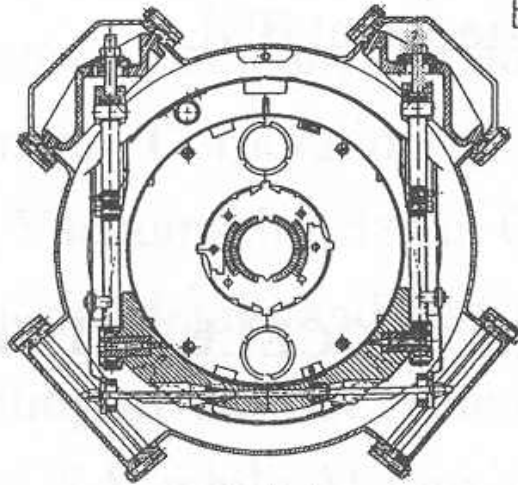


Tevatron  
4T, 90mm



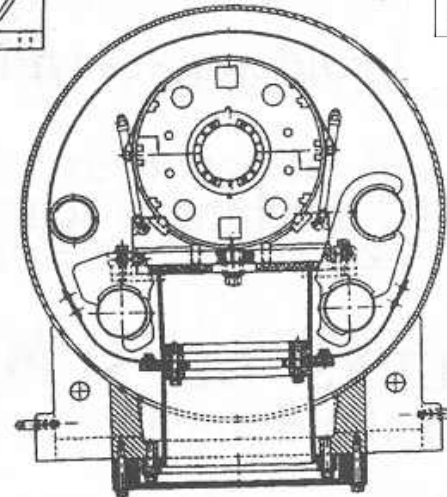
SSC

6.8 T, 50 mm



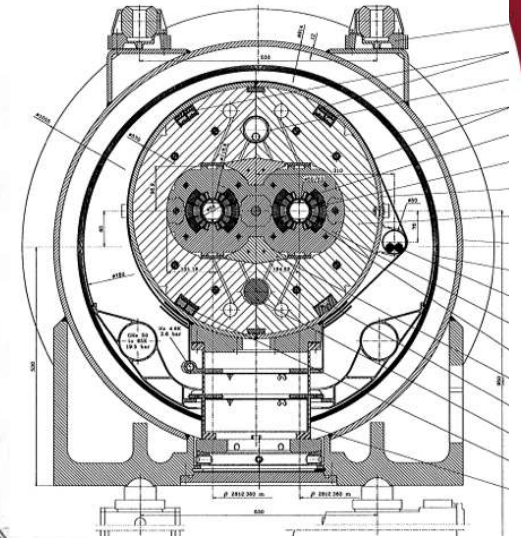
HERA

4.7T, 75 mm



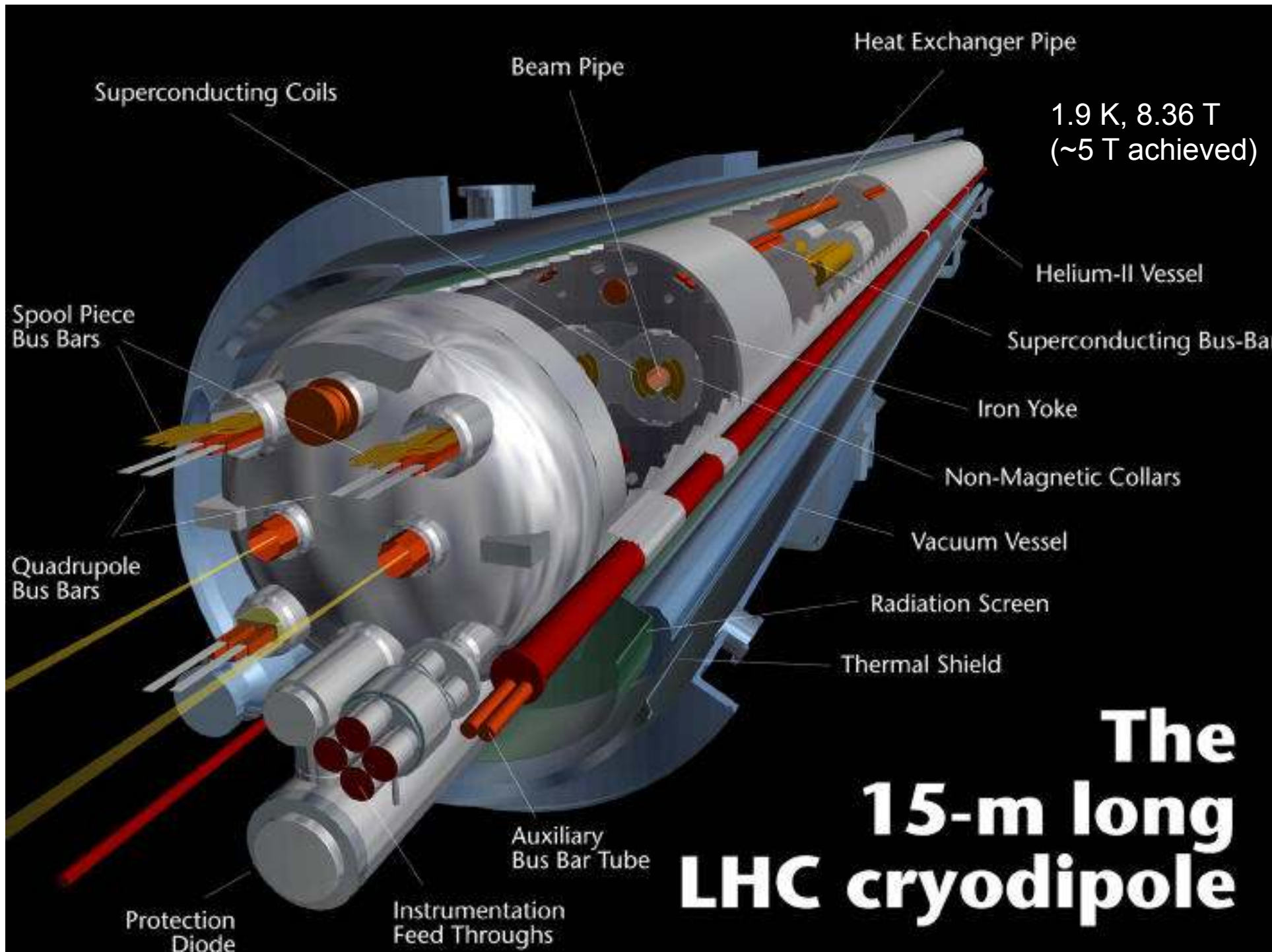
RHIC

3.4T, 80mm

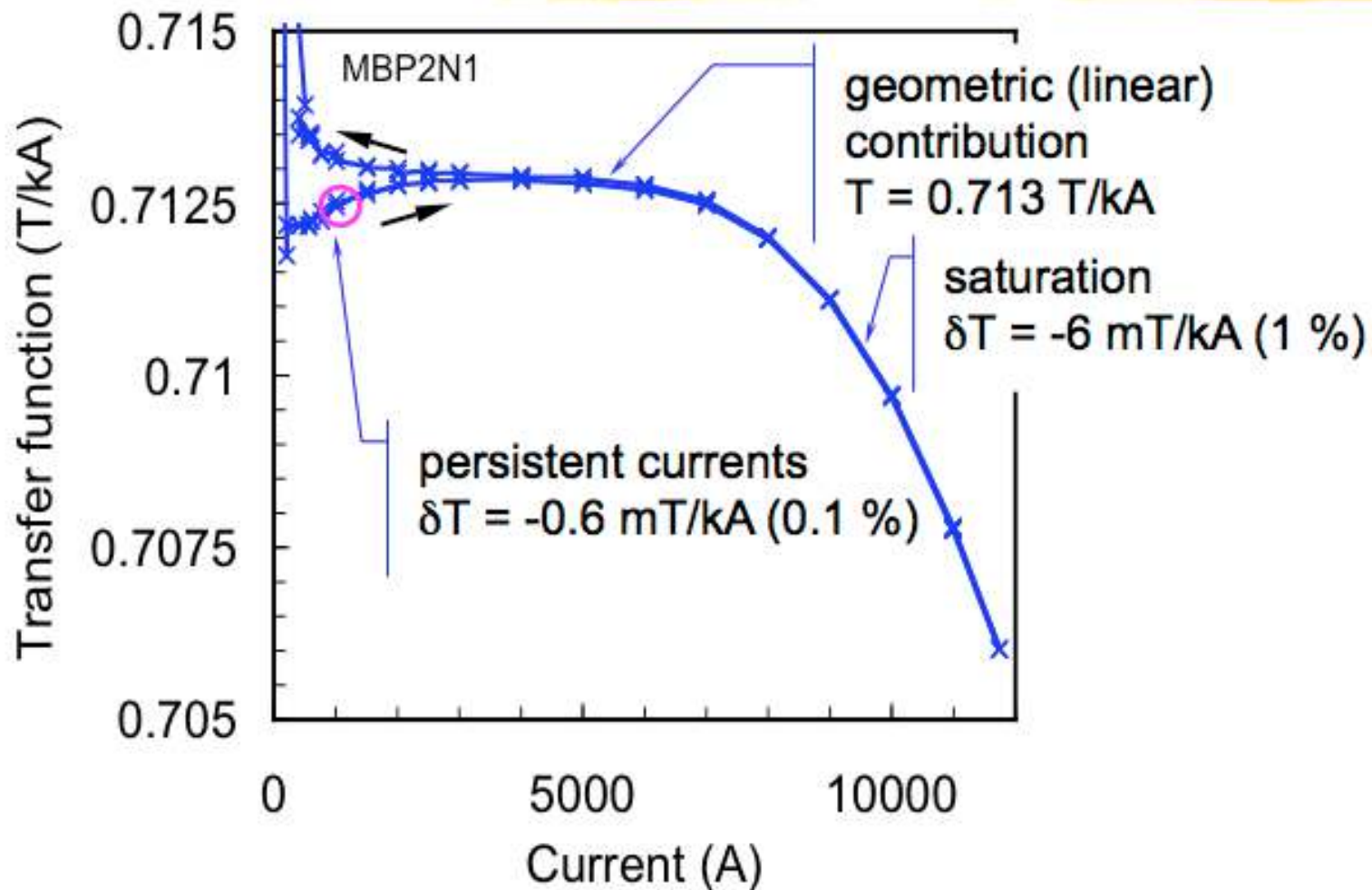


LHC

8.36T, 56mm



# Superconducting Magnet Transfer Function



- Transfer function: relationship between current/field
  - Persistent currents: surface currents during magnet ramping



# Quenching

- Magnetic stored energy

$$E = \frac{B^2}{2\mu_0}$$

$$B = 5 \text{ T}, \quad E = 10^7 \text{ J/m}^3$$

- LHC dipole

$$E = \frac{LI^2}{2} \quad L = 0.12 \text{ H} \quad I = 11.5 \text{ kA}$$

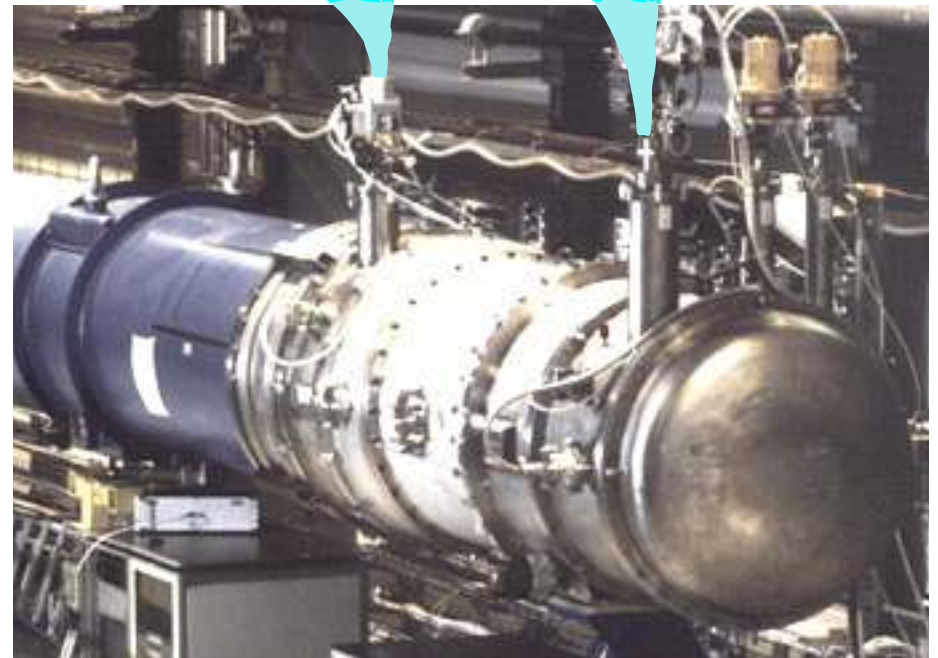
$$\Rightarrow E = 7.8 \times 10^6 \text{ J}$$

22 ton magnet

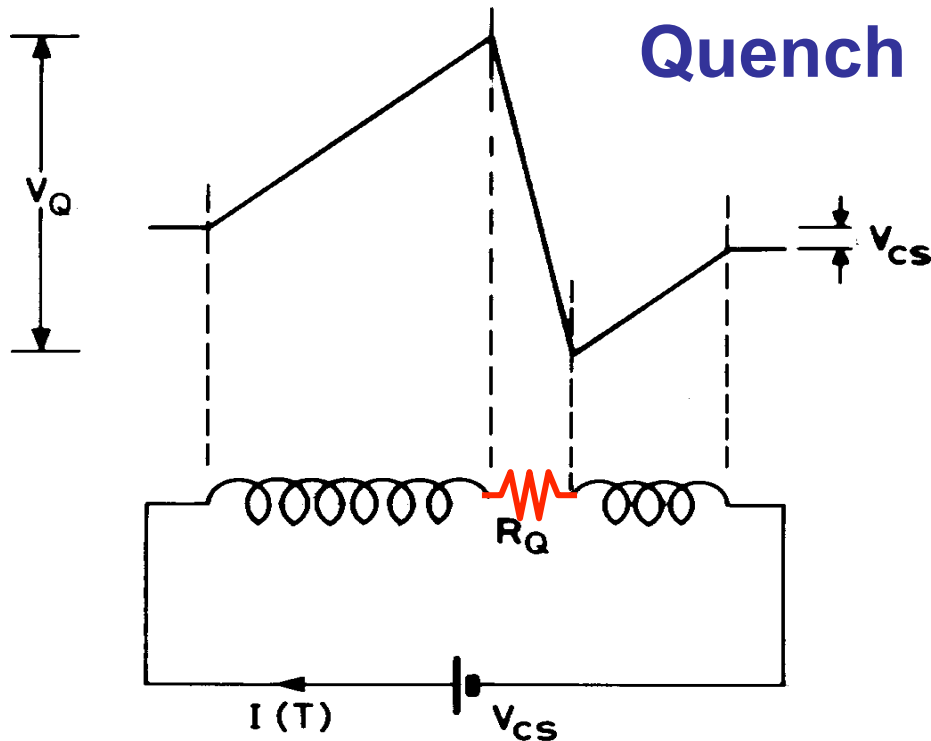
$\Rightarrow$  Energy of 22 tons,  $v = 92 \text{ km/hr!}$



*the most likely  
cause of **death**  
for a  
superconducting  
magnet*



# Quench Process

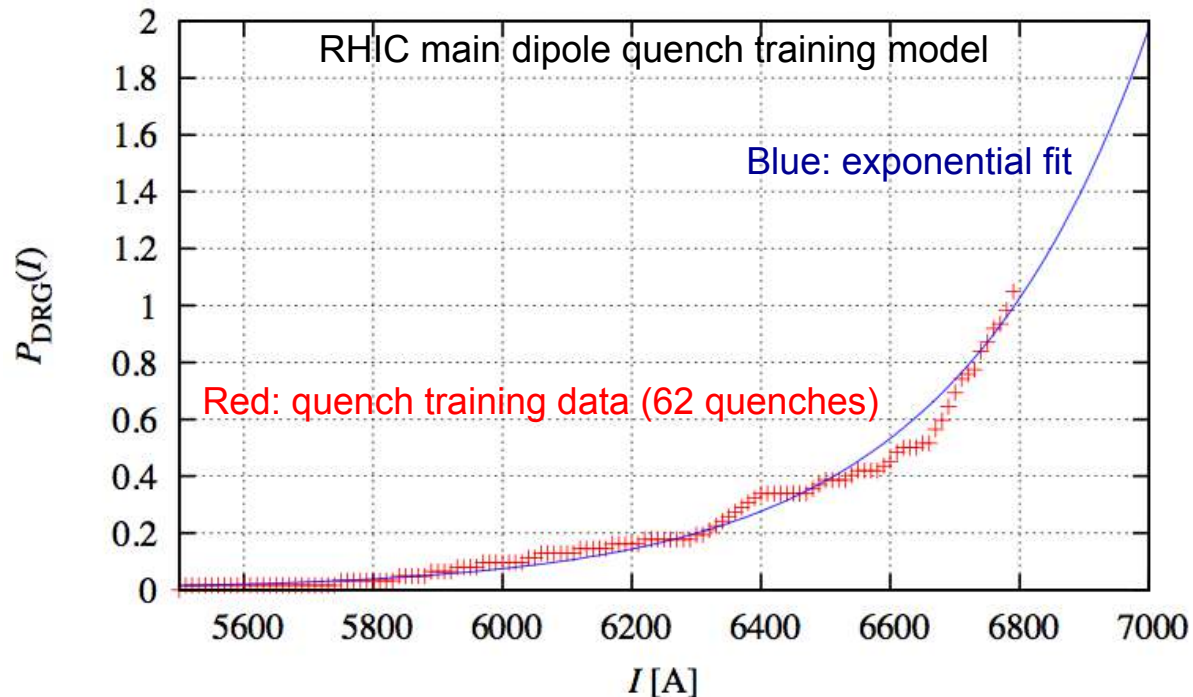


- Resistive region starts somewhere in the winding at a point: A problem!
  - Cable/insulation slipping
  - Inter-cable short; insulation failure
- Grows by thermal conduction
- Stored energy  $\frac{1}{2}LI^2$  of the magnet is dissipated as heat
- Greatest integrated heat dissipation is at localized point where the quench starts
- Internal voltages **much** greater than terminal voltage ( $= V_{CS}$  current supply)
  - Can profoundly damage magnet
  - Quench protection is important!



# Quench Training

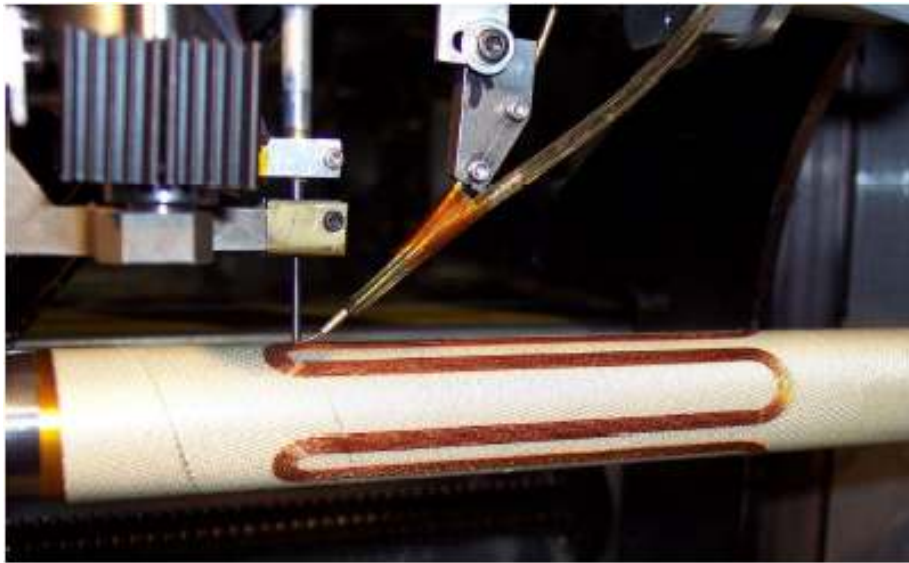
- Intentionally raising current until magnet quenches
  - Later quenches presumably occur at higher currents
    - Compacts conductors in cables, settles in stable position
  - Sometimes necessary to achieve operating current



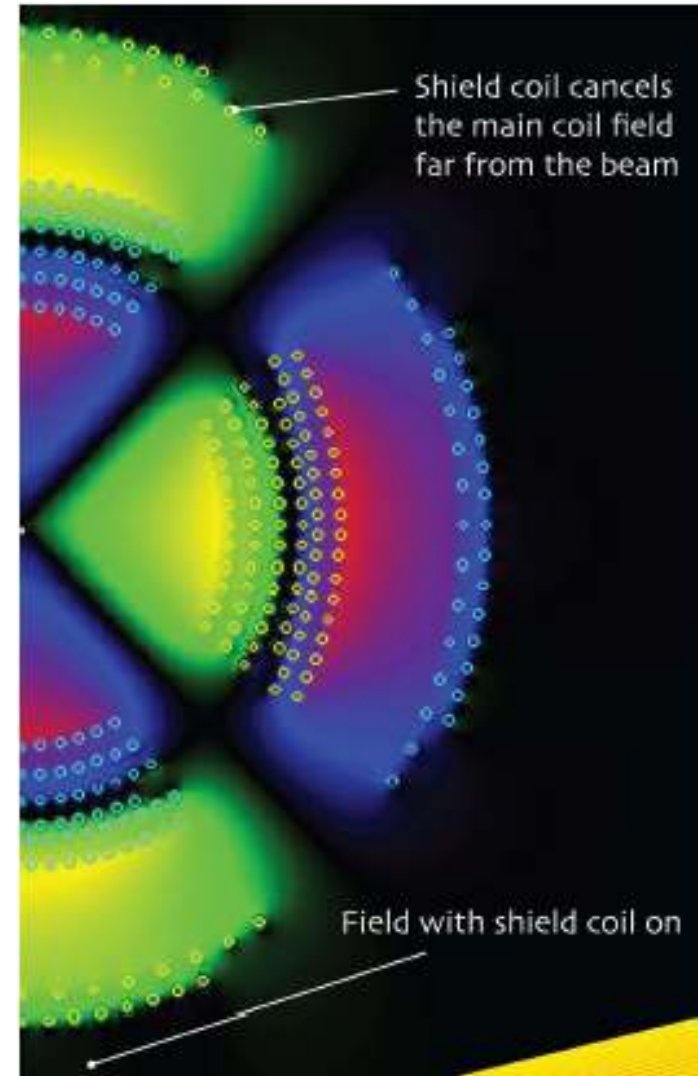
“Energy Upgrade as Regards Quench Performance”, W.W. MacKay and S. Tepikian, on class website

# Direct-Wind Superconducting Magnets (BNL)

- 6T Iron-free (superconducting)
- Solid state coolers (no Helium)
- Field containment (LC magnet)
- “Direct-wind” construction

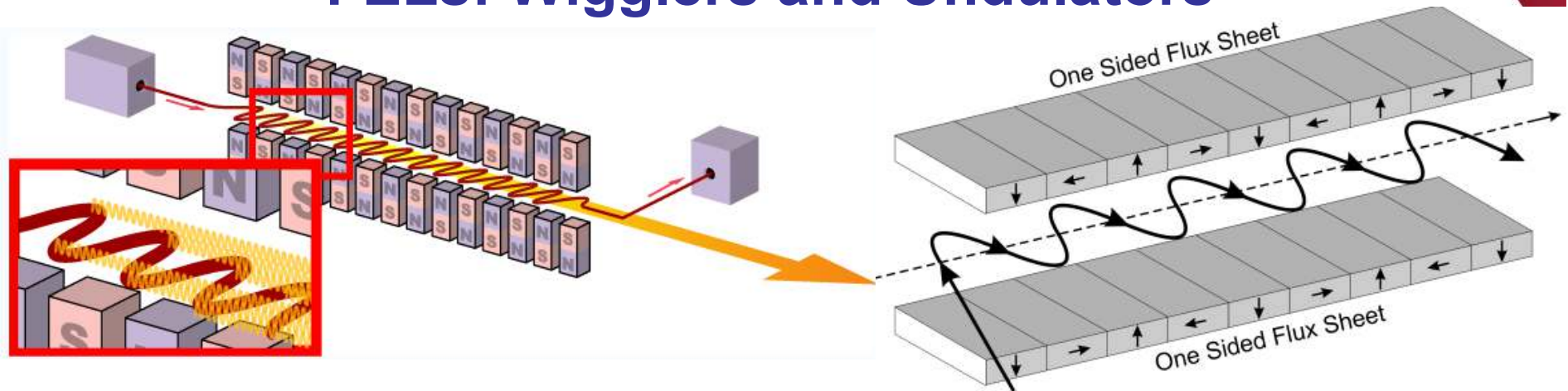


World's first “direct wind” coil machine at BNL



Linear Collider magnet

# FELs: Wigglers and Undulators



- Used to produce synchrotron radiation for FELs
  - Often rare earth permanent magnets in Halbach arrays
  - Adjust magnetic field intensity by moving array up/down
  - **Undulators:** produce nm wavelength FEL light from ~cm magnetic periods ( $\gamma^2$  leverage in undulator equation)
    - Narrow band high spectral intensity
  - **Wigglers:** higher energy, lower flux, more like dipole synchrotron radiation
    - More about synchrotron light and FELs etc next week
    - LCLS: 130+m long undulator!

# Feedback to Magnet Builders

[http://www.agsrhichome.bnl.gov/AP/ap\\_notes/RHIC\\_AP\\_80.pdf](http://www.agsrhichome.bnl.gov/AP/ap_notes/RHIC_AP_80.pdf)

## FEEDBACK BETWEEN ACCELERATOR PHYSICISTS AND MAGNET BUILDERS

S. PEGGS

*Relativistic Heavy Ion Collider, Brookhaven National Laboratory,  
Upton, New York 11973, USA*

*Submitted to the proceedings of the LHC Single Particle Dynamics Workshop, Montreux, 1996.*

### 1 PHILOSOPHY

Our task is not to record history but to change it. *K. Marx (paraphrased)*

How should Accelerator Physicists set magnet error specifications? In a crude social model, they place tolerance limits on undesirable nonlinearities and errors (higher order harmonics, component alignments, et cetera). The Magnet Division then goes away for a suitably lengthy period of time, and comes back with a working magnet prototype that is reproduced in industry.