

University Physics 226N/231N Old Dominion University

Introductions, Units, Measurements, Vectors

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<http://www.toddsatogata.net/2016-ODU>

Monday, August 29 2016

Happy Birthday to Liam Payne, Noah Syndergaard, Leah Michele, Michael Jackson,
Stephen Wolfram (Mathematica), and Arthur McDonald (Nobel Prize, 2015)

Happy International Day Against Nuclear Tests!

Please set your cell phones to “vibrate” or “silent” mode. Thanks!

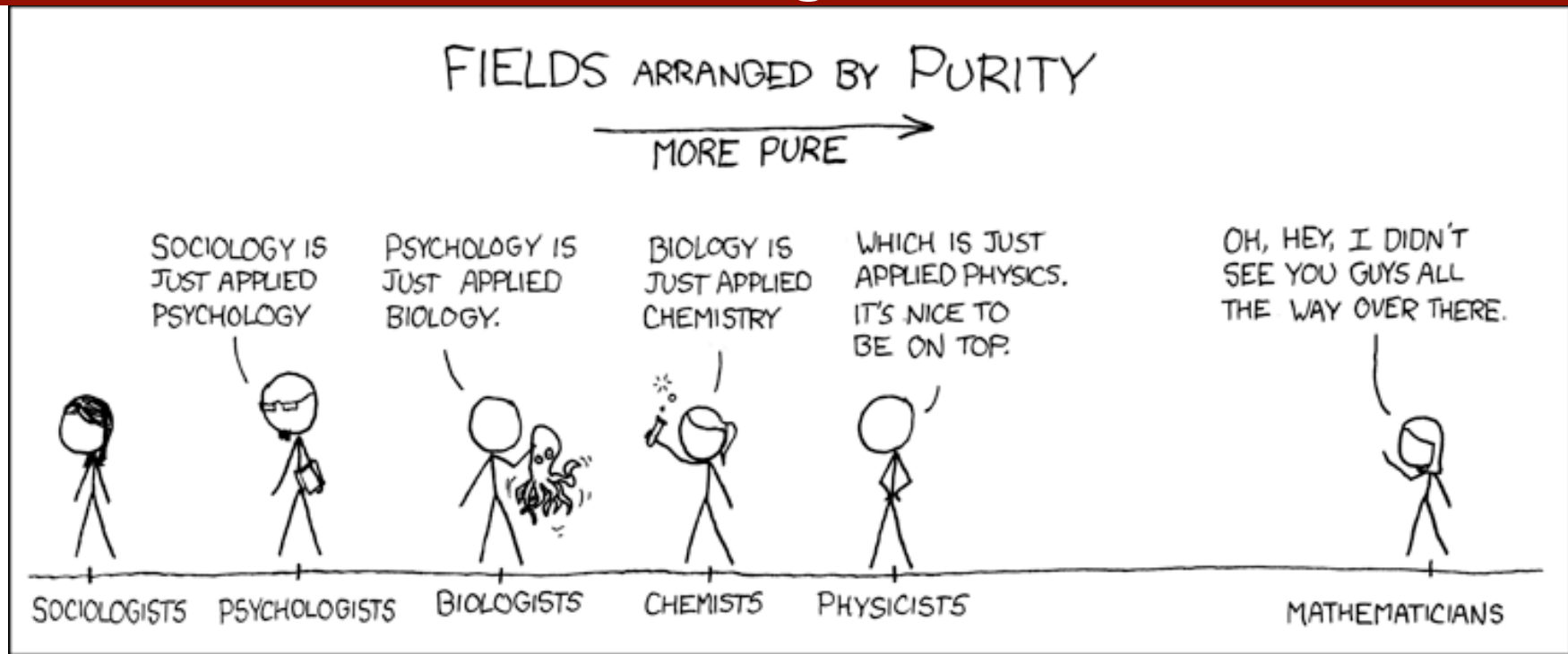


What is physics?

- You're in a (another?) physics class!
- So what is physics?
 - An experimental science that seeks to quantitatively connect all observed natural physical phenomena with predictive, falsifiable models.
- **Experimental:** based on observations and measurements.
- **Quantitative:** based on numbers and mathematical formulae.
- **Predictive:** support predictions of future measurements.
- **Falsifiable:** are the model predictions consistent with previous and future observations and measurements?



According to xkcd



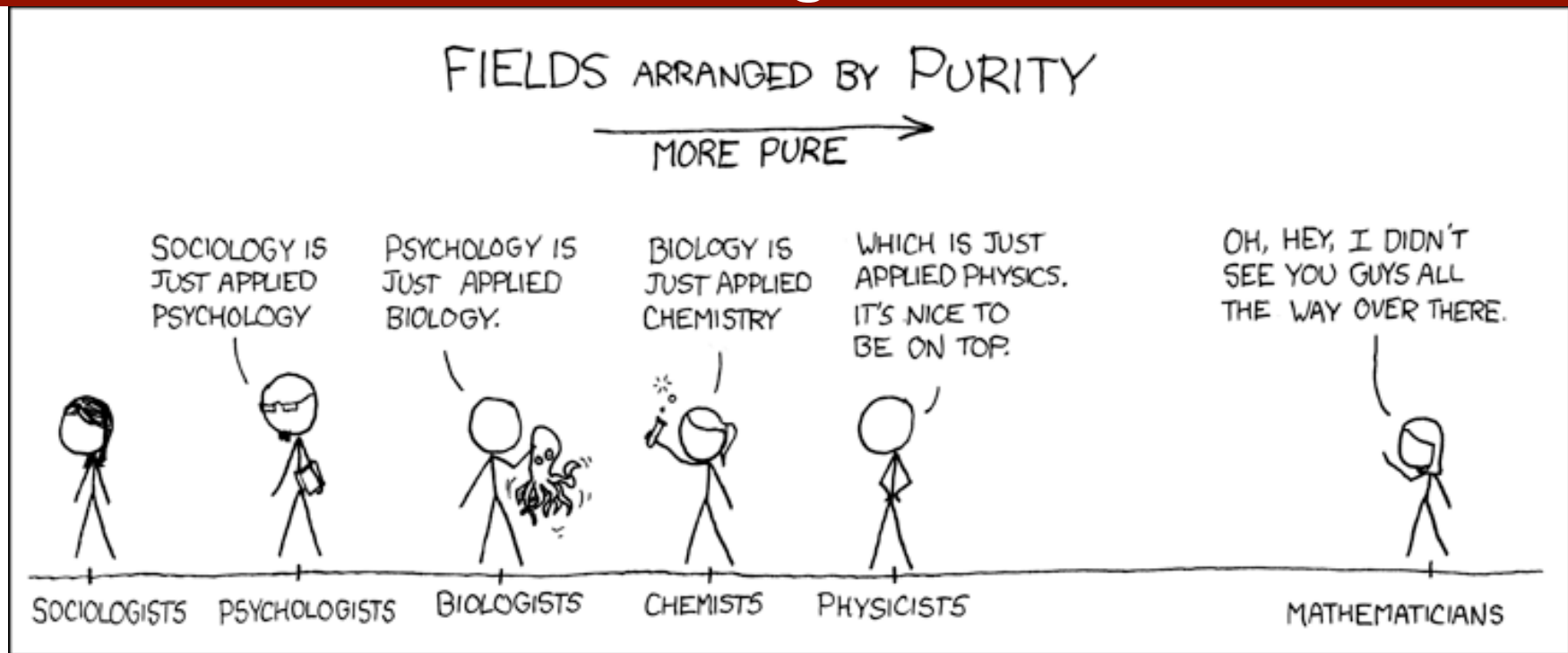
I'M GLAD I BUILT
A BOX TO HOUSE
ALL THOSE LOONS



Engineers



According to xkcd



STAND BACK



**I'M GOING TO TRY
SCIENCE**

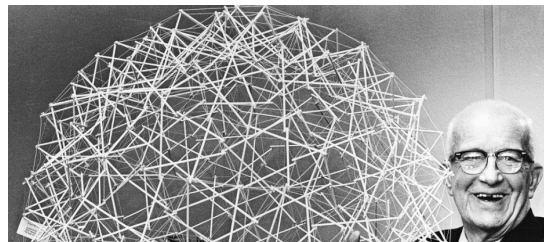


Limits of physics

- The previous definition describes pretty much all sciences that follow the scientific method (Bacon, Popper, et al.)
 - We cannot **prove** that a theory is correct, only show when it is not
 - We can support, however, support theory with overwhelming evidence
- Physics only concerns itself with models that have observable, calculable, verifiable consequences
- **Truth?** It depends who you ask...
 - The math techniques you learn and use this semester are true
 - The physical principles, however, are **only good models**

“Everything you've learned in school as "obvious" becomes less and less obvious as you begin to study the universe. For example, there are no solids in the universe. There's not even a suggestion of a solid. There are no absolute continuums. There are no surfaces. There are no straight lines.”

– R. Buckminster Fuller



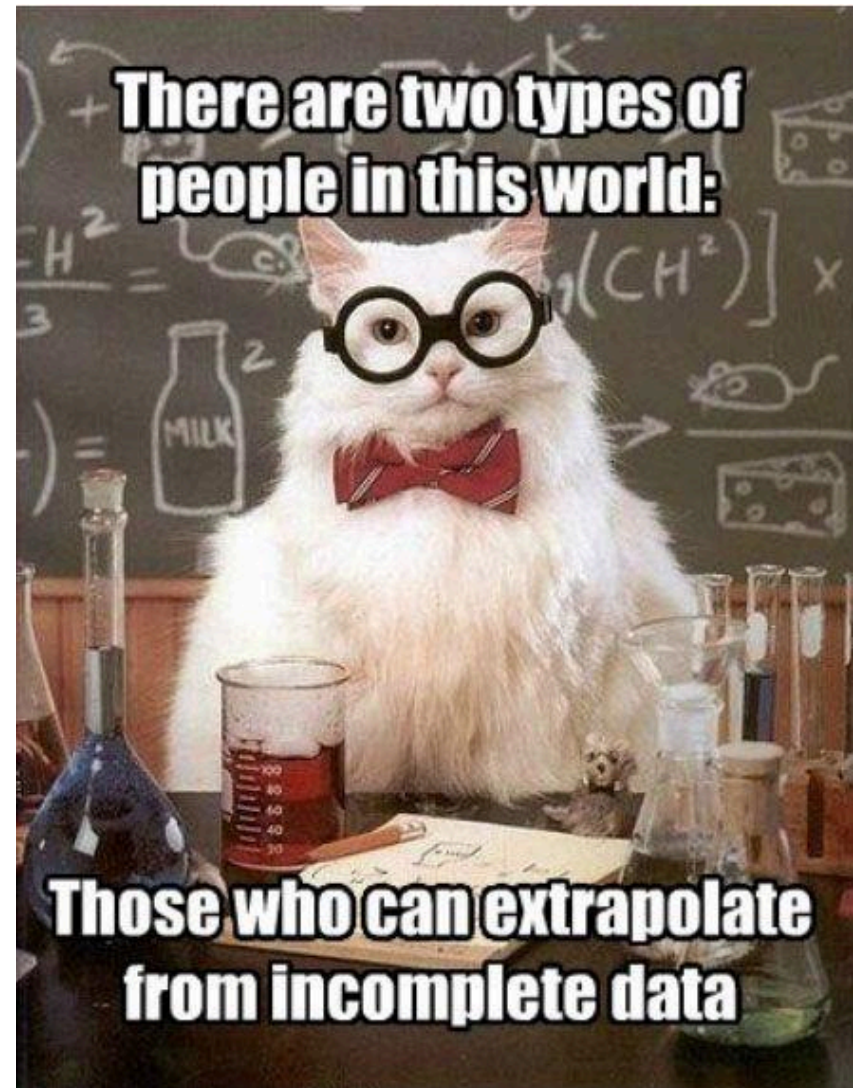
What is SCALE-UP?

- **Observation:** you're not seated in a lecture-style hall
 - Lectures are boring (yawn)
 - Lectures let you (the folks who are learning) become passive
 - Lectures isolate you from your peers
- In this class, we'll do it a bit differently
 - **Me:** (a bit) less lecturer - more moderator, coach, mentor
 - **You:** regular, active, group participants in class activities
- So what do we do?
 - **(Some) Lectures:** Setting up the day's topics, and summaries
 - **Observations:** Related to the current class topic
 - **Ponderables:** Conceptual or calculating group exercises
 - **Tangibles:** Try it out and see group exercises
 - (And, most importantly, Preparation **outside** of class)



Preparation Outside of Class

- **Me:** Post reading, (some of) the class slides, and Mastering Physics homework
 - By evening after previous class
- **You**
 - Look through the material before class
 - You can't engage a topic you haven't at least briefly thought about
 - Do the homework
 - ~half review, half new
 - Keep notebook (see syllabus)
 - ~Maybe 6-12 hours/week



Class Website and Syllabus

<http://www.toddsatogata.net/2016-ODU>

ODU University Physics PHYS 226N/231N/261N

CRNs 12743/10067/19775 (lab is a separate course)
Dr. Todd Satogata / satogata@jlab.org or tsatogat@odu.edu
TAs: Jiwan Poudel (jpoud001@odu.edu) and Bhawin Dhital (bdhit001@odu.edu)
Fall Semester, Aug 27-Dec 09 2016

• Class Information

- **Class Syllabus and Information** [last updated 27 Aug 2016] ← **Syllabus**
- **Archive of all class-wide emails** sent through Blackboard (last email sent Aug 17)
- **Textbook:** University Physics with Modern Physics (14th Ed), Young/Freedman (strongly recommended but not required)
- **Homework:** Online through **Mastering Physics** (required!) ← **Mastering Physics**
Some instructions for using MasteringPhysics are [located here](#).
- **Class:** MW 9:00-10:50 in OOCNPS 142-144 (SCALE-UP Classroom)
- **Lab Schedule:** Labs are a separate class in OCNPS138 on Fridays. [Schedule is here](#).
- A useful physics website with demos and explanations of many of this semester's concepts is [The Physics Classroom](#).
- Another useful physics website with some problems and worked-out solutions is the [University of Oregon Physics Student Page](#).

• Class Materials

Date	Chapter	Slides [pdf]	Topic and other notes
M Aug 29	1	[?.? Mb]	← Class Materials
W Aug 31	2	[?.? Mb]	
M Sep 5	--	--	Labor Day, No class

NOTE that labs do not start until Friday Sep 12!!!!

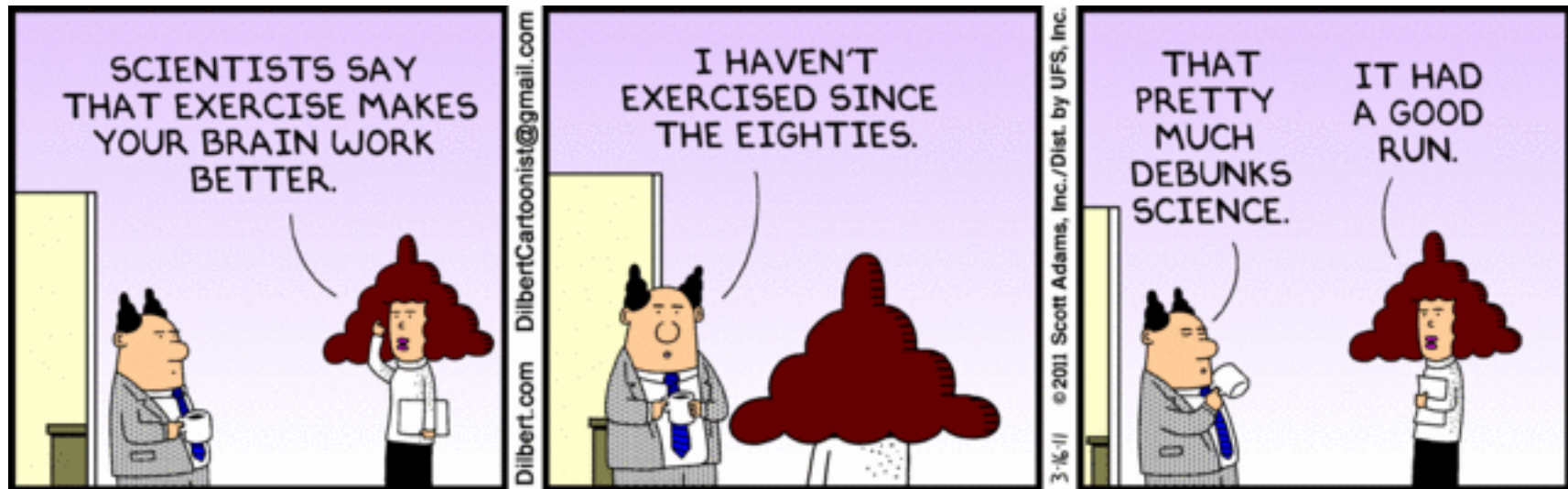


Mastering Physics

- <http://masteringphysics.com> should take you there
- All class homework is **required** to be in Mastering Physics

The screenshot shows the MasteringPhysics website. At the top is a black header with 'PEARSON' on the left and 'ALWAYS LEARNING' on the right. Below this is the 'MasteringPhysics™' logo. A yellow banner contains a message: 'Students, do you need help registering or enrolling in your Mastering course? [Get help now >](#)'. The main content area features a large image of a smiling student. On the left, the text 'BREAKTHROUGH' is displayed in large, bold letters, followed by 'To improving results'. Below this, a small bar chart icon is next to the text: 'Our goal is to help every student succeed. We're working with educators and institutions to improve results for students everywhere. [Learn more >](#)'. On the right side of the main area, there is a 'Sign In' section with the text 'Already registered? Sign in with your Pearson account.' and a pink button labeled 'SIGN IN' with a user icon. Below this is a link 'Forgot username or password?'. Further down is a 'Register Now' section with the text 'Need access? Start here!' and two pink buttons: 'STUDENT' and 'EDUCATOR', each with a pencil icon. At the bottom left, there are two vertical pink menus. The first menu is for 'EDUCATORS & ADMINISTRATORS' and includes links for 'Results', 'Features', and 'Training & Support'. The second menu is for 'STUDENTS >' and includes links for 'Get Registered', 'Support', and 'More...'. A small logo is visible in the bottom left corner of the screenshot.

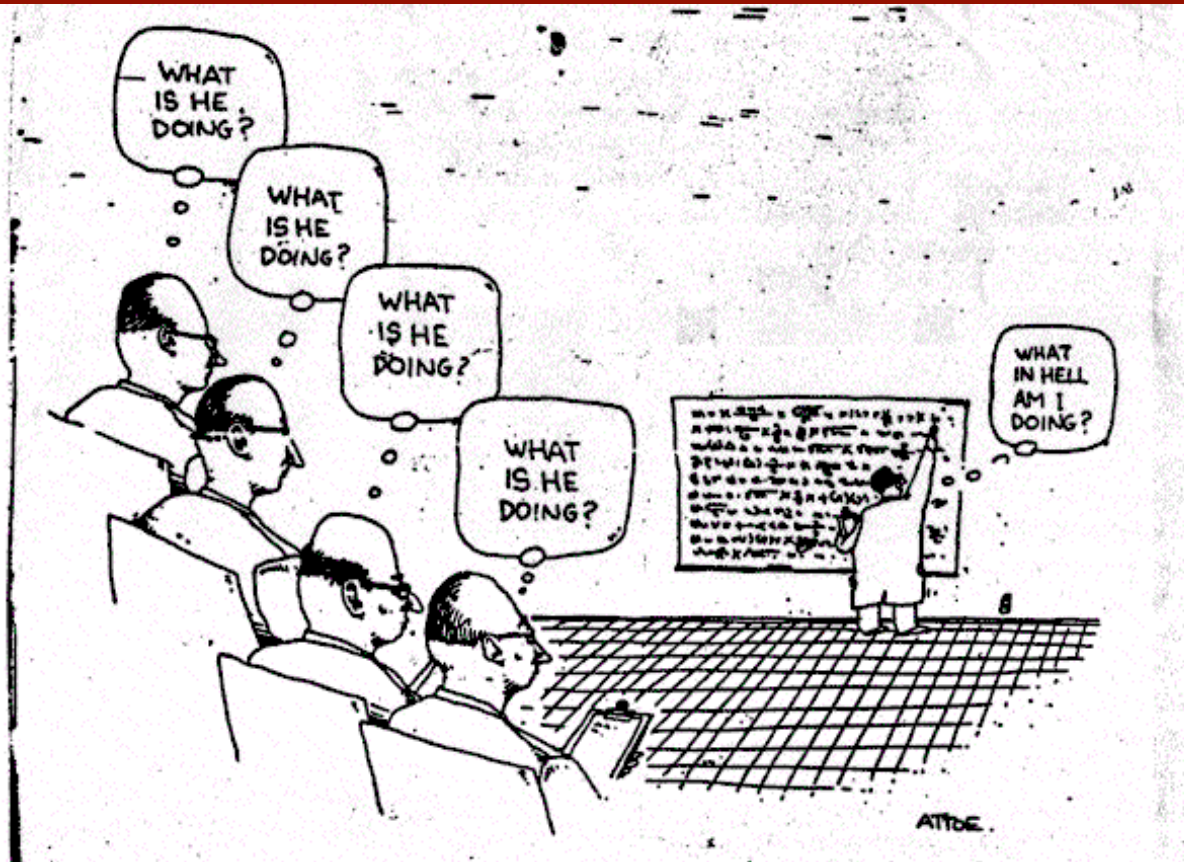
Homework: Brain Exercise



- **Do the homework.** You won't do well if you don't. Really.
- By this Wednesday, at least do the [Introduction to Mastering Physics](#) section.
 - It doesn't count towards your grade, but everything following it does, and you'll be a lot less frustrated if you get this out of the way.
- I'll have homework up for 1D motion on Wednesday morning.



Tangible (5-10 minutes)

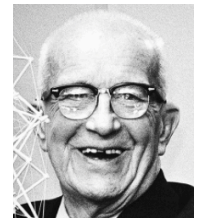


- Who are you? Why are (or aren't ☺) you here?
- Introduce yourself to those around you and say hi!
 - Use boards and markers in creative ways if you like.
 - Enter your birthday into <http://toddsatogata.net/cgi-bin/birthday.pl> and work with those around you to enter theirs too.



Experiment and The Birthday Paradox

- We just did our first class “experiment”!
- We collected data and can now analyze it to learn something
- A bit of thinking and math can produce some expectations
 - In a class of 70+ people, there is a 99.995%+ chance that at least two people in this class have the same birthday
 - Even in a group of 23 people, the probability of two having the same birthday is (slightly) over 50%. (A useful bar bet.)
- Physics (and science) is not always obvious
 - It should challenge and structure the way you look and think about all physical phenomena
 - The more you learn, the more consistent your thought processes (and expectations and expertise) will be



Physics and Homework (1.2)

- At some point you will probably get frustrated
 - At this point in your education, **concepts** are often easier to grasp than **technique**



I've read over all the concepts and understand the reading, but I don't know which formula to use or where to start to solve this problem!

- It's okay – we're going to cover a lot of material in one semester.
- Technique, unlike raw information, cannot be “crammed”
 - You have to practice regularly until it becomes second nature
 - Math and physics are best learned like a language – practice!!



I SEE (1.2)

- Most textbooks try to help by providing a structure, or recipe, that you can use when you are solving problems
- Our text uses the mnemonic “**I SEE**”
 - **I: Identify** the relevant concepts
 - What are the physical quantities are known, and unknown?
 - What is a possible model or approach to “tell the story”?
 - **S: Set Up** the problem
 - Tell the story: Draw a picture and choose equations to solve.
 - **E: Execute** the solution
 - “Do the math” or “crunch the numbers”.
 - Remember to always use units on all physical quantities!
 - **E: Evaluate** your answer
 - Does your answer make sense? Check the units.



Modeling and Physics are Creative Endeavors (1.2)

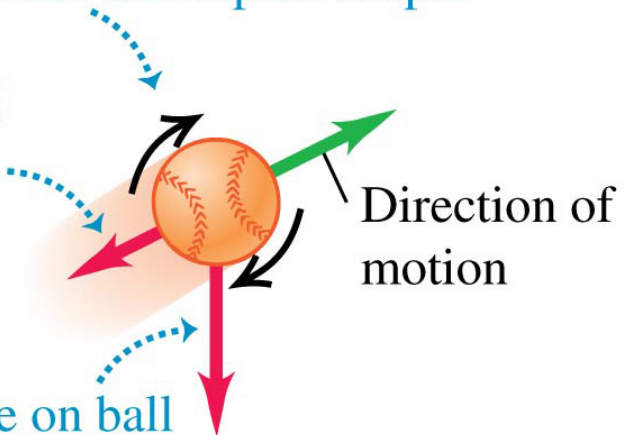
- We simplify analysis of complicated problems by making **idealized models**
- All our models in this class are idealized
 - Keep what is important (e.g. gravity)
 - Neglect what is expected to be much less important (e.g. air resistance)
 - Idealize what is left (e.g. point particle, gravitational acceleration is constant)

(a) A real baseball in flight

Baseball spins and has a complex shape.

Air resistance and wind exert forces on the ball.

Gravitational force on ball depends on altitude.

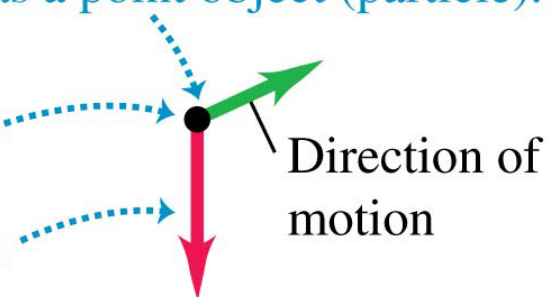


(b) An idealized model of the baseball

Treat the baseball as a point object (particle).

No air resistance.

Gravitational force on ball is constant.



Measurements, Experiments, Observation (1.3)

- Physics is rooted in observations and measurements
- **Physical quantities** can be measured directly or derived
 - Distance is usually measured directly
 - Compare to a known well-defined distance
 - Average speed \bar{v} is derived
 - Measure distance Δx and time Δt , then average speed = $\bar{v} = \frac{\Delta x}{\Delta t}$
- For much of our initial studies, there are three fundamental quantities: **length, time, and mass**
 - Time: 1 s is defined by transition frequency of ^{133}Cs electrons
 - Length: 1 m is defined by distance light travels in vacuum in 1/299792458 second
 - Mass: 1 kg is defined by a reference mass in a lab near Paris



Measurements and Units (1.3)

- Nearly all physical quantities have units
 - Technique: always, **always** write units whenever you write a number for a physical quantity that has units
- Units follow the laws of simple algebra
 - Only two quantities with the **same units** can be added or subtracted
 - Any quantities with **any units** can be multiplied or divided
 - Example: I traveled an average speed of 30 mph for 0.2 hours
 - How far did I travel?

$$\bar{v} = \frac{\Delta x}{\Delta t} \quad \Delta x = \bar{v}(\Delta t) = \left(30 \frac{\text{miles}}{\text{hr}} \right) (0.2 \text{ hr}) = 6 \text{ miles}$$



SI and Metric Units (1.3)

- The modern, scientific and engineering form of the metric system
 - You should get in the habit of writing units for **all** physical quantities!

- Length: meter [m]
- Time: second [s]
- Mass: kilogram [kg]

We'll deal with these for most of the first semester

1 inch = 2.54 cm

1 m ~ 3.281 feet ~ is "approximately"

1 km ~ 0.6214 miles

$(100 \text{ m})^2 \sim 2.471 \text{ acres}$

$(10 \text{ cm})^3 = 1 \text{ liter} \sim 0.264 \text{ gal}$

1 kg ~ 2.205 lbs

1 g ~ 0.0353 oz

1 year ~ $\pi \times 10^7 \text{ s}$
(or "525600 minutes")

1 Hz = 1 s^{-1}

- *Electric current: ampere [A]*
- *Temperature: kelvin [K]*
- *Amount of a substance: mole [mol]*
- *Luminous intensity: candela [cd]*
- *Plane angle: radian [rad]*
- *Solid angle: steradian [sr]*



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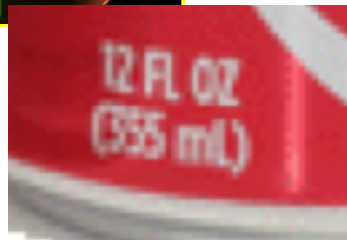
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Google

1 year in seconds



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Converting Units (1.4)

- **Units are important.** Measures of physical quantities must always have the correct units so we can compare them properly.
- Conversion tables or Google can be used to convert:
 - Example: What is the length of the Olympic 200 meter dash in feet?

$$(200 \text{ m}) \left(\frac{3.281 \text{ feet}}{1 \text{ m}} \right) = 656.2 \text{ feet}$$

Multiplying by “1”

- Example: What is 60 mph in feet/s? in m/s?

$$(60 \text{ miles/hr}) \left(\frac{5280 \text{ feet}}{1 \text{ mile}} \right) \left(\frac{1 \text{ hr}}{3600 \text{ s}} \right) = \left(\frac{5280}{60} \right) \text{ ft/s} = 88 \text{ ft/s}$$

$$(60 \text{ miles/hr}) \left(\frac{5280 \text{ feet}}{1 \text{ mile}} \right) \left(\frac{1 \text{ hr}}{3600 \text{ s}} \right) \left(\frac{1 \text{ m}}{3.281 \text{ feet}} \right) = 27 \text{ m/s}$$

- Units are a useful double-check to your physics calculations.



Significant Figures (1.5)

- The answer to the last example is 27 m/s, **not** 26.86 m/s or 26.8617 m/s or 2.68617×10^1 (units!) as a calculator might show.
 - That's because the given quantity, 60 mph, has only two **significant figures**.
 - That means we know that the actual value is closer to 60 mph than to 61 mph or 59 mph. (This matters when, say, you're speeding!)
 - If we had been given 60.00 mph, we would know that the value is closer to 60.0 mph than to 60.1 mph or 59.9 mph.
 - In that case, the number given has three significant figures.
 - Significant figures tell how **precisely** we know the values of physical quantities based on our observations (measurements).
 - Calculations cannot increase that precision, so it's important to report the **results** of calculations with the correct number of significant figures (but carry all digits **through** calculations).



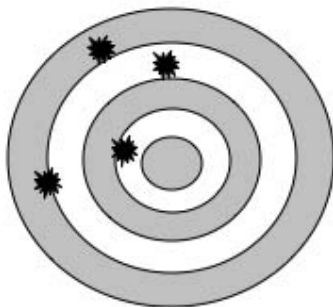
Ponderable (10 minutes)

- Note that I was very careful with my language there!
 - Significant figures tell how **precisely** we know the values of physical quantities based on our observations (measurements).
 - Calculations can't increase that **precision**, so it's important to report the results of calculations based on measurements with the correct number of significant figures.
 - I intentionally did **not** use the words “accurately” and “accuracy”
- Ponderables (10 minutes)
 - Is there a difference between precision and accuracy?
 - Can a set of measurements be precise without being accurate?
 - Can a set of measurements be accurate without being precise?

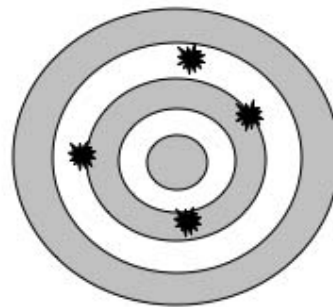


Measurements: Precision and Accuracy

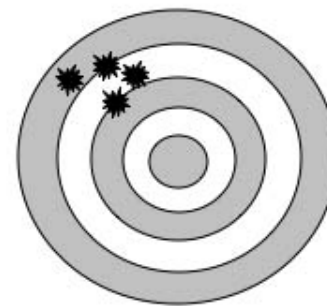
- All measurements (and experiments) have both precision and accuracy. These are independent descriptions of uncertainty (technically random and systematic uncertainty).
 - **Precision:** what is the smallest difference we can measure, or the random variation among many measurements?
 - **Accuracy:** how close is the average of many measurements to the actual value?



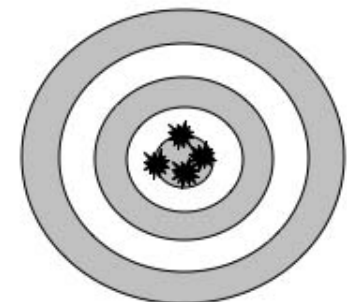
Not Accurate
Not Precise



Accurate
Not Precise



Not Accurate
Precise



Accurate
Precise



Rules for Significant Figures

- In multiplication and division, the answer should have the same number of significant figures as the **least** accurate of the quantities entering the calculation.
 - Example: $(3.1416 \text{ N})(2.1 \text{ m}) = 6.6 \text{ N}\cdot\text{m}$
 - Note the centered dot, often used when units are multiplied (though there are historical exceptions like kWh).
- In addition and subtraction, the answer should have the same number of digits to the right of the decimal point as the term in the sum or difference that has the smallest number of digits to the right of the decimal point.
 - Example: $3.2492 \text{ m} - 3.241 \text{ m} = 0.008 \text{ m}$
 - Note the loss of precision, with the answer having only one significant figure.
 - This includes **explicit** trailing zeros: 1.1×10^3 is **not** the same as 1.10×10^3 !



Scientific Notation (1.3ish)

- The vast range of quantities that occur in physics are best expressed with ordinary-sized numbers multiplied by powers of 10:

- $31416.5 = 3.14165 \times 10^4$
- $0.002718 = 2.718 \times 10^{-3}$

- SI prefixes describe powers of 10:

- Every three powers of 10 gets a different prefix
- Examples with units:

- $1.21 \times 10^9 \text{ W} = 1.21 \text{ GW}$
(1.21 gigawatts!)
- $1.6 \times 10^{-8} \text{ m} = 16 \text{ nm}$
(16 nanometers)
- $10^{12} \text{ kg} = 1 \text{ Pg}$
(1 petagram)



Table 1.1 SI Prefixes

Prefix	Symbol	Power
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
hecto	h	10^2
deca	da	10^1
—	—	10^0
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

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Rules for Scientific Notation

- You should be comfortable with rules of exponents when handling numbers with scientific notation
 - It's all just convenient bookkeeping for powers of 10

$$x \times 10^a + y \times 10^a = (x + y) \times 10^a$$

$$(x \times 10^a) \times (y \times 10^b) = (x \times y) \times 10^{(a+b)}$$

- Example: How many feet are in a light-year?
- Answer: Light travels about 3×10^8 m/s, and a light-year is the distance light travels in one year. So

$$1 \text{ light year} \approx (3 \times 10^8 \text{ m/s})(\pi \times 10^7 \text{ s}) \left(\frac{3.3 \text{ feet}}{\text{m}} \right) \approx \pi \times 10^{16} \text{ feet}$$



Quick Question

- Choose the sequence that correctly ranks the numbers according to the number of significant figures. (Rank from fewest to most.)

A. 0.041×10^9 , 3.14×10^7 , 2.998×10^{-9} , 0.0008.

B. 3.14×10^7 , 0.041×10^9 , 0.0008, 2.998×10^{-9} .

C. 2.998×10^{-9} , 0.041×10^9 , 0.0008, 3.14×10^7 .

D. 0.0008, 0.041×10^9 , 3.14×10^7 , 2.998×10^{-9} .

E. 0.0008, 0.041×10^9 , 2.998×10^{-9} , 3.14×10^7 .



Quick Question

- Choose the sequence that correctly ranks the numbers according to the number of significant figures. (Rank from fewest to most.)

- A. 0.041×10^9 , 3.14×10^7 , 2.998×10^{-9} , 0.0008. 2,3,4,1 significant figures
- B. 3.14×10^7 , 0.041×10^9 , 0.0008, 2.998×10^{-9} . 3,2,1,4 significant figures
- C. 2.998×10^{-9} , 0.041×10^9 , 0.0008, 3.14×10^7 . 4,2,1,3 significant figures
- D. 0.0008, 0.041×10^9 , 3.14×10^7 , 2.998×10^{-9} . 1,2,3,4 significant figures
- E. 0.0008, 0.041×10^9 , 2.998×10^{-9} , 3.14×10^7 . 1,2,4,3 significant figures



Estimation

- You should often estimate an answer to a given problem
 - It can provide substantial insight into a problem or physical situation
 - It's a good "reality check" or "gut check" to compare to calculation
 - It's good mental exercise in rough calculation of various quantities
- Example: What's the United States' yearly automobile gasoline consumption?
 - There are about 300 million people in the U.S., so perhaps about 100 million cars (10^8 cars).
 - A typical car goes about 10,000 miles per year (10^4 miles).
 - A typical car gets about 20 miles per gallon.
 - So in a year, a typical car uses $(10^4 \text{ miles}) / (20 \text{ miles/gallon}) = 500$ gallons
 - So the United States' yearly gasoline consumption is about $(500 \text{ gal/car})(10^8 \text{ cars}) = 5 \times 10^{10}$ gallons.
 - That's about 20×10^{10} liters or 200 Gl.
 - That's also only about a factor of 2-3 too low from the actual number!



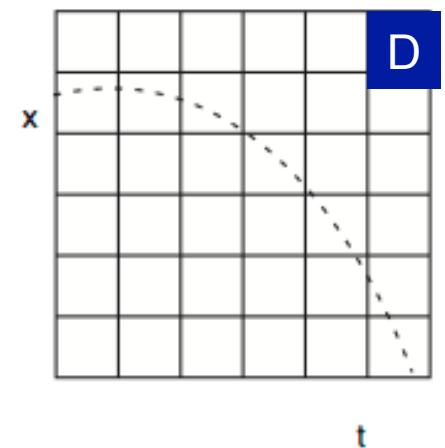
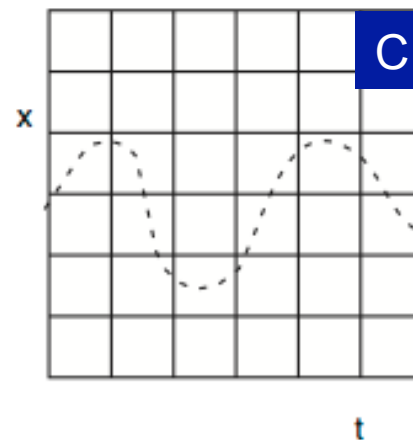
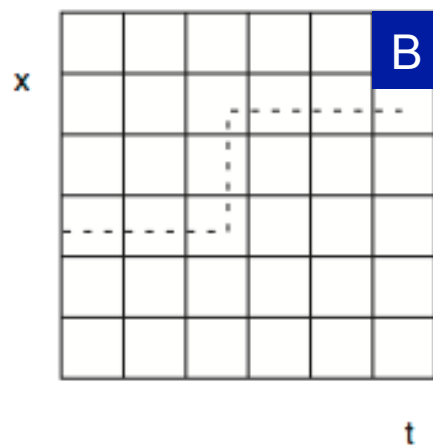
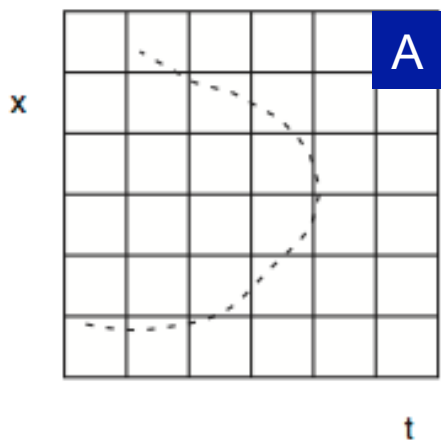
(Honors Estimation)

- These are extra problems if the honors students are feeling up to the challenge. I'll try to include at least one in every class.
- About how far away is the Earth's horizon...
 - For a 6' tall person standing at ground level?
 - For someone looking out the window of a plane flying at 30,000 feet?
 - Assume a perfectly spherical Earth, of course... ☺
 - (Hint: To within a few percent, each time zone of the Earth is about 1000 miles at the equator so you can estimate the Earth's radius quickly. We'll use this later in the semester.)



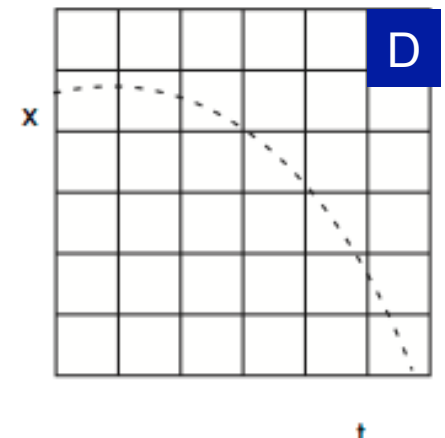
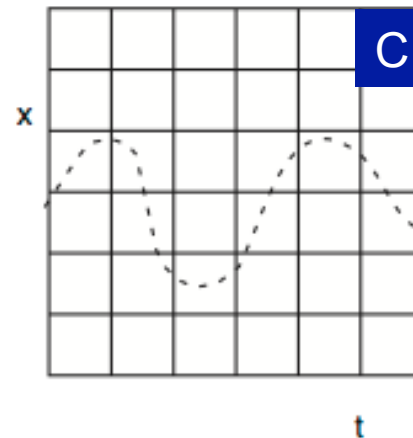
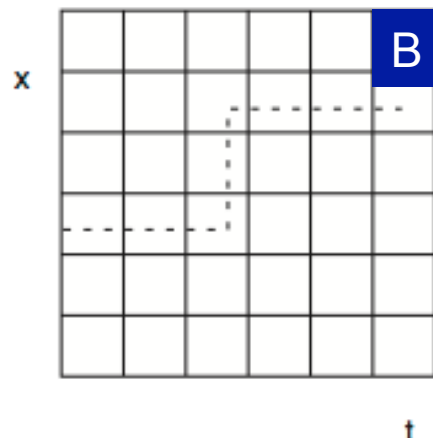
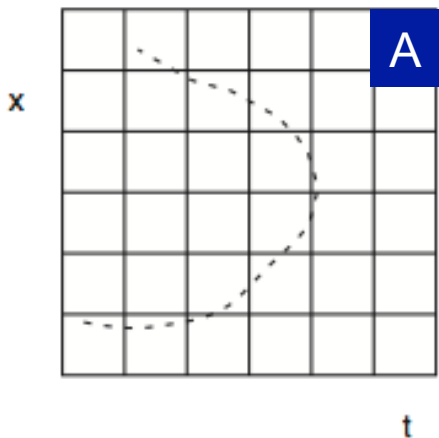
Ponderable: Graphs and Observation (10 minutes)

- Physics involves relating physical quantities together over ranges of their possible values
 - Graphs are an excellent way of visualizing relationships between numerical quantities
- Ponderables (10 minutes)
 - These plots depict an object's position x as a function of time t .
 - For each plot, describe the motion in words or explain why it is not a type of motion we see in normal everyday objects.

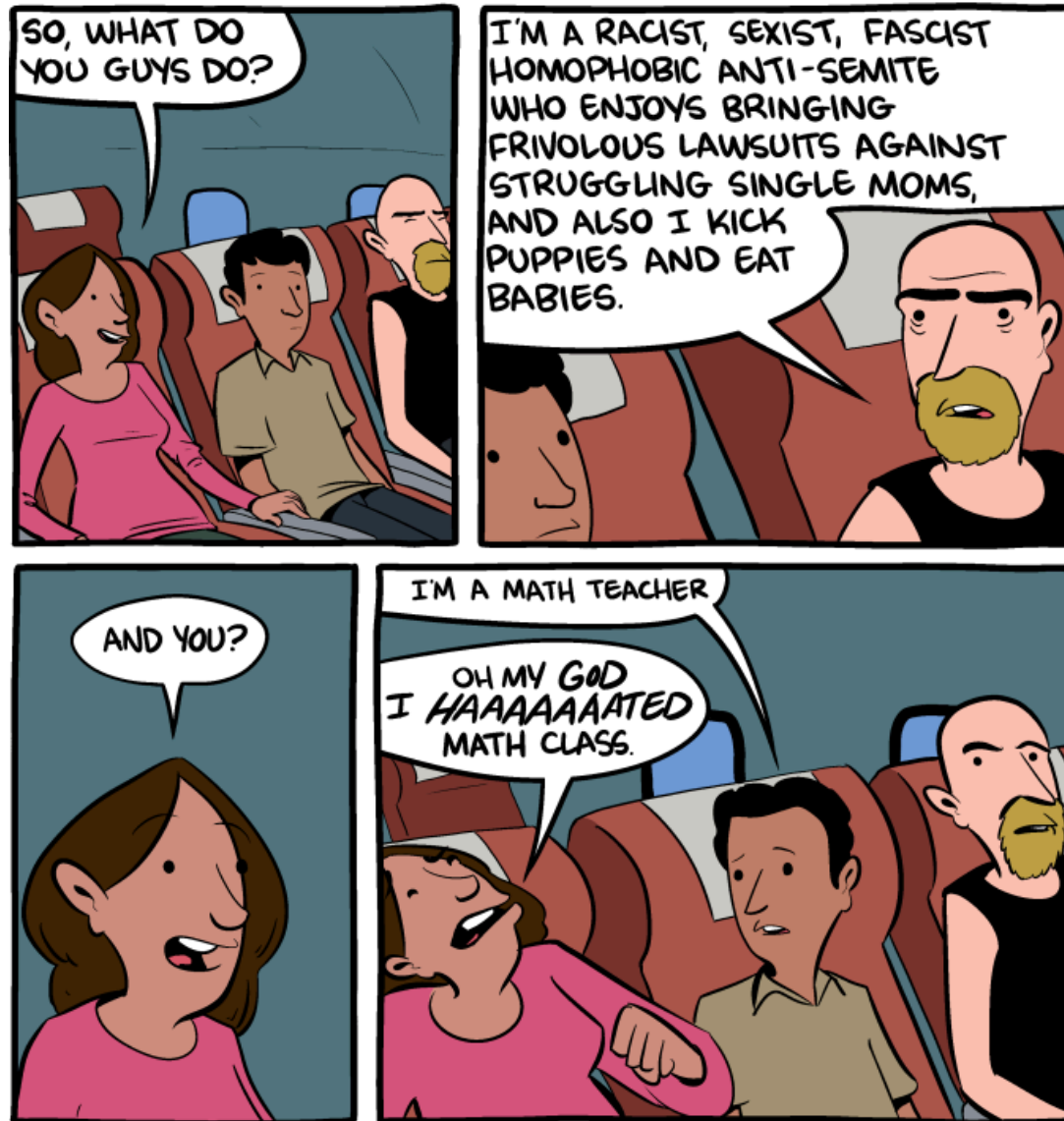


Ponderable: Graphs and Observation

- A: Normal objects are usually not in two places at once, and they usually have a well-defined position for all time.
- B: Normal objects usually don't suddenly jump from one location to another.
- C: Normal objects can oscillate in time, like a mass on a spring or a pendulum (we cover those later this semester)
- D: Normal objects can move parabolically (we cover that soon!)
 - I emphasize “normal” to avoid questions of quantum mechanics.
 - More on velocity (dx/dt) and acceleration (d^2x/dt^2) next class.



(Break time)



smbc-comics.com

Coming up: Vectors, vector algebra, components



Jefferson Lab

Prof. Satogata / Fall 2016

ODU University Physics 226N/231N 33



Vectors and Scalars

- A **scalar quantity** can be described by a *single number*, and the associated physical unit if appropriate.
 - Example: speed = 30 mph (you don't specify direction)
 - Example: temperature = 293 K (about room temperature)
 - Example: circumference of a circle
- A **vector quantity** has both a *magnitude* and a *direction* in space, and the associated physical unit if appropriate.
 - Example: displacement = 2m away from me
 - Example: velocity = 100 mph heading east on Main
 - Example: wind velocity
- In this class and the textbook, a vector quantity is represented in italic type with an arrow over it: \vec{A}
- The length or magnitude of \vec{A} is written as A or $|\vec{A}|$



A Note About Math (including Vectors)

- Most of the remainder of this lecture concerns **math technique**
 - That is, how do we **do** simple math with vectors
 - Addition, subtraction, and three kinds of multiplication
 - You don't have to worry about vector division in this course!
 - Interchanging vector length/direction and vector components
 - This is all called “vector algebra”
 - We will be doing a LOT of vector algebra in this course
- The best way to learn technique is to practice
 - I have put a Vector math review on Mastering Physics for you to practice if you so desire



[Introduction to MasteringPhysics](#)

09/02/16



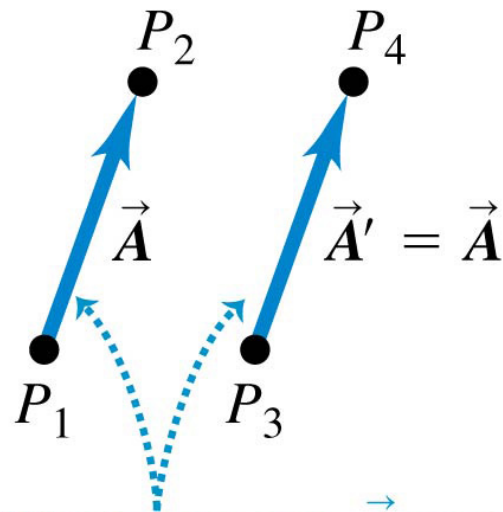
[Vector math review \(practice!\)](#)

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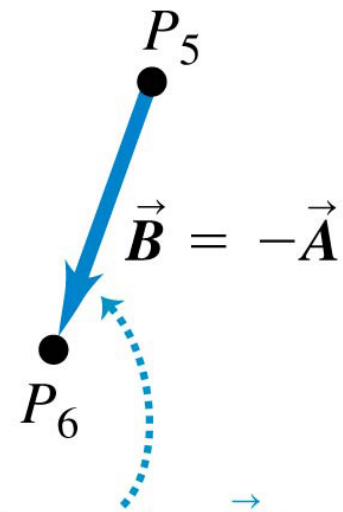


Drawing vectors

- Draw a vector as a line with an arrowhead at its tip.
- The *length* of the line shows the vector's *magnitude*.
- The *direction* of the line shows the vector's *direction*.



Displacements \vec{A} and \vec{A}' are equal because they have the same length and direction.



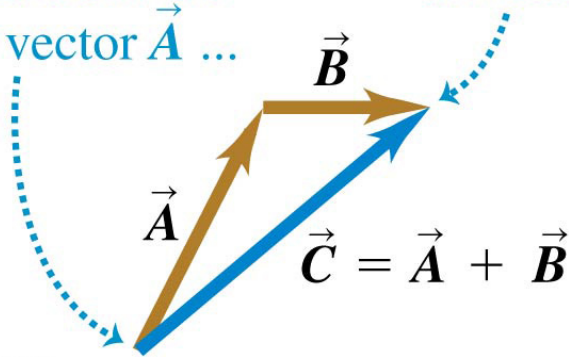
Displacement \vec{B} has the same magnitude as \vec{A} but opposite direction; \vec{B} is the negative of \vec{A} .

Adding two vectors graphically

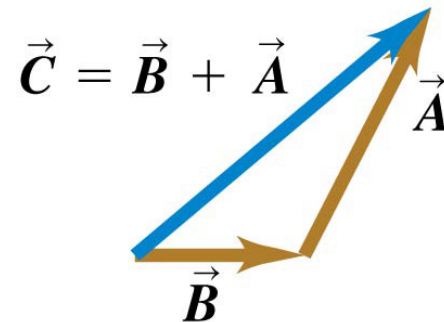
(a) We can add two vectors by placing them head to tail.

The vector sum \vec{C} extends from the tail of vector \vec{A} ...

... to the head of vector \vec{B} .



(b) Adding them in reverse order gives the same result: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. The order doesn't matter in vector addition.

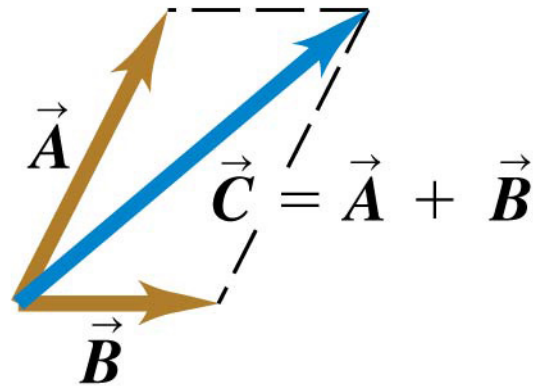


Vector addition is “commutative”



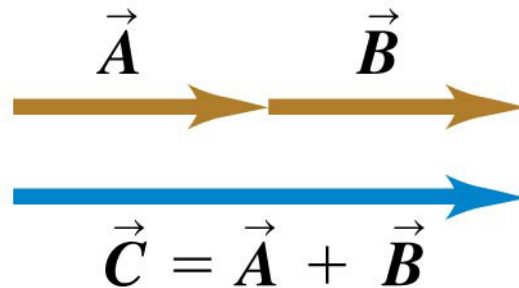
Adding two vectors graphically

(c) We can also add two vectors by placing them tail to tail and constructing a parallelogram.

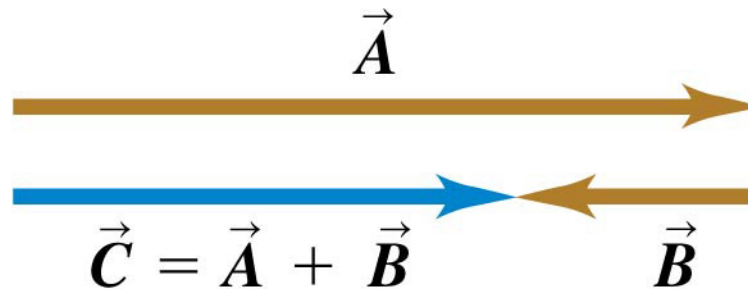


Adding two vectors graphically

(a) Only when vectors \vec{A} and \vec{B} are parallel does the magnitude of their vector sum \vec{C} equal the sum of their magnitudes: $C = A + B$.



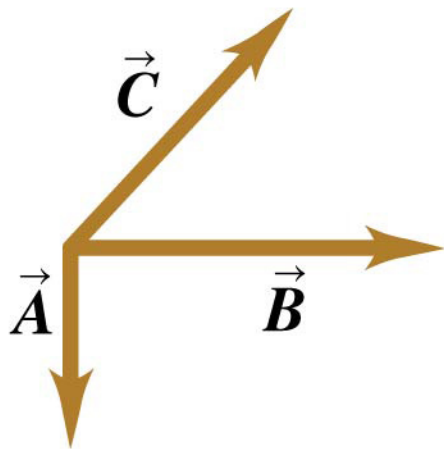
(b) When \vec{A} and \vec{B} are antiparallel, the magnitude of their vector sum \vec{C} equals the *difference* of their magnitudes: $C = |A - B|$.



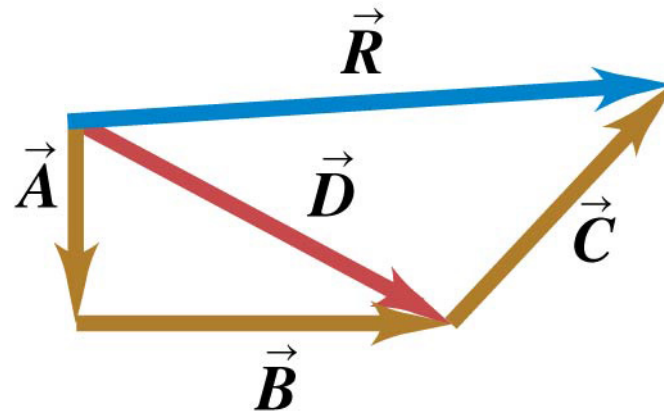
Adding more than two vectors graphically

- To add several vectors, use the head-to-tail method.
- The vectors can be added in any order.

(a) To find the sum of these three vectors ...



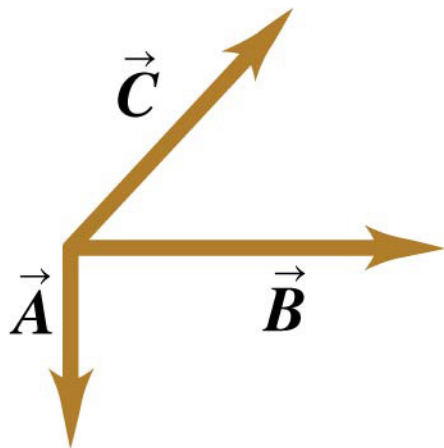
(b) ... add \vec{A} and \vec{B} to get \vec{D} and then add \vec{C} to \vec{D} to get the final sum (resultant) \vec{R} ...



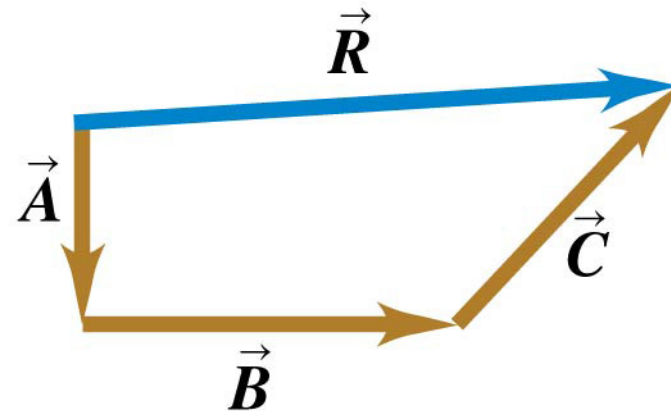
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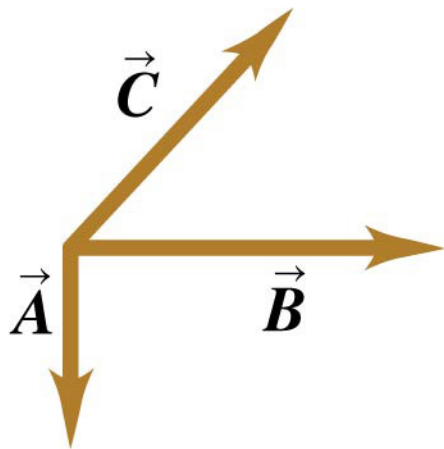
(d) ... or add \vec{A} , \vec{B} , and \vec{C} to get \vec{R} directly ...



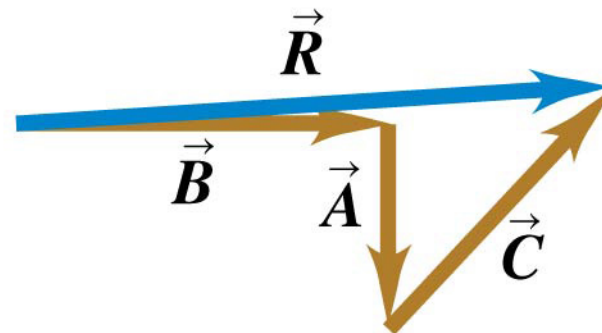
Adding more than two vectors graphically

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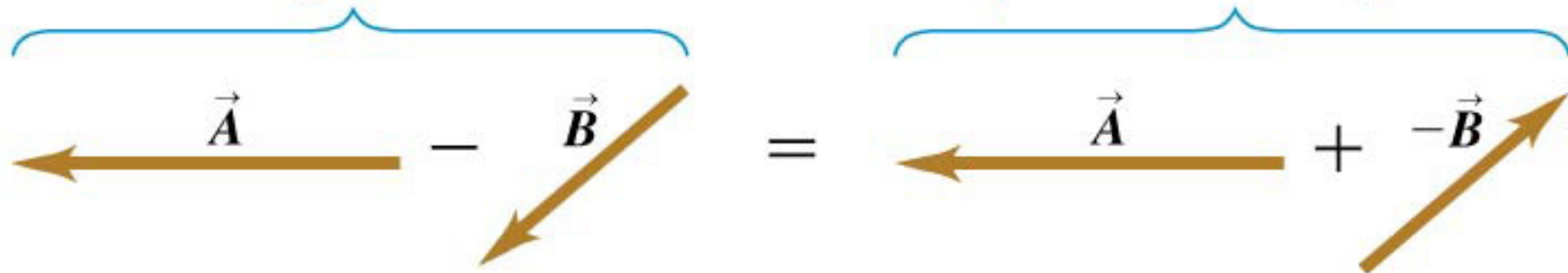
(e) ... or add \vec{A} , \vec{B} , and \vec{C} in any other order and still get \vec{R} .



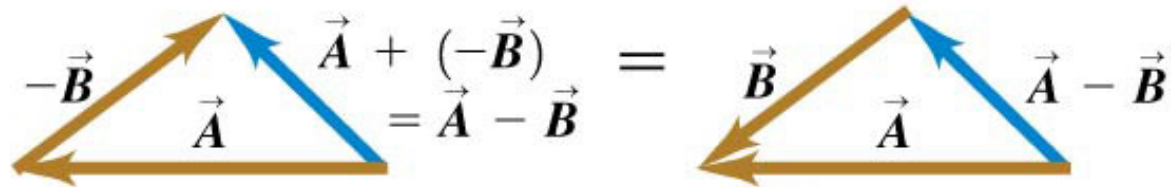
Subtracting vectors

Subtracting \vec{B} from \vec{A} ...

... is equivalent to adding $-\vec{B}$ to \vec{A} .



$$\vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$$



With \vec{A} and $-\vec{B}$ head to tail,
 $\vec{A} - \vec{B}$ is the vector from the
 tail of \vec{A} to the head of $-\vec{B}$.

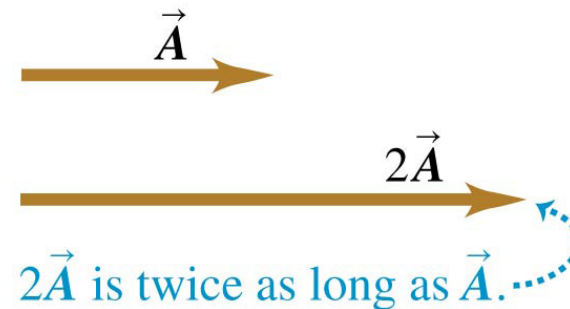
With \vec{A} and \vec{B} head to head,
 $\vec{A} - \vec{B}$ is the vector from the
 tail of \vec{A} to the tail of \vec{B} .



Multiplying a vector by a scalar

- How do we multiply vectors?
- The first multiplication to learn is multiplying a vector by a scalar.
- This gives a new vector. If c is a scalar, the product $c\vec{A}$ has magnitude $|c|A$.
- The figure illustrates multiplication of a vector by (a) a positive scalar and (b) a negative scalar.

(a) Multiplying a vector by a positive scalar changes the magnitude (length) of the vector but not its direction.



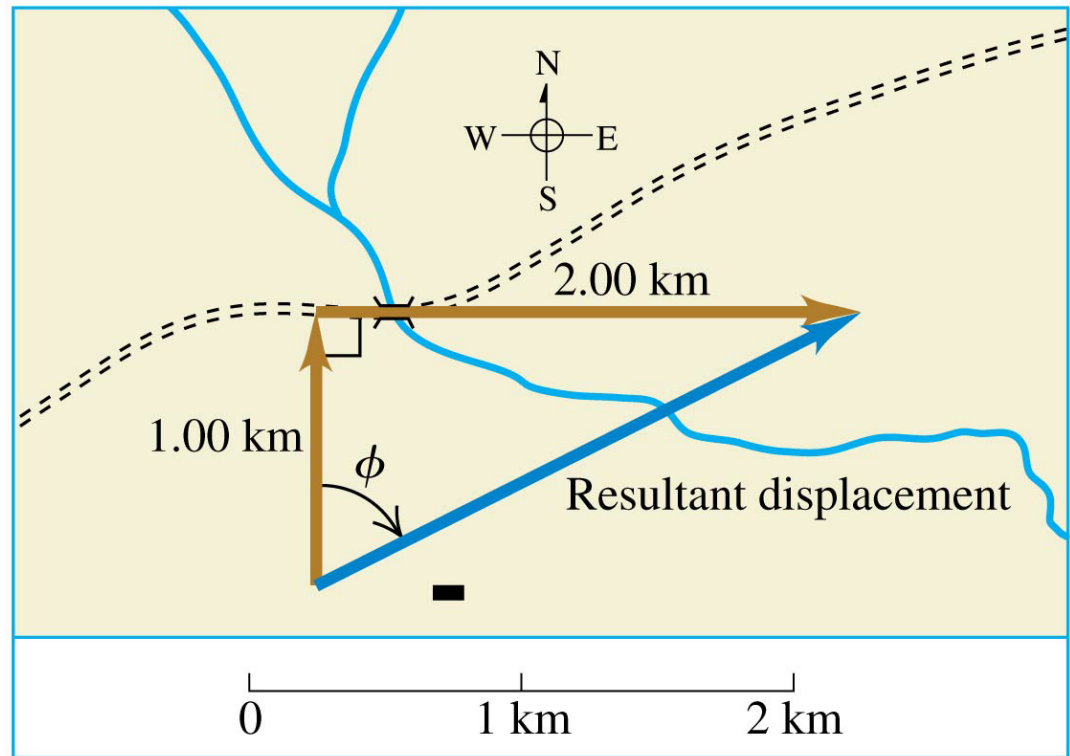
(b) Multiplying a vector by a negative scalar changes its magnitude and reverses its direction.



Addition of two vectors at right angles

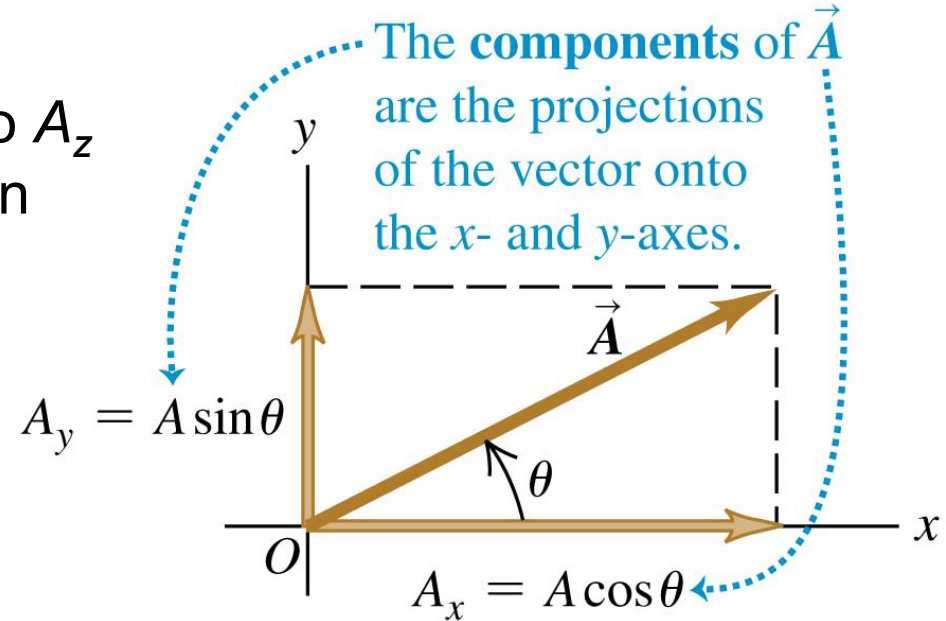
- To add two vectors that are at right angles, first add the vectors graphically.
- Then use trigonometry to find the magnitude and direction of the sum: “soh-cah-toa”
- In the figure, a cross-country skier ends up 2.24 km from her starting point, in a direction of 63.4° east of north.

$$\phi = \text{atan} \left(\frac{2 \text{ km}}{1 \text{ km}} \right)$$



Components of a vector

- Adding vectors graphically provides limited accuracy. Vector components provide a general method for adding vectors.
- Any vector can be represented by an x-component A_x and a y-component A_y – again use right-triangle trigonometry to get components.
- We can even extend this to A_z if we are doing a problem in three dimensions.
- Components of a vector always have the **same units** as the vector.

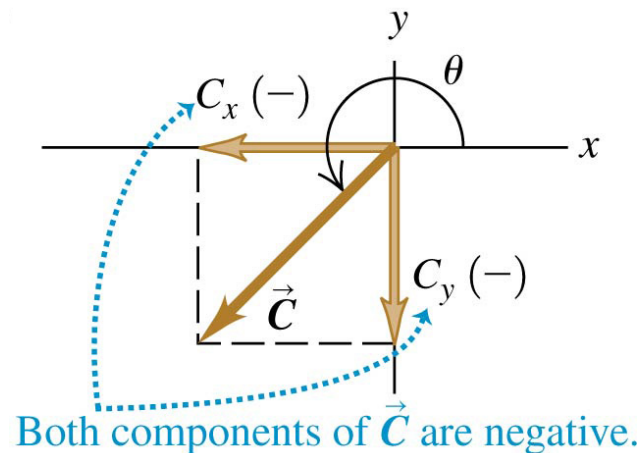
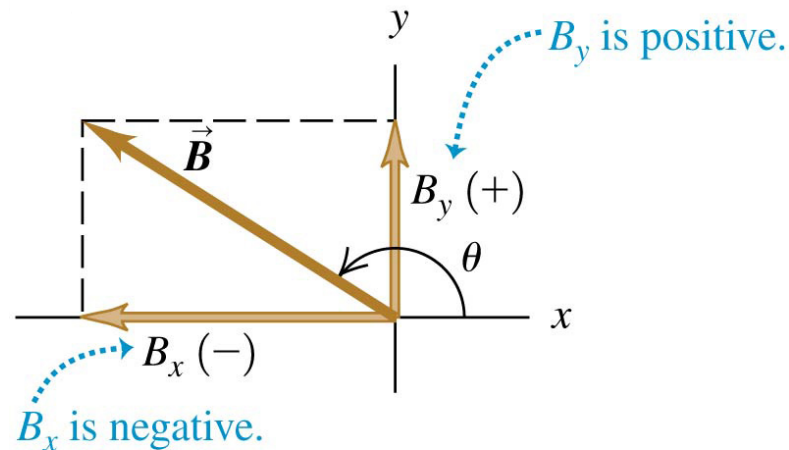


In this case, both A_x and A_y are positive.



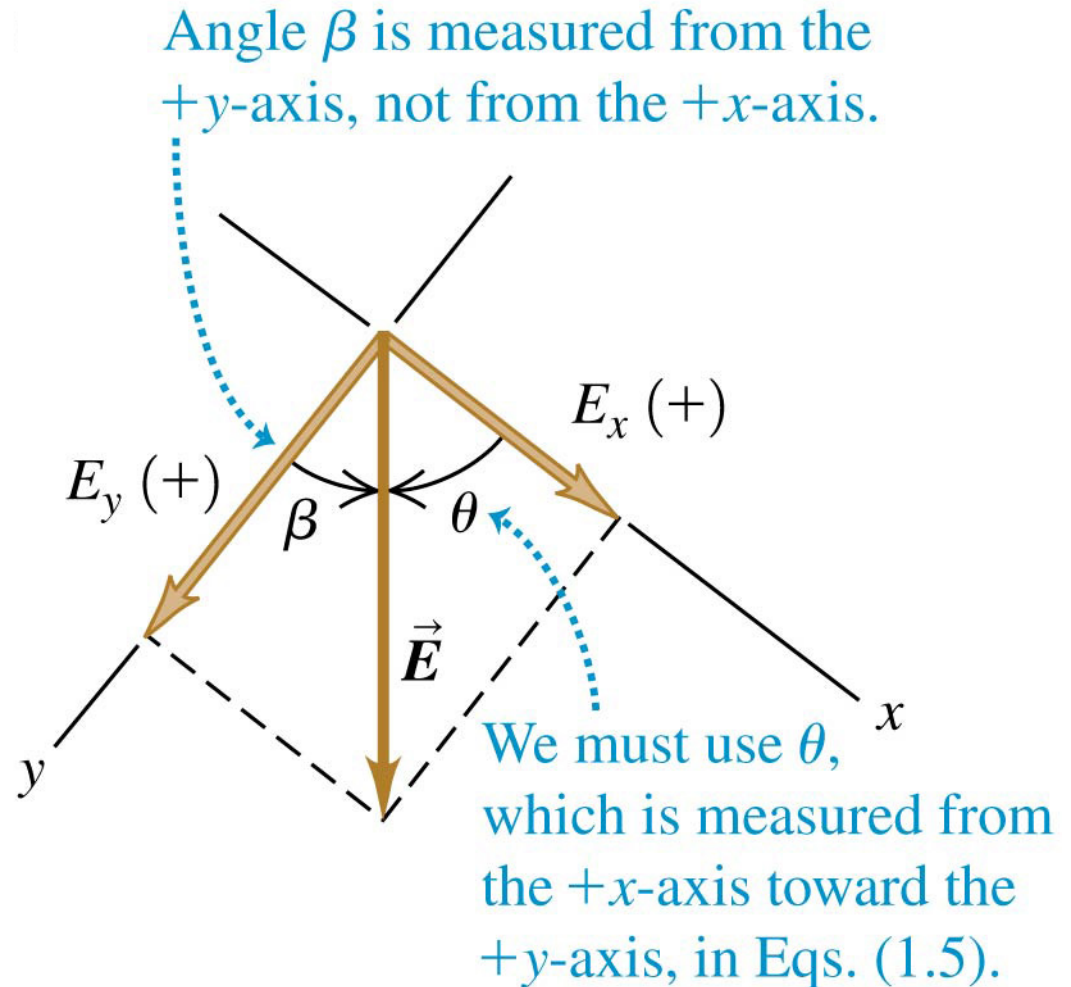
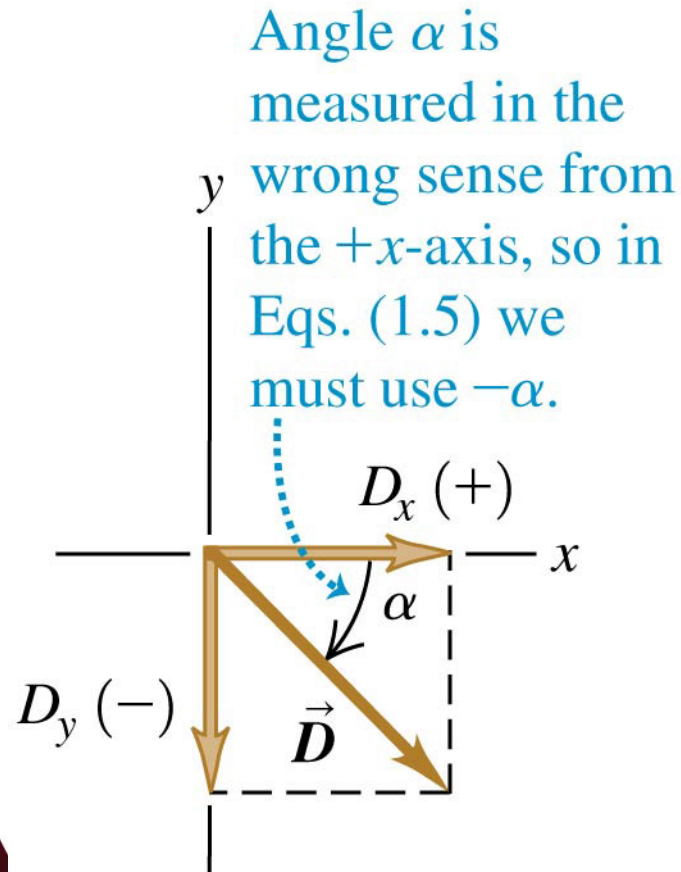
Positive and negative components

- The components of a vector may be positive or negative numbers, as shown in the figures.



Finding components

- We can calculate the components of a vector from its magnitude and direction.



Calculations using components

- We can use the components of a vector to find its magnitude and direction:

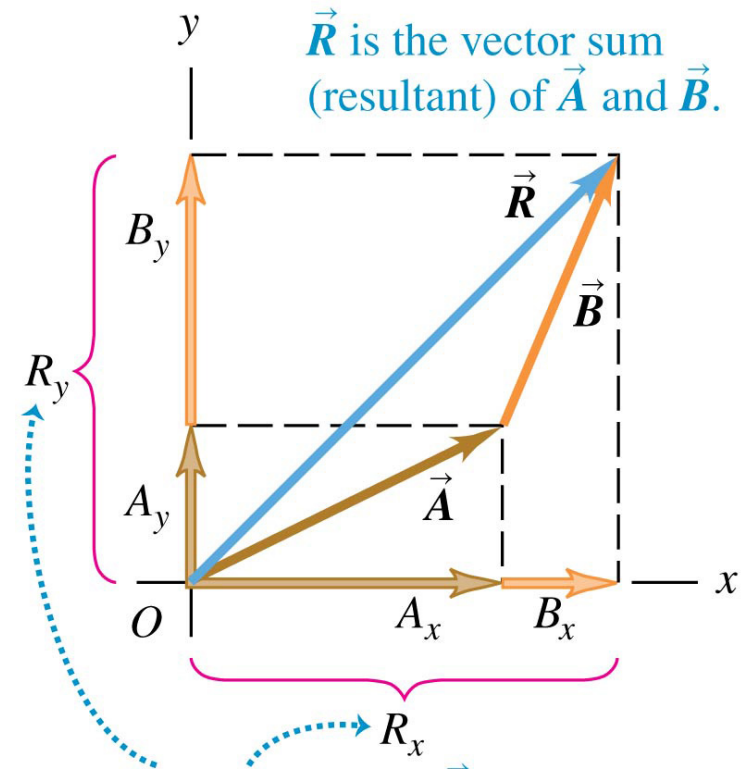
$$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

- We can use the components of a set of vectors to find the components of the resultant sum

$$R_x = A_x + B_x + C_x + \dots$$

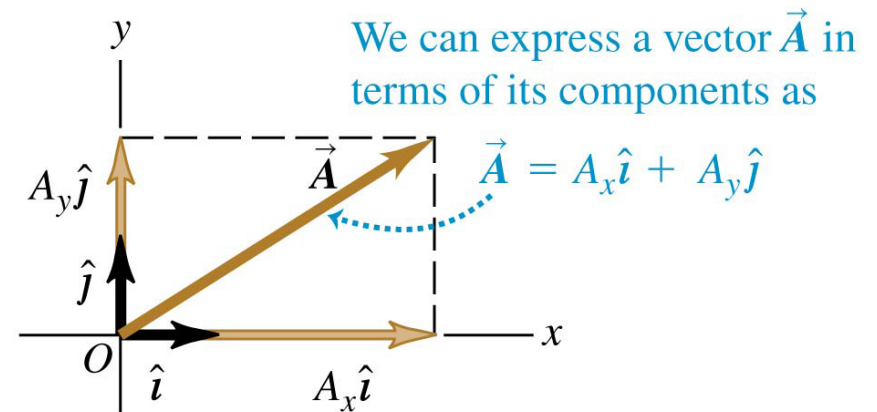
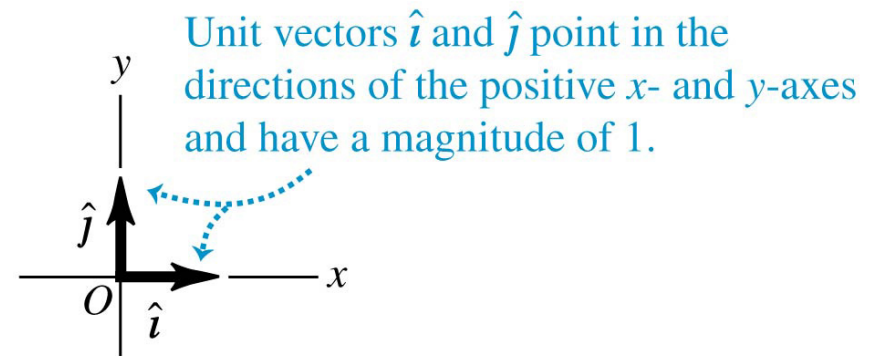
$$R_y = A_y + B_y + C_y + \dots$$



Unit vectors

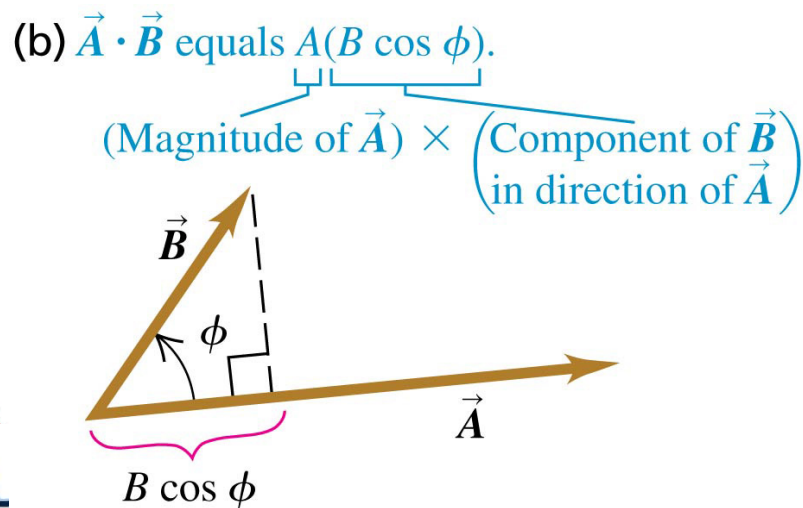
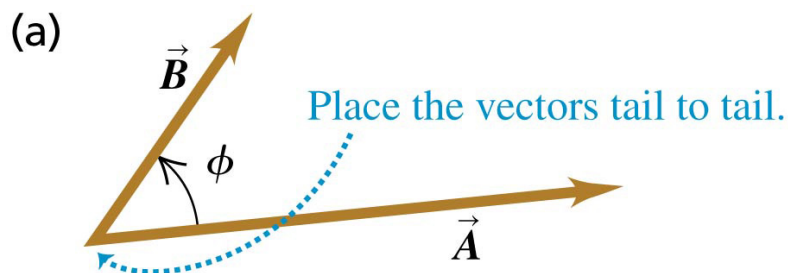
- A **unit vector** has a magnitude of 1 with **no units**.
- It is simply a useful notation to indicate vector components.
- By convention unit vectors are lower case letters with a $\hat{}$
- The unit vector \hat{i} points in the +x-direction, \hat{j} points in the +y-direction, and \hat{k} points in the +z-direction.
- Any vector can be expressed in terms of its components as

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$



The scalar product

- The scalar product is a way of “multiplying” two vectors to get a scalar. The units are multiplied too!
- One interpretation: “component in the direction of”.
- The scalar product is commutative.

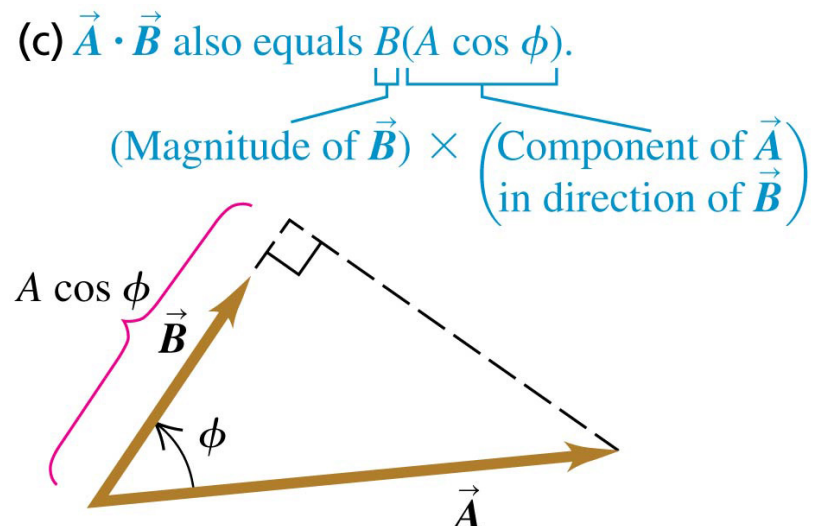


Scalar (dot) product of vectors \vec{A} and \vec{B}

Magnitudes of \vec{A} and \vec{B}

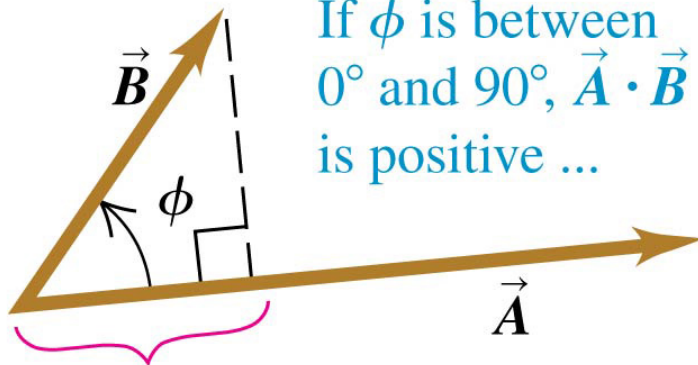
$$\vec{A} \cdot \vec{B} = AB \cos \phi = |\vec{A}| |\vec{B}| \cos \phi$$

Angle between \vec{A} and \vec{B} when placed tail to tail



The scalar product

(a)

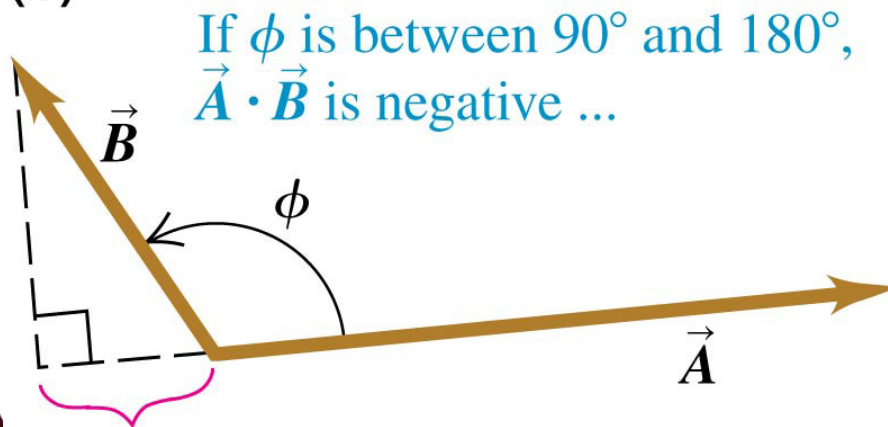


If ϕ is between 0° and 90° , $\vec{A} \cdot \vec{B}$ is positive ...

... because $B \cos \phi > 0$.

The scalar product can be positive, negative, or zero, depending on the angle between \vec{A} and \vec{B} .

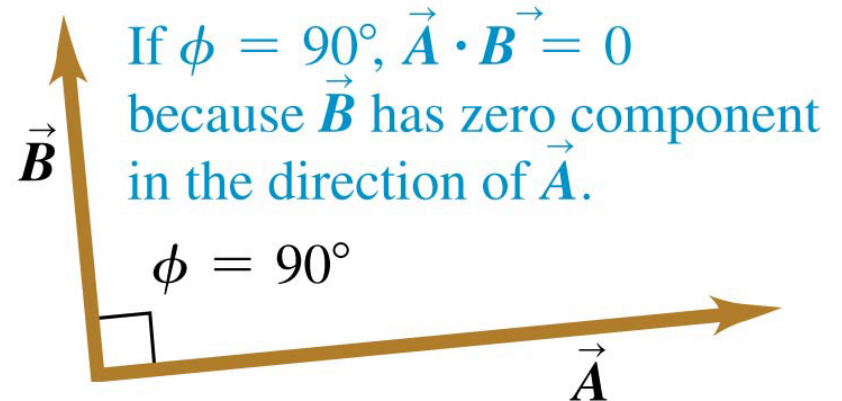
(b)



If ϕ is between 90° and 180° , $\vec{A} \cdot \vec{B}$ is negative ...

... because $B \cos \phi < 0$.

(c)



If $\phi = 90^\circ$, $\vec{A} \cdot \vec{B} = 0$ because \vec{B} has zero component in the direction of \vec{A} .

$\phi = 90^\circ$



Calculating a scalar product using components

- The scalar product of two vectors is the sum of the products of their respective components.

Scalar (dot) product
of vectors \vec{A} and \vec{B}

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Components of \vec{A}

Components of \vec{B}

$$\vec{A} \cdot \hat{i} = (A_x)(1) + (A_y)(0) + (A_z)(0) = A_x$$

- Taking a dot product with a unit vector gives the component of the original vector in the direction of the unit vector.



The vector product

If the vector product (“cross product”) of two vectors is $\vec{C} = \vec{A} \times \vec{B}$ then:

Magnitude of **vector (cross) product** of vectors \vec{B} and \vec{A}

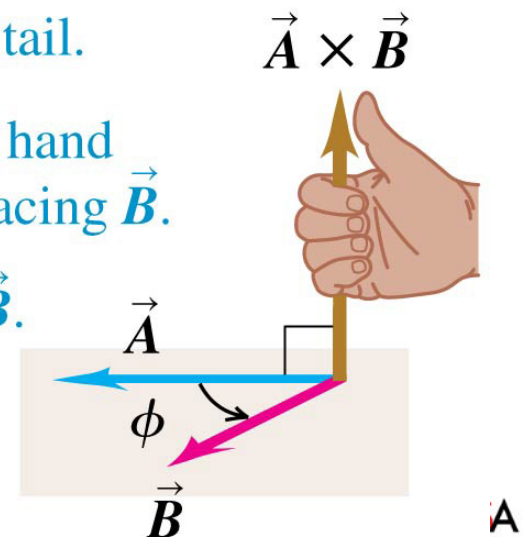
$$C = AB \sin \phi$$

Magnitudes of \vec{A} and \vec{B}

Angle between \vec{A} and \vec{B}
when placed tail to tail

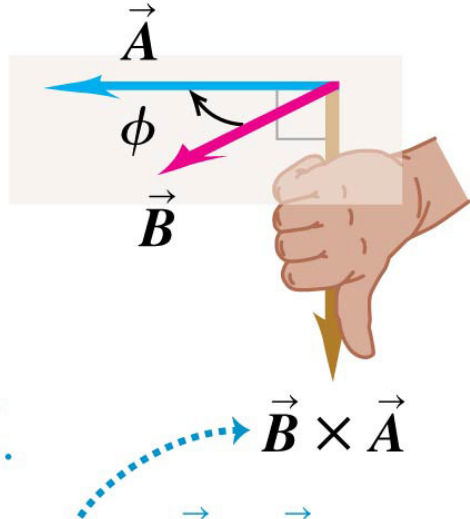
The direction of the vector product can be found using the right-hand rule:

- ① Place \vec{A} and \vec{B} tail to tail.
- ② Point fingers of right hand along \vec{A} , with palm facing \vec{B} .
- ③ Curl fingers toward \vec{B} .
- ④ Thumb points in direction of $\vec{A} \times \vec{B}$.



The vector product is anticommutative

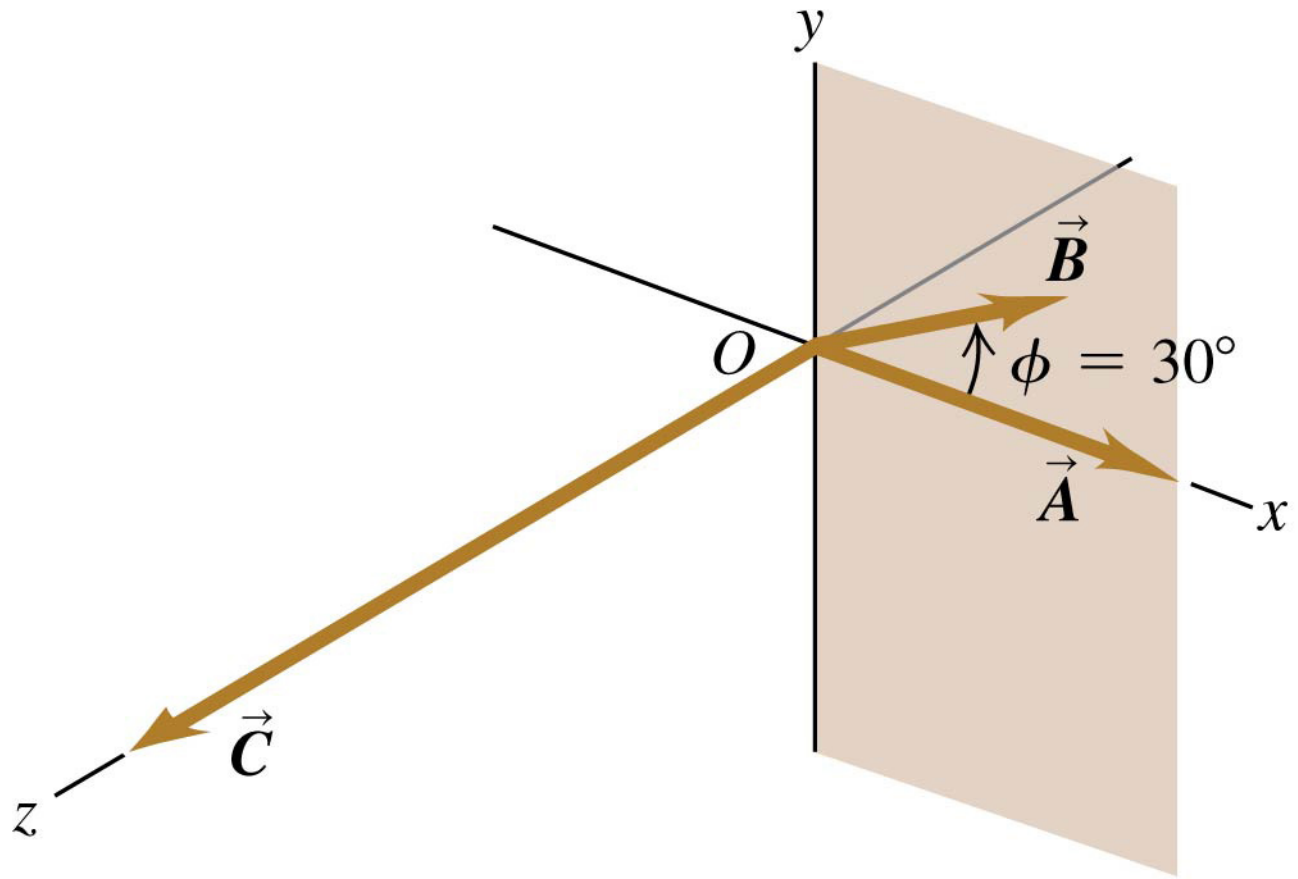
$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

- ① Place \vec{B} and \vec{A} tail to tail.
 - ② Point fingers of right hand along \vec{B} , with palm facing \vec{A} .
 - ③ Curl fingers toward \vec{A} .
 - ④ Thumb points in direction of $\vec{B} \times \vec{A}$.
 - ⑤ $\vec{B} \times \vec{A}$ has same magnitude as $\vec{A} \times \vec{B}$ but points in opposite direction.
- 



Calculating the vector product

- Use $AB\sin\phi$ to find the magnitude and the right-hand rule to find the direction.



===== Extra Slides =====



Operational Definitions

- Of the three most basic units— length, time, and mass — two are defined operationally, so their definitions can be re-created in another laboratory.
- The meanings of both these definitions will become clearer as you advance in your studies:
 - The **meter** is the length of the path traveled by light in vacuum during a time interval of $1/299,792,458$ of a second.
 - The **second** is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium-133 atom.
- The standard of mass is less satisfactory:
 - The **kilogram** is defined by the international prototype kilogram kept at the International Bureau of Weights and Measures at Sèvres, France.

