

University Physics 226N/231N Old Dominion University

Work, Energy, and Conservation

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Reminder: The Second Midterm will be Weds Oct 19 2016

Happy Birthday to Stevie Ray Vaughan, Gwen Stefani, Lena Headey,
India Arie, Alicia Vikander, and James Buchanan (1986 Nobel)!

Happy [Blue Shirt Day](#) and National Techies Day!

Please set your cell phones to “vibrate” or “silent” mode. Thanks!



I SEE (1.2)

- Most textbooks try to help by providing a structure, or recipe, that you can use when you are solving problems
- Our text uses the mnemonic “**I SEE**”
 - **I: Identify** the relevant concepts
 - What are the physical quantities are known, and unknown?
 - What is a possible model or approach to “tell the story”?
 - **S: Set Up** the problem
 - Tell the story: Draw a picture and choose equations to solve.
 - **E: Execute** the solution
 - “Do the math” or “crunch the numbers”.
 - Remember to always use units on all physical quantities!
 - **E: Evaluate** your answer
 - Does your answer make sense? Check the units.

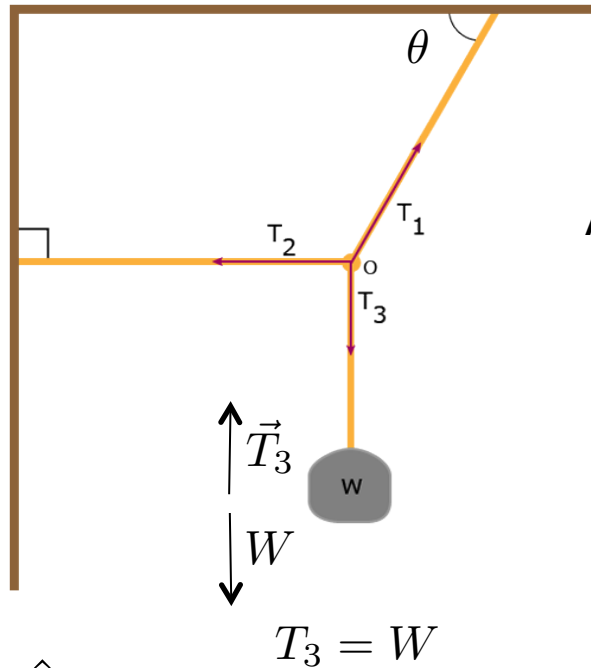


I SEE (“Statics”)

- **I: Identify** the relevant concepts
 - What are the physical quantities are known, and unknown?
 - “Statics” problems involve **forces acting on objects**: $\vec{F}_{\text{net}} = m\vec{a}$
 - Usually those objects are in equilibrium ($\vec{a} = 0$) but not always
- **S: Set Up** the problem
 - Tell the story: Draw a picture and choose equations to solve.
 - Draw your coordinate system. Decompose all vectors into components.
 - Write out Newton’s Second Law in each relevant dimension.
- **E: Execute** the solution
 - “Do the math” or “crunch the numbers”.
 - Can involve solving “multiple equations in multiple unknowns”
- **E: Evaluate** your answer
 - Does your answer make sense? Check the units.

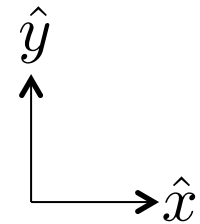
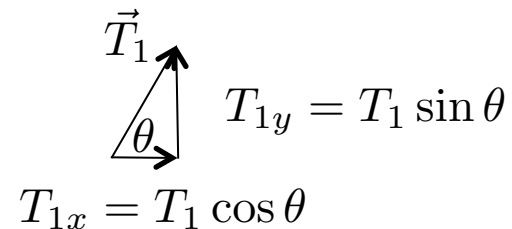
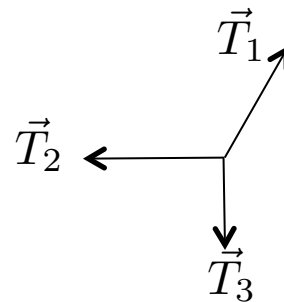


Example Statics Problem



Can apply Newton's 2nd Law to any point
(but it is "trivial" for points along the ropes)

At point "o" where tensions meet, draw forces and decompose forces into their x,y components:



$$\vec{F}_{\text{net}} = m\vec{a}$$

Then use Newton's 2nd Law in each of x and y directions:

$$\textcircled{x} \quad F_{\text{net},x} = T_1 \cos \theta - T_2 = 0$$

$$\textcircled{y} \quad F_{\text{net},y} = T_1 \sin \theta - T_3 = T_1 \sin \theta - W = 0$$

Here you are done with the physics: the rest is math

$$\textcircled{y} \Rightarrow T_1 \sin \theta = W \quad \Rightarrow \quad T_1 = \frac{W}{\sin \theta}$$

$$\textcircled{x} \Rightarrow T_2 = T_1 \cos \theta = W \frac{\cos \theta}{\sin \theta} = W \cot \theta$$



Example Statics Problem (Photo from class)

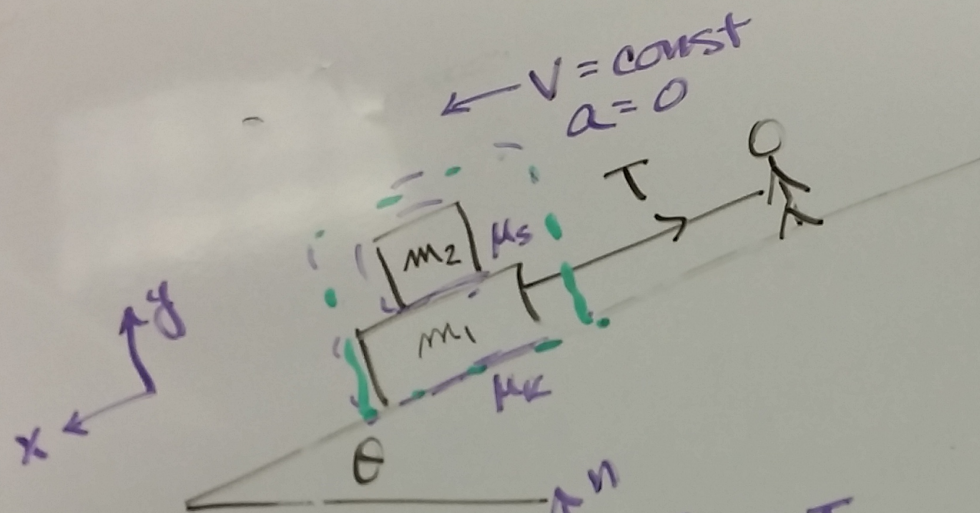
$\vec{F}_{\text{net}} = m\vec{a}$
 \hat{y}
 \hat{x}

$\theta = 0$
 θ
 T_1
 T_2
 T_3
 w
 w
 $T_2?$

STATICS
 T_1
 T_2
 T_3
 $T_1 \sin \theta$
 $T_1 \cos \theta$
 $F_{\text{net } y} = T_3 - w = 0$
 $T_3 = w$

(x) $F_{\text{net } x} = T_1 \cos \theta - T_2 = 0$
 (y) $F_{\text{net } y} = T_1 \sin \theta - T_3 = 0$
 Done with physics (u) $T_1 = \frac{w}{\sin \theta}$
 $T_2 = T_1 \cos \theta = w \cdot \frac{\cos \theta}{\sin \theta} = w \cdot \cot \theta = T_2$





"STATICS"

① Identify statics

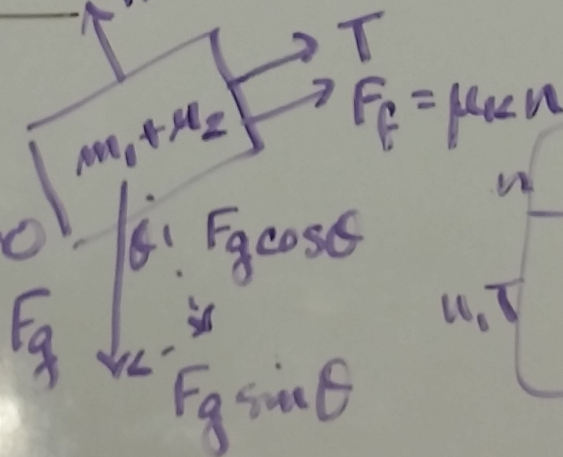
② Setup
given velocity
 μ_k μ_s coeff of friction
 m_1 m_2 θ

③ Evaluate N2 Law

$$n = F_g \cos \theta$$

$$\Rightarrow F_g \sin \theta - T - \mu_k F_g \cos \theta = 0$$

$$T = F_g \sin \theta - \mu_k F_g \cos \theta$$



$$n \quad F_{\text{net } y} = n - F_g \cos \theta = 0$$

$$n, T \quad F_{\text{net } x} = F_g \sin \theta - T - \mu_k n = 0$$

I SEE (Energy and Work)

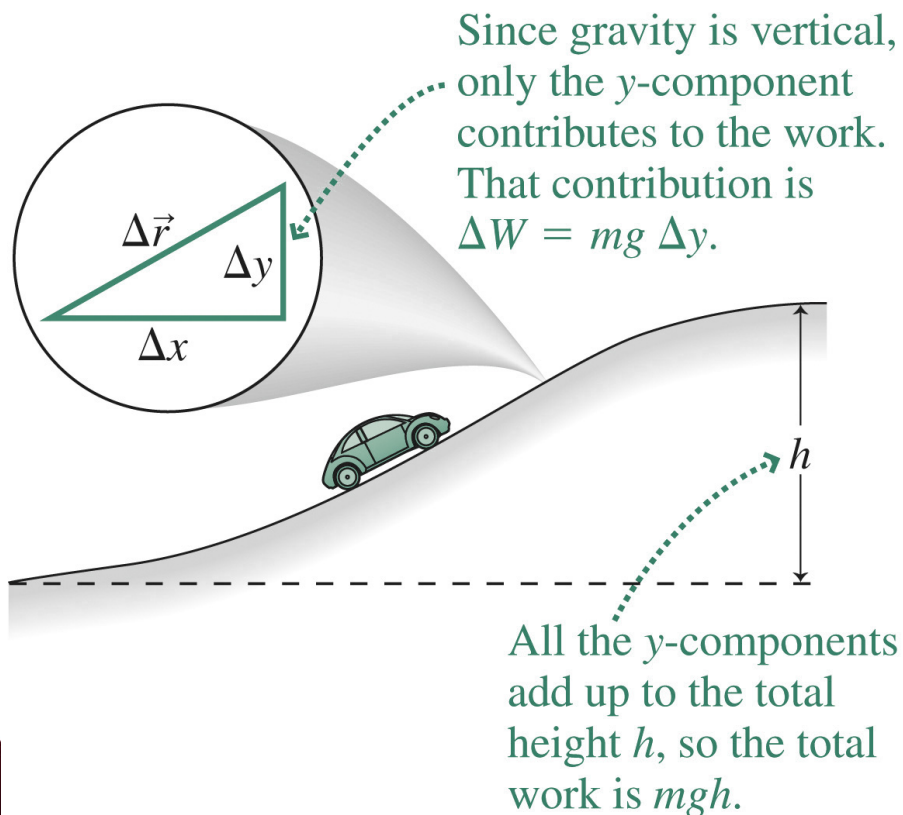
- **I: Identify** the relevant concepts
 - What are the physical quantities are known, and unknown?
 - Energy problems involve **comparing instances of time**: $\vec{E}_{\text{net}} = \text{constant}$
 - Often those objects are moving at some point (kinetic energy)
- **S: Set Up** the problem
 - Tell the story: Draw a picture and choose equations to solve.
 - Write out an energy accounting spreadsheet. Fill in known energies.
- **E: Execute** the solution
 - “Do the math” or “crunch the numbers”.
 - Can involve solving “multiple equations in multiple unknowns”
- **E: Evaluate** your answer
 - Does your answer make sense? Check the units.



Work Done Against Gravity

- The work done by an agent lifting an object of mass m against gravity depends only on the vertical distance h :

$$W = mgh$$



- The work is **positive** if the object is raised (moved against the force of gravity) and **negative** if it's lowered (moved with the force of gravity).
- The horizontal motion



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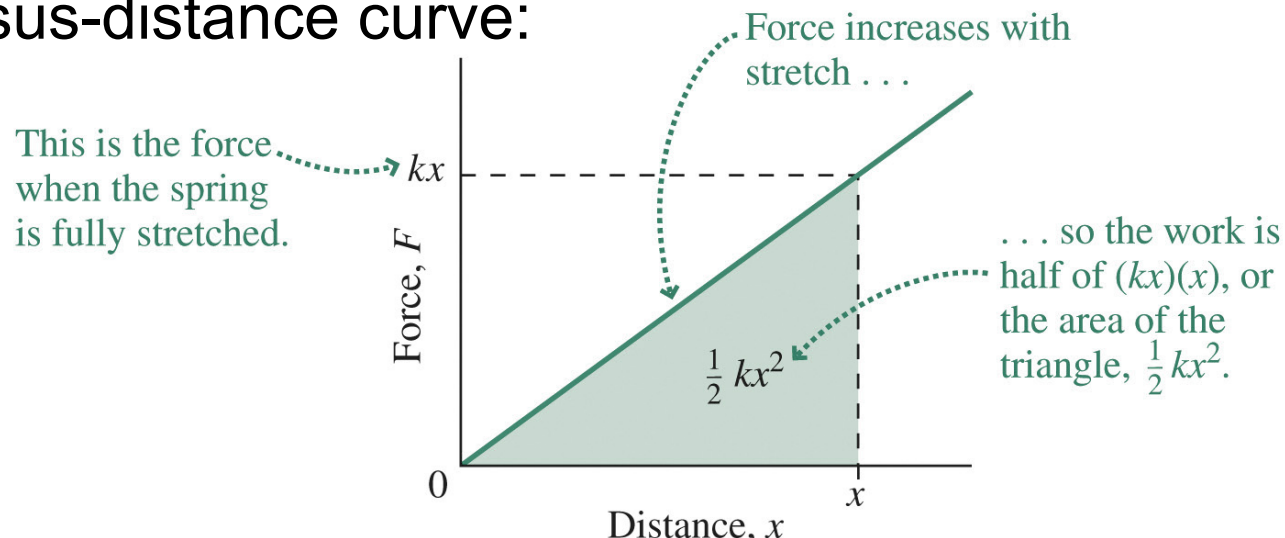


Work Done in Stretching a Spring

- A spring exerts a force $F_{\text{spring}} = -kx$
- Someone stretching a spring exerts a force $F_{\text{stretch}} = +kx$, and the work done is

$$W = \int_0^x F(x) dx = k \int_0^x x dx = \left(\frac{1}{2} kx^2 \right) \Big|_0^x = \boxed{\frac{1}{2} kx^2 = W}$$

- In this case the work is the area under the triangular force-versus-distance curve:



Energy

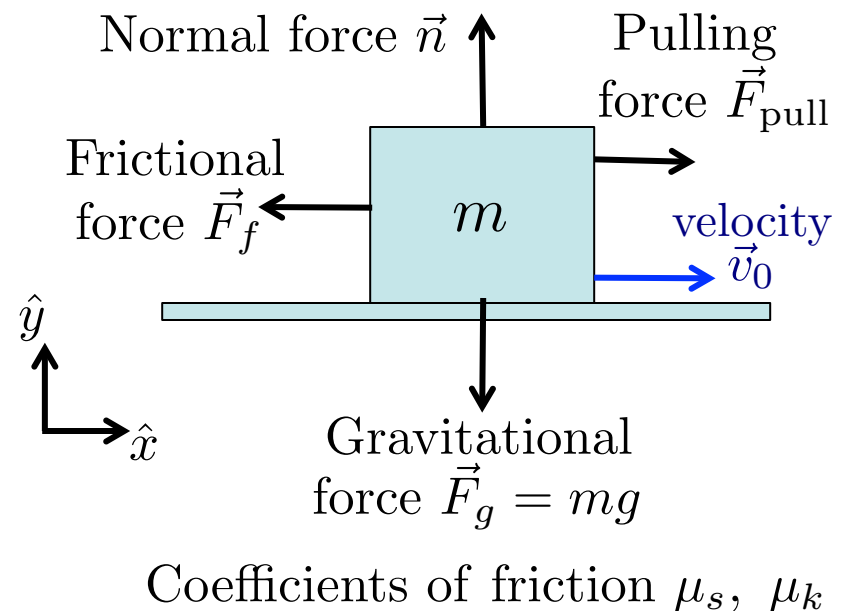
- **Energy**: the capacity of an object to perform work
 - **Energy** is what we add up when we do our bookkeeping
 - **Work** is how energy moves through application of forces
- How do we do the energy bookkeeping for a system?
 - Add up energy from a variety of different sources and things that we know can do work
 - **Conservation of energy**: total energy for a system is constant
- Kinetic energy: energy of object's motion, $KE = \frac{1}{2} mv^2$
- Gravitational potential energy: energy from the potential of falling a certain distance under constant gravity: $PE_g = mg\Delta y$
- Spring potential energy: $PE_s = \frac{1}{2} kx^2$
- Energy lost to friction over distance Δx : $E_f = \mu_x n \Delta x$
- Chemical energy, nuclear energy, and others...



Work and Net Work

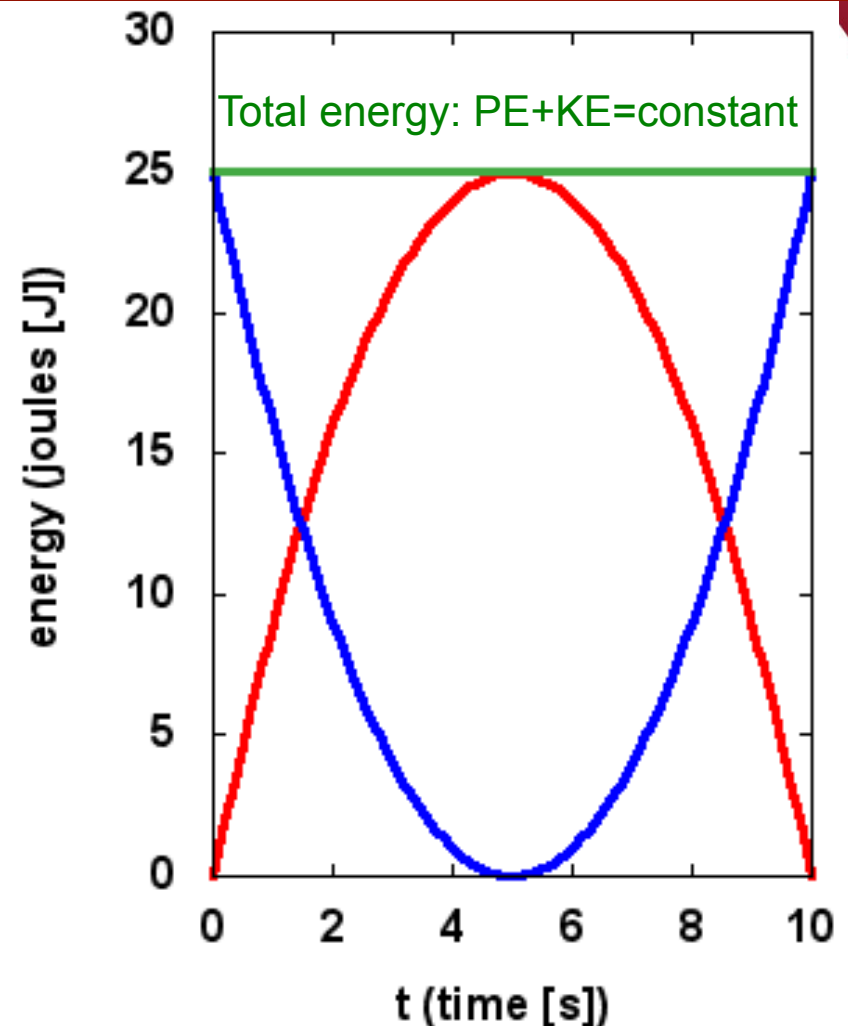
- **Energy**: the capacity of an object to perform work
 - **Energy** is what we add up when we do our bookkeeping
 - **Work** is how energy moves through application of forces
- Since work involves transfer of energy, and we want to account for all energy, it's important to account for all forces

- Example: Pulling a box against friction at constant velocity
 - Net sum of forces on box is zero
 - So **work done on box** is zero
 - But I still do work (I'm exerting a force over a distance)
 - The energy of my work goes into **frictional losses**



Example: Ball Toss

- Consider your professor tossing a juggling ball upwards
 - I do some work on it to add energy to the system
 - The system is now the ball!
- At start, $h=0$ m and all energy is kinetic energy
- As the ball moves up, potential energy grows and kinetic energy goes down
- At top, all energy is potential energy since $v=0$ m/s and $KE=0$ J
- As the ball comes back down, potential energy is released and kinetic energy grows again



Kinetic energy: $KE = \frac{1}{2} mv^2$

Potential energy: $PE_g = mgh$

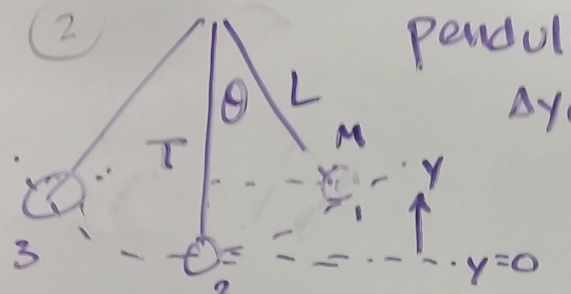


Energy Conservation: Pendulum

2

pendulum

$\Delta y_1 = L - L \cos \theta$



3

2

$y=0$

y

T

mg

$a_y = \frac{v_2^2}{L}$

| Energy term | 1 | 2 |
|-----------------|-----------------|-----------------------|
| PE _g | $mg \Delta y_1$ | 0 |
| KE | 0 | $\frac{1}{2} m v_2^2$ |

$\Sigma_1 = \Sigma_2$

$mg \Delta y_1 = \frac{1}{2} m v_2^2$

$v_2 = \sqrt{2g \Delta y_1}$

$v_2 = \sqrt{2gL(1 - \cos \theta)}$

$F_{net,y} = ma_y = T - mg = \frac{mv_2^2}{L} = \frac{2mg(1 - \cos \theta)}{1} = 2mg(1 - \cos \theta)$

$T = mg + 2mg(1 - \cos \theta) = mg[1 + 2(1 - \cos \theta)] = mg(3 - 2\cos \theta)$

Energy

PE_g = $mg \Delta y$

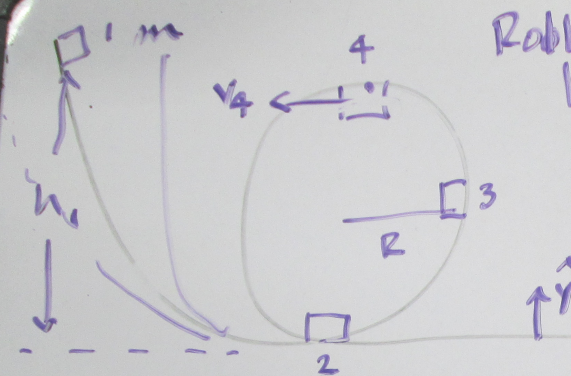
PE_s = $\frac{1}{2} k x^2$

KE = $\frac{1}{2} m v^2$

$E_f = \mu_k n \Delta x$

Energy Conservation: Loop the Loop

Roller coaster
loop : start $v = 0$
what h_1 necessary to just barely make it around the loop?
 $n_4 = 0N$



| Energy term | 1 | 4 |
|-------------|---------|---------------------|
| PEg | mgh_1 | $mg \cdot 2R$ |
| KE | 0 | $\frac{1}{2}mv_4^2$ |

$\Rightarrow mgh_1 = mg \cdot 2R + \frac{1}{2}mv_4^2$
2 unknowns! v_4, h_1
 $gh_1 = 2Rg + \frac{1}{2}v_4^2$

Circular Motion
 $v_4 = \text{constant}$
 $a = \frac{v_4^2}{R}$
 $F_{\text{net}} = mg = m/a = \frac{mv_4^2}{R}$
 $\Rightarrow v_4 = \sqrt{Rg}$

$gh_1 = 2Rg + \frac{1}{2} \cdot Rg = \frac{5}{2}Rg$
 $\Rightarrow h_1 = \frac{5}{2}R$

Energy Conservation: With Friction and Spring

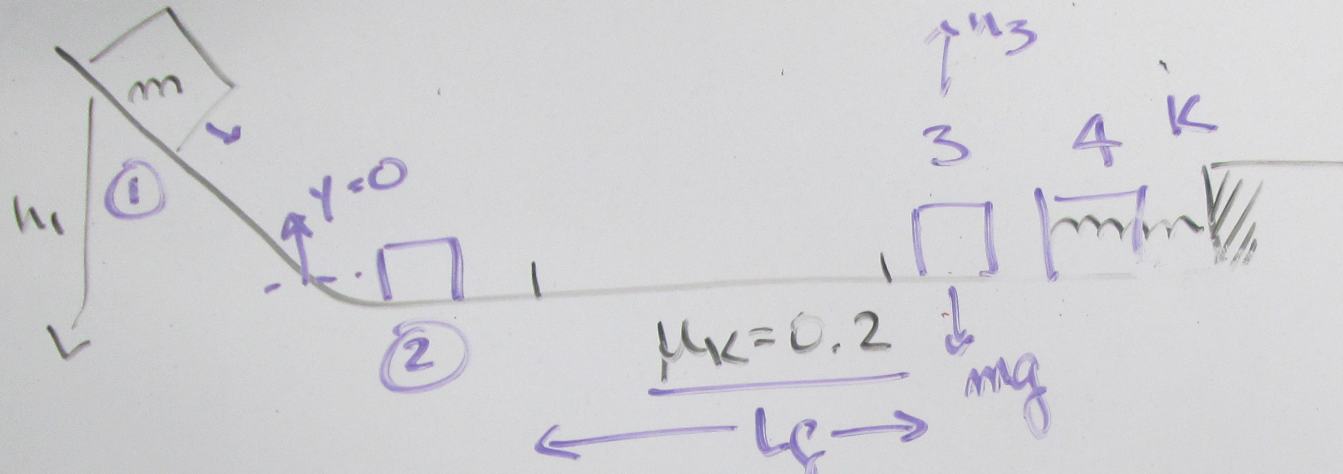


Diagram labels: 1 (start on incline), 2 (start of horizontal surface), 3 (end of frictionless section), 4 (spring compression).
 Incline height: h .
 Horizontal surface length: L_f .
 Coefficient of friction: $\mu_k = 0.2$.
 Forces: n (normal), mg (weight), f (friction).

Equations:

$$PE_g = mgh$$

$$PE_s = \frac{1}{2}k\Delta x^2$$

$$KE = \frac{1}{2}mv^2$$

$$E_f = \mu_k \cdot n \cdot \Delta x$$

$$E_f = \underline{F \cdot \Delta x}$$

| Energy | 1 | 2 | 3 | 4 |
|--------|---|---------------------|---------------------|----------------------------|
| PE_g | mgh_1 | 0 | 0 | 0 |
| KE | $\frac{1}{2}mv_1^2$ | $\frac{1}{2}mv_2^2$ | $\frac{1}{2}mv_3^2$ | 0 |
| PE_s | 0 | 0 | 0 | $\frac{1}{2}k\Delta x_4^2$ |
| E_f | 0 | 0 | $\mu_k n_3 L_f$ | $\mu_k n_3 L_f$ |