

University Physics 226N/231N Old Dominion University

Energy Conservation, Potentials, Energy Diagrams

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Reminder: The Second Midterm will be Weds Oct 19 2016

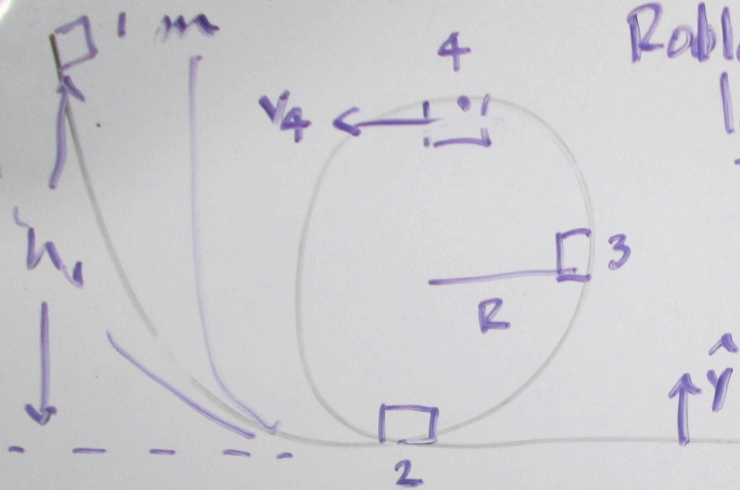
Happy Birthday to Neil deGrasse Tyson, Sean Carroll,
Robert Goddard, Kate Winslet, Ray Kroc, and Vaclav Havel!
The [2016 Physics Nobel prize](#) was awarded yesterday!!

Please set your cell phones to “vibrate” or “silent” mode. Thanks!



Reminder: Loop the Loop

Roller coaster
loop : start $v = 0$
what h_1 necessary to just barely make it around the loop?
 $v_4 = 0 \text{ m/s}$



Energy term	1	4
PEg	mgh_1	$mg \cdot 2R$
KE	0	$\frac{1}{2}mv_4^2$

$\Rightarrow mgh_1 = mg \cdot 2R + \frac{1}{2}mv_4^2$
2 unknowns! v_4, h_1
 $gh_1 = 2Rg + \frac{1}{2} \cdot Rg = \boxed{h_1 = \frac{5}{2}R}$

→ Circular Motion
 $v_4 = \text{constant}$
 $a = \frac{v_4^2}{R}$
 $F_{\text{net}} = mv_4^2/R = mg = \frac{mv_4^2}{R}$
 $\boxed{v_4 = \sqrt{Rg}}$

Loop the Loop Demo (Hot Wheels!)

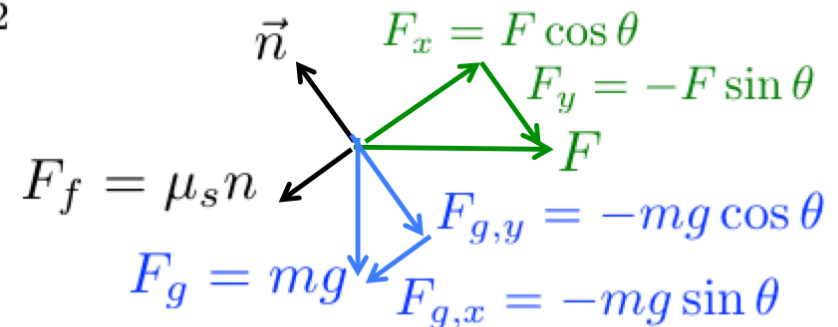
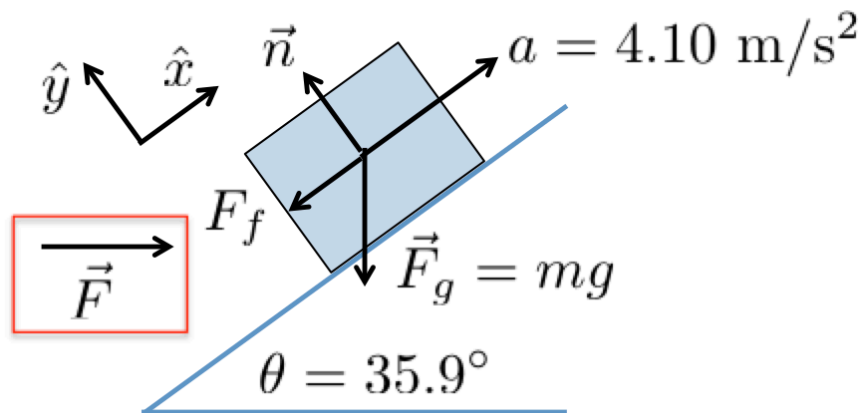


Review: Solution to Homework 5.72

A 5.00-kg box sits on a ramp that is inclined at 35.9° above the horizontal. The coefficient of kinetic friction between the box and the ramp is 0.27.

Part A

What horizontal force is required to move the box up the incline with a constant acceleration of 4.10 m/s^2 ?



Newton's 2nd law y:

$$n + F_{g,y} + F_y = 0 \quad n = mg \cos \theta + F \sin \theta$$

Newton's 2nd law x:

$$F_x + F_{g,x} + F_f = ma$$

$$F \cos \theta - mg \sin \theta = \mu_s n = ma$$

$$F \cos \theta - mg \sin \theta - \mu_s (mg \cos \theta + F \sin \theta) = ma$$

$$F(\cos \theta - \mu_s \sin \theta) - mg \sin \theta - \mu_s mg \cos \theta = ma$$

$$F = \frac{ma + mg \sin \theta + \mu_s mg \cos \theta}{\cos \theta - \mu_s \sin \theta} = 91.98 \text{ N}$$



Example: Car Going Around a Banked Turn

Car going around banked turn: how fast can I go?

μ_s known
 θ bank known

R
 v
 $a = \frac{v^2}{R}$
 (circular motion)

$F_f = \mu_s n$
 $a_x = -a \cos \theta$
 $a_y = a \sin \theta$
 $F_{gy} = -Mg \cos \theta$
 $F_{gx} = -Mg \sin \theta$

x: $F_{\text{net } x} = F_f + F_{gx} = m a_x \Rightarrow F_f - Mg \sin \theta = m(-a \cos \theta)$
 y: $F_{\text{net } y} = n + F_{gy} = m a_y \Rightarrow n - Mg \cos \theta = m a \sin \theta$

$F_f = \mu_s n = -m a \cos \theta + Mg \sin \theta$

Example: Block and Tackle

Block & Tackle:

Fixed

Frictionless Massless pulleys

Moving with W

$T = W$

Δh

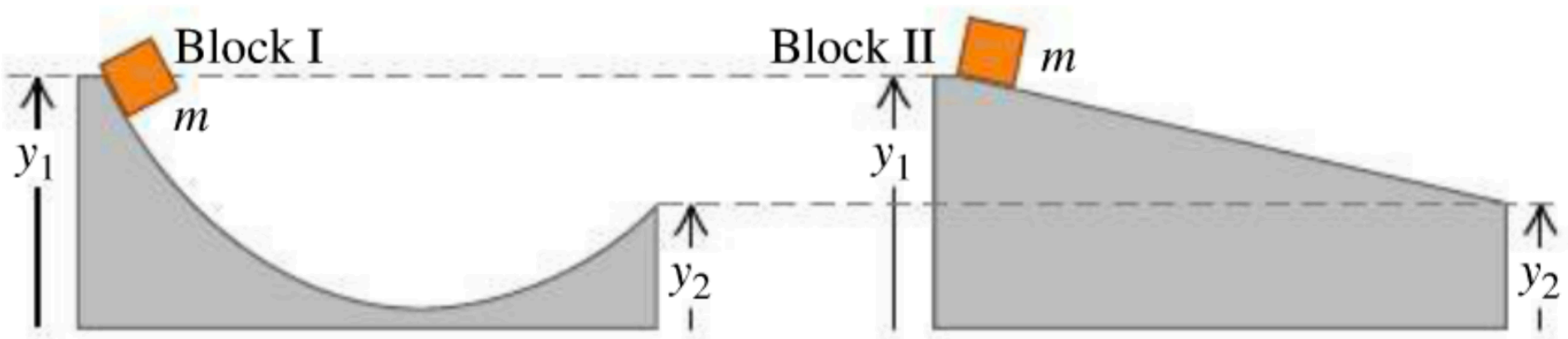
$E = F \cdot \Delta x_{\text{rope}}$
 $= T \cdot 3\Delta h$

How far I pulled rope to lift W by Δh

$\Delta PE_g = \underline{W \cdot \Delta h}$
 $= 3T \cdot \Delta h$

Review: Energy Conservation

- Assume a frictionless system
 - Is the block's speed at y_2 the same in both cases?



- Now add a bit of friction...
 - Is the block's speed at y_2 the same in both cases this time?
 - Why?



Conservative and NonConservative Forces

Friction : Energy loss is path dependent

$$E_f = \vec{F}_f \cdot \Delta \vec{x}$$
$$= \vec{n}(x) \cdot \mu_k \cdot \Delta \vec{x}$$

General term : Nonconservative

Conservative forces : change in energy
is not path dependent

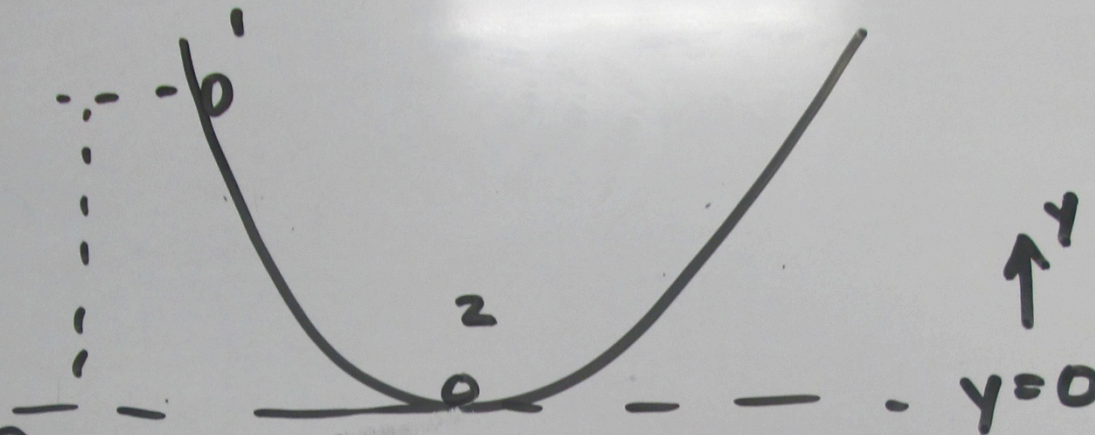
- Gravity
- Elec & Magnetism
- Ideal springs

\Rightarrow Potential function



Potential Energy Relationship to Forces

Potential function related to forces



No
Forces
(☺)

$$\left[\begin{array}{l} E_1 = mgy_1 \text{ (no KE)} \\ E_2 = \frac{1}{2} M V_2^2 \text{ (no PEG)} \end{array} \right]$$

Remember $W = \Delta E = \int \vec{F} \cdot d\vec{x}$

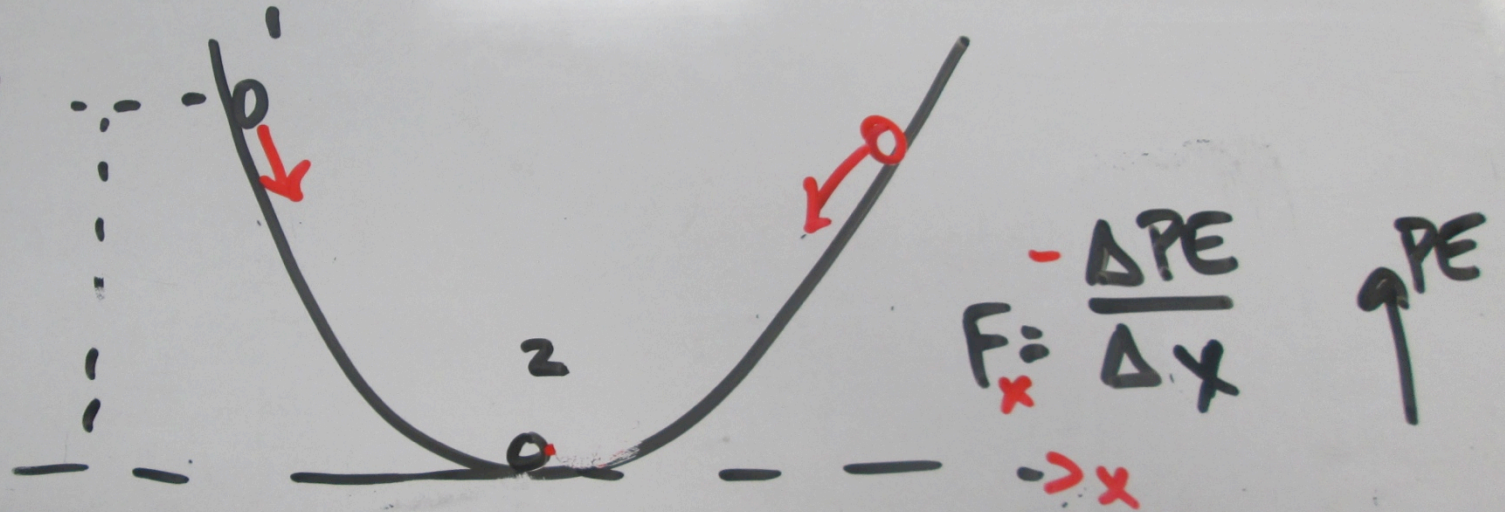
$$\vec{F} = \frac{\Delta E}{\Delta \vec{x}} \text{ (in a vector way)}$$



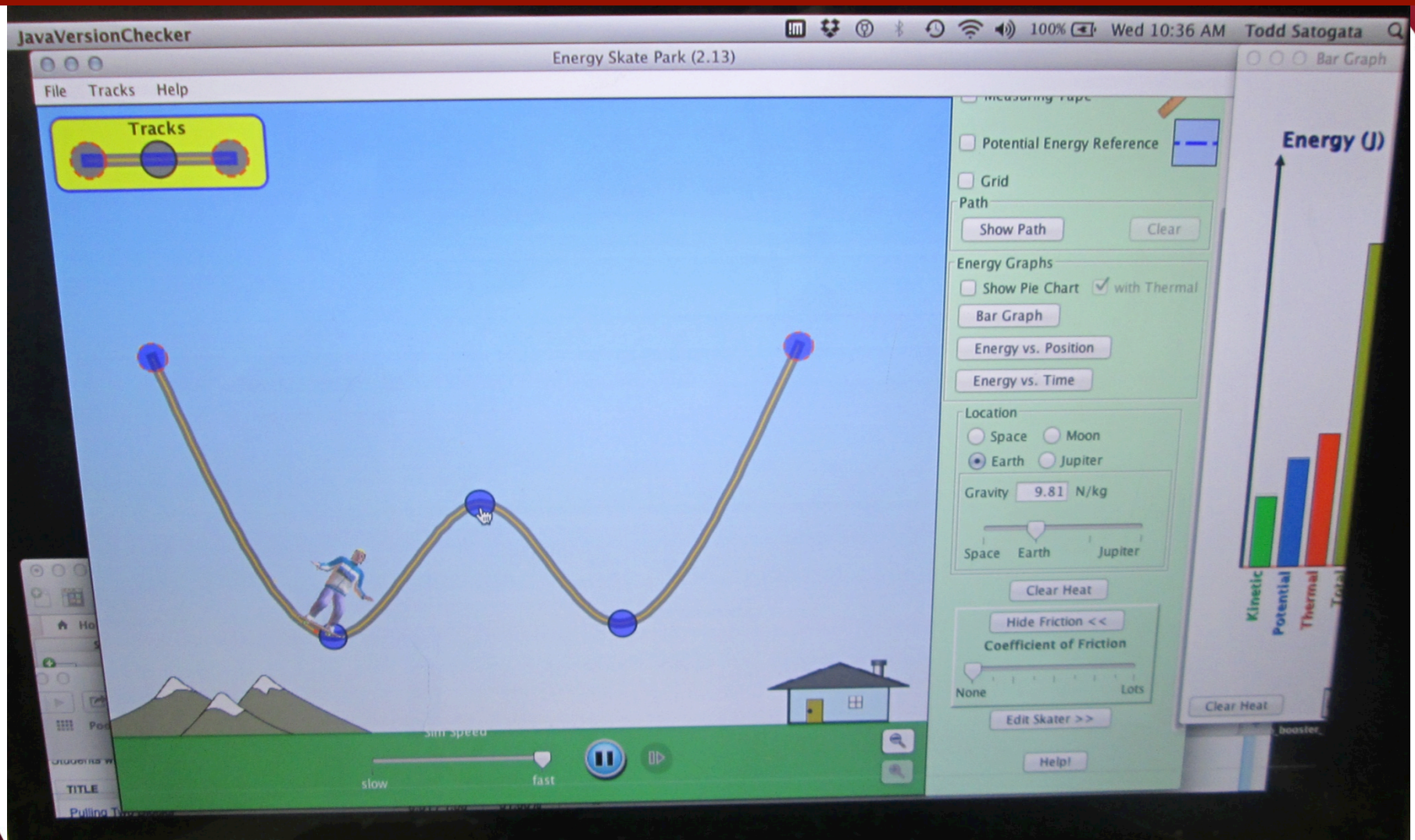
Potential Function Relationship to Force: 1D

Potential function related to forces

$$PE_g = mgy$$



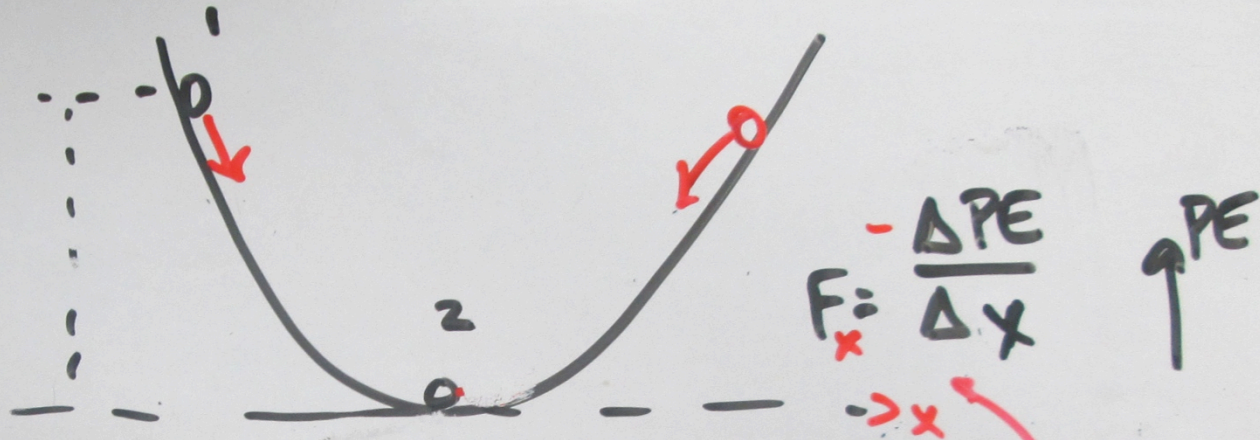
Skate Park Demo



Back to Potentials and Forces

Potential function related to forces

$$PE_g = mgy$$

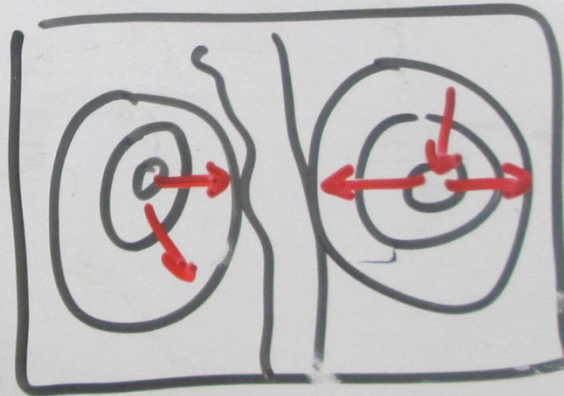


Local 3D slope:
gradient
scalar \Rightarrow vector

$$\vec{F}_g = -\vec{\nabla} \phi(x,y)$$

potential energy

Can only write this for conservative forces



$$\vec{\nabla} \equiv \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix}$$

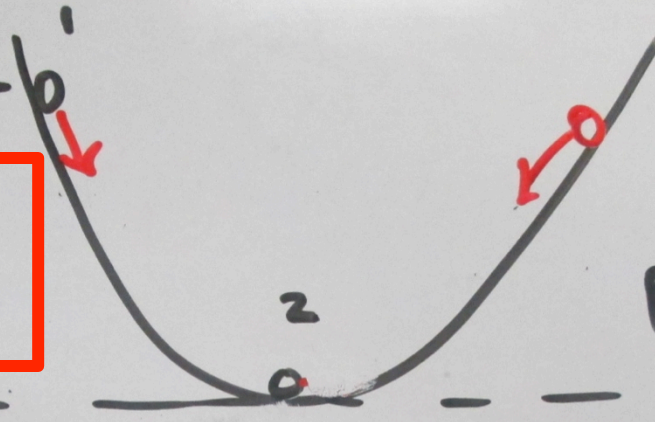


Back to Potentials and Forces

Potential function related to forces

$$PE_g = mgy$$

$$F_g = -\frac{\partial}{\partial y}(mgy) = -mg$$



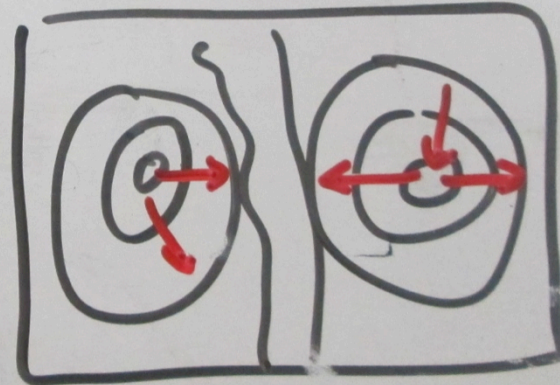
$$F_x = -\frac{\Delta PE}{\Delta x}$$

Local 3D slope:
gradient
scalar \Rightarrow vector

$$\vec{F}_g = -\vec{\nabla} \phi(x,y)$$

potential energy

Can only write this for conservative forces



$$\vec{\nabla} \equiv \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix}$$



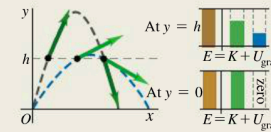
Energy Diagrams

■ Skate park example

When total mechanical energy is conserved:

The total potential energy U is the sum of the gravitational and elastic potential energies:
 $U = U_{\text{grav}} + U_{\text{el}}$. If no forces other than the gravitational and elastic forces do work on a particle, the sum of kinetic and potential energies is conserved. This sum $E = K + U$ is called the total mechanical energy. (See Examples 7.1, 7.3, 7.4, and 7.7.)

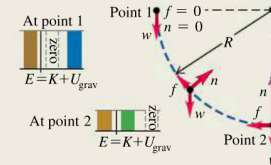
$$K_1 + U_1 = K_2 + U_2 \quad (7.4), (7.12)$$



When total mechanical energy is not conserved:

When forces other than the gravitational and elastic forces do work on a particle, the work W_{other} done by these other forces equals the change in total mechanical energy (kinetic energy plus total potential energy). (See Examples 7.2, 7.5, 7.6, 7.8, and 7.9.)

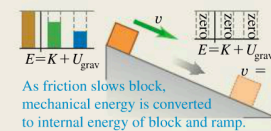
$$K_1 + U_1 + W_{\text{other}} = K_2 + U_2 \quad (7.14)$$



Conservative forces, nonconservative forces, and the law of conservation of energy:

All forces are either conservative or nonconservative. A conservative force is one for which the work-kinetic energy relationship is completely reversible. The work of a conservative force can always be represented by a potential-energy function, but the work of a nonconservative force cannot. The work done by nonconservative forces manifests itself as changes in the internal energy of bodies. The sum of kinetic, potential, and internal energies is always conserved. (See Examples 7.10–7.12.)

$$\Delta K + \Delta U + \Delta U_{\text{int}} = 0 \quad (7.15)$$



Determining force from potential energy: For motion along a straight line, a conservative force $F_x(x)$ is the negative derivative of its associated potential-energy function U . In three dimensions, the components of a conservative force are negative partial derivatives of U . (See Examples 7.13 and 7.14.)

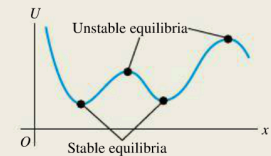
$$F_x(x) = -\frac{dU(x)}{dx} \quad (7.16)$$

$$F_x = -\frac{\partial U}{\partial x} \quad F_y = -\frac{\partial U}{\partial y} \quad (7.17)$$

$$F_z = -\frac{\partial U}{\partial z}$$

$$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right) \quad (7.18)$$

$$= -\vec{\nabla}U$$



Another Homework Solution

