University Physics 226N/231N
Old Dominion University

Momentum, Impulse, Elastic Collisions

Dr. Todd Satogata (ODU/Jefferson Lab)

satogata@jlab.org

http://www.toddsatogata.net/2016-ODU

Monday, October 17, 2016

Reminder: The Second Midterm will be Weds Oct 19 2016

(Yes, still open book, open computer, etc.)

Happy Birthday to Eminem, Felicity Jones, Evil Knievel,
Rita Hayworth, Mike Judge, and Randall Munroe (CNU physics, of xkcd fame)!
Happy National Pasta Day and National Mulligan Day!

Please set your cell phones to “vibrate” or “silent” mode. Thanks!

What We Have Covered So Far

- Physical quantities and vectors
- Kinematics in one and two dimensions
- Centripetal acceleration
- Newton's laws
  - First law
  - Second Law
  - Third Law
  - Application of Newton's Laws
- Work and Energy
  - Friction:
  - Conservation of energy
  - Potential energy and potentials
- Momentum and Impulse
  - Definitions
  - Conservation of momentum

What We Have Covered So Far

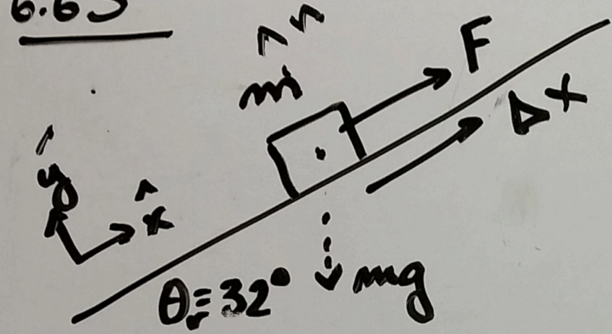
- Physical quantities and vectors
- Kinematics in one and two dimensions
- Centripetal acceleration
- Newton's laws
  - First law
  - Second Law
  - Third Law
  - Application of Newton's Laws
- Work and Energy
  - Friction:
  - Conservation of energy
  - Potential energy and potentials
- Momentum and Impulse
  - Definitions
  - Conservation of momentum

} Second Midterm

Second Midterm "Cheat Sheet"

- Force of friction: $F_\text{f} = \underset{\text{Static}}{\mu_s n}$ or $\underset{\text{kinetic}}{\mu_k n}$ opposing direction of motion
- Newton's second law: $\vec{F}_\text{net} = m\vec{a}$ **vector equation**
- Work done by force over a distance: $W = \vec{F} \cdot \Delta\vec{x} = F\,\Delta x\,\cos\theta$
  - Gravitational potential energy: $E_g = mgh$
  - Kinetic energy: $E_k = \frac{1}{2}mv^2$
  - Spring stored energy: $E_s = \frac{1}{2}k\Delta x^2$
  - Energy lost to friction: $E_f = \mu_k n\,\Delta x$
  - Conservation of energy: **total** energy of a closed system is conserved
- Momentum: $\vec{p} = m\vec{v}$
  - Conservation of momentum: **total** momentum of a closed system is conserved
- Collisions:
  - **Elastic collisions:** conserve energy **and** momentum
  - **Inelastic collisions:** conserve momentum but not (calculable) energy
    - **Examples:** explosions, deformations, things that stick to each other

6.63



$F = 160\,N$

$m = 20.0\,kg$

$\mu_K = 0.3$

$\Delta x = 3.80\,m$

a) work on m by F?

b) work on m by gravity?

c) work on m by normal force?

d) work on m by friction?

e) Total work? f) v_f?

$$W = \vec{F} \cdot d\vec{x} = F \cdot dx \cdot \cos\theta$$

a) $\theta = 0°$ $W = F\Delta x = \ldots$

b) $\theta = 90° + 32° = 122°$

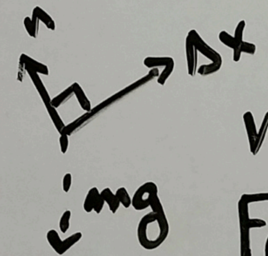
$W = mg\,(3.8\,m)\cos 122° < 0$

c) $\theta = 90° \Rightarrow W = 0\,N \cdot m$

d) $\theta = 180° \Rightarrow \cos\theta = -1$

$W = -F_f\,\Delta x = \ldots$

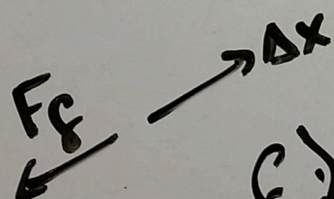
$= -mg\cos 32° \cdot \Delta x\,\mu_K =$



$n = mg\cos 32°$

d) $F_f = n\,\mu_K$

down plane

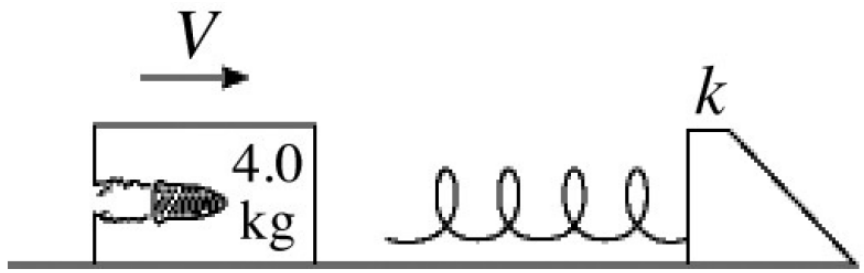


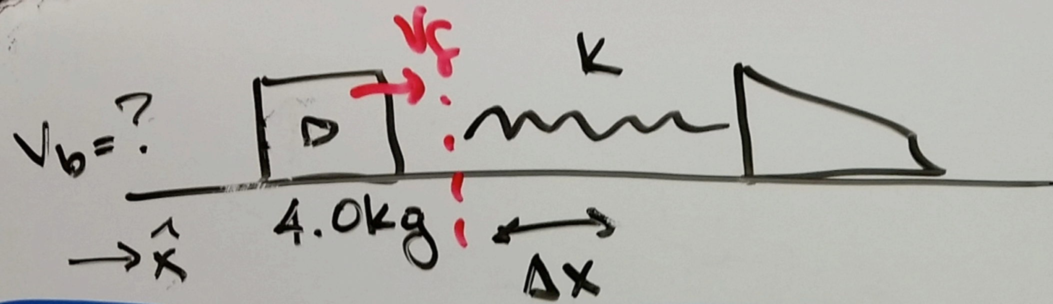
f) $F_{net} = ma$ in x direction & use kinematics $\rightarrow v_f$

f) Use cons of energy $\rightarrow \frac{1}{2} m v_f^2$

Conservation of Momentum Example 5

- An 8.0-g bullet is shot into a 4.0-kg block, at rest on a frictionless horizontal surface. The bullet remains lodged in the block. The block moves into an ideal massless spring and compresses it by 8.7 cm. The spring constant of the spring is 2400 N/m.
  - What is the initial velocity of the bullet?





$m_b = 8.0g = 0.008\,kg$

$M_B = 4.0\,kg >> M_b$

$\Delta x = 8.7\,cm = 0.087\,m$

$K = 2.4 \times 10^3\ N/m$

Note: deformation of block

$\Rightarrow$ INELASTIC $\Rightarrow$ cons of Momentum, NOT energy (during collision of bullet & block)

$p_i = m_b v_b + m_B v_B^{\,\rightarrow 0} = m_b v_b$

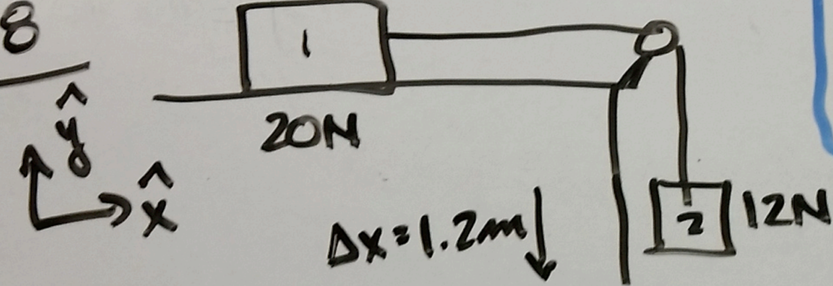
$p_f = \boxed{(M_b + M_B) v_f = m_b v_b}$ 2 unknowns: v_b v_f

Then apply cons of energy during spring compression

$E_i = \frac{1}{2}(M_b + M_B) v_f^2$ $E_f = \frac{1}{2} k x^2$ (M_b & M_B stopped)

$\Rightarrow \boxed{(M_b + M_B) v_f^2 = k \Delta x^2} \Rightarrow v_f$

6.98



a) Find T in rope

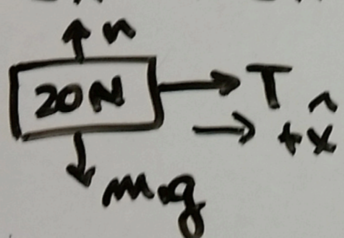
b) W on 20N block (total)

c) W on 12N block (")

d) Total W done on system

$W = \vec{F} \cdot \Delta\vec{x} = F \Delta x \cos\theta$

accelerating? Could be!

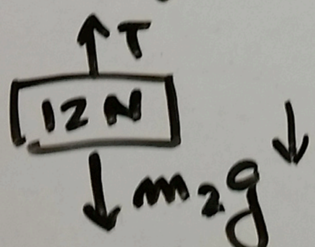
① 

Newton:

y1: $n = M_1 g$ ($a_y = 0\ m/s$)

x1: $F_{net} = \boxed{T = m_1 a_x}$ $a_{x1} = a_{y2}$

y2: $F_{net} = \boxed{T - M_2 g = -M_2 a_y}$

② 

x1: $a = T/M_1$ → $T - M_2 g = -M_2 T/M_1$

$M_1 T - M_1 M_2 g = -M_2 T$

$M_1 T + M_2 T = M_1 M_2 g$

$$\boxed{T = \frac{M_1 M_2 g}{M_1 + M_2}}$$

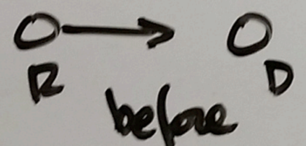
b) $W = F \Delta x \cos\theta$ $\theta = 0°$

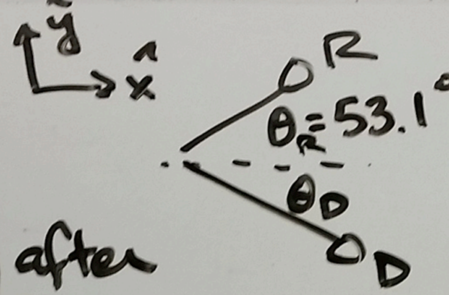
$\Rightarrow T \Delta x = W(\text{net})_1$

c) $W = (T - M_2 g)\cos 180° \Delta x = W(\text{net})_2$

④

8.27





$m_R = 45.0\,kg$

$M_D = 65.0\,kg$ $\quad$ V_{Df} ?

$V_{Ri} = 13.0\,m/s$ $\quad$ θ_D ?

$V_{Rf} = 8.0\,m/s$

⇒ inelastic collision ? Probably

cons of momentum ⇒ 2 equations x, y

$P_{ix} = M_R V_{Ri}$ (D at rest)

$P_{fx} = M_R V_{Rfx} + M_D V_{Dfx}$

$$= \boxed{M_R V_{Rf} \cos\theta_R + M_D V_{Df} \cos\theta_D = M_R V_{Ri}}$$

P_{Dx}

$P_{iy} = 0$

$P_{fy} = M_R V_{Rfy} + M_D V_{Dfy}$

$$= \boxed{M_R V_{Rf} \sin\theta_R - M_D V_{Df} \sin\theta_D = 0}$$

P_{Dy}

physics is done!

How to Solve Those Equations

From the class slide, the two equations to solve for two unknowns v_{Df} and θ_D are:

$$m_R v_{Ri} = m_R v_{Rf} \cos\theta_R + m_D v_{Df} \cos\theta_D \quad (0.1)$$

$$0 = m_R v_{Rf} \sin\theta_R - m_D v_{Df} \sin\theta_D \quad (0.2)$$

It is helpful to note that only the rightmost terms in each equation contain unknowns, so you can write

$$m_D v_{Df} \cos\theta_D = m_R v_{Ri} - m_R v_{Rf} \cos\theta_R \quad (0.3)$$

$$m_D v_{Df} \sin\theta_D = m_R v_{Rf} \sin\theta_R \quad (0.4)$$

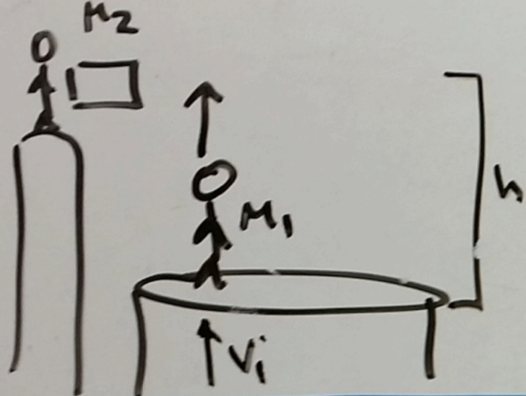
Taking the ratio of the second equation over the first equation conveniently cancels out the unknown v_{Df}:

$$\tan\theta_D = \frac{m_R v_{Rf} \sin\theta_R}{m_R v_{Ri} - m_R v_{Rf} \cos\theta_R} \quad (0.5)$$

$$\tan\theta_D = \frac{v_{Rf} \sin\theta_R}{v_{Ri} - v_{Rf} \cos\theta_R} \quad (0.6)$$

From there you can get the angle θ_D, and you can use that to get v_{Df} from either equation (0.3) or (0.4).

Girl on trampoline ☺ ⑤



$m_1 = 60.0\,kg$

$m_2 = 15.0\,kg$

$h = 2.0\,m$

$v_i = 8.0\,m/s$

$g = 9.80\,m/s^2$

v (grab box)? before $= v_{f1}$

v (after grab box)? $\rightarrow v_{f2}$

	KE	PE_g	Tot
E_i	$\frac{1}{2} M_1 v_i^2$	0	→
E_f	$\frac{1}{2} M_1 v_{f1}^2$	mgh	→

$$\frac{1}{2} M_1 v_i^2 = \frac{1}{2} M_1 v_{f1}^2 + M_1 g h$$

$$v_{f1} = \sqrt{v_i^2 - 2gh} = 4.98\,m/s$$

cons of E a) initial h = 0

b) ⇒ cons of momentum

$$p_i = M_1 v_{f1}$$

$$p_f = (M_1 + M_2) v_{f2} \Rightarrow v_{f2} = v_{f1} \left(\frac{M_1}{M_1 + M_2} \right) = 3.98\,m/s$$

d) max height?

	KE	PE_g	Tot
grab	$\frac{1}{2}(M_1+M_2) v_{f2}^2$	$(M_1+M_2) g h$	→ . . .
top	0	$(M_1+M_2) g h_{max}$	→ . . .

$$h_{max} = h + \frac{1}{2} \frac{v_{f2}^2}{g}$$

$$\boxed{h_{max} = 2.80\,m}$$

Conservation of Momentum and Relativity

- Momentum is conserved regardless of your frame of reference
  - Remember that **momentum** is defined as $\vec{p} = m\vec{v}$
  - Velocity depends on how you and the object are moving **relative** to each other

- As long as you do not change your velocity between "before" and "after", conservation of momentum still works
  - You are adding or subtracting the total mass of the system times your extra velocity to both "before" and "after" total momenta

- This observation has deep and subtle connections to Einstein's theory of special relativity