University Physics 226N/231N
Old Dominion University

Review and
Ch 11: Equilibrium and (More) Statics

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Wednesday, November 2, 2016

Reminder: The Third Midterm will be Mon Nov 21 2016

Happy Birthday to Burt Lancaster, Marie Antoinette, k.d. lang,
David Schwimmer, and Richard Taylor (1990 Nobel)!
Happy National Deviled Egg Day and All Souls' Day!

Please set your cell phones to "vibrate" or "silent" mode. Thanks!

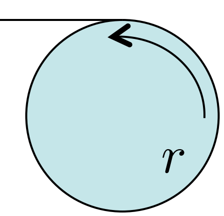
Linear and Angular Summary Comparison

Linear Quantity	Angular Quantity
Position x	Angular position θ
Velocity $v = \dfrac{dx}{dt}$	Angular velocity $\omega = \dfrac{d\theta}{dt}$
Acceleration $a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$	Angular acceleration $\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$

$x = r\theta$

$v = r\omega$

$a = r\alpha$



Equations for Constant Linear Acceleration	Equations for Constant Angular Acceleration
$\bar{v} = \frac{1}{2}(v_0 + v)$	$\bar{\omega} = \frac{1}{2}(\omega_0 + \omega)$
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

"movie equation"

"equation that doesn't involve time"

Forces	Torques: $\vec{\tau} = \vec{r} \times \vec{F}$
Newton's 2nd Law: $\vec{F}_{\mathrm{net}} = m\vec{a} = d\vec{p}/dt$	$\vec{\tau}_{\mathrm{net}} = I\vec{\alpha} = d\vec{L}/dt$
Momentum: $\vec{p} = m\vec{v}$	$\vec{L} = I\vec{\omega}$
Kinetic Energy: $\mathrm{KE} = \dfrac{1}{2}mv^2$	$\mathrm{KE}_{\mathrm{rot}} = \dfrac{1}{2}I\omega^2$
(inertial) Mass: m	Moment of Inertia: $I = \sum m_i r_i^2$

Parallel Axis Theorem: $I = I_{\mathrm{cm}} + Md^2$

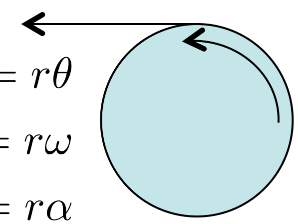
Linear and Angular Motion Combined

- Like many other Newton’s Law problems
  - Draw free body diagrams of each of the components and label coordinate systems, forces, accelerations, and any other vectors
  - Break vectors down into components if necessary
    - Just like inclined plane problems from earlier in class
  - Write Newton’s 2nd law for each interesting coordinate direction for each individual piece of the system

$$\vec{F}_{\text{net}} = m\vec{a} = d\vec{p}/dt \qquad \vec{\tau}_{\text{net}} = I\vec{\alpha} = d\vec{L}/dt$$

    - Be careful about signs!!!
  - When the rotational and translational motions are connected (e.g. rope that is moving but not stretching, object that is rolling), angular and linear quantities are related:
    - Be careful about signs!!!
  - Physics is done
    - Solve the resulting equations for unknowns

$$x = r\theta$$
$$v = r\omega$$
$$a = r\alpha$$



Combining Linear and Rotational Motion

- We now have the tools to do some interesting problems…
  - Bucket of mass m unrolls from a solid cylinder of mass M and radius R into a well.
  - What is the bucket's (linear) acceleration?

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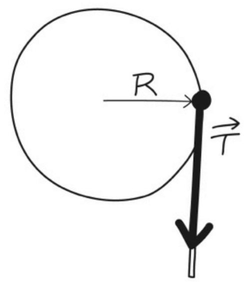
$\hat{y}$

Be careful about signs!

bucket



cylinder

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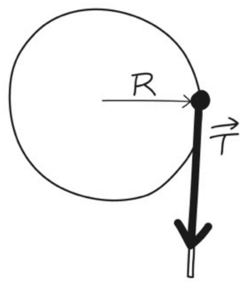

cylinder

$$\text{Cylinder}: \sum \tau = I\alpha$$

$$\text{Bucket}: \quad \sum F = mg - T = ma$$

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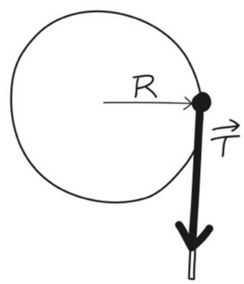

cylinder

$$\text{Bucket}: \quad \sum F = mg - T = ma$$

$$\text{Cylinder}: \quad \sum \tau = I\alpha$$

$$I_{\text{cylinder}} = \frac{1}{2}MR^2$$

$$\tau = TR\sin\theta = TR$$

$$\alpha = a/R$$

$$\Rightarrow TR = \left(\frac{1}{2}MR^2\right)\frac{a}{R}$$

$$\Rightarrow \quad T = Ma/2$$

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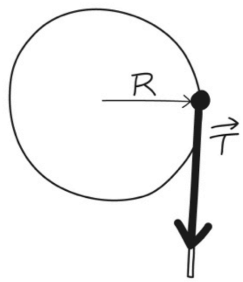

cylinder

$\hat{y}$

Be careful about signs!

$$\text{Bucket}: \quad \sum F = mg - T = ma$$

$$mg - Ma/2 = ma$$

$$\boxed{a = \frac{mg}{M/2 + m}}$$

$$\text{Cylinder}: \quad \sum \tau = I\alpha$$

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Combining Linear and Rotational Motion

- Similar to a homework problem
  - Bucket of mass m unrolls from a solid cylinder of mass M and radius R into a well.
  - How fast is the bucket moving when it hits the bottom if the well is D meters deep?

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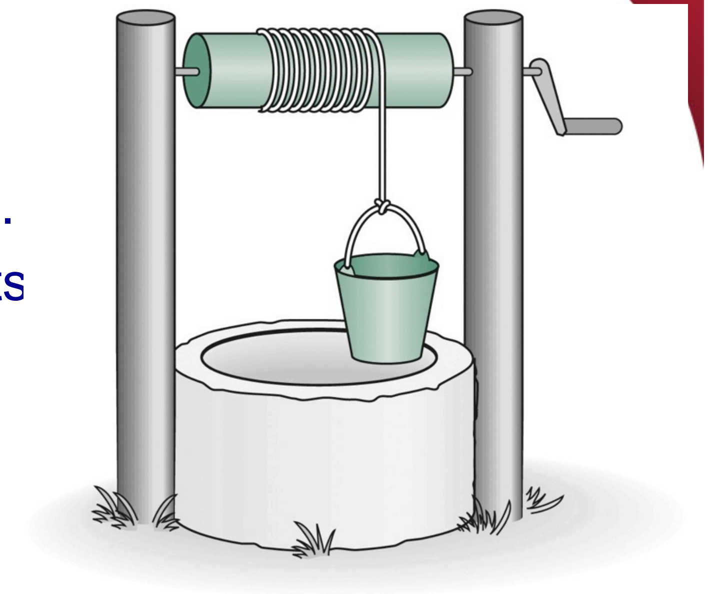

One option: Use kinematics and previous result

$$a = \frac{mg}{M/2 + m}$$

$$v_0 = 0 \text{ m/s}$$

$$v^2 = v_0^2 + 2a\Delta y$$

$$\boxed{v = \sqrt{2a\Delta y} = \sqrt{\frac{2mgD}{M/2 + m}}}$$

Combining Linear and Rotational Motion

- Similar to a homework problem
  - Bucket of mass m unrolls from a solid cylinder of mass M and radius R into a well.
  - How fast is the bucket moving when it hits the bottom if the well is D meters deep?

Another option: Use conservation of energy

$$\mathrm{E}_{\mathrm{init}} = \mathrm{PE}_{\mathrm{g,init}} = mgD$$

$$\begin{aligned}\mathrm{E}_{\mathrm{final}} &= \mathrm{KE}_{\mathrm{bucket,final}} + \mathrm{KE}_{\mathrm{cylinder,final}} \\ &= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}mv^2 + \frac{1}{4}Mv^2 \\ &= \frac{1}{2}v^2\left(m + \frac{1}{2}M\right)\end{aligned}$$

$$I_{\mathrm{cylinder}} = \frac{1}{2}MR^2$$

$$\omega = v/R$$

$$\frac{1}{2}I\omega^2 = \frac{1}{4}MR^2\left(\frac{v}{R}\right)^2 = \frac{1}{4}Mv^2$$

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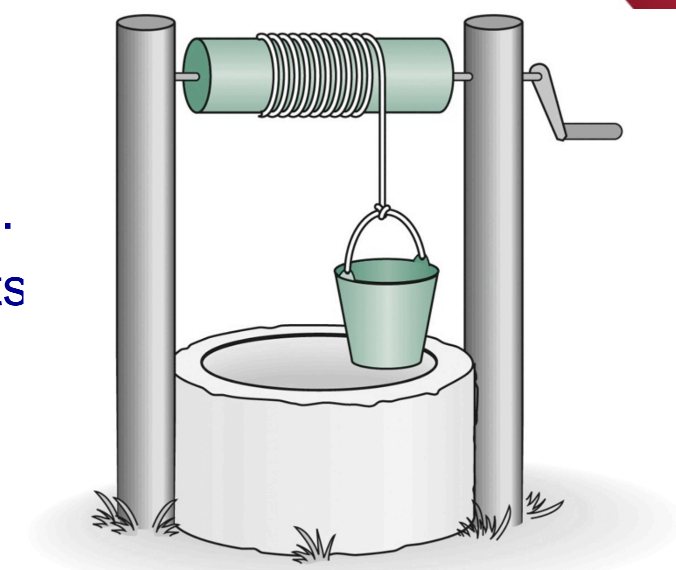

Another option: Use conservation of energy

$$\mathrm{E_{init}} = \mathrm{PE_{g,init}} = mgD$$

$$\mathrm{E_{final}} = \mathrm{KE_{bucket,final}} + \mathrm{KE_{cylinder,final}}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{4}Mv^2$$

$$= \frac{1}{2}v^2\left(m + \frac{1}{2}M\right)$$

$$I_{\mathrm{cylinder}} = \frac{1}{2}MR^2$$

$$\omega = v/R$$

$$mgD = \frac{1}{2}v^2\left(m + \frac{1}{2}M\right)$$

$$\boxed{v = \sqrt{\frac{2mgD}{m + \frac{1}{2}M}}}$$

$$\frac{1}{2}I\omega^2 = \frac{1}{4}MR^2\left(\frac{v}{R}\right)^2 = \frac{1}{4}Mv^2$$

Ball and Inclined Plane

- Consider a solid sphere of mass M and radius R rolling down an inclined plane of angle θ without slipping, starting from rest
  - What is the sphere's linear acceleration down the plane?

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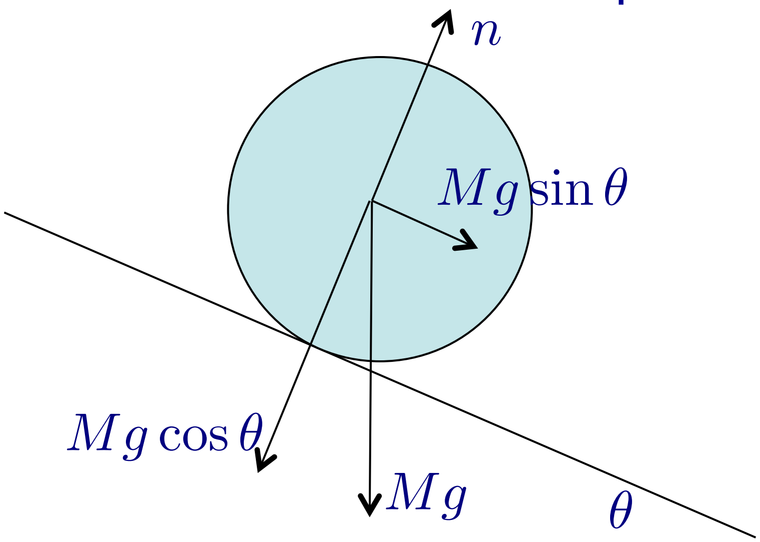

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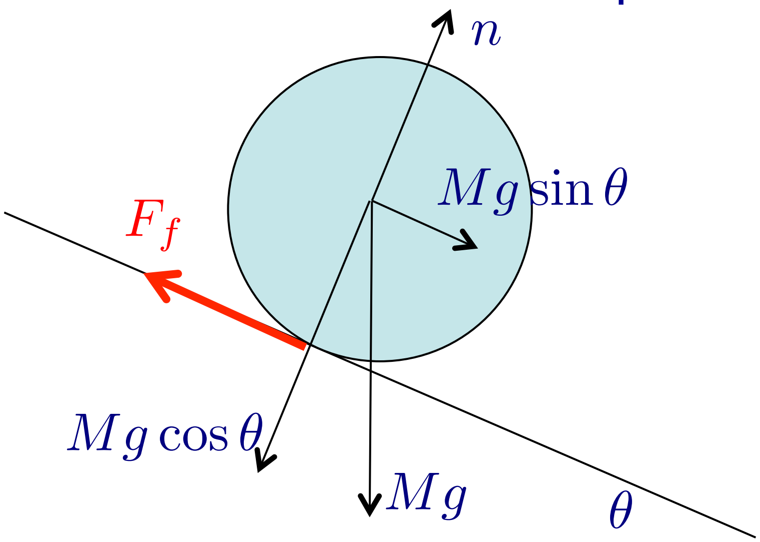

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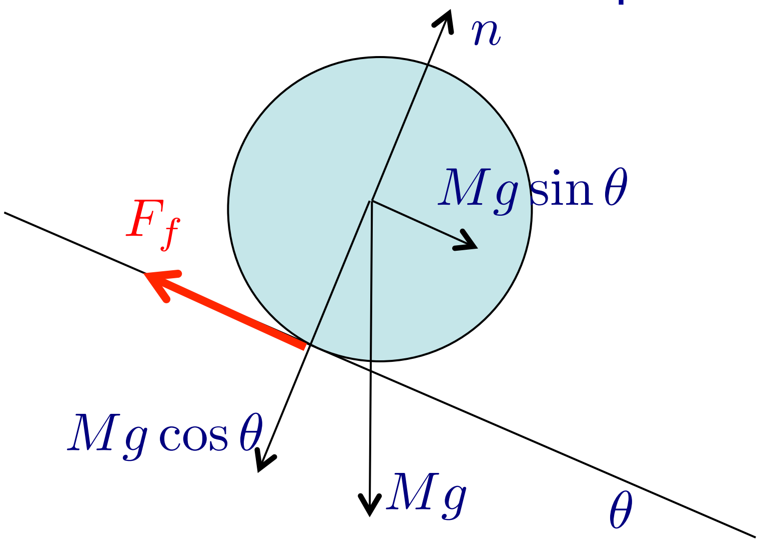

Newton's 2nd law (linear) down plane:

$$F_{\mathrm{net}} = Mg\sin\theta - F_f = Ma$$

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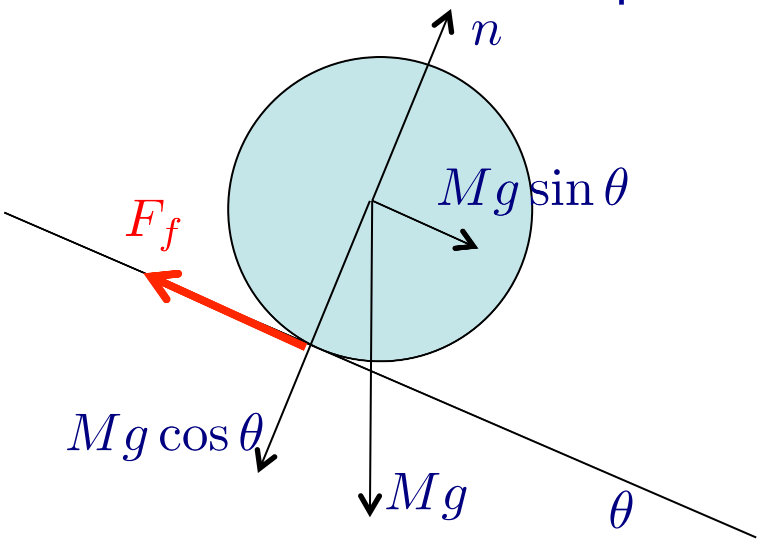

Newton's 2nd law (linear) down plane:

$$F_{\text{net}} = Mg\sin\theta - F_f = Ma$$

Newton's 2nd law (angular):

$$I = \frac{2}{5}MR^2$$

$$a = R\alpha$$

$$\begin{aligned}\tau_{\text{net}} = F_f R = I\alpha \\ &= \frac{2}{5}MR^2\left(\frac{a}{R}\right) \\ &= \frac{2}{5}MRa \qquad \Rightarrow F_f = \frac{2}{5}Ma\end{aligned}$$

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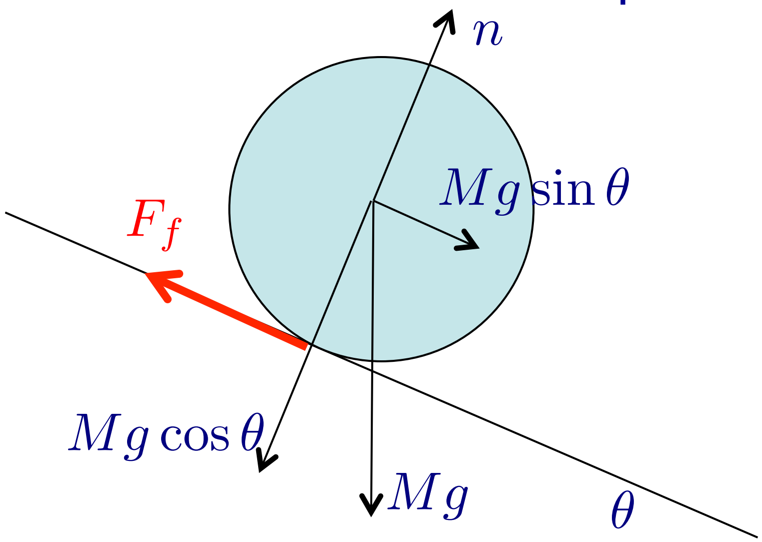

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$$\tau_{\text{net}} = F_f R = I\alpha$$

$$= \frac{2}{5}MR^2\left(\frac{a}{R}\right)$$

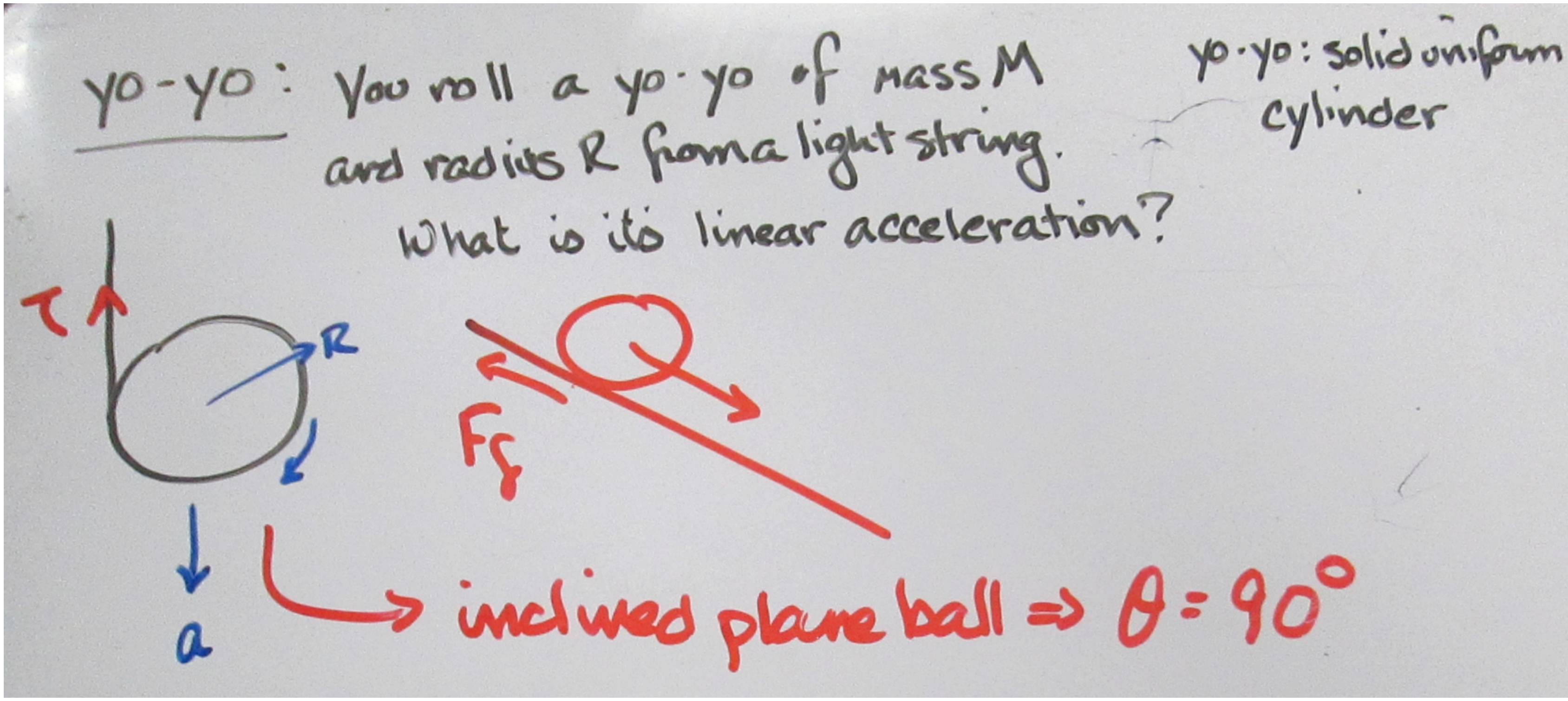
$$= \frac{2}{5}MRa \quad \Rightarrow \boxed{F_f = \frac{2}{5}Ma}$$

$$\cancel{M}g\sin\theta - \frac{2}{5}\cancel{M}a = \cancel{M}a$$

$$g\sin\theta = \frac{7}{5}a$$

$$\boxed{a = \frac{5}{7}g\sin\theta}$$

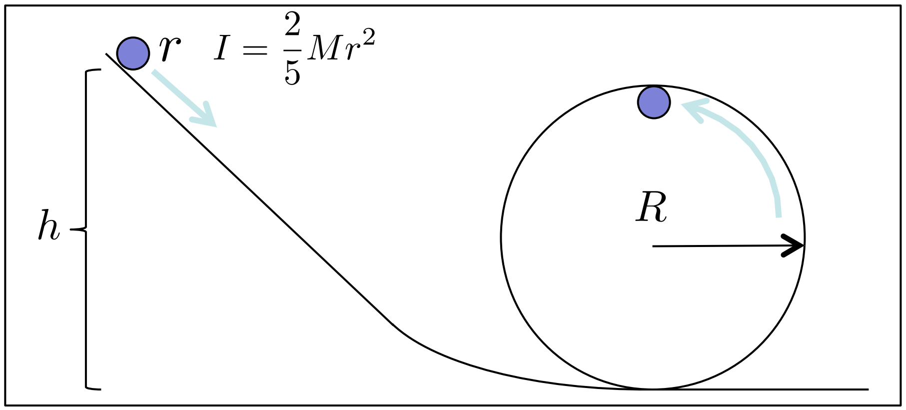
Yo-Yo Homework: Like Ball Rolling Down Ramp



Energy Conservation Example

A rework of a problem I tried on the board last class…

Consider the loop the loop problem, now with a solid ball rolling down a track and barely making it around the loop. What is the minimum height that the ball needs to roll from (without slipping) to make it around a loop of radius R?



State	$KE_{translation}$	$KE_{rotation}$	PE_g
Top of ramp	0	0	Mgh
Top of loop	$\frac{1}{2}Mv^2$	$\frac{1}{2}I\omega^2$	$Mg(2R)$

Rotating without slipping: $\omega = \frac{v}{r}$

Solid uniform ball: $I = \frac{2}{5}Mr^2$

Barely make it through the loop:

$$a_{\text{centrip}} = \frac{v^2}{R} = g \quad \Rightarrow \quad v^2 = Rg$$

$$r^2\omega^2 = v^2 = Rg$$

Conservation of energy:

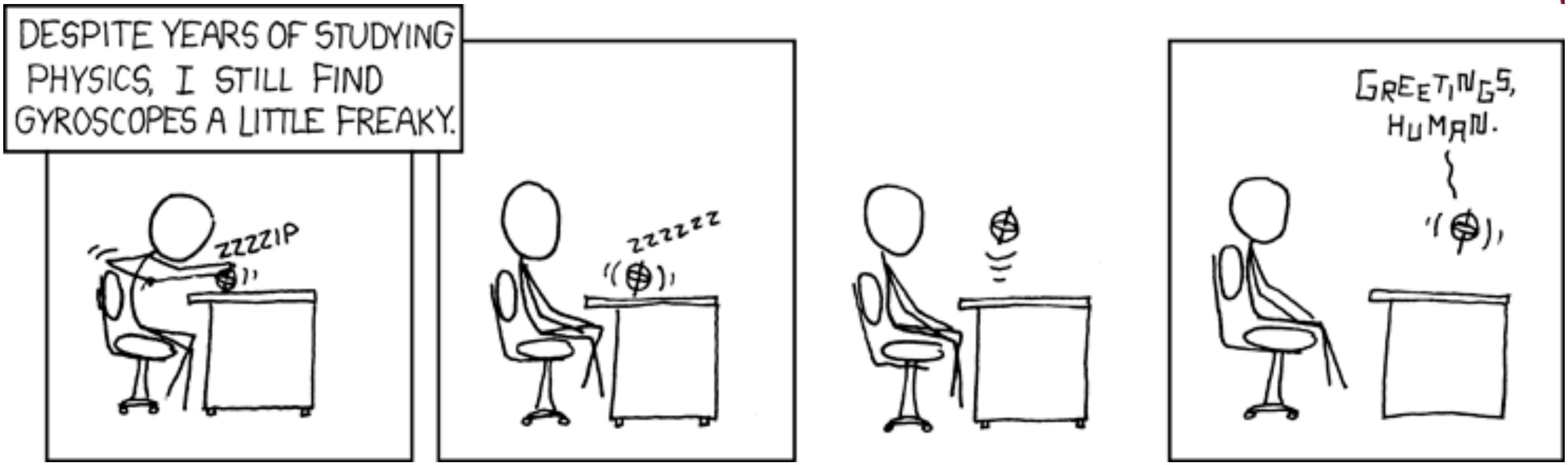
$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + 2MgR$$

$$\cancel{M}gh = \frac{1}{2}\cancel{M}v^2 + \frac{1}{5}\cancel{M}r^2\omega^2 + 2\cancel{M}gR$$

$$\cancel{g}h = \frac{R\cancel{g}}{2} + \frac{R\cancel{g}}{5} + 2\cancel{g}R \quad \Rightarrow \quad h = 2.7R$$

Watch:
https://www.youtube.com/watch?v=4fCFxD_Ud9E

Phew!



https://xkcd.com/332/

Equilibrium And Statics

- Recall that in translational statics, we studied systems that have all forces acting on them balanced so the objects do not accelerate

$$\vec{F}_{\text{net}} = 0$$

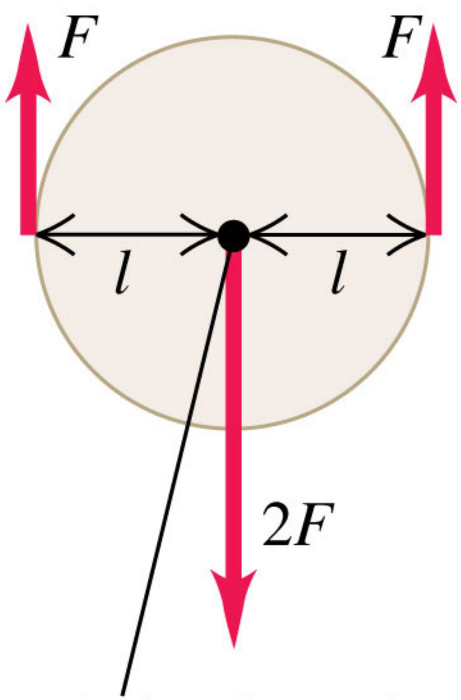
- We can also apply this to potentially rotating systems where the forces aren't all acting on the object's center of mass
  - All the torques have to balance to keep the object from rotating or angularly accelerating too

$$\vec{\tau}_{\text{net}} = 0$$

- Next week we'll look at what happens when objects are deformed by tension, compression, shear, and stress
  - For the rest of today, let's look at equilibrium examples

Conditions for equilibrium: Example 1

(a) This body is in static equilibrium.



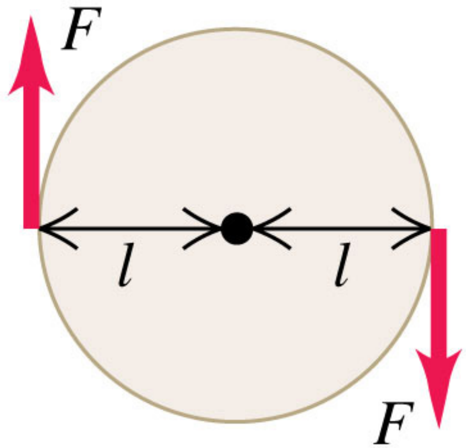
Equilibrium conditions:

First condition satisfied:
Net force = 0, so body at rest has no tendency to start moving as a whole.

Second condition satisfied:
Net torque about the axis = 0, so body at rest has no tendency to start rotating.

Conditions for equilibrium: Example 2

(b) This body has no tendency to accelerate as a whole, but it has a tendency to start rotating.

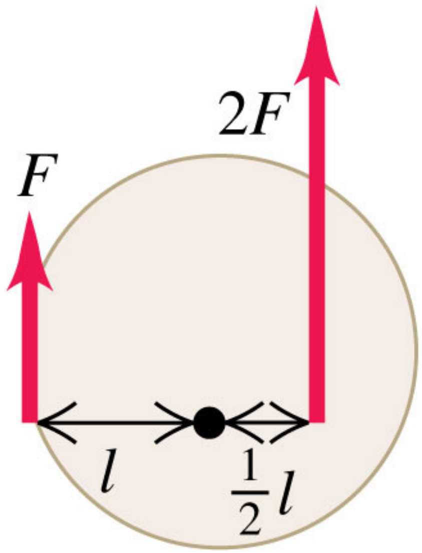


First condition satisfied: Net force = 0, so body at rest has no tendency to start moving as a whole.

Second condition NOT satisfied: There is a net clockwise torque about the axis, so body at rest will start rotating clockwise.

Conditions for equilibrium: Example 3

(c) This body has a tendency to accelerate as a whole but no tendency to start rotating.



First condition NOT satisfied: There is a net upward force, so body at rest will start moving upward.

Second condition satisfied: Net torque about the axis = 0, so body at rest has no tendency to start rotating.

Problem-solving strategy for static equilibrium

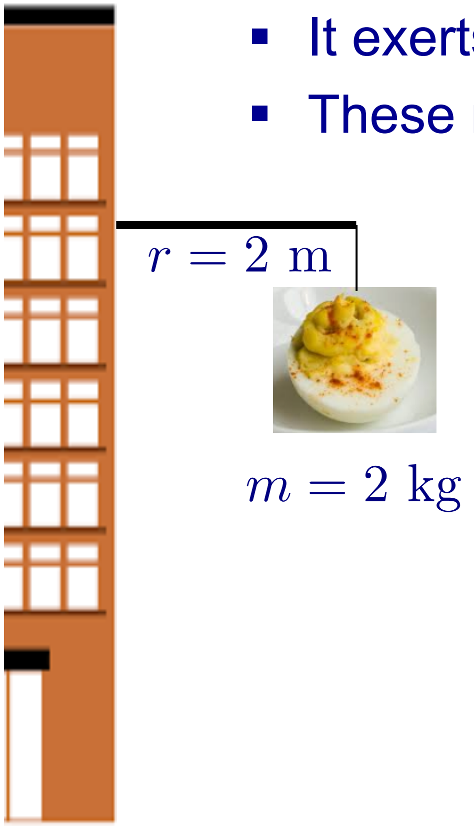
- *Identify* the relevant concepts: The first and second conditions for equilibrium are $\sum F_x = 0$, $\sum F_y = 0$, and $\sum \tau_z = 0$.
- *Set up* the problem by using the following steps:
  1. Sketch the physical situation and identify the body in equilibrium to be analyzed.
  2. Draw a free-body diagram showing all forces acting on the body. Show the point on the body at which each force acts.
  3. Choose coordinate axes and specify their direction. Specify a positive direction of rotation for torques.
  4. Choose a reference point about which to compute torques.
     1. Almost always this point is the center of mass of the object

Problem-solving strategy for static equilibrium

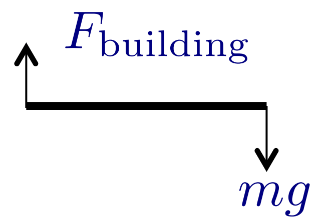
- *Execute* the solution as follows:
  1. Write equations expressing the equilibrium conditions. Remember that $\sum F_x = 0$, $\sum F_y = 0$, and $\sum \tau_z = 0$ are separate equations.
  2. To obtain as many equations as you have unknowns, you may need to compute torques with respect to two or more reference points.
- *Evaluate* your answer: Check your results by writing $\sum \tau_z = 0$ with respect to a different reference point. You should get the same answers.

Example: Deviled Egg Advertising

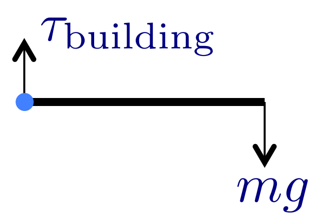
- You are hanging a sign from a light rod extending from the side of a building
  - It exerts a net force downward and a net torque clockwise
  - These must be counteracted by a force and torque acting close to the contact point with the building:



Net forces:



Net torques:

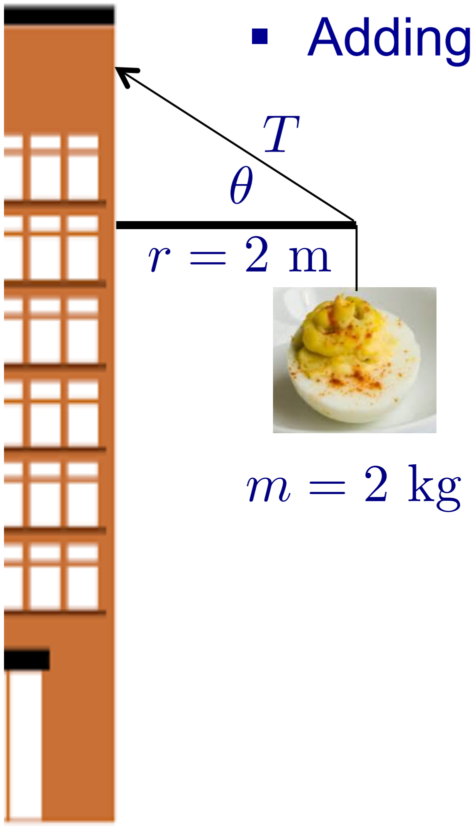


$$\tau = mgr = 39.2\ \mathrm{N-m\ clockwise}$$

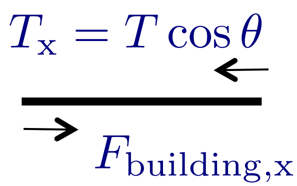
Building torque has very small radius to act!

Example: : Deviled Egg Advertising

- You are hanging a sign from a light rod extending from the side of a building
  - Adding a thin support wire that can support tension T helps

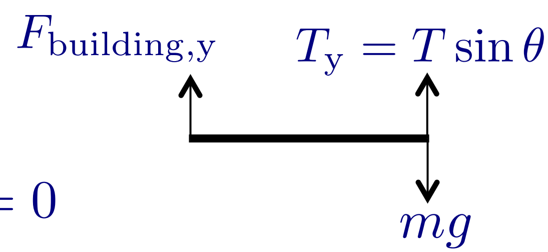


Net horizontal forces acting on rod:



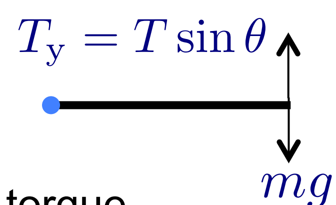
$$F_{\mathrm{x,tot}} = F_{\mathrm{building,x}} - T\cos\theta = 0$$

Net vertical forces acting on rod:



$$F_{\mathrm{y,tot}} = F_{\mathrm{building,y}} + T\sin\theta - mg = 0$$

Net torques acting on rod:



$$\tau_{\mathrm{tot}} = mgr - T\sin\theta\, r = 0$$

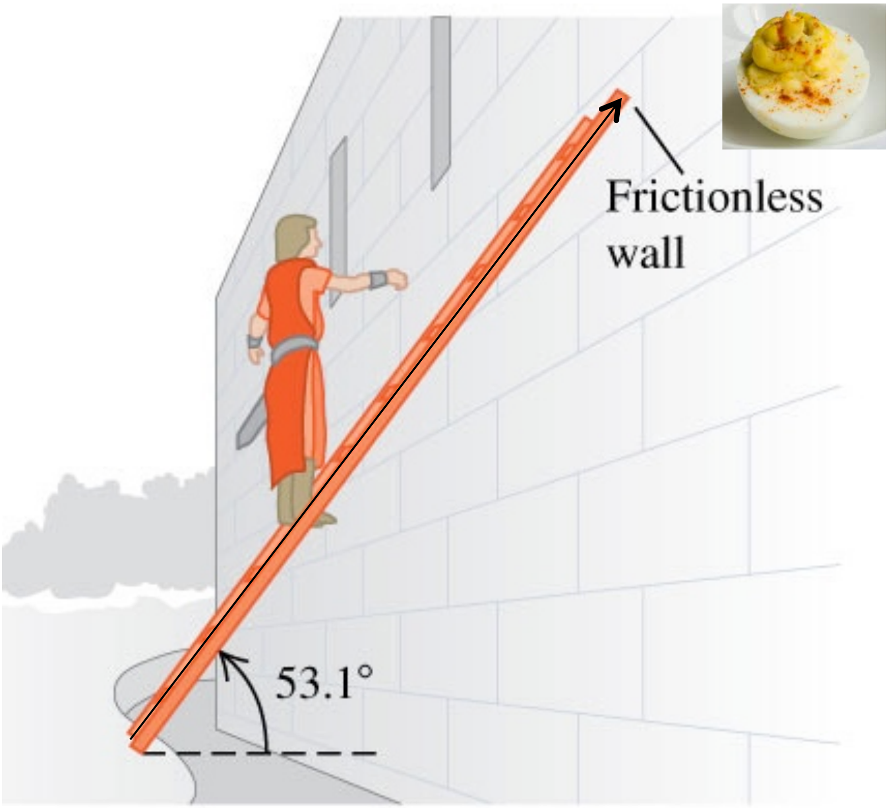
No torque from building

Given θ , solve for $T, F_{\mathrm{building,x}}, F_{\mathrm{building,y}}$

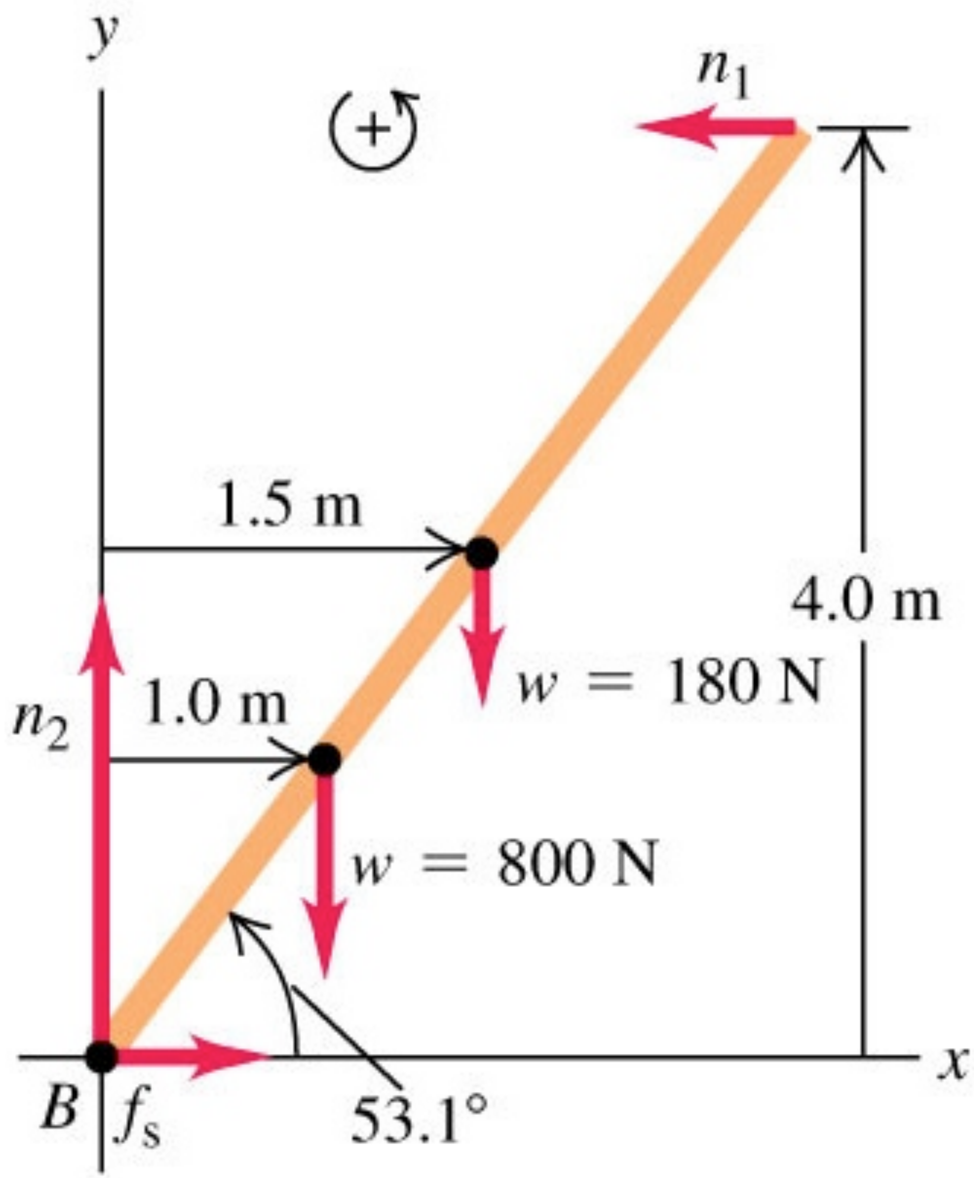
Example: Lancelot and the Ladder

- Sir Lancelot weighs 800N, and is climbing a 53.1 degree ladder that is 5.0m long and weighs 180N to raid a castle for its deviled eggs. The top of the ladder leans against a frictionless wall. The bottom of the ladder encounters friction on the ground. Lancelot is 1/3 of the way up the ladder.

  - Draw the force diagrams
  - Find normal and friction forces on the bottom of the ladder
  - Find the minimum coefficient of friction necessary to keep the ladder from slipping.

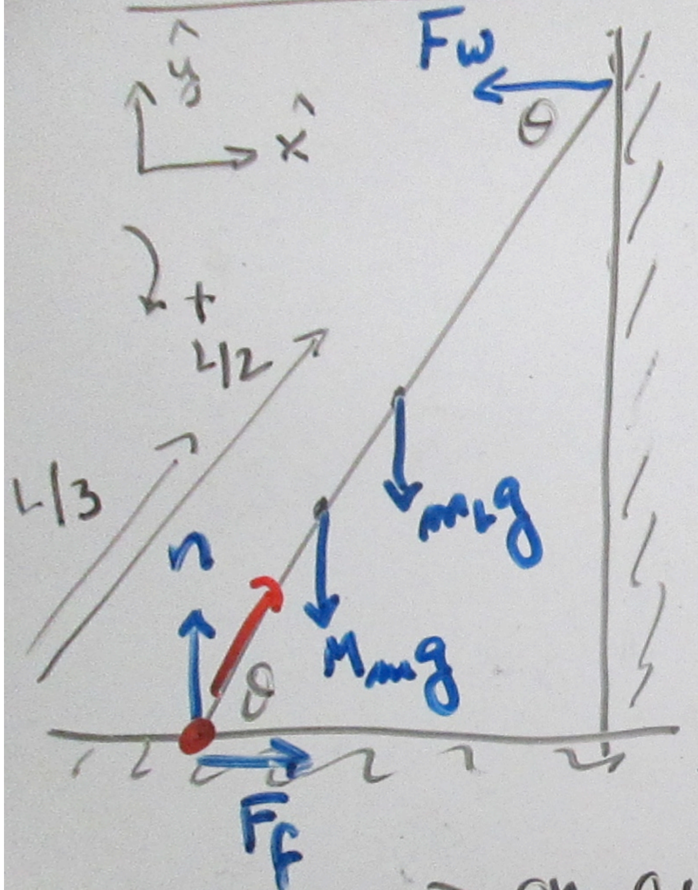


Lancelot and the Ladder: Force Diagrams



Lancelot and the Ladder

Ladder & wall



L = length of ladder

M_L = mass of ladder

M_M = mass of man

Draw forces ON LADDER ☺

Newton: linear

$$F_{net,x} = 0 = F_f - F_W \Rightarrow \boxed{F_f = F_W} = \mu_s n$$

$$F_{net,y} = 0 = n - m_m g - m_L g$$

$$\Rightarrow \boxed{n = (m_m + m_L) g}$$

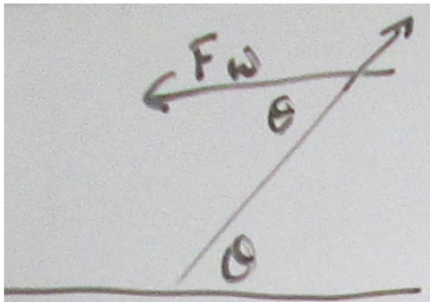
Newton: angular

$$\tau_{net} = 0 = \tau_M + \tau_L - \tau_W$$

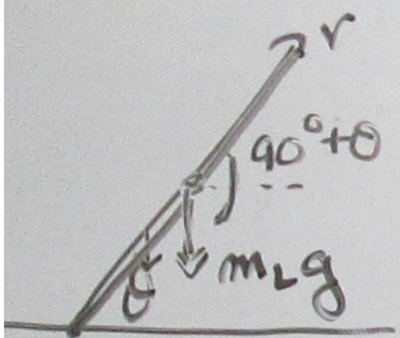
Def$^{\underline{n}}$ of torque: $\vec{\tau} = \vec{r} \times \vec{F}$ or $\tau = rF \sin\theta (r \& F)$

$$\Rightarrow 0 = \frac{L}{3} M_M g \cos\theta + \frac{L}{2} M_L g \cos\theta - L F_W \sin\theta$$

Lancelot and the Ladder



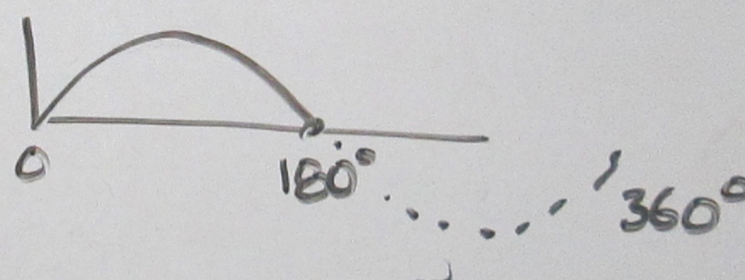
$$\tau_{wall} = L F_W \sin\theta \text{ (ladder \& wall)}$$
$$= L F_W \sin(\underline{180^\circ - \theta}) \quad \text{ccw}$$



$$\tau_{Ladder} = \left(\frac{L}{2}\right)(m_L g)\sin(\underline{90^\circ + \theta})$$

$$\tau_{Man} = \left(\frac{L}{3}\right)(M_m g)\sin(\underline{90^\circ + \theta})$$

sin



$$\Rightarrow \sin(180^\circ - \theta) = \sin\theta$$
$$\sin(90^\circ + \theta) = \cos\theta$$