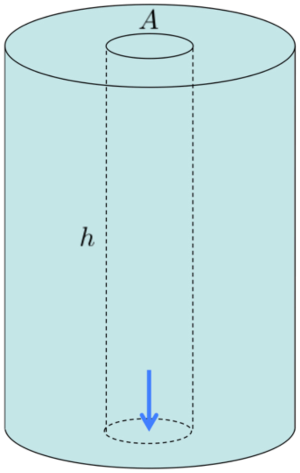
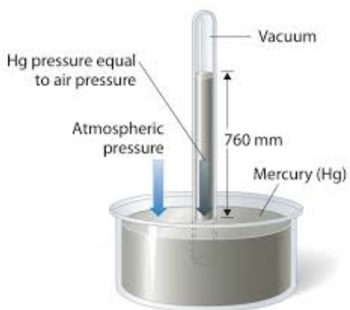
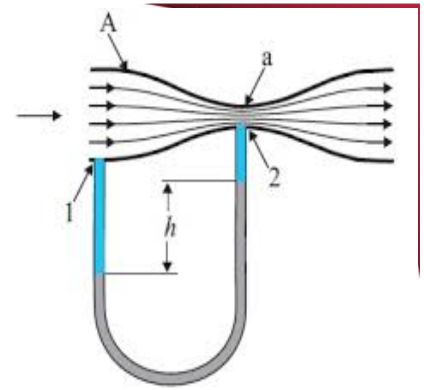


$F = mg = (\rho h A)g$



University Physics 226N/231N
Old Dominion University

Ch 12: Finish Fluid Mechanics
Exam Review



Dr. Todd Satogata (ODU/Jefferson Lab)

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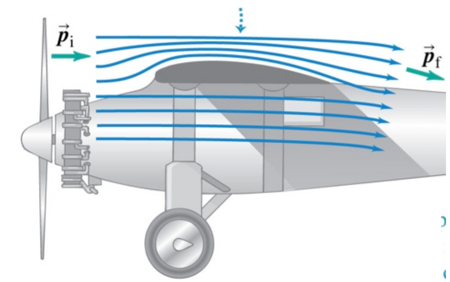


Wednesday, November 16, 2016

Reminder: The Third Midterm will be Mon Nov 21 2016

Homework Notebooks are also due

when you turn in your exam!

Happy Birthday to Maggie Gyllenhaal, Diana Krall, Andy Dalton,
Allison Crowe, and Gene Amdahl!
Happy Fast Food Day and Have A Party With Your Bear Day!

Please set your cell phones to “vibrate” or “silent” mode. Thanks!

Midterm #3

- **Midterm #3:**
  - Rotational kinematics and rotational motion
    - Including conservation of energy and angular momentum
  - Rotational equilibrium
  - Stress/Strain/Shear
  - Fluid mechanics
  - I will prepare a handout with all cheat sheat materials
    - Includes tables of material constants, moments of inertia, etc
  - I'll also post something like a sample midterm later today
- **Homework notebook**
  - Your homework notebook is due **when you turn in your test**
    - Hopefully you find it useful while you are taking the test
  - It will be graded on a 3- or 4-point scale and counts for about 5% of your final grade

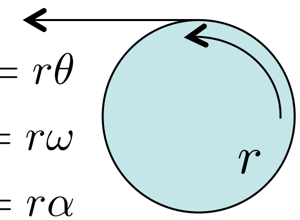
(Cheat Sheet: Linear and Angular Summary Comparison)

Linear Quantity	Angular Quantity
Position x	Angular position θ
Velocity $v = \dfrac{dx}{dt}$	Angular velocity $\omega = \dfrac{d\theta}{dt}$
Acceleration $a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$	Angular acceleration $\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$

$x = r\theta$

$v = r\omega$

$a = r\alpha$



Equations for Constant Linear Acceleration		Equations for Constant Angular Acceleration
$\bar{v} = \frac{1}{2}(v_0 + v)$		$\bar{\omega} = \frac{1}{2}(\omega_0 + \omega)$
$v = v_0 + at$		$\omega = \omega_0 + \alpha t$
$x = x_0 + v_0 t + \frac{1}{2}at^2$	"movie equation"	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	"equation that doesn't involve time"	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Forces		Torques: $\vec{\tau} = \vec{r} \times \vec{F}$
Newton's 2nd Law:	$\vec{F}_{\text{net}} = m\vec{a} = d\vec{p}/dt$	$\vec{\tau}_{\text{net}} = I\vec{\alpha} = d\vec{L}/dt$
Momentum:	$\vec{p} = m\vec{v}$	$\vec{L} = I\vec{\omega}$
Kinetic Energy:	$\text{KE} = \dfrac{1}{2}mv^2$	$\text{KE}_{\text{rot}} = \dfrac{1}{2}I\omega^2$
(inertial) Mass:	m	Moment of Inertia: $I = \sum m_i r_i^2$

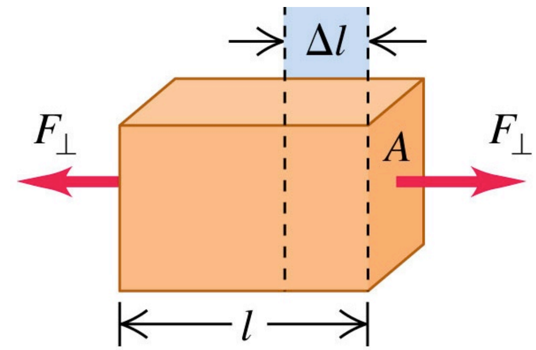
Parallel Axis Theorem: $I = I_{\text{cm}} + Md^2$

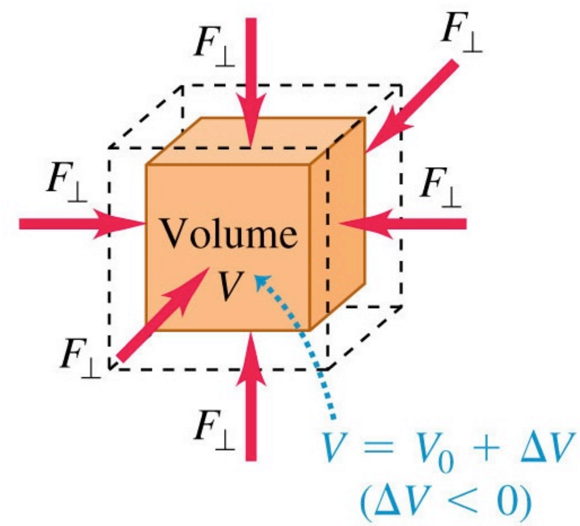
Review: Elastic Moduli Definitions

- Tensile stress: $F_\perp/A$
- Tensile strain: $\Delta l/l_0$
- Young's modulus: $Y \equiv \dfrac{F_\perp/A}{\Delta l/l_0}$

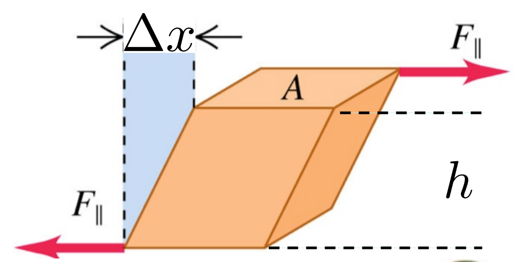
- **Pressure** or volume stress: $p = F_\perp/A$
- Volume strain: $\Delta V/V_0$
- Bulk modulus: $B \equiv -\Delta p/(\Delta V/V_0)$

- Shear stress: $F_\parallel/A$
- Shear strain: $\Delta x/h$
- Shear modulus: $S \equiv \dfrac{F_\parallel/A}{(\Delta x/h)}$





- All moduli are **material properties**
  - All moduli assume **elastic deformation**
  - All moduli have units of pressure [Pa]



Review: The Limits of Hooke's Law

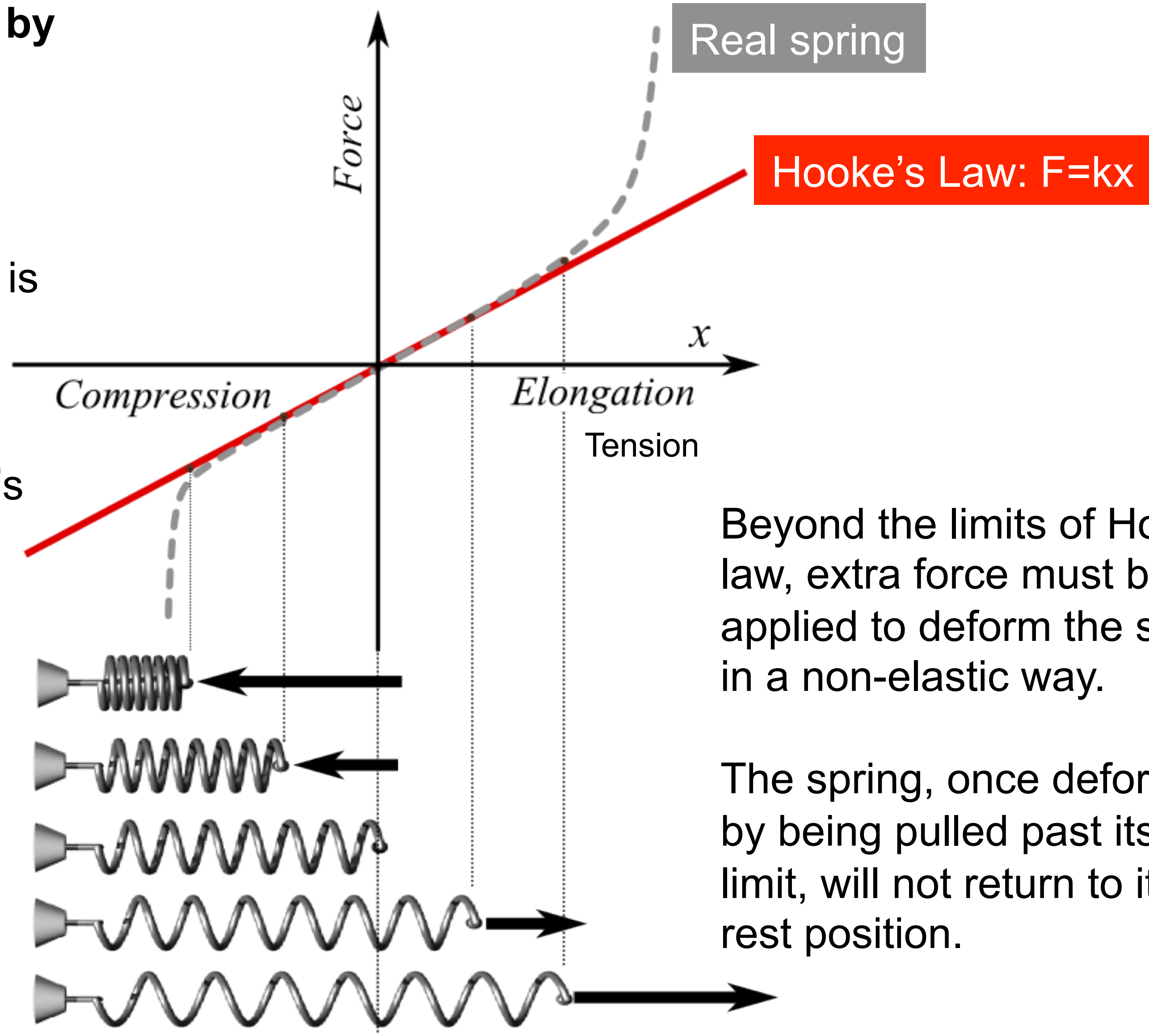
Recall force exerted **by** a spring:

$$F = -kx$$

So force **to** stretch a spring in elastic limit is

$$F = kx$$

(because of Newton's Third law)

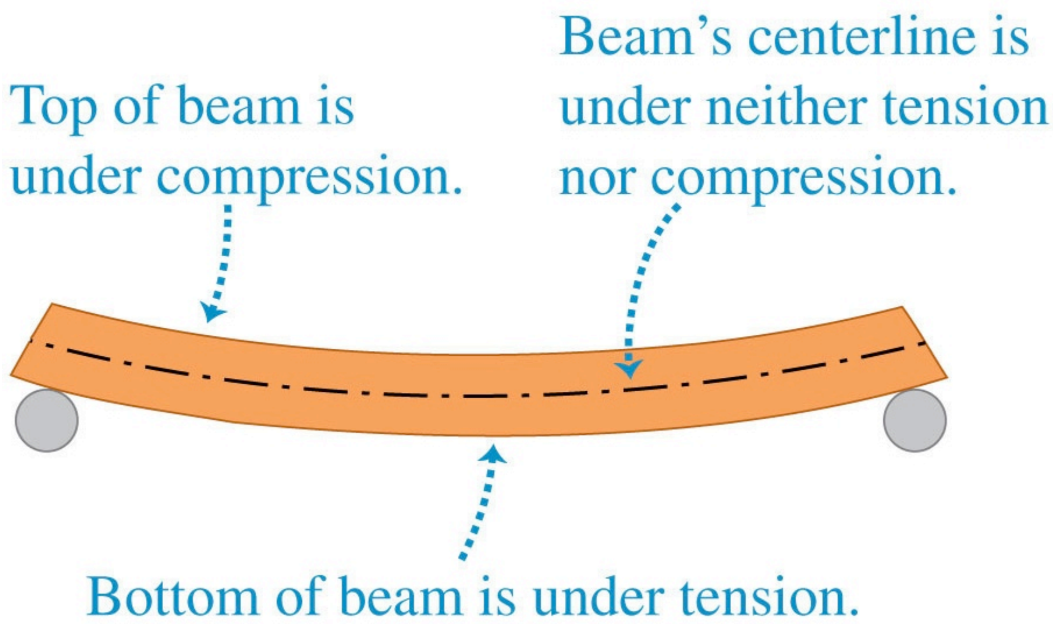


Beyond the limits of Hooke's law, extra force must be applied to deform the spring in a non-elastic way.

The spring, once deformed by being pulled past its elastic limit, will not return to its initial rest position.

Review: Compression and Tension

- In many situations, objects can experience both tensile and compressive stresses at the same time.
- For example, a horizontal beam supported at each end sags under its own weight.



- Which engineering materials are stronger under compression? Which are stronger under tension?

Chapter 12: Fluid Mechanics (Statics and Dynamics)

- We consider fluids that are **nearly incompressible**
  - **Fluids** are materials that cannot sustain a shear force
    - They basically have **zero shear modulus** S
  - We assume here that fluid volume doesn't change under pressure
    - They basically have **infinite bulk modulus** B
  - We saw on Monday this is a reasonable approximation for water
  - **Gases** are fluids that cannot maintain a well-defined surface
  - **Liquids** are fluids that can maintain a well-defined surface

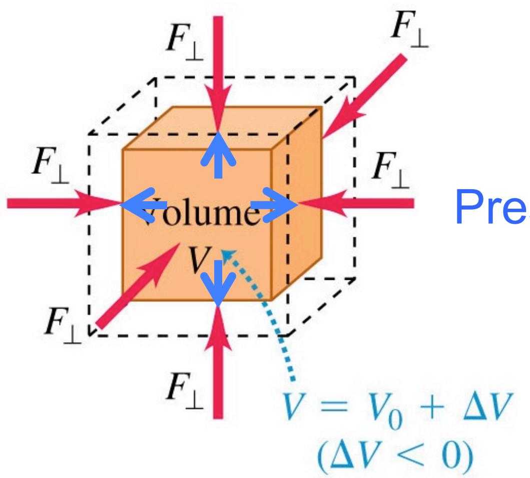
- This is equivalent to assuming that the fluid has nearly constant **density** ρ

$$\rho \equiv \frac{m}{V}$$

  - Density is a material property for solids and incompressible liquids
  - For gases ~~and compressible liquids~~, it can vary by quite a lot

Review: Pressure In A Fluid

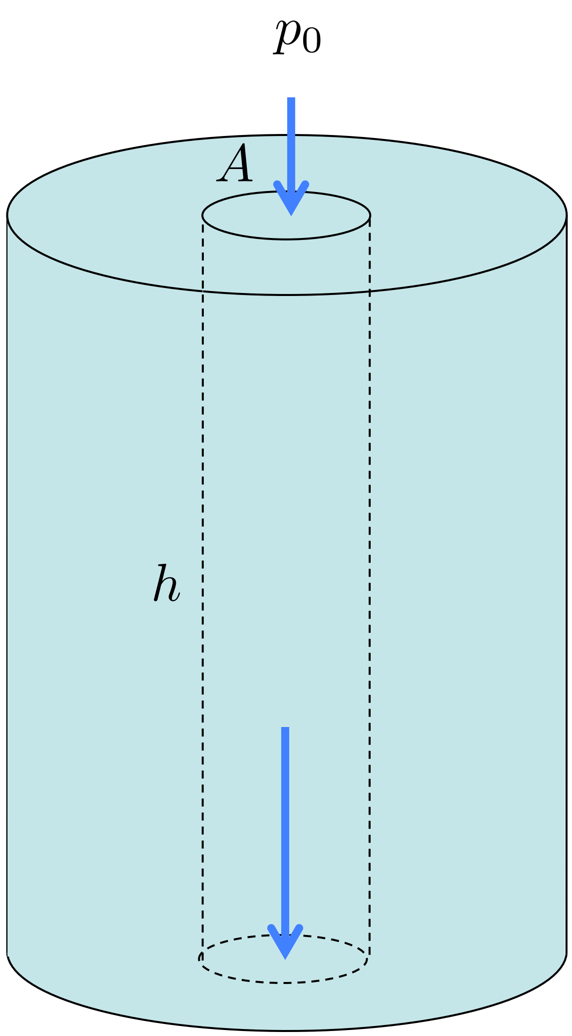
- A fluid exerts a force perpendicular to any surface in contact with it, such as a container wall or a body immersed in the fluid.
  - This is an example of Newton's Third Law



Pressure $p \equiv \frac{F_\perp}{A}$

- Pressure in a small volume of fluid is constant over all surfaces of the volume
  - Pressure certainly varies with height in a fluid column though
- Pressure is a scalar! (Always perpendicular to the area *A*)

Review Example: Fluid Column with External Pressure



$$F = p_0 A + mg = p_0 A + \rho h A g$$

$$p = F/A$$

- In atmosphere, there is additional air pressure p_0 acting at the top of the liquid column
  - Mass of liquid in column is

$$m = \rho h A$$

- So the total pressure at a depth h below the surface of the fluid is

$$p = p_0 + \rho g h$$

Example

- Typical household water pressure is 60 psi in first floor plumbing. How much pressure is lost at a second floor faucet that is 14 feet higher than the first floor faucet?

$$p - p_0 = \rho g h = (10^3\ \mathrm{kg/m^3})(9.8\ \mathrm{m/s^2})(14\ \mathrm{feet})$$

$$p - p_0 = 6.06\ \mathrm{psi} = 4.18 \times 10^4\ \mathrm{Pa}$$

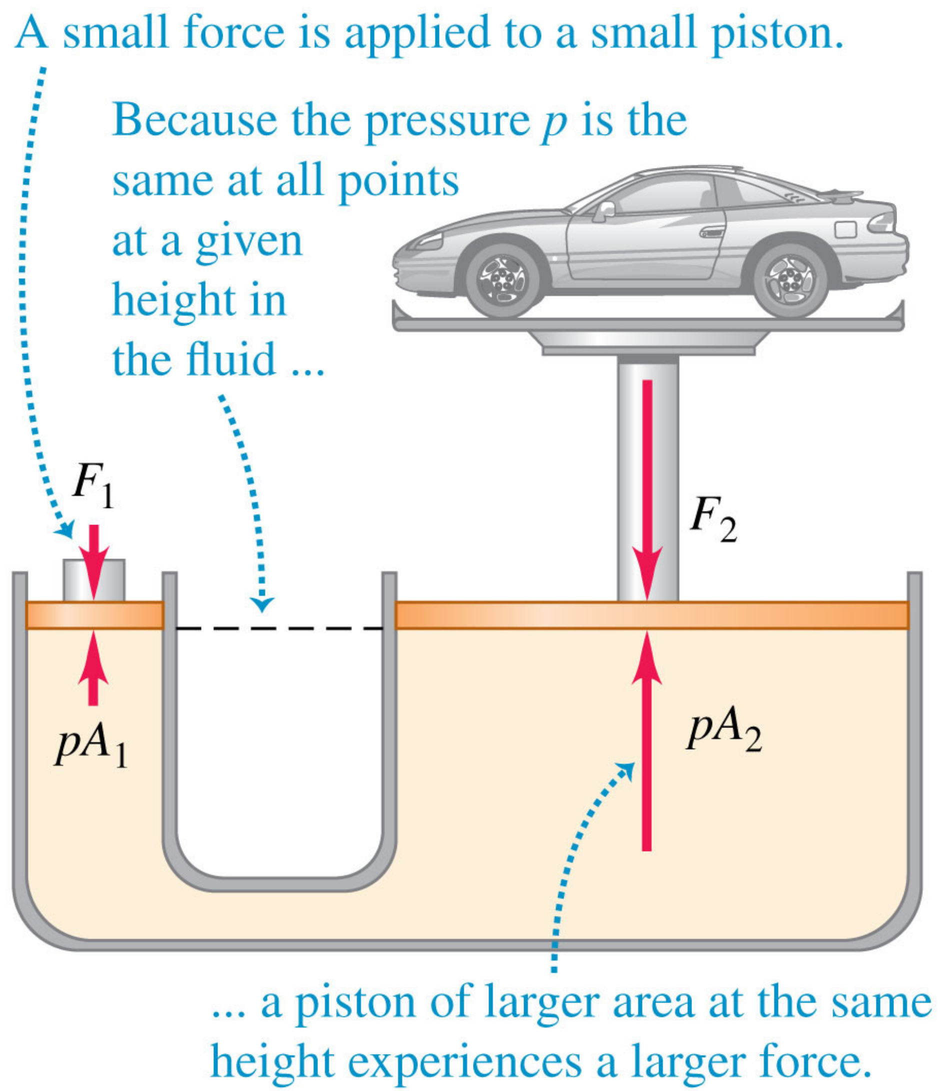
You lose about 10% of your water pressure per floor without additional pumps.

Review: Pascal's law

- Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel:

$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

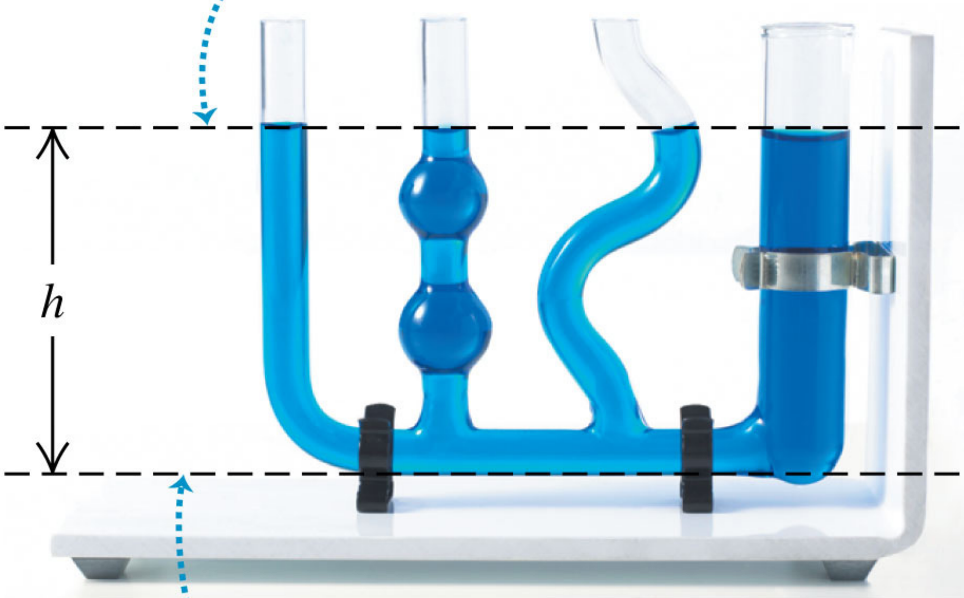
- Why hydraulics work
- Acts like a liquid lever
- Energy (force times distance) is conserved



The beer bottle trick is cavitation shock, NOT Pascal's Law: https://www.youtube.com/watch?v=lj3x2U4CaEs

Pressure at depth in a fluid

The pressure at the top of each liquid column is atmospheric pressure, p_0.



The pressure at the bottom of each liquid column has the same value p.

The difference between p and p_0 is $\rho g h$, where h is the distance from the top to the bottom of the liquid column. Hence all columns have the same height.

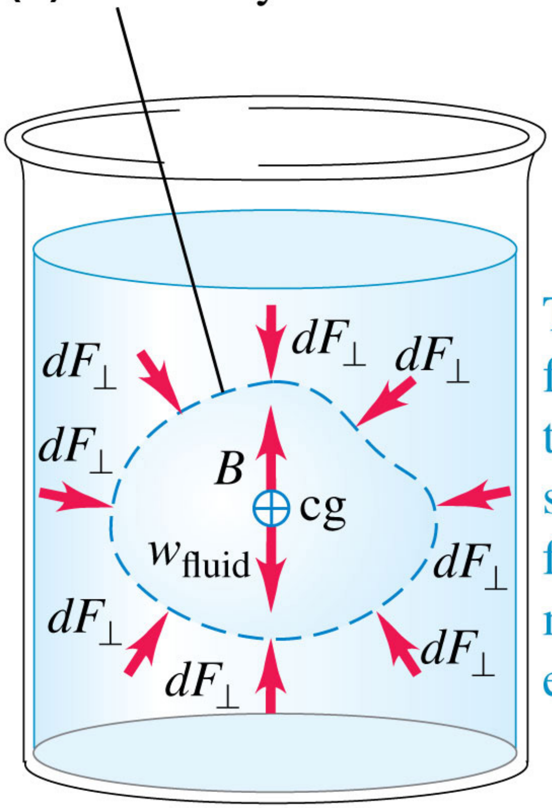
- Each fluid column has the same height, no matter what its shape.
- The pressure at any depth *h* within each column is the same:

$$p = p_0 + \rho g h$$

- This is a statics problem, where all forces balance on every small volume of fluid.

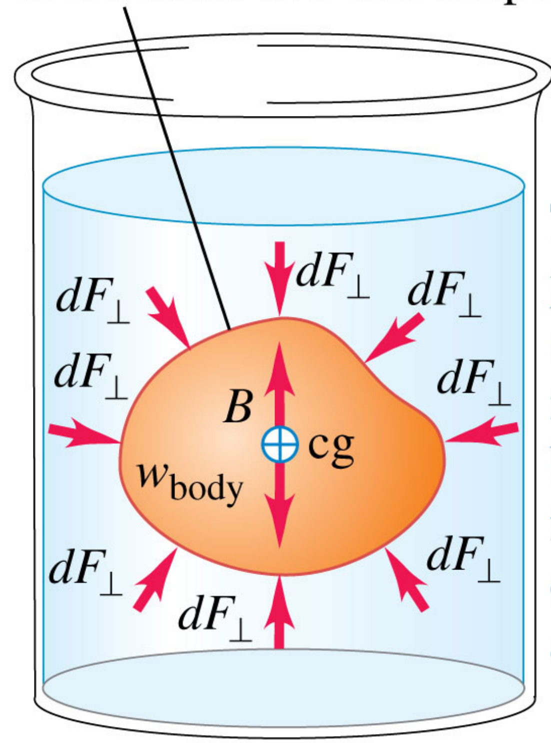
Archimedes's Principle

(a) Arbitrary element of fluid in equilibrium



The forces on the fluid element due to pressure must sum to a buoyant force equal in magnitude to the element's weight.

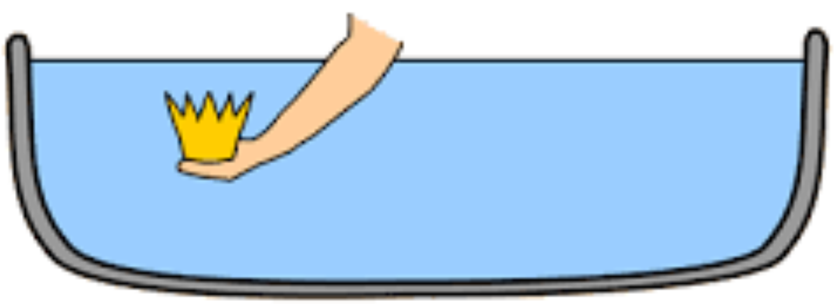
(b) Fluid element replaced with solid body of the same size and shape



The forces due to pressure are the same, so the body must be acted upon by the same buoyant force as the fluid element, *regardless of the body's weight.*

Archimedes's Principle

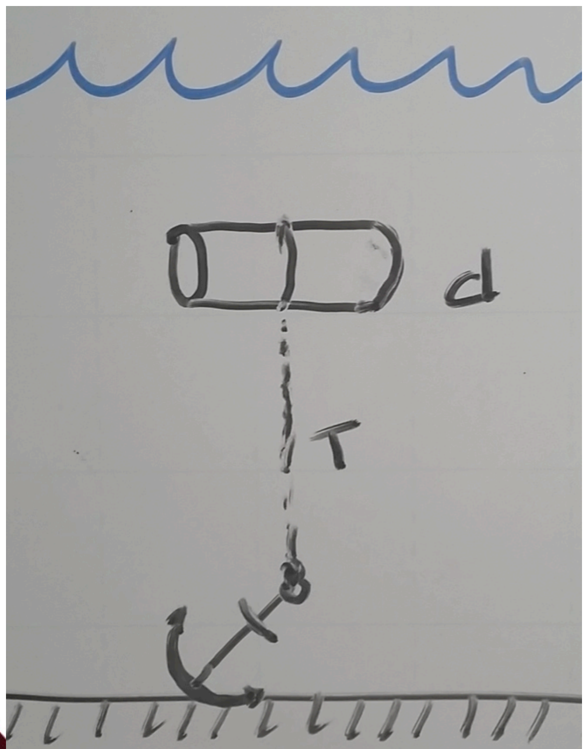
- A body immersed in water seems to weigh less than when it is in air.
- When the body is less dense than the fluid, it floats.
- The human body usually floats in water, and a helium-filled balloon floats in air.
- These are examples of buoyancy, a phenomenon described by *Archimedes's principle*:
  - **When a body is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.**

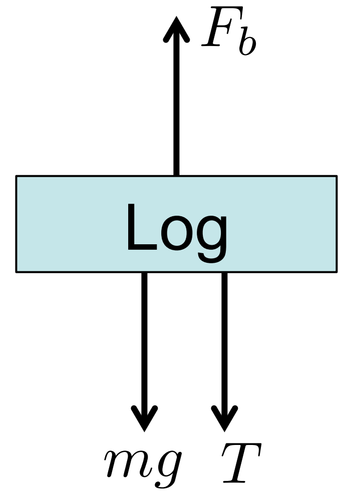


Weight (air) – Weight (submerged) = (Fluid density) * (Object volume)

Example

- A cylindrical m=12 kg wooden log of diameter 20 cm and length 50 cm is tied to a heavy anchor and dropped over the side of a boat into a moderately deep fresh water lake. What is the tension in the rope when the anchor comes to rest?





$$F_{\text{net}} = 0 = F_b - mg - T$$

$$T = F_b - mg$$

$$F_b = g\, V_{\text{log}}\, \rho_{\text{water}}$$

$$V_{\text{log}} = 0.0157\ \text{m}^3$$

$$F_b = 154\ \text{N}$$

$$T = 154\ \text{N} - 118\ \text{N} = 36\ \text{N}$$

Example

2. **Fluid Mechanics** (15 total points)

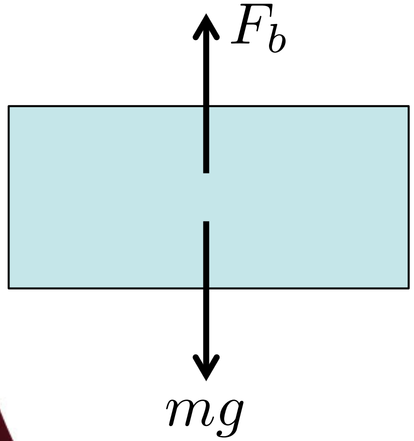
A slab of ice 2.0 m per side and 1.0 m thick floats on a pure freshwater lake.

(a) (5 points) How much thickness of the ice slab is exposed above the lake's surface?

(b) (5 points) A 75 kg woman gently steps onto the ice slab and stands on it, pushing the ice down into the water a bit. How much thickness of the ice slab is exposed above the lake's surface in this case?

(c) (5 points) If the woman quickly jumps to accelerate upwards at 3.0 m/s^2, does the ice slab bob below the surface of the water? Why or why not?

$$\rho_{\text{ice}} = 917 \text{ kg/m}^3 \qquad \rho_{\text{water}} = 1000 \text{ kg/m}^3$$



Example

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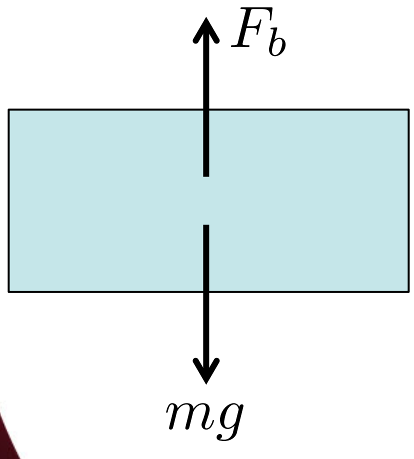


$$F_{\text{net}} = F_b - mg = 0 \qquad \Rightarrow \qquad F_b = mg$$

$$V_{\text{ice}} = (2.0 \text{ m})^2(1.0 \text{ m}) = 4.0 \text{ m}^3 \qquad m = V_{\text{ice}}\, \rho_{\text{ice}} = 3668 \text{ kg}$$

$$F_b = g\, V_{\text{submerged}}\, \rho_{\text{water}} = mg \quad \Rightarrow \quad V_{\text{submerged}} = \frac{m}{\rho_{\text{water}}} = 3.7 \text{ m}^3$$

$$h_{\text{submerged}} = \frac{V_{\text{submerged}}}{(2.0 \text{ m})^2} = 0.92 \text{ m}$$

$$h_{\text{exposed}} = 1.0 \text{ m} - 0.92 \text{ m} = 0.08 \text{ m}$$

With the woman, $h_{\text{submerged}}$ goes up by 0.019 m

The Continuity Equation

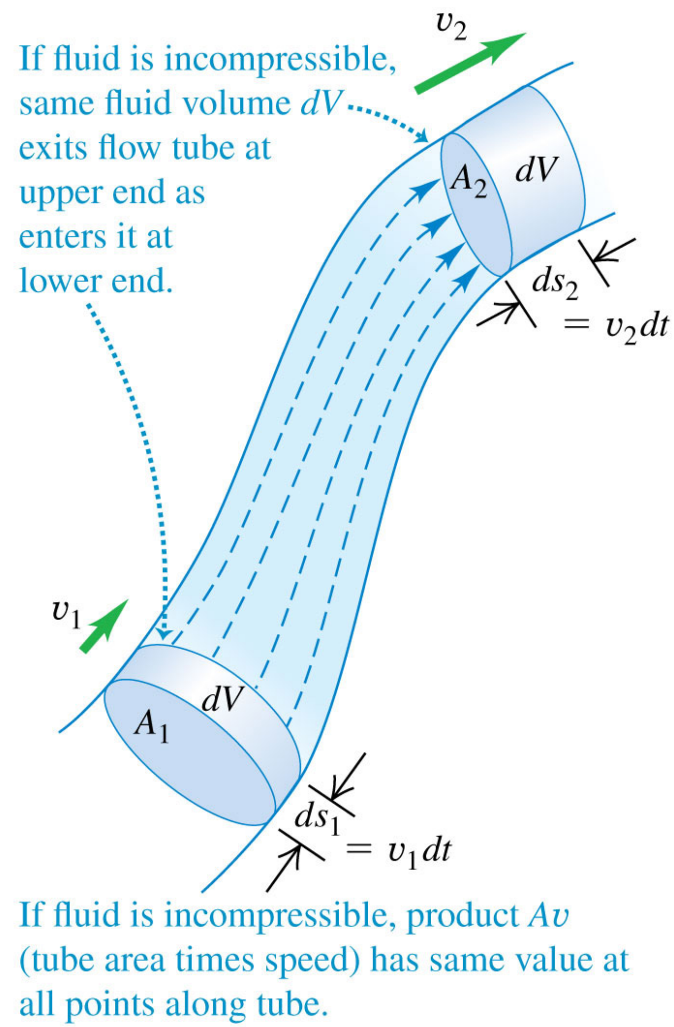
- The figure at the right shows a flow tube with changing cross-sectional area.
- The **continuity equation** for an incompressible fluid is

$$A_1 v_1 = A_2 v_2$$

- The **volume flow rate** is

$$\frac{dV}{dt} = Av$$

  - Describes how incompressible fluid flows through area *A* with velocity *v*



This is a mathematical way of saying "fluid volume is conserved": what goes in must come out

Example

- Your kitchen faucet produces a volume flow rate of 2.0 gallons/minute through a faucet nozzle of area 0.10 in^2.
  - What is the velocity of water exiting out the nozzle?

$$\frac{dV}{dt} = Av \qquad \Rightarrow \qquad v = \frac{dV/dt}{A} = \frac{(2.0\ \mathrm{gal/min})}{(0.1\ \mathrm{in}^2)} = 1.96\ \mathrm{m/s} = 4.4\ \mathrm{mi/hr}$$

  - What is the water pressure?

$$\Delta p = p - p_0 = \frac{1}{V}\left(\frac{1}{2}mv^2\right) = \frac{1}{2}\rho v^2 \qquad \rho = 10^3\ \mathrm{kg/m^3}$$

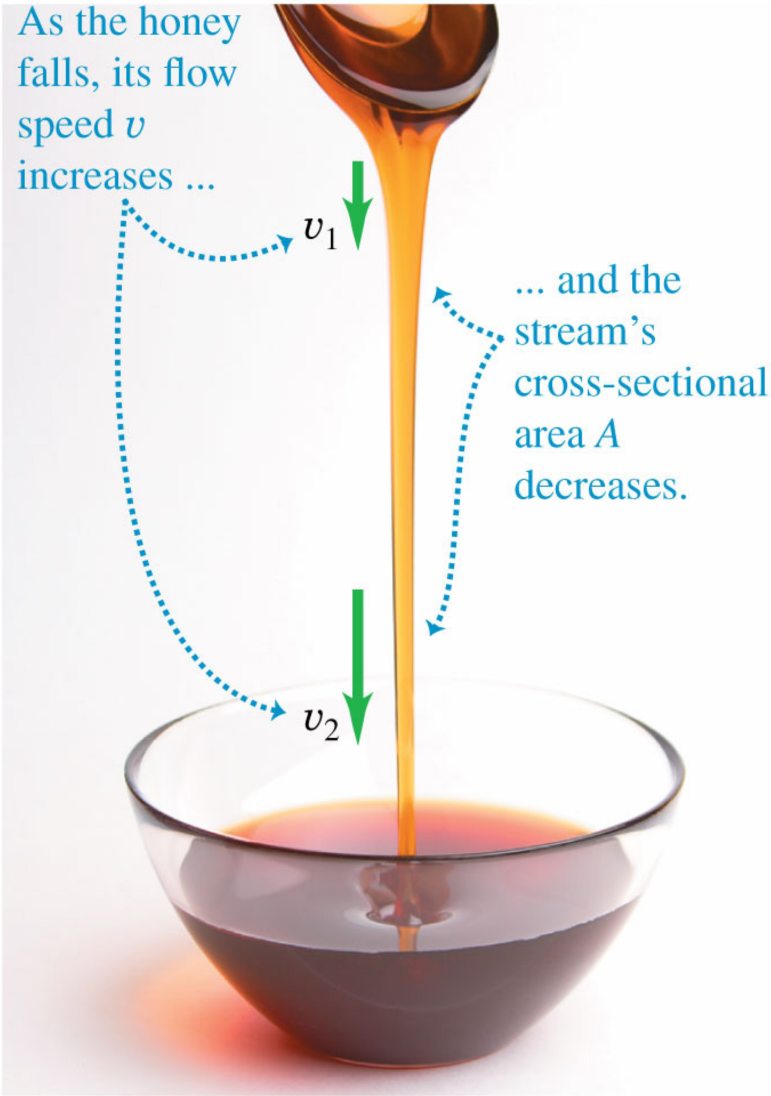
$$\Delta p = 1900\ \mathrm{Pa\ above\ air\ pressure}$$

The Continuity Equation

- The continuity equation helps explain the shape of a stream of honey poured from a spoon.

$$A_1 v_1 = A_2 v_2$$

- As velocity *v* increases, area *A* must decrease to keep the product *Av* constant



The volume flow rate $dV/dt = Av$ remains constant.

Bernoulli's equation

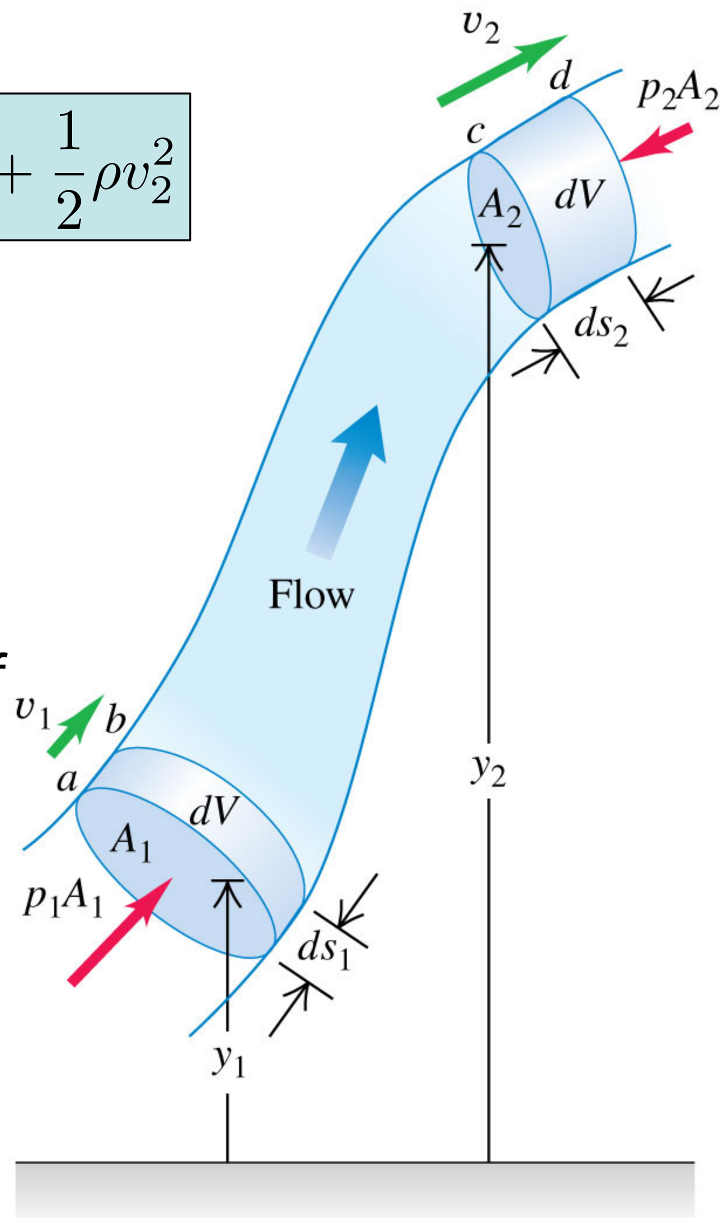
- Bernoulli's equation is:

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

Work/volume

Potential energy/volume

Kinetic energy/volume



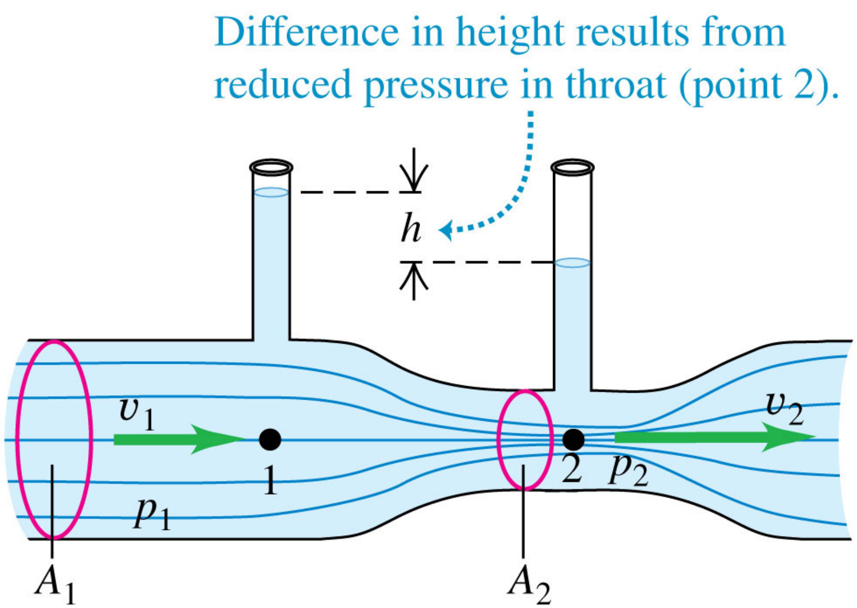
- It is due to the fact that the work done on a unit volume of fluid by the surrounding fluid is equal to the sum of the changes in kinetic and potential energies per unit volume that occur during the flow.
  - It is really just a fancy conservation of energy statement for incompressible flow.

The Venturi Meter

- Bernoulli applied at a constant height gives a relationship between the fluid velocity and pressure at any point:

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

  - Higher velocity means lower pressure and vice-versa

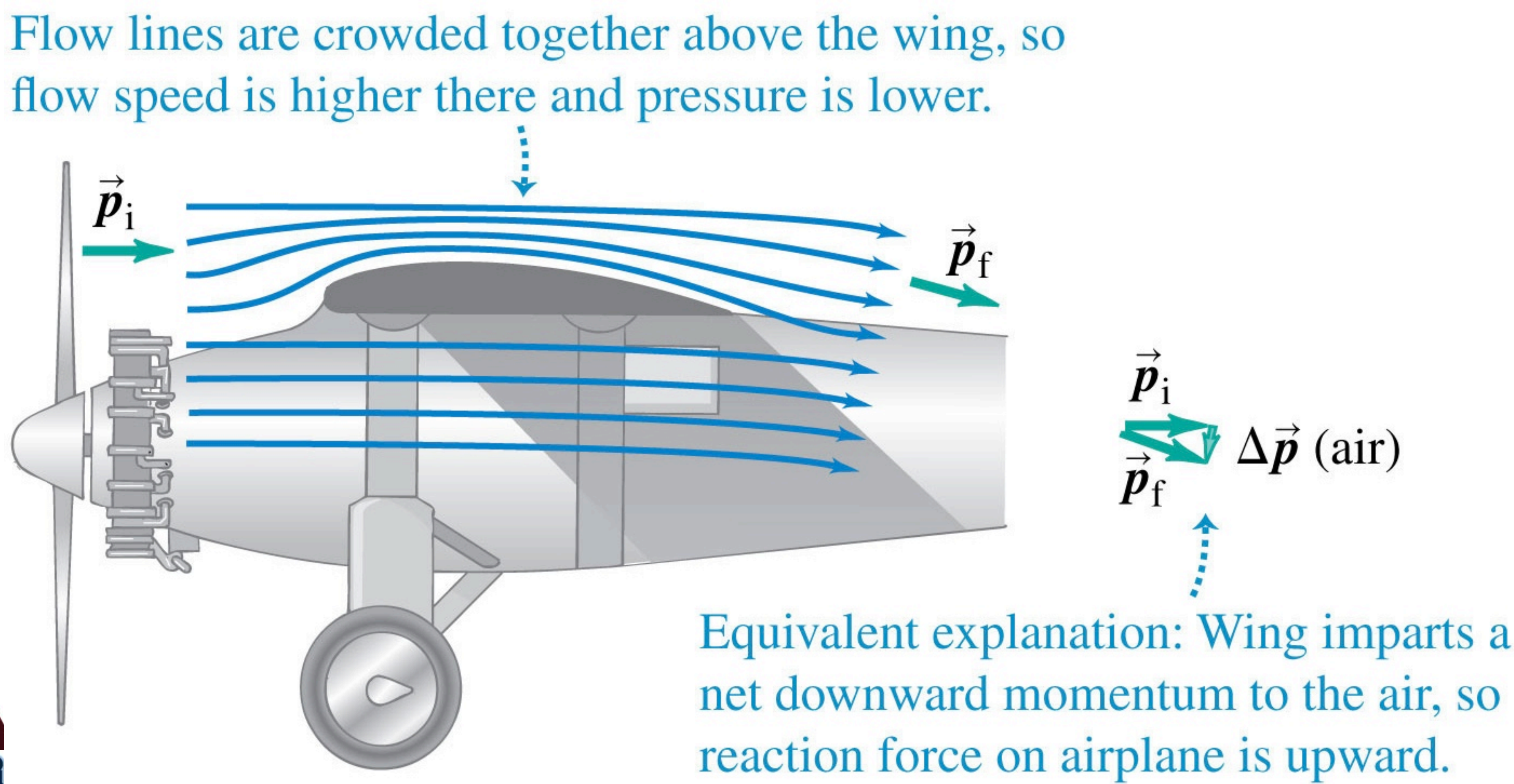


The pressure is less at point 2 because the fluid velocity is greater due to the constriction.

Again, higher velocity means lower pressure.

Lift on an airplane wing

- Bernoulli's principle helps to explain how airplanes fly.

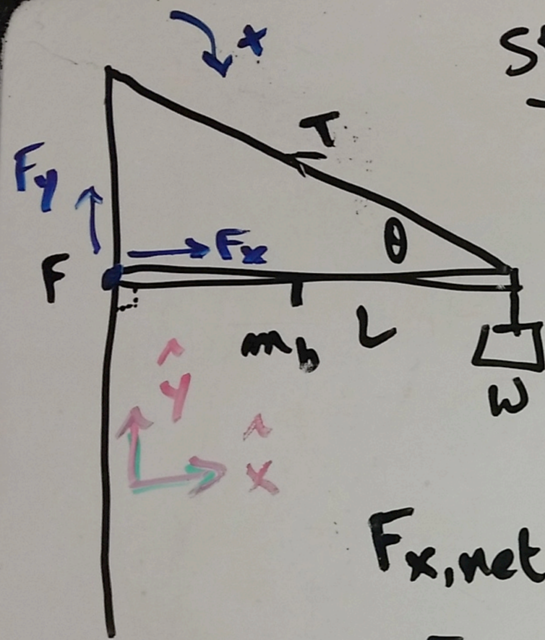


Statics example: know W of sign ①

θ

m_b - mass of bar

L = length of bar



$\Rightarrow$ Find out T, F_x, F_y $\Rightarrow$ 3 unknowns

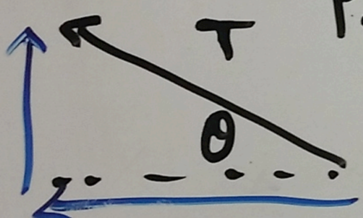
Statics: $F_{x,net} = 0$ $F_{net,y} = 0$ $\tau_{net} = 0$

$F_{x,net}\,(\text{bar}) = 0 = F_x - T\cos\theta \Rightarrow \boxed{F_x = T\cos\theta}$

$F_{y,net}\,(\text{bar}) = \boxed{0 = F_y + T\sin\theta - m_b g - W}$

$\tau_{net} = \boxed{0 = WL + m_b g\,L/2 - TL\sin(180^\circ - \theta)}$

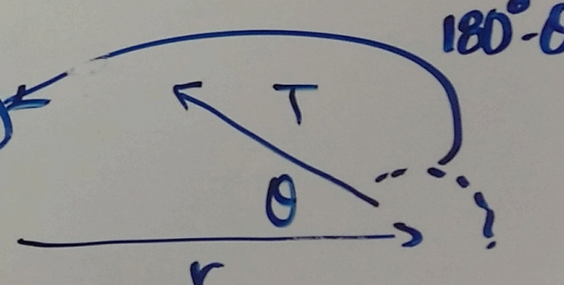
Physics is done $\sin(180^\circ - \theta) = \sin\theta$



$T_x = T\cos\theta$

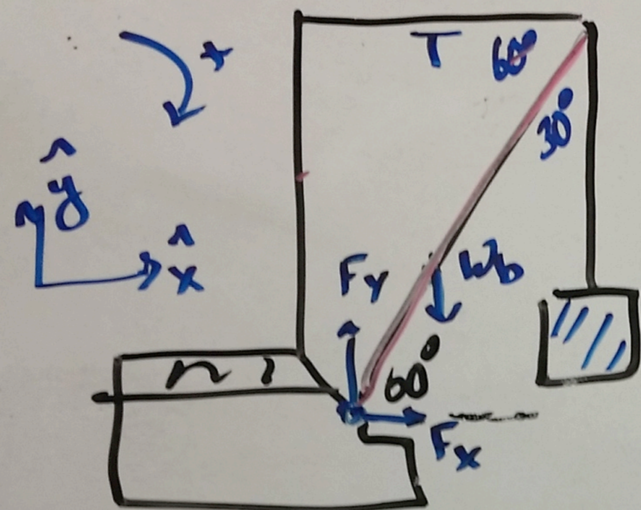
$T_y = T\sin\theta$

$\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta\,(r \to F)$



$$T = \frac{W + m_b g/2}{\sin\theta}$$

(2)



$W_{boom} = 2750N$

c.g. at 37% of length

Statics: $F_{net,x} = F_{net,y} = \tau_{net} = 0$

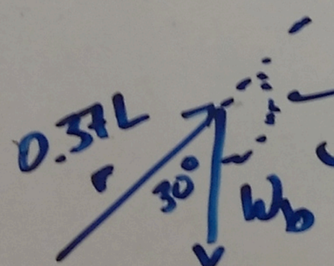
3 unknowns: F_x, F_y, T

Forces on boom

$$F_{net,x} = 0 = F_x - T \Rightarrow \boxed{F_x = T}$$

$$F_{net,y} = 0 = F_y - W_b - 5000N \Rightarrow \boxed{F_y = W_b + 5000N = 7750N}$$

$$\tau_{net} = \boxed{(0.37L)W_b \sin(150^\circ) + L(5000N)\sin(150^\circ) - LT\sin 120^\circ = 0}$$



$$\tau = \vec{r} \times \vec{F} = rF\sin\theta\,(r \rightarrow F)$$

$$= (0.37L)W_b \sin(180^\circ - 30^\circ)$$