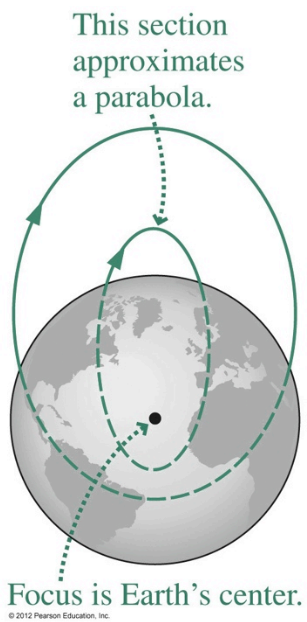
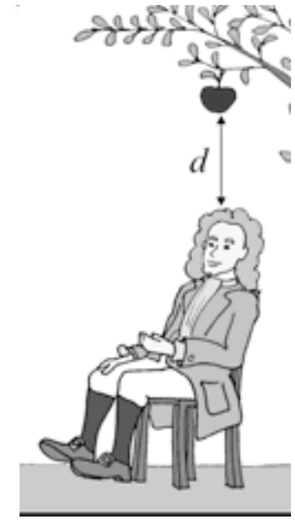


University Physics 226N/231N
Old Dominion University

Chapter 13: Gravity (and then some)



Dr. Todd Satogata (ODU/Jefferson Lab)

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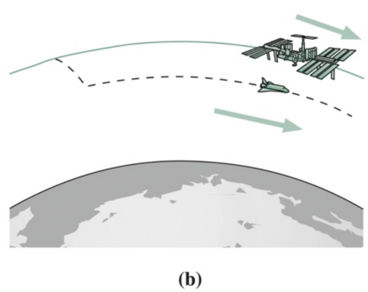
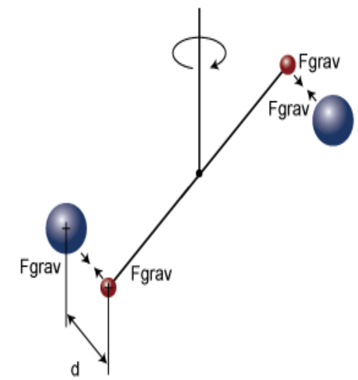




Monday, November 28, 2016

There is NO CLASS this Wednesday, November 30

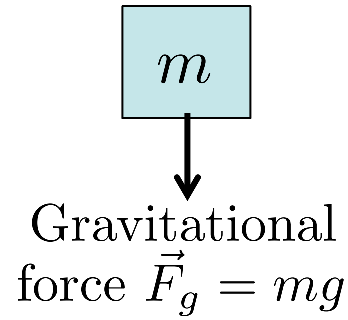
Homework due Monday December 5

Happy Birthday to Jon Stewart, Randy Newman,
Alan Lightman, and Russell Alan Hulse (1993 Nobel, binary pulsars)!
Happy National French Toast Day and Cyber Monday!

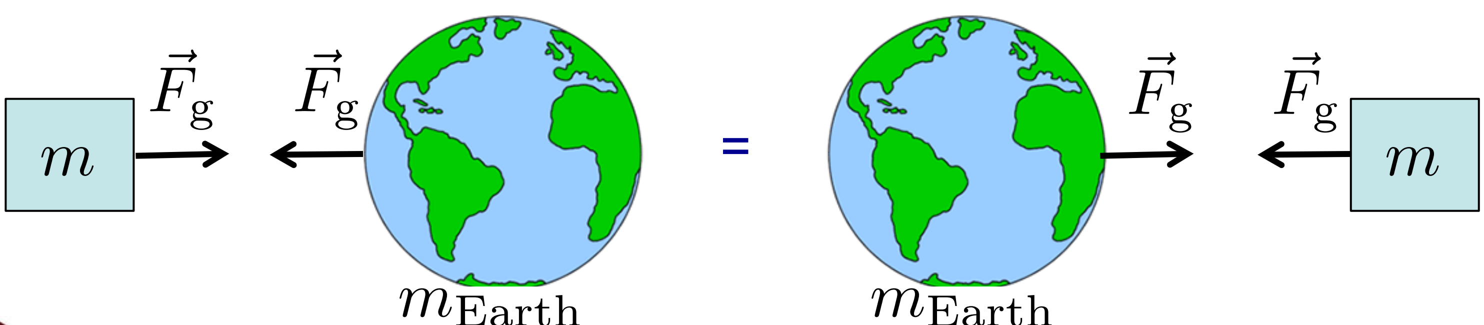
Please set your cell phones to “vibrate” or “silent” mode. Thanks!

Gravity Reconsidered

- We've learned a lot about gravity so far
  - But what we've learned about it **must be an approximation**
  - We've assumed the gravitational force is independent of height and the mass of the earth, $m_{\rm Earth}$



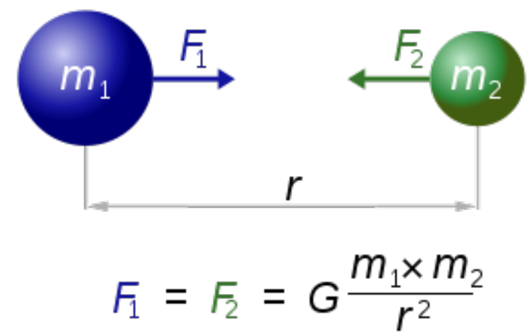
- But Newton's third law tells us that the box also exerts an equal and opposite force on the earth
  - Gravitational forces should be the same if we interchange the masses of the earth and box: g is proportional to $m_{\rm Earth}$!



Gravity Reconsidered

- The gravitational force must also get weaker when objects are further apart
  - Otherwise we'd live in a very crowded little universe
  - Some experiments showed that our original approximation of the gravitational force between two objects is more generally

$$F_{\mathrm{g}} = \frac{G m_1 m_2}{r^2}$$



$$G = 6.67384 \times 10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}$$

  - This becomes our more familiar $F_{\mathrm{g}} = m_1 a = m_1 g$ when

$$m_2 = m_{\mathrm{Earth}} = 5.972 \times 10^{24}\ \mathrm{kg}$$

and $$r = r_{\mathrm{Earth}} \approx 6371\ \mathrm{km}$$

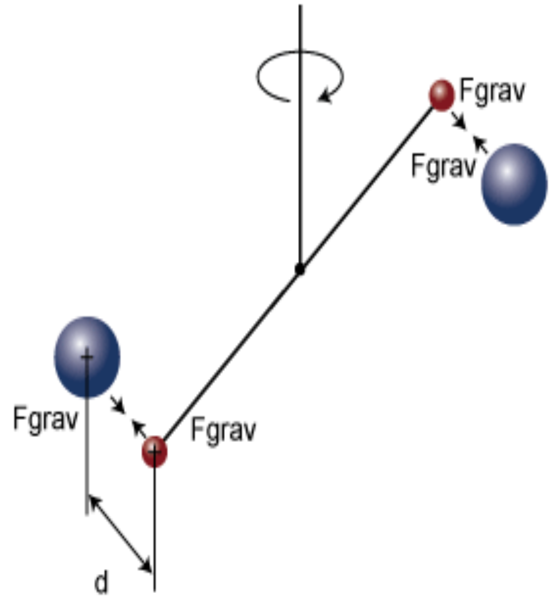
Ponderable

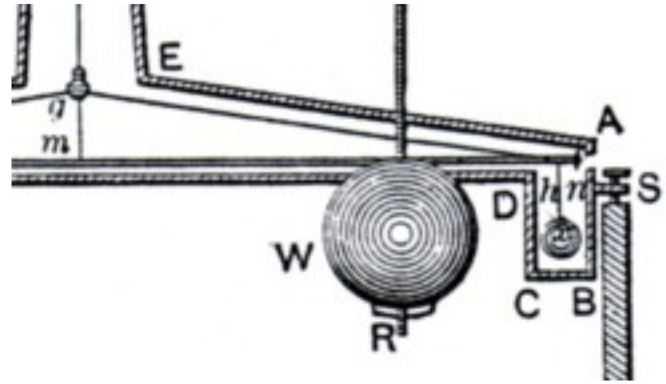
- How do you devise an **experiment** to measure a force that's this weak?
  - For two 1 kg objects 1m apart, $F_g = 6.67 \times 10^{-11}\ \mathrm{N}$
  - Earth's gravity on one of those is $W = mg = 9.8\ \mathrm{N}$
  - That's a huge difference!

- And yet Henry Cavendish devised an experiment to measure the force of gravity between a 158 kg ball and a 0.73 kg ball.

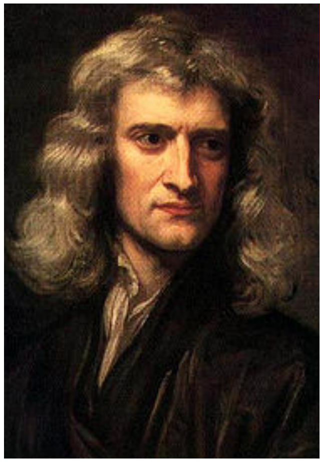
… in 1797.

1797: The Cavendish Experiment





- A balance used to measure force or weight has friction
- Cavendish invented the **torsion balance** to measure gravity
  - The twisting angle on a string is proportional to the force applied times the distance the force is applied at
  - His balance was exquisite for its time, with a sensitivity of less than 10^{-8} N, or the weight of a very small grain of sand



A Law of Universal Gravitation

$$F_{\mathrm{g}} = \frac{G m_1 m_2}{r^2}$$

$$G = 6.67384 \times 10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}$$

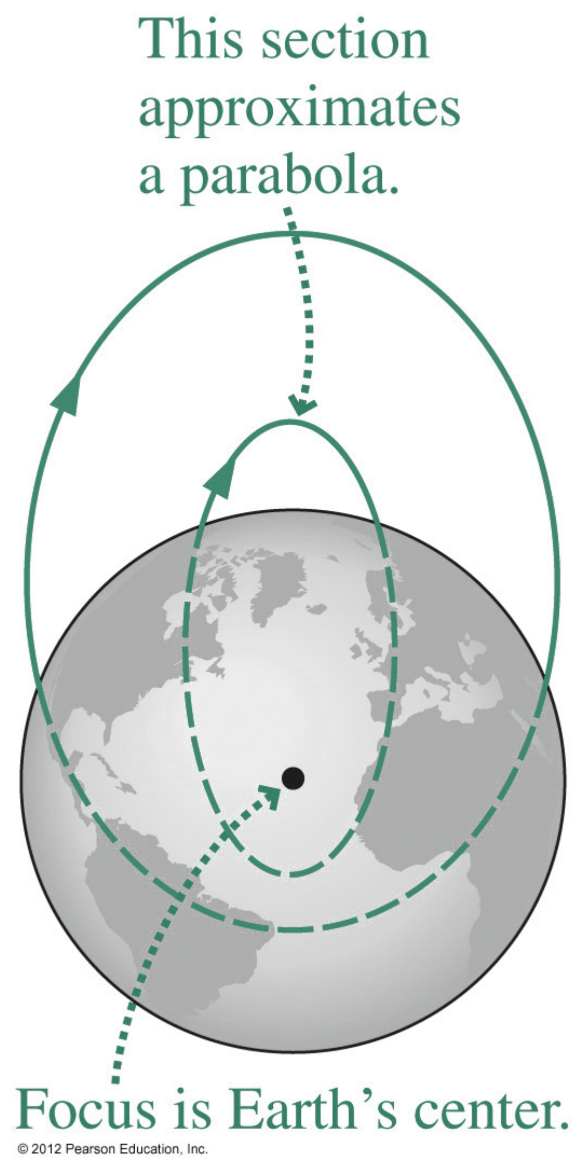
- This formula for the general gravitational force was developed by Newton (1687)
  - It's a much much better approximation, but again only an approximation!
  - Superceded by Einstein's general theory of relativity (1916)
  - But this equation works well enough to pretty much all everyday phenomena
- Technically for point particles, but works perfectly well for spheres where centers are distance r apart
  - Gravity is **always attractive along the vector** $\vec{r}$
  - We know of no classic examples of anti-gravity
    - Not even anti-matter, which still has positive mass: $E = mc^2$

Orbits: Our Old Ponderable

- Newton explained orbits using universal gravitation and his laws of motion:
  - Bound orbits are generally elliptical.
  - In the special case of a circular orbit, the orbiting object "falls" around a gravitating mass, always accelerating toward its center with the magnitude of its acceleration remaining constant.
  - Unbound orbits are hyperbolic or (borderline case) parabolic.

Projectile Motion and Orbits

- The "parabolic" trajectories of projectiles near Earth's surface are actually sections of elliptical orbits that intersect Earth.
- The trajectories are parabolic only in the approximation that we can neglect Earth's curvature and the variation in gravity with distance from Earth's center.



Circular Orbits

- In a circular orbit, gravity provides the centripetal force of magnitude $F_{\text{centrip}} = mv^2/r$ needed to keep an object of mass *m* in its circular path about a much more massive object of mass M:

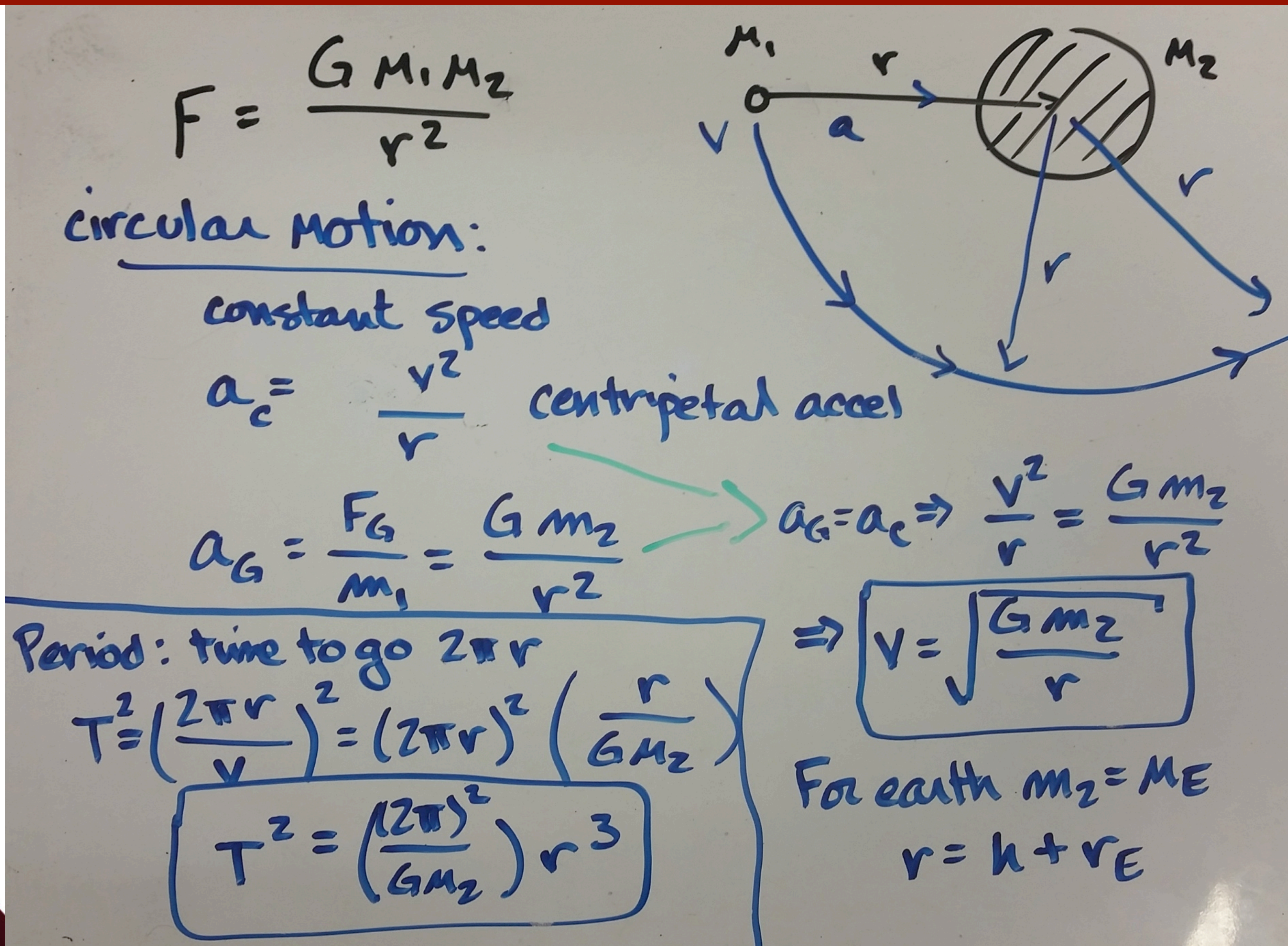
$$F_{\text{g}} = F_{\text{centrip}} \quad \Rightarrow \quad \frac{GMm}{r^2} = \frac{mv^2}{r}$$

- Orbital speed v (solve for v): $v = \sqrt{GM/r}$
- Orbital period (time to go 2πr with velocity v): $T^2 = \frac{4\pi^2 r^3}{GM}$
  - This is a derivation of Kepler's **observation** for planetary motion that the period squared goes like the orbit radius cubed:

$$T^2 \propto r^3$$

  - For satellites in low-Earth orbit, the period T is about 90 minutes.

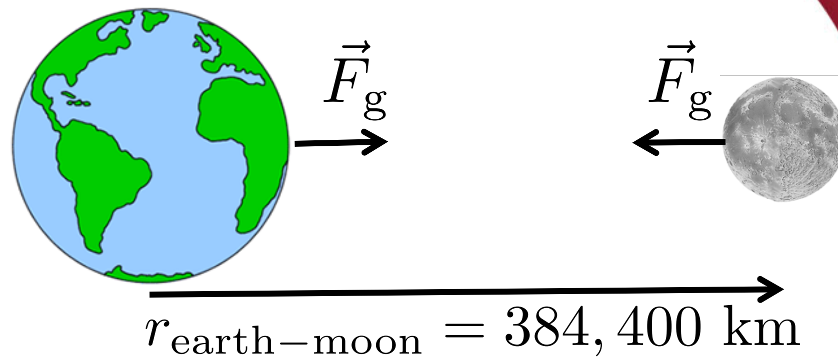
Board Work for Circular Orbits



Ponderable

$$F_g = \frac{G m_1 m_2}{r^2}$$

$$G = 6.67384 \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$$



- What is the gravitational force between the Earth and its moon?

$$m_{\mathrm{Earth}} = 5.9722 \times 10^{24}\ \mathrm{kg} \qquad m_{\mathrm{moon}} = 7.3477 \times 10^{22}\ \mathrm{kg}$$

- What is the gravitational force between Todd and his laptop?

$$m_{\mathrm{Todd}} = 100\ \mathrm{kg} \qquad m_{\mathrm{laptop}} = 1.8\ \mathrm{kg} \qquad r = 0.5\ \mathrm{m}$$

- What is the altitude of geosynchronous satellites above Earth?

$$r_{\mathrm{Earth}} = 6370\ \mathrm{km} \qquad T^2 = \frac{4\pi^2 r^3}{GM}$$

Board work for Geosynchronous Satellites

Geosync satellite: $T = 24\,\text{hr}\,\frac{3600\,\text{s}}{\text{hr}} = 86400\,\text{s}$

$$T^2 = \frac{4\pi^2}{GM} r^3$$

$G = 6.67\times 10^{-11}$ ..... (units)

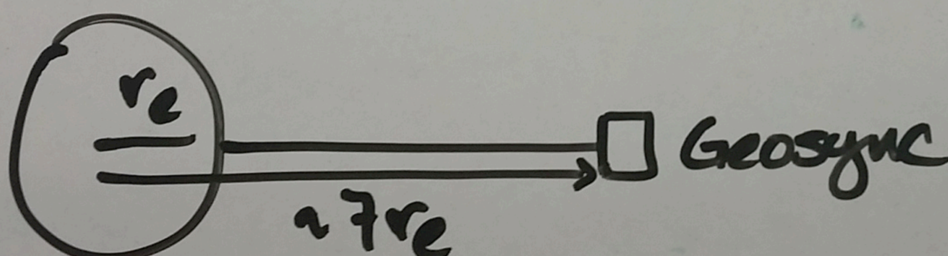
$M = 5.97\times 10^{24}\,\text{kg}$

$$r = \sqrt[3]{\frac{GM}{4\pi^2} T^2}$$

$$= \sqrt[3]{7.46\times 10^{22}\,\text{m}^3} = 4.2\times 10^{7}\,\text{m}$$

$$= \boxed{4.2\times 10^{4}\,\text{km} = r}$$

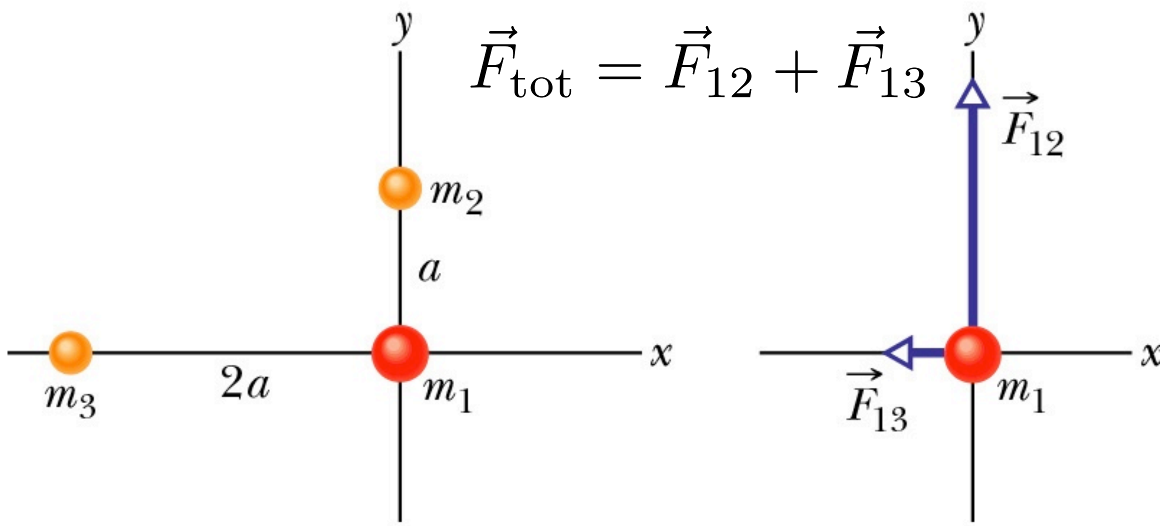
$$r_e = 0.64\times 10^{4}\,\text{km}$$



Multiple Objects

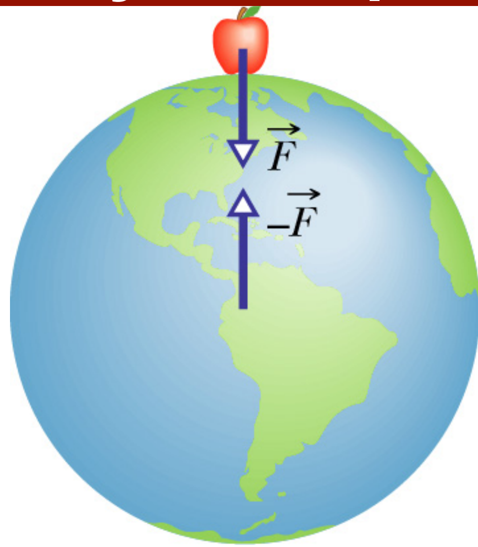
- To calculate the total gravitational force from many objects, we add together the vectors of their individual forces
  - This is known as the **principle of superposition**
  - This is (mostly) a general rule for vectors that you already know

$$\vec{F}_{\text{tot}} = \vec{F}_{12} + \vec{F}_{13}$$



  - This summation extends to any number of forces: integral!
    - We can calculate the gravitational force between any two oddly shaped objects by adding up (integrating) gravitational forces between all the combinations of all their pieces

Gravity and Spheres



- Newton had invented calculus so he could do these sums for simple geometric objects
- One cool result was **Newton's Shell Theorem**
  - A uniform spherical shell of matter attracts an object that is outside the shell as if all the shell's mass was concentrated at its center
- This is why we can dodge calculus and simplify calculations of the force of gravity for spherical objects (like planets)

Revisiting g

- Recall that we have $F_{\mathrm{g}} = mg$ near Earth's surface
- And gravity says $$F_{\mathrm{g}} = \frac{G m m_{\mathrm{Earth}}}{r^2}$$
- So $g = \dfrac{G m_{\mathrm{Earth}}}{r_{\mathrm{Earth}}^2}$ on the surface of the Earth

- This applies to any nearly spherical cosmic body!
  - Again, an approximation (spherical, uniform mass, rotation…)

- On Mars:

$$m_{\mathrm{Mars}} = 6.4 \times 10^{23}\ \mathrm{kg}$$

$$r_{\mathrm{Mars}} = 3400\ \mathrm{km}$$

$$g_{\mathrm{Mars}} = 3.7\ \mathrm{m/s^2} = 0.38\ g_{\mathrm{Earth}}$$

On Moon:

$$m_{\mathrm{Moon}} = 7.3 \times 10^{22}\ \mathrm{kg}$$

$$r_{\mathrm{Moon}} = 1737\ \mathrm{km}$$

$$g_{\mathrm{Moon}} = 1.6\ \mathrm{m/s^2} = 0.16\ g_{\mathrm{Earth}}$$

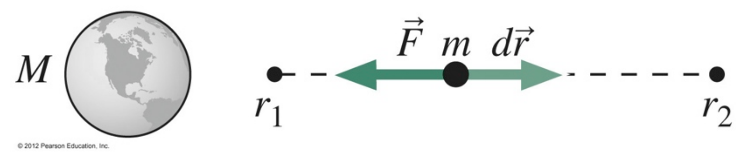
Gravitational Potential Energy

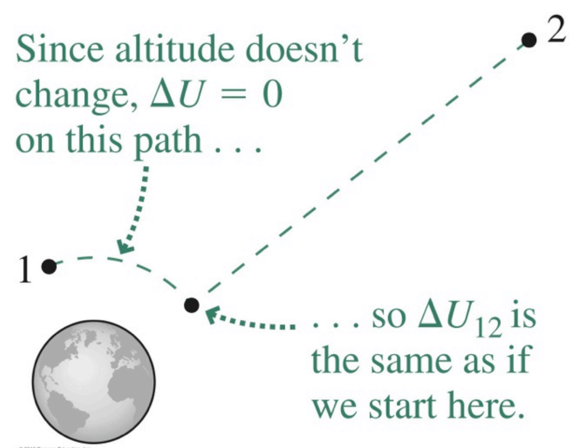
- Because the gravitational force changes with distance, it's necessary to integrate to calculate how gravitational potential energy U changes over large distances. This integration gives

$$\Delta U_{12} = G m_1 m_2 \int_{r_1}^{r_2} r^{-2} dr = G m_1 m_2 (-r^{-1}) \Big|_{r_1}^{r_2} = G m_1 m_2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

- This result holds regardless of whether the two points are on the same radial line.
- It's convenient to take the zero of gravitational potential energy at infinity. Then the gravitational potential energy becomes

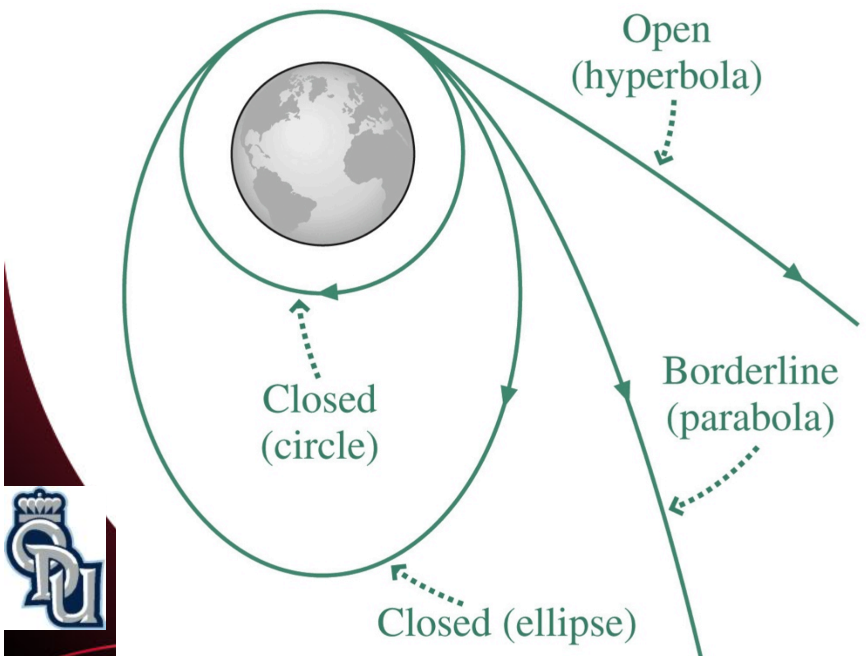
$$U(r) = -\frac{G m_1 m_2}{r}$$

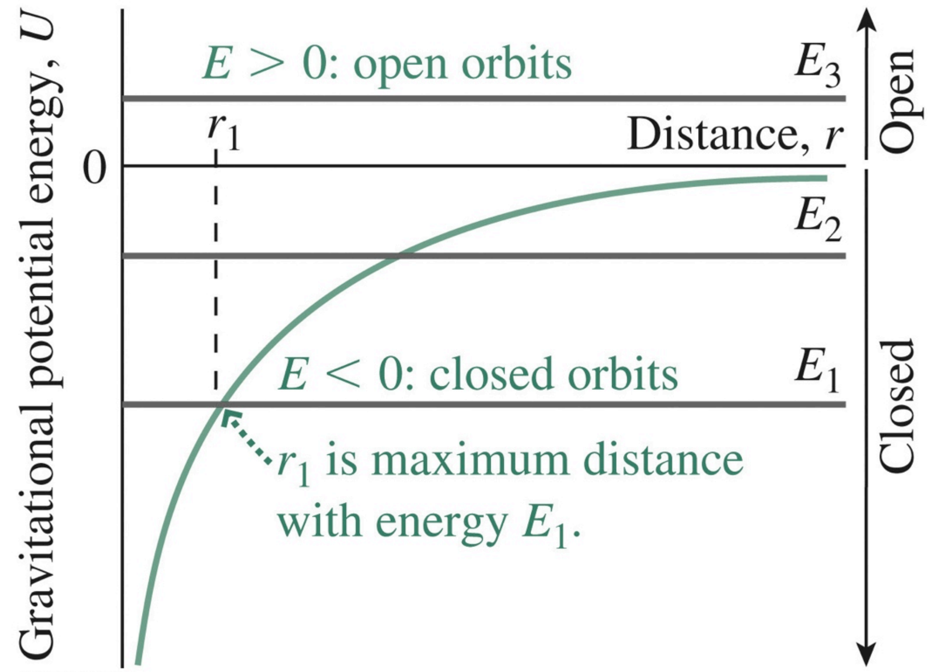




Energy and Orbits

- The total energy $E = K + U$, the sum of kinetic energy K and potential energy U, determines the type of orbit:
  - $E < 0$: The object is in a bound, elliptical orbit.
    - Special cases include circular orbits and the straight-line paths of falling objects.
  - $E > 0$: The orbit is unbound and hyperbolic.
  - $E = 0$: The borderline case gives a parabolic orbit.





Escape Velocity

- An object with total energy *E* less than zero is in a bound orbit and can't escape from the gravitating center.
- With energy *E* greater than zero, the object is in an unbound orbit and can escape to infinitely far from the gravitating center.
- The minimum speed (launching directly away from the center of the earth) required to escape is given by

$$0 = K + U = \tfrac{1}{2} m v^2 - \frac{GMm}{r}$$

- Solving for *v* gives the escape velocity:

$$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

  – Escape velocity from Earth's surface is about 11 km/s.

Energy in Circular Orbits

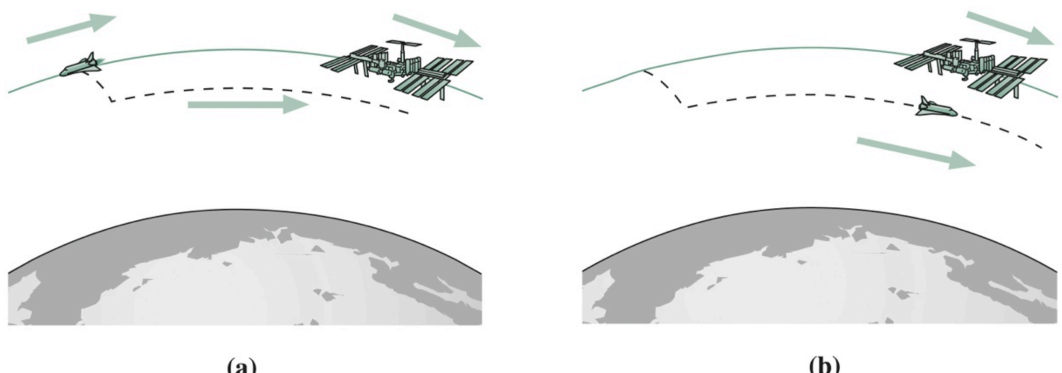
- In the special case of a circular orbit, kinetic energy and potential energy are precisely related:

$$U = -2K$$

- Thus in a circular orbit the total energy is

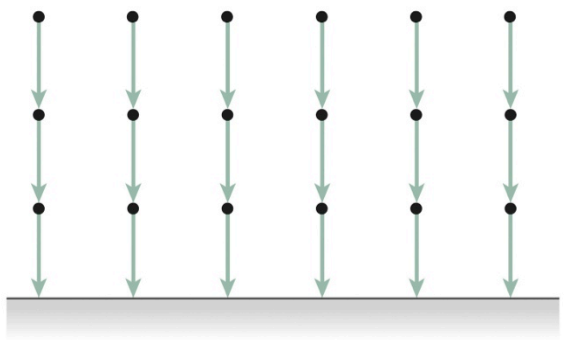
$$E = K + U = -K = \tfrac{1}{2}U = -\frac{GMm}{2r}$$

  - This negative energy shows that the orbit is bound.
  - The lower the orbit, the lower the total energy—but the faster the orbital speed.
    - This means an orbiting spacecraft needs to lose energy to gain speed.

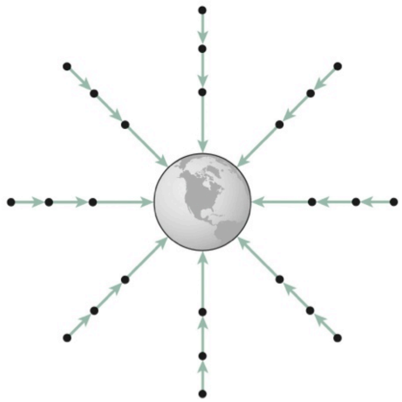


The Gravitational Field

- It's convenient to describe gravitation not in terms of "action at a distance" but rather in terms of a **gravitational field** that results from the presence of mass and that exists at all points in space.
  - A massive object creates a gravitational field in its vicinity, and other objects respond to the field *at their immediate locations.*
  - The gravitational field can be visualized with a set of vectors giving its strength (in N/kg; equivalently, m/s^2) and its direction.



Near Earth's surface



On a larger scale

Millennium Simulation

- http://www.mpa-garching.mpg.de/galform/press/

- The biggest and most detailed supercomputer simulation of the evolution of the Universe from a few hundred thousand years after the Big Bang to the present day.

- The Millennium Simulation used 10 billion particles to track the evolution of 20 million galaxies over the history of the universe.

- A 3-dimensional visualization of the Millennium Simulation. The movie shows a journey through the simulated universe. During the two minutes of the movie, we travel a distance for which light would need more than 2.4 billion years.