University Physics 226N/231N
Old Dominion University

Chapter 14: Oscillatory Motion

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Monday, November 5, 2016

The Final Exam is next Monday, December 12 08:30-11:30

Last Homework Due Monday December 12

Happy Birthday to Arnold Sommerfeld, Sheldon Glashow (1979 Nobel),
CF Powell (1950 Nobel), and Werner Heisenberg (1932 Nobel)!
Happy National Sacher Torte Day and International Ninja Day!

Please set your cell phones to “vibrate” or “silent” mode. Thanks!

Lab Info

- In case you didn't see my email from Friday
  - **The lab section contributes to your grade for this course**
  - I was mistaken when I disseminated info to some students that the lab is a separately graded course

  - **I've updated the class grading in the syllabus**
  - 15% contribution from lab to final grade is mandated
  - Final exam is now worth 30% of your grade, not 40%
  - Midterms now count 25% of your grade, not 35%

Course Grades The final grade is calculated on an absolute scale. There are 100 points possible for this course:

25 points – Three Midterm Exams (lowest dropped)
30 points – Final Exam
15 points – Lab
30 points – Homework Assignments (including journal)

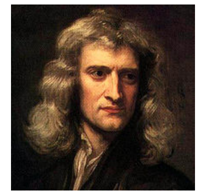
Exam Info

- **The final exam is next Monday, December 12**
  - **NOTE THE TIME**: 8:30-11:30, not 9 AM! Don't arrive late!
- There will be eight equally scored questions (17.5 pts each)
  - Two from material from midterm 1
  - Two from material from midterm 2
  - Two from material from midterm 3
  - Two from material covered since midterm 3
  - **"Pick six"**: The top six contribute to your exam score
  - This gives a top exam score of 105 points (e.g. a 5 pt bonus)

Putting Some Things Together

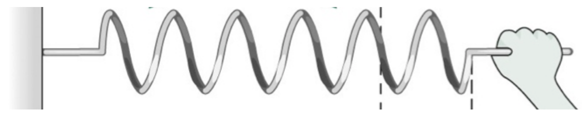
- Put **Newton's Second Law** together with **Springs**
  - And apply a bit of calculus and trig to find something new

$$F = ma = m\frac{d^2x}{dt^2}$$



Newton's 2^{nd} Law

$$F = -kx(t)$$

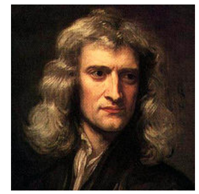


Springs

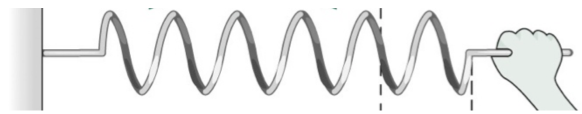
Putting Some Things Together

- Put **Newton's Second Law** together with **Springs**
  - And apply a bit of calculus and trig to find something new

$$
\begin{cases}
F = ma = m\dfrac{d^2x}{dt^2} \\
F = -kx(t)
\end{cases}
$$



Newton's 2nd Law



Springs

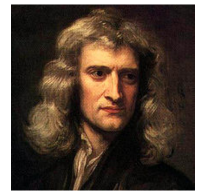
$$
\frac{d^2x(t)}{dt^2} = -\left(\frac{k}{m}\right)x(t)
$$

- What kind of function *x(t)* is a solution of this equation?

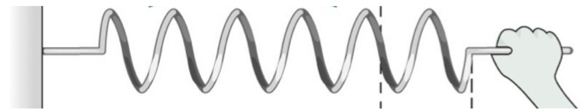
Putting Some Things Together

- Put **Newton's Second Law** together with **Springs**
  - And apply a bit of calculus and trig to find something new

$$\begin{cases} F = ma = m\dfrac{d^2x}{dt^2} \\ F = -kx(t) \end{cases}$$



Newton's 2^{nd} Law



Springs

$$\frac{d^2x(t)}{dt^2} = -\left(\frac{k}{m}\right)x(t)$$

- What kind of function *x(t)* is a solution of this equation?

$$x(t) = A\cos(\omega t + \phi_0) \qquad \text{or } x(t) = Ae^{i(\omega t+\phi_0)}$$

- Let's check it to be sure…

Checking Our Solution

$$\frac{d^2x(t)}{dt^2} = -\left(\frac{k}{m}\right)x(t)$$

$$x(t) = A\cos(\omega t + \phi_0)$$

$$\frac{dx}{dt} = -A\omega\sin(\omega t + \phi_0)$$

$$\frac{d^2x}{dt^2} = -A\omega\cos(\omega t + \phi_0) = -\omega^2\, x$$

Checking Our Solution

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- So things under the influence of spring forces move **periodically**, like an amplitude times a sine or cosine
  - Or some constant phase shift ϕ_0 from a sine or cosine wave
  - Here

$$\omega^2 = \left(\frac{k}{m}\right) \qquad \omega = \sqrt{\frac{k}{m}}$$

Checking Our Solution

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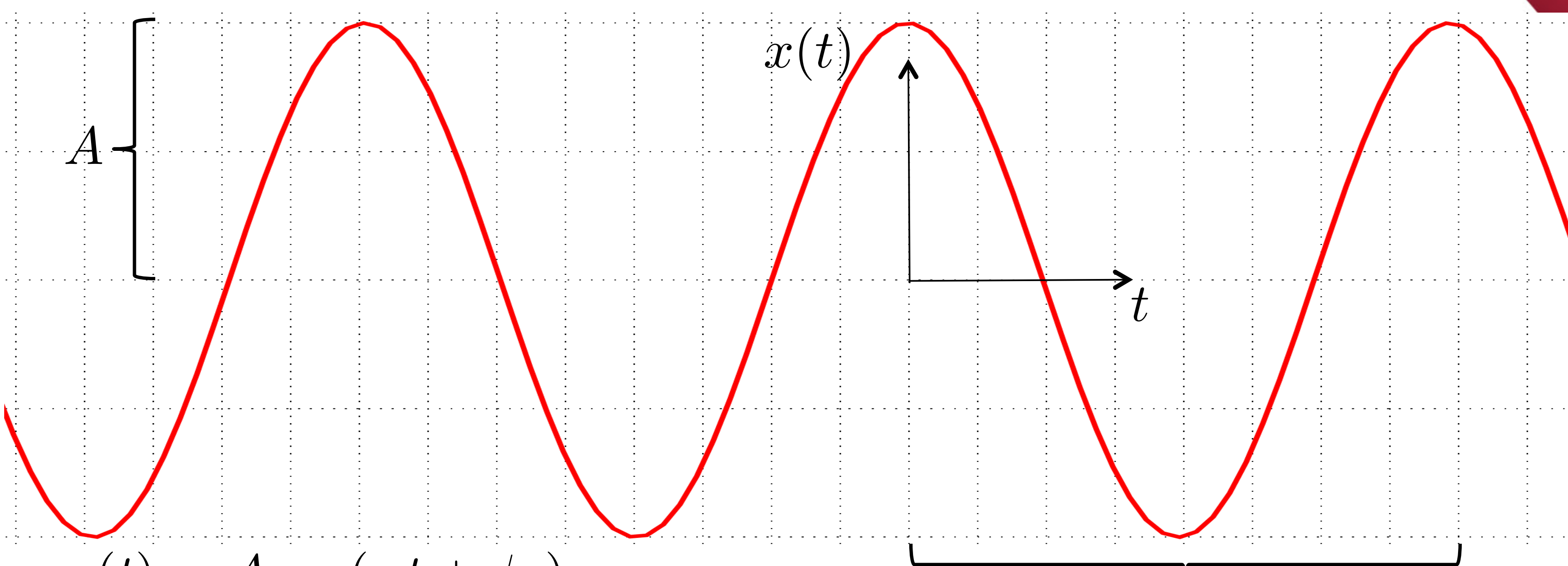
$$\omega^2 = \left(\frac{k}{m}\right) \qquad \boxed{\omega = \sqrt{\frac{k}{m}}}$$

Plotting Periodic Motion



$$x(t) = A\cos(\omega t + \phi_0)$$

$$\text{period } T = \frac{2\pi}{\omega} = \frac{1}{f}$$

- **Amplitude *A*:** oscillates between positions +/-A
- **Angular frequency ω:** how fast is oscillation, in radians/s
- **Frequency** of oscillation is $f = \omega/(2\pi)$ Units: Hz=1/s
- **Period** of the periodic motion is time *T* for one oscillation

Characteristics of SHM

- For a body of mass *m* vibrating by an ideal spring with a force constant *k*:

Angular frequency for simple harmonic motion

$$\omega = \sqrt{\frac{k}{m}}$$

Force constant of restoring force

Mass of object

Frequency for simple harmonic motion

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Angular frequency

Force constant of restoring force

Mass of object

Period for simple harmonic motion

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

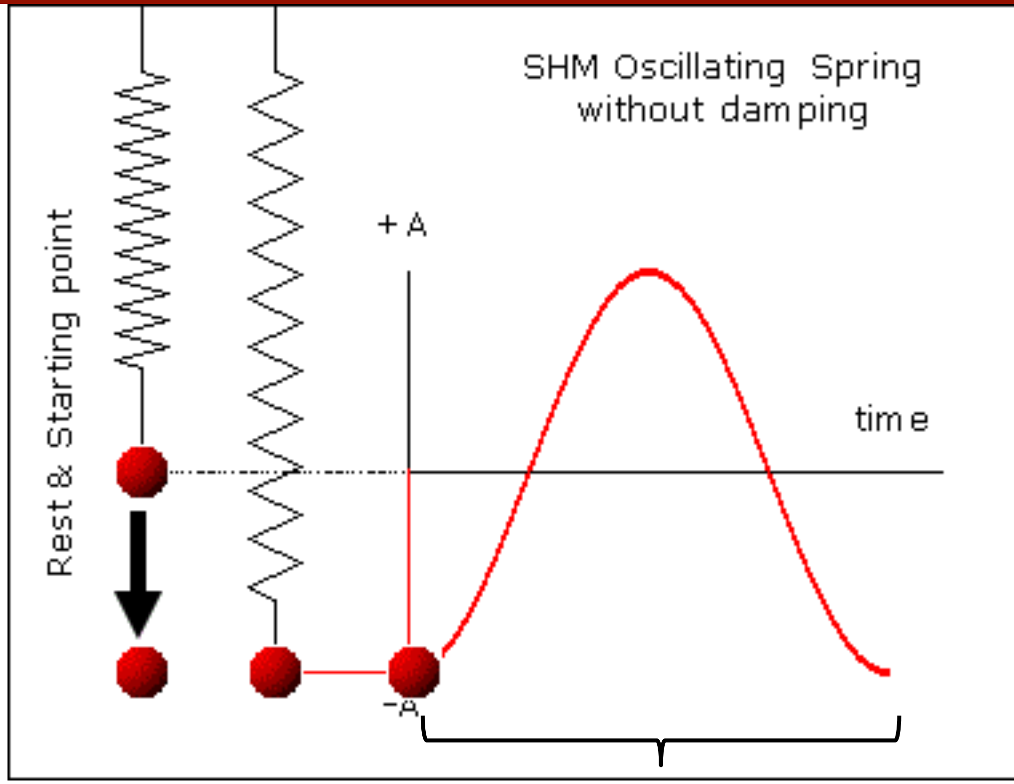
Mass of object

Force constant of restoring force

Frequency

Angular frequency

Simple Harmonic Motion

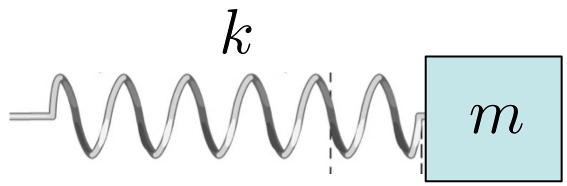


$$\text{period } T = \frac{2\pi}{\omega} = \frac{1}{f}$$

- Wave motion is best explained with visual aids
  - An excellent source of several animations is here:
    - http://www.animations.physics.unsw.edu.au/jw/oscillations.htm
  - I'll be showing some of them and talking through them

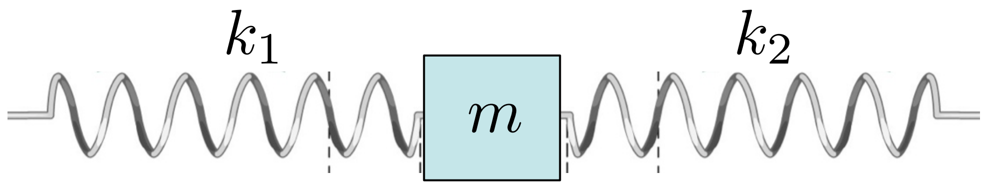
Multiple Springs

- One spring, one mass:



$$F = -kx$$

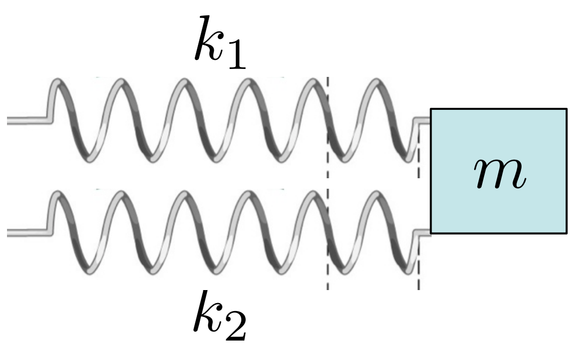
- Two springs, one mass:



$$F_1 = -k_1 x \qquad F_2 = -k_2 x$$

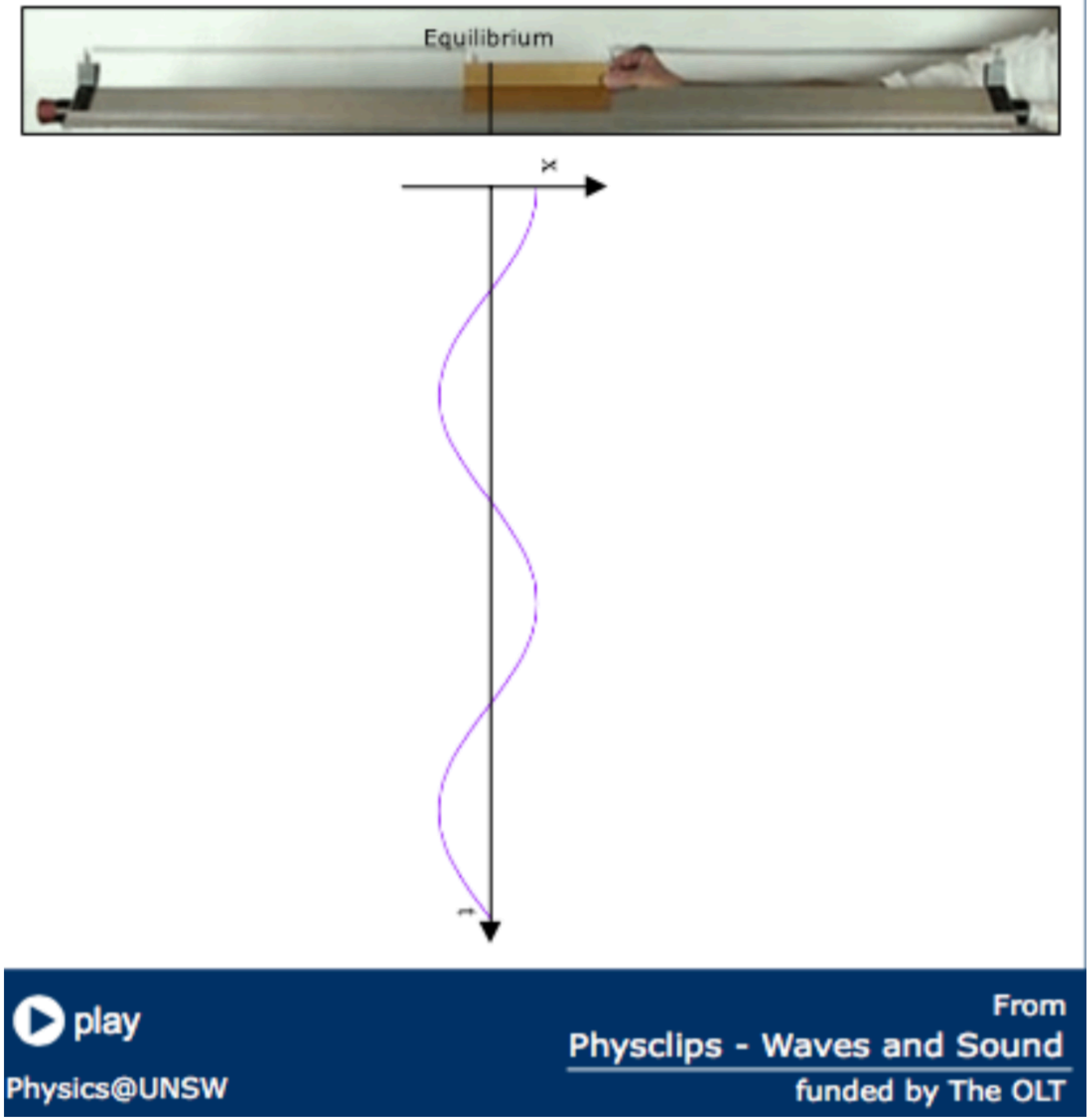
$$F_{\text{tot}} = -(k_1 + k_2)x$$

  - Acts like single spring of effective strength $k=k_1+k_2$
  - Doesn’t matter which side the springs are on. For example

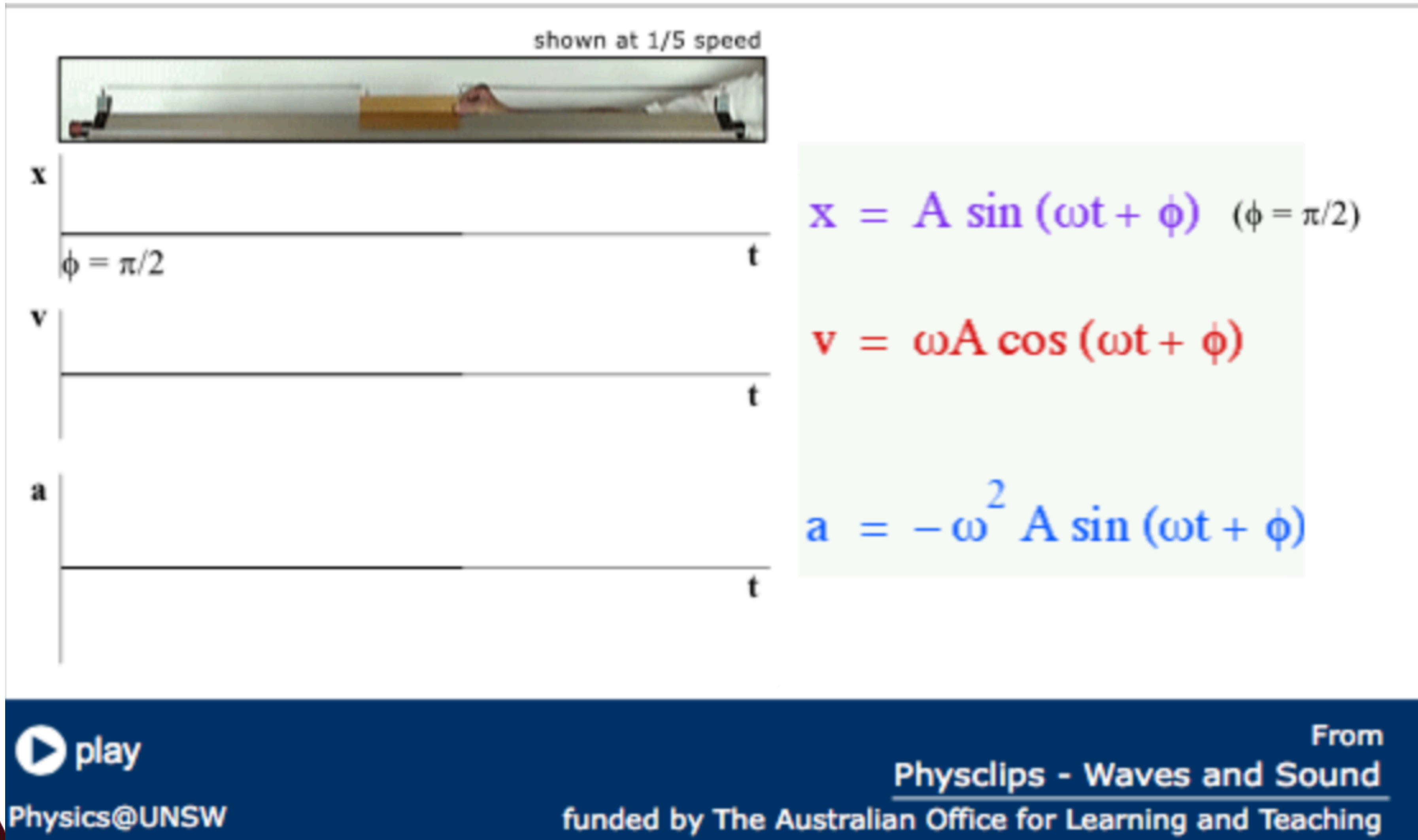


$$F_{\text{tot}} = -(k_1 + k_2)x$$

x(t) and the Mechanics of Simple Harmonic Motion

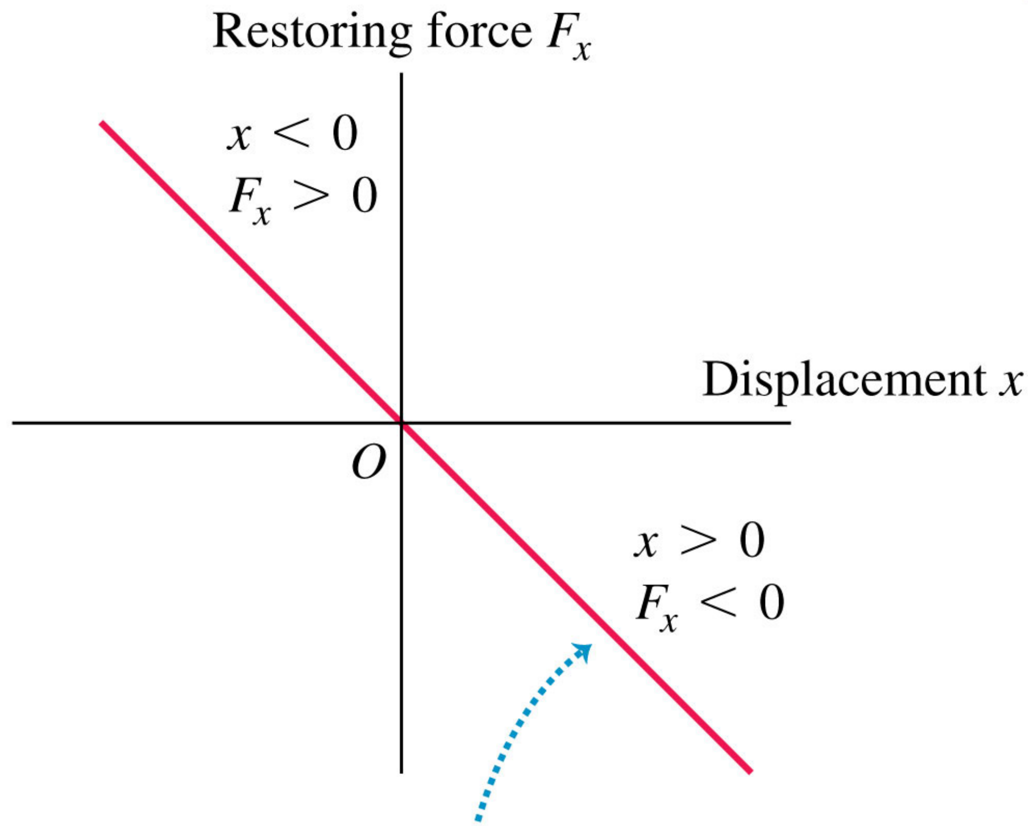


x(t), v(t), and a(t) for Simple Harmonic Motion



Simple Harmonic Motion (SHM)

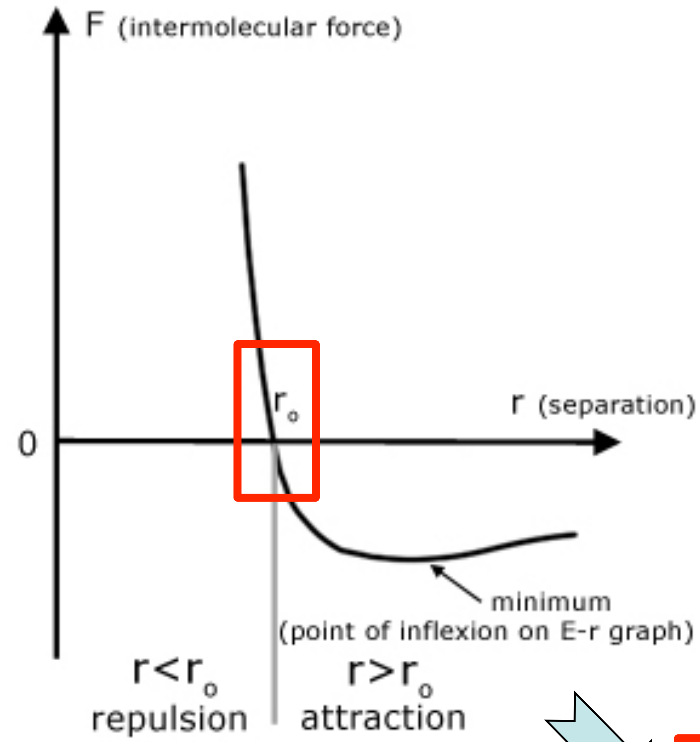
- When the restoring force is *directly proportional* to the displacement from equilibrium, the resulting motion is called **simple harmonic motion** (SHM).

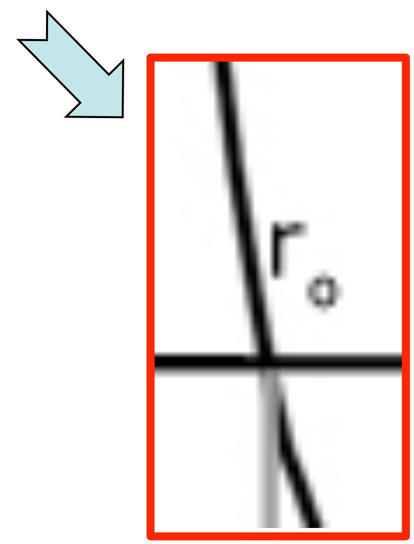


The restoring force exerted by an idealized spring is directly proportional to the displacement (Hooke's law, $F_x = -kx$): the graph of F_x versus x is a straight line.

Universality of Simple Harmonic Motion (SHM)

- Nearly **ANY physical force** looks pretty close to linear over a small enough variation of distance as it crosses zero
- Oscillations near that equilibrium are well described by SHM

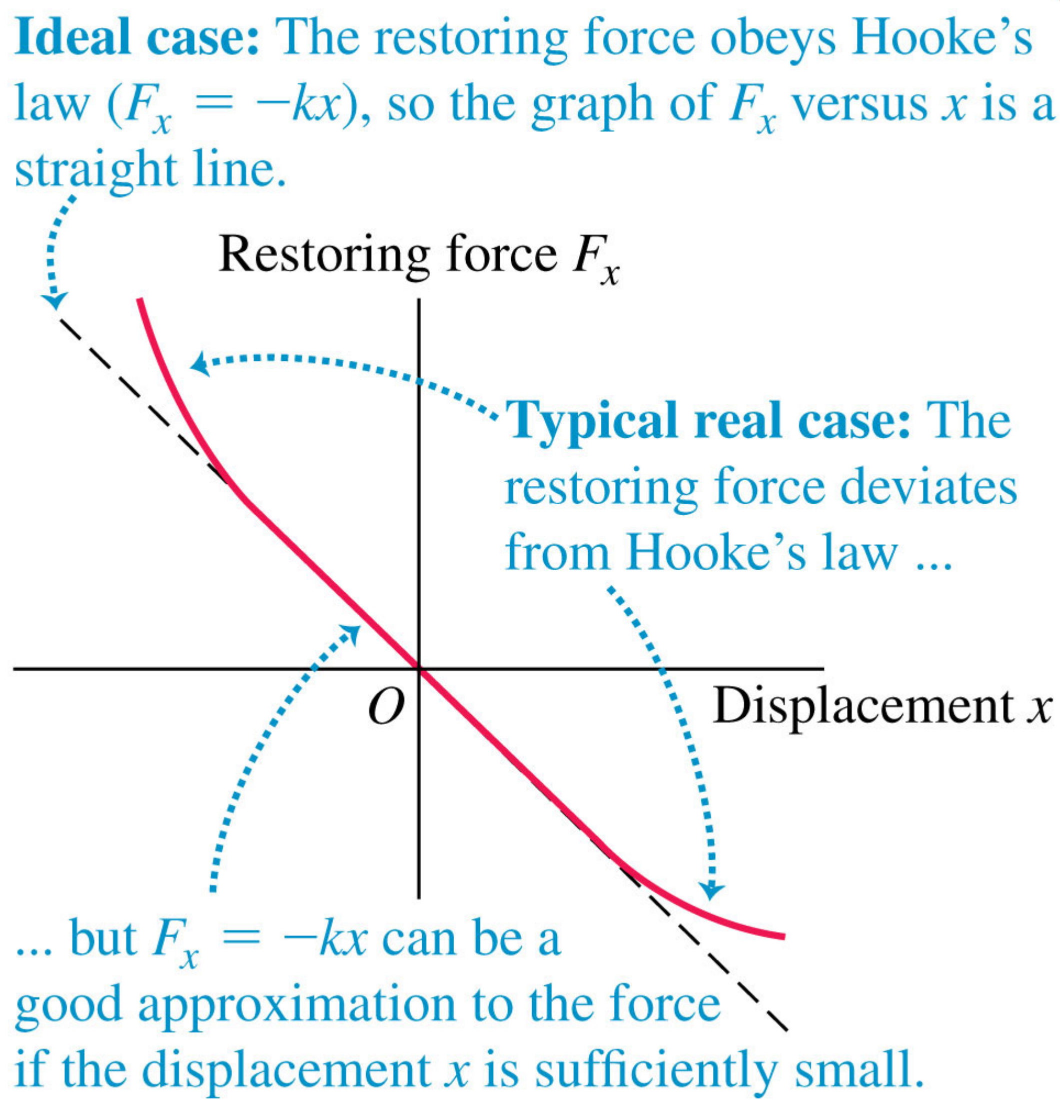




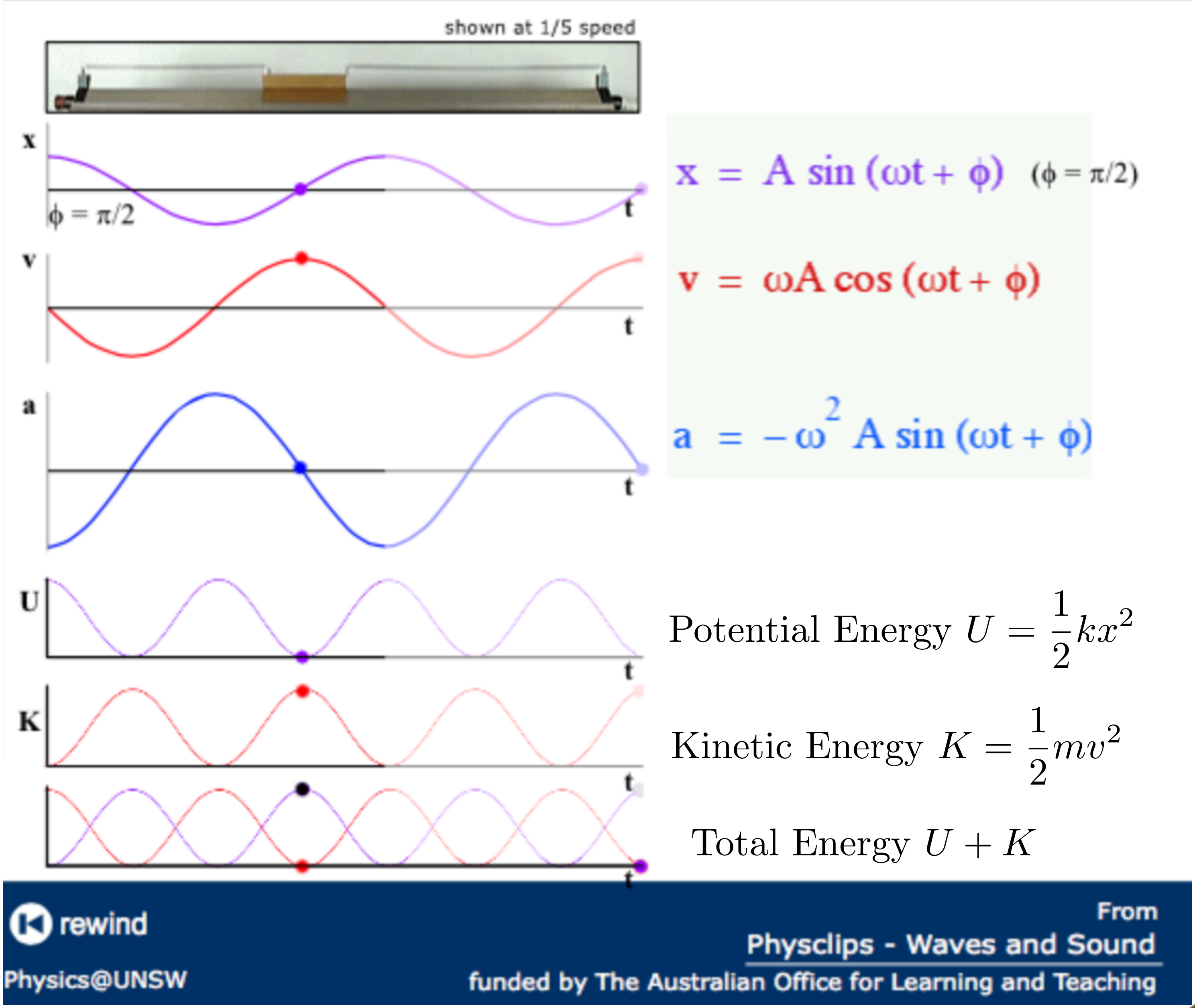
- Examples include
  - Pendulums
  - Tuning forks
  - Oscillating springs (in clocks)
  - Intramolecular forces in solids
  - Molecular vibrations
  - Electrical RC circuits

Simple Harmonic Motion (SHM)

- In many systems the restoring force is approximately proportional to displacement if the displacement is sufficiently small.
- That is, if the amplitude is small enough, the oscillations are approximately simple harmonic.



Energy in Simple Harmonic Motion



Potential Energy $U = \frac{1}{2}kx^2$

Kinetic Energy $K = \frac{1}{2}mv^2$

Total Energy $U + K$

Friction and SHMs

- The simple harmonic oscillator equation is

acceleration
$$\frac{d^2x(t)}{dt^2} = -\omega^2\, x(t)$$
F/m

- In practice, **frictional forces** are also present
  - These are usually proportional to velocity, *v=dx/dt*

$$F_{\text{friction}} = -bv = -b\,\frac{dx}{dt}$$

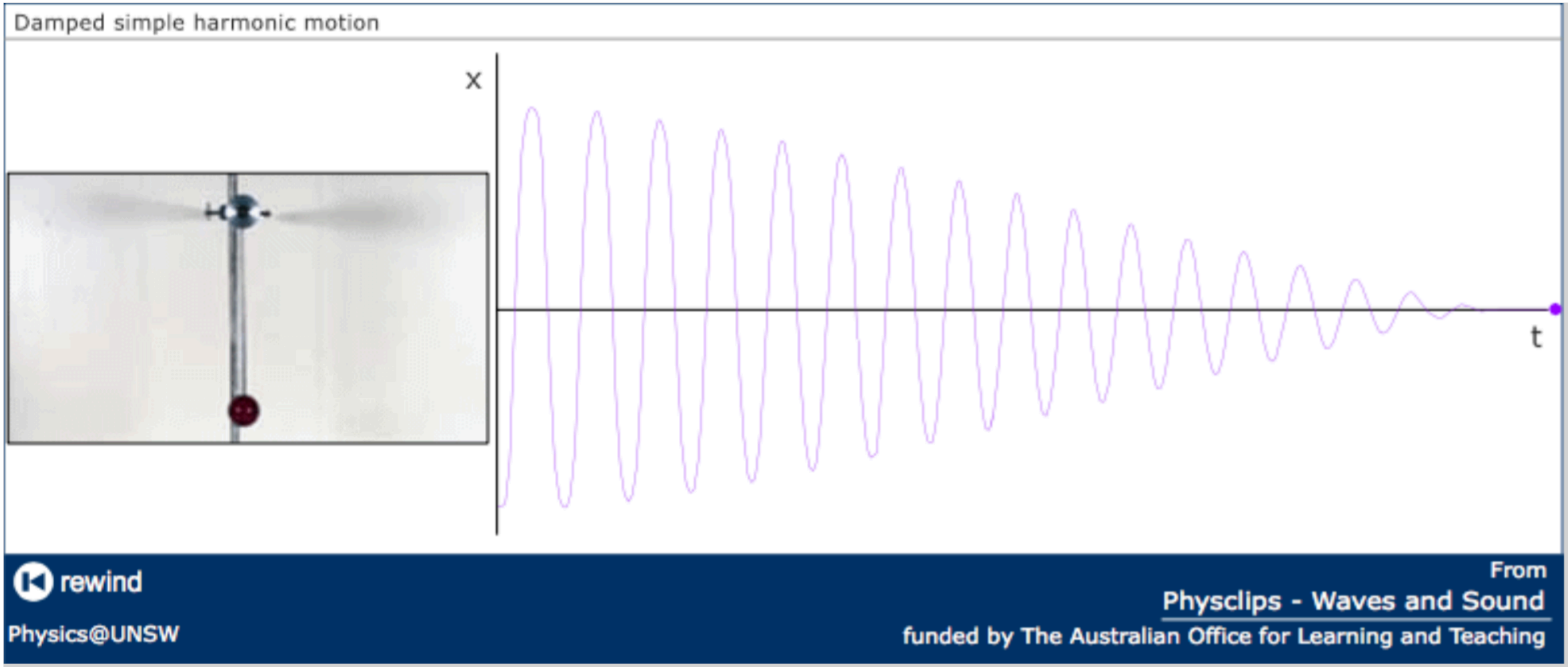
  - Adding this in as an extra force, with $\beta \equiv b/2m$, gives

acceleration
$$\boxed{\frac{d^2x(t)}{dt^2} = -\omega^2 x(t) - 2\beta\frac{dx(t)}{dt}}$$
F/m

  - Now the solution is not a simple sine or cosine.
    - You can verify this by taking derivatives and plugging in

$$\boxed{x(t) = A\,\mathrm{e}^{-\beta t}\,\cos(\omega_f t + \phi_0) \qquad \text{where } \omega_f^2 = \omega^2 - \beta^2}$$

Damped Oscillations



$$x(t) = A\,\mathrm{e}^{-\beta t}\,\cos(\omega_f t + \phi_0) \qquad \text{where } \omega_f^2 = \omega^2 - \beta^2$$

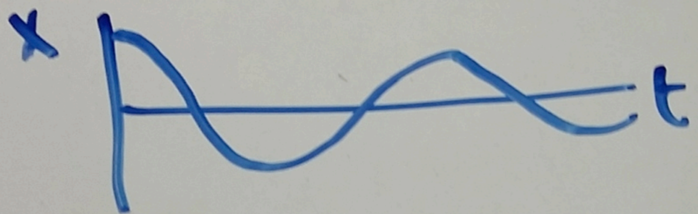
Oscillation amplitude decreases exponentially, depending on damping
Oscillation frequency is also slightly reduced compared to the case of the undamped "free" oscillator
Depending on β, the oscillation can be **overdamped for underdamped**

Under- and Overdamped Oscillations

Damped SHM: $\omega_d^2 = \omega^2 - \beta^2$ $\beta \equiv \frac{b}{2m}$ (2)

$\omega^2 \equiv \frac{k}{m}$

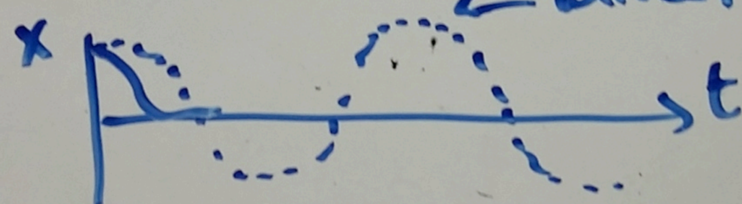
$\beta \ll \omega$: underdamped



→ Takes many oscillations to die out

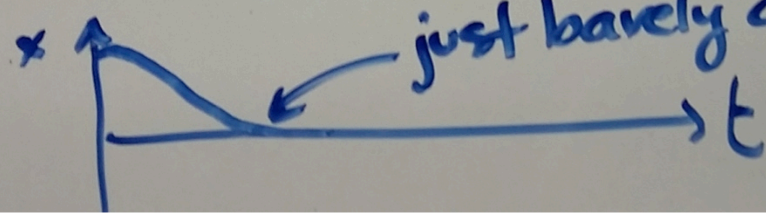
$\beta \gg \omega$: overdamped

← without damping



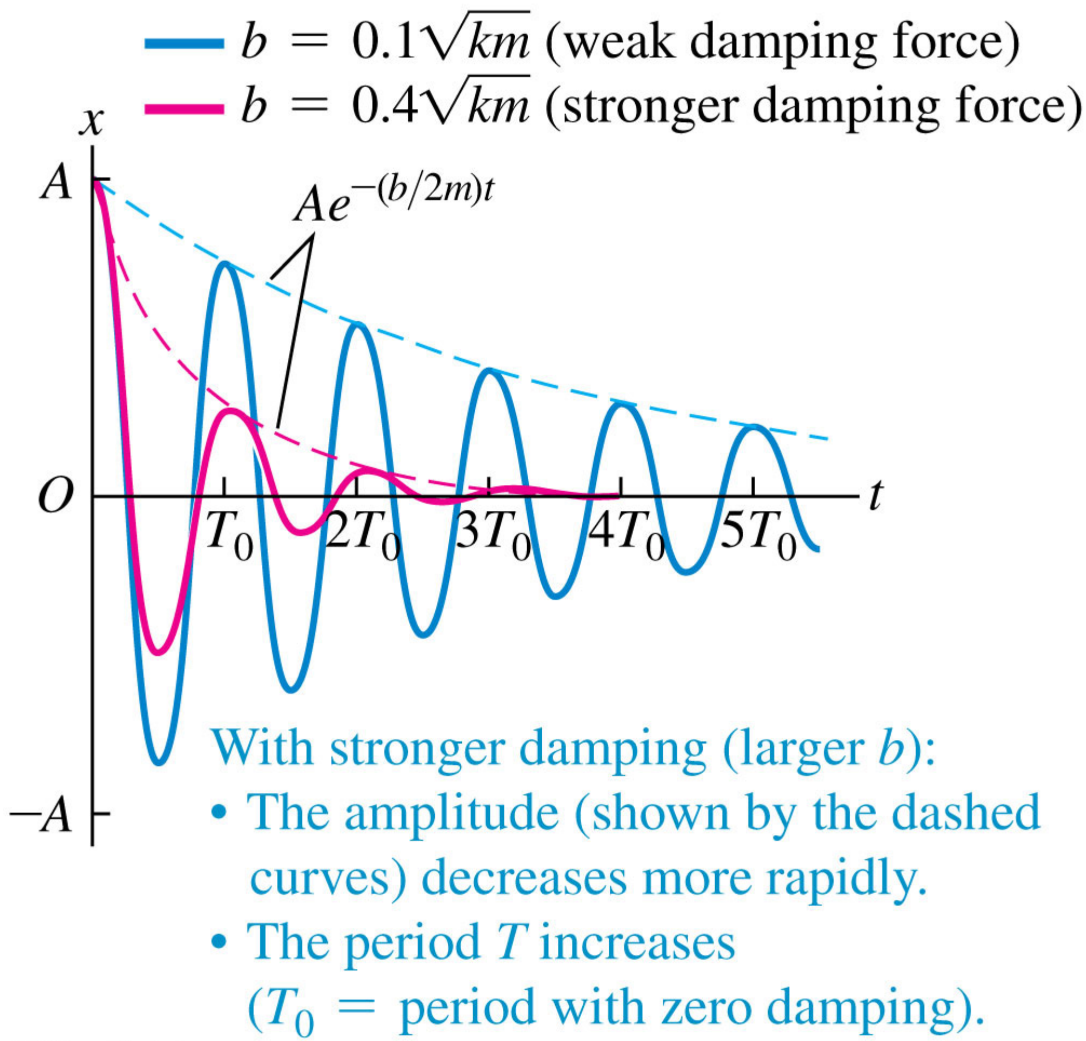
$\beta = \omega$: critically damped

just barely dont overshoot



Damped Oscillations

- Real-world systems have some dissipative forces that decrease the amplitude.
- The decrease in amplitude is called **damping** and the motion is called **damped oscillation**.



Forced oscillations and resonance

- A damped oscillator left to itself will eventually stop moving.
- But we can maintain a constant-amplitude oscillation by applying a force that varies with time in a periodic way.
- We call this additional force a **driving force**.
- If we apply a periodic driving force with angular frequency ω_d to a damped harmonic oscillator, the motion that results is called a forced oscillation or a driven oscillation.

Maximum value of driving force

Amplitude of a forced oscillator

$$A = \frac{F_{max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$$

Damping constant

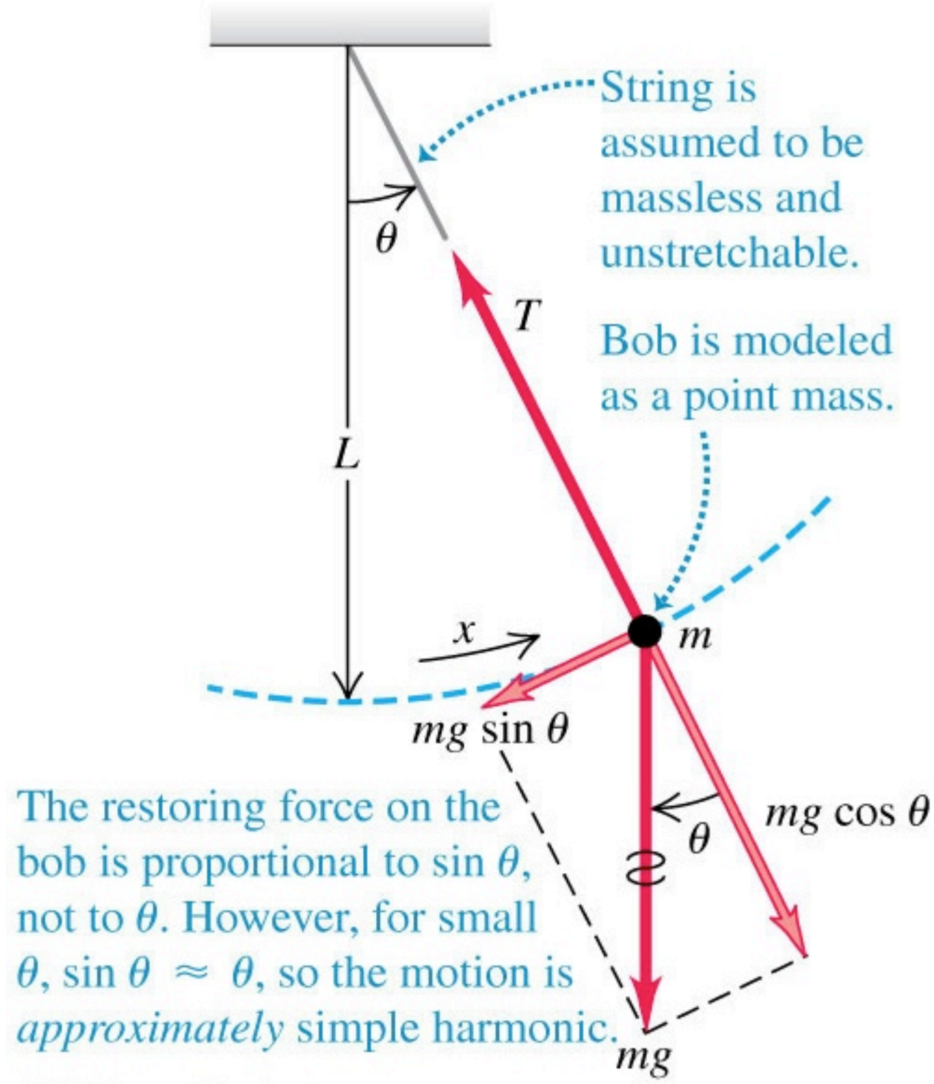
Force constant of restoring force

Mass

Driving angular frequency

The simple pendulum

- A **simple pendulum** consists of a point mass (the bob) suspended by a massless, unstretchable string.
- If the pendulum swings with a small amplitude θ with the vertical, its motion is simple harmonic.



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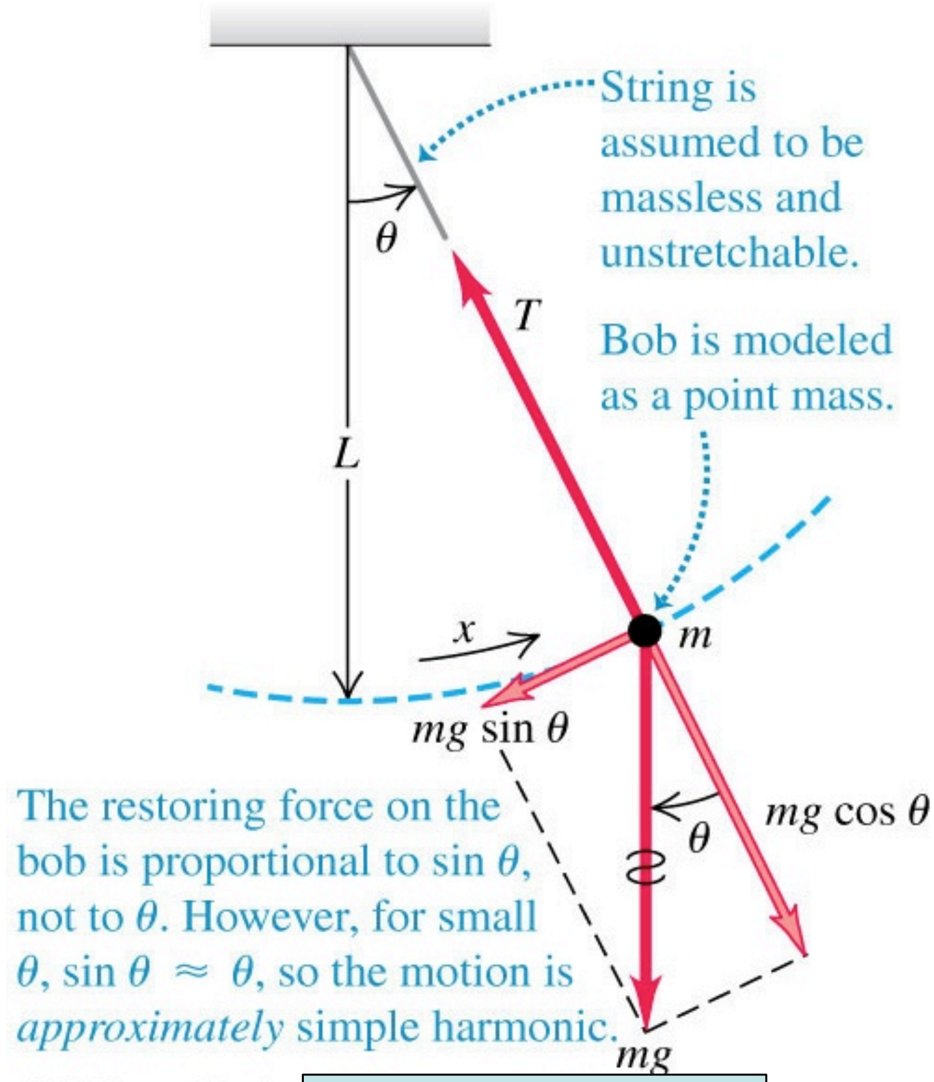


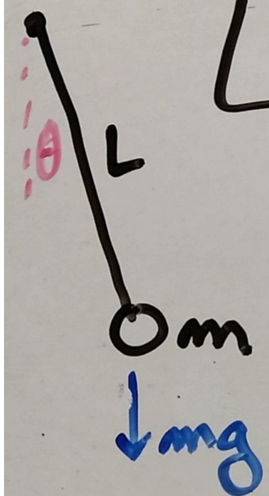
$$\tau = I\alpha = I\frac{d^2\theta}{dt^2}$$

$$I = mL^2 \quad \tau = Fr\sin\theta = -(mg)(L)\sin\theta$$

$$\frac{d^2\theta}{dt^2} = -\left(\frac{g}{L}\right)\sin\theta \approx -\left(\frac{g}{L}\right)\theta \quad \text{for small } \theta$$

$$\omega^2 = \left(\frac{g}{L}\right) \quad \Rightarrow \quad \boxed{\omega = 2\pi f = \sqrt{\frac{g}{L}}}$$

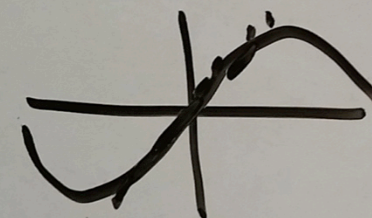
Simple Pendulum on the White Board



$\boxed{\tau = I\alpha}$ $\quad \alpha = \frac{d^2\theta}{dt^2}$ (like $a = \frac{d^2x}{dt^2}$) $F = ma$

$I = \sum_i m_i r_i^2 = mL^2$

$\tau = -rF\sin\theta = -L(mg)\sin\theta$ makes sense



For $\theta \ll 1\,\text{rad}$ $\sin\theta \sim \theta$

$\Rightarrow \tau \sim -L\,mg\,\theta$

$-\cancel{L}\,\cancel{m}g\theta = \cancel{m}L^{\cancel{2}}\frac{d^2\theta}{dt^2} \Rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\left(\frac{g}{L}\right)\theta}$

$\boxed{\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{g}{L}}}$ independent of m!

Example

- You want to build a grandfather clock with a pendulum that swings through one cycle in two seconds. How long does the pendulum need to be?

$$\boxed{\omega = 2\pi f = \sqrt{\frac{g}{L}}}$$

Example

- You want to build a grandfather clock with a pendulum that swings through one cycle in two seconds. How long does the pendulum need to be?

$$\boxed{\omega = 2\pi f = \sqrt{\frac{g}{L}}}$$

$$f = 1/(2 \text{ seconds}) = 0.5 \text{ Hz}$$

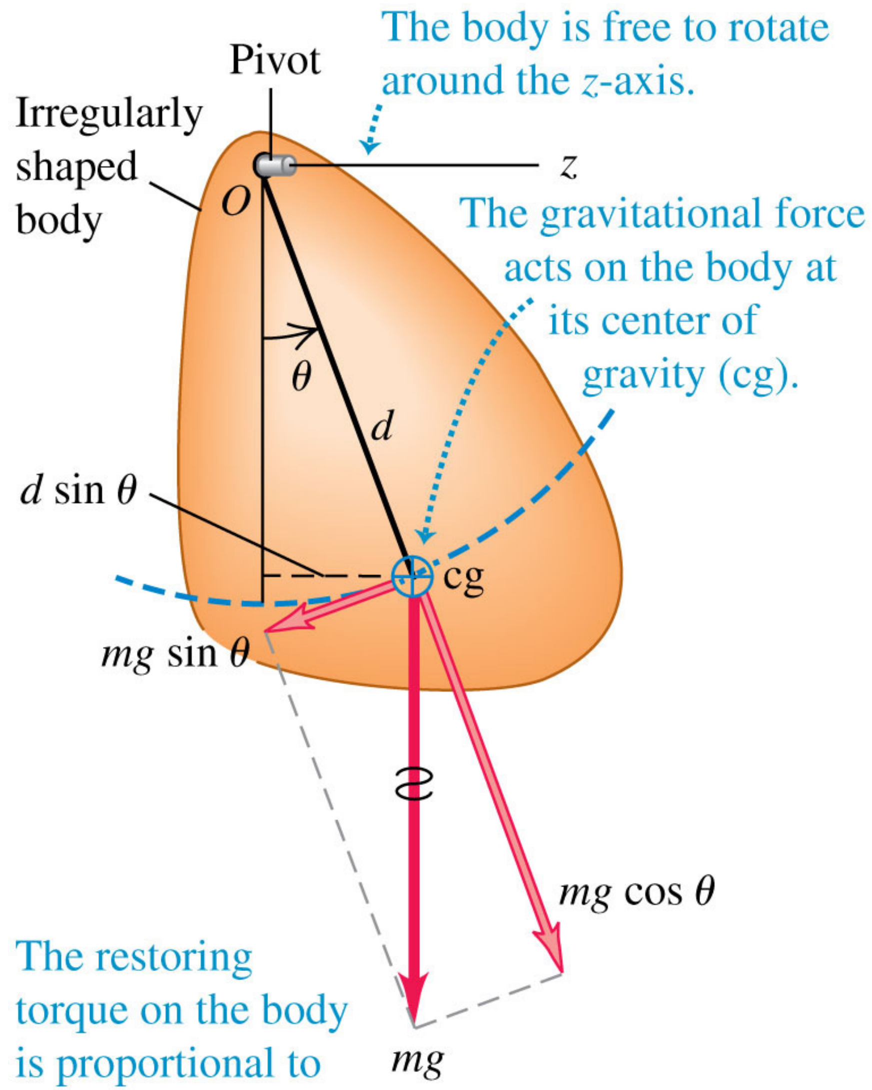
$$L = \frac{g}{(2\pi f)^2} = \frac{(9.8 \text{ m/s}^2)}{(2\pi(0.5/\text{second}))^2} = 0.993 \text{ m}$$

This is known as a seconds pendulum:
https://en.wikipedia.org/wiki/Seconds_pendulum

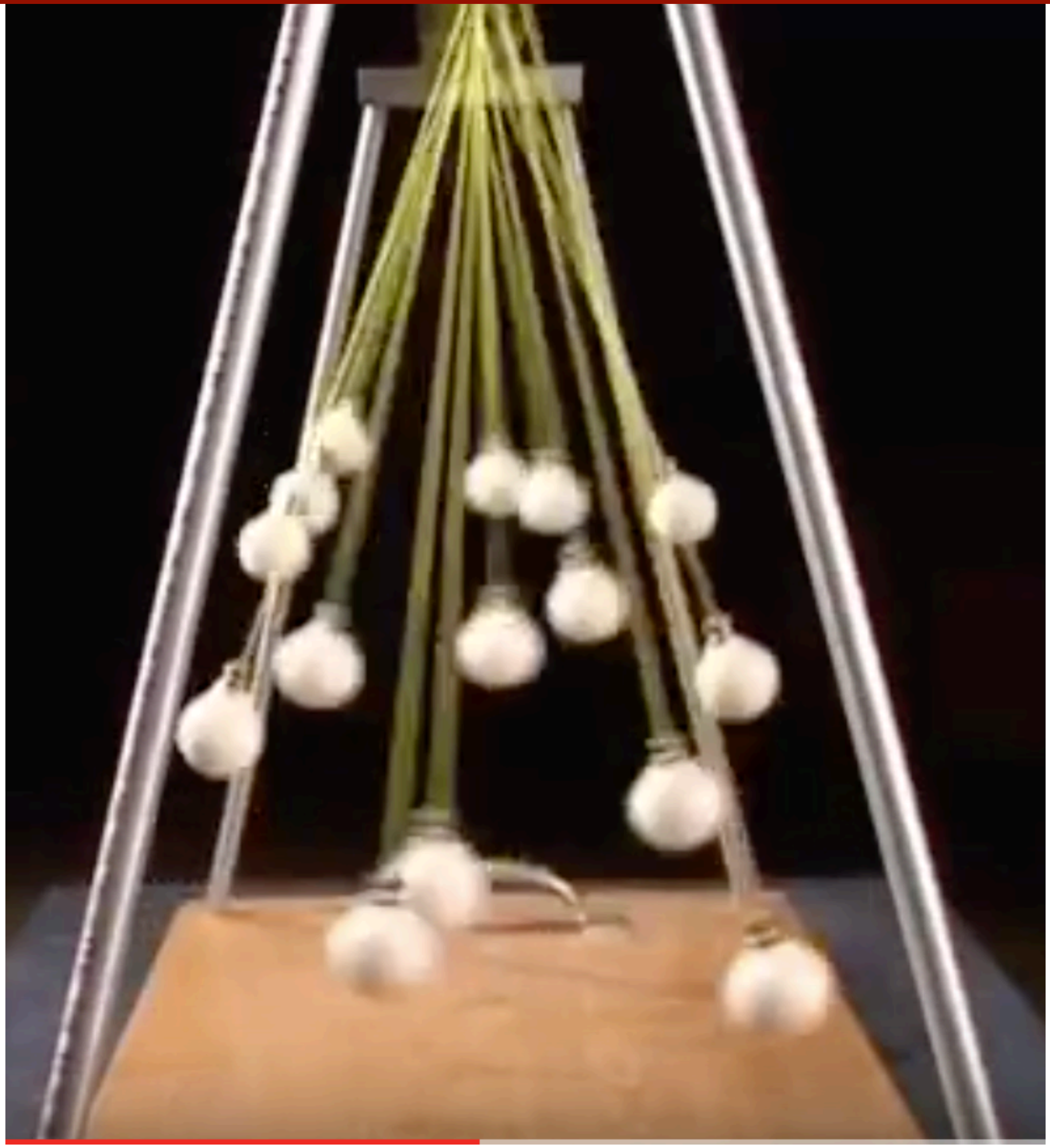
And this was the standard unit of length for a while!

The physical pendulum

- A **physical pendulum** is any real pendulum that uses an extended body instead of a point-mass bob.
- For small amplitudes, its motion is simple harmonic.
- The length L is effectively the length to the center of mass of the object
    - So for example, a meter stick swings with the frequency of a pendulum of about length 0.5m



Pendulum Waves Video



https://www.youtube.com/watch?v=yVkdfJ9PkRQ