USPAS Accelerator Physics 2017 University of California, Davis

Lattice Extras: Linear Errors, Doglegs, Chicanes, Achromatic Conditions, Emittance Exchange

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Review

Hill's equation x'' + K(s)x = 0quasi – periodic ansatz solution $x(s) = A\sqrt{\beta(s)}\cos[\Psi(s) + \Psi_0]$

$$\beta(s) = \beta(s+C) \quad \gamma(s) \equiv \frac{1+\alpha(s)^2}{\beta(s)}$$
$$\alpha(s) \equiv -\frac{1}{2}\beta'(s) \quad \Psi(s) = \int \frac{ds}{\beta(s)}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_0+C} = \begin{pmatrix} \cos \mu + \alpha(0) \sin \mu & \beta(0) \sin \mu \\ -\gamma(0) \sin \mu & \cos \mu - \alpha(0) \sin \mu \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0}$$

betatron phase advance

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$$\mu = \int_{s_0}^{s_0 + C} \frac{ds}{\beta(s)}$$

$$\operatorname{Tr} M = 2\cos\mu$$

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$$M = I\cos\mu + J\sin\mu \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad J(s_0) \equiv \begin{pmatrix} \alpha(0) & \beta(0) \\ -\gamma(0) & -\alpha(0) \end{pmatrix}$$

 $J^2 = -I \quad \Rightarrow \quad M = e^{J(s)\mu}$

General Non-Periodic Transport Matrix

• We can parameterize a general non-periodic transport matrix from s₁ to s₂ using lattice parameters and $\Delta \Psi = \Psi(s_2) - \Psi(s_1)$

$$M_{s_1 \to s_2} = \begin{pmatrix} \sqrt{\frac{\beta(s_2)}{\beta(s_1)}} [\cos \Delta \Psi + \alpha(s_1) \sin \Delta \Psi] & \sqrt{\beta(s_1)\beta(s_2)} \sin \Delta \Psi \\ -\frac{[\alpha(s_2) - \alpha(s_1)] \cos \Delta \Psi + [1 + \alpha(s_1)\alpha(s_2)] \sin \Delta \Psi}{\sqrt{\beta(s_1)\beta(s_2)}} & \sqrt{\frac{\beta(s_1)}{\beta(s_2)}} [\cos \Delta \Psi - \alpha(s_2) \sin \Delta \Psi] \end{pmatrix}_{(C\&M \text{ Eqn } 5.52)}$$

• This does not have a pretty form like the periodic matrix However both can be expressed as $M = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix}$

where the C and S terms are cosine-like and sine-like; the second row is the s-derivative of the first row!

A common use of this matrix is the m_{12} term:

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$$\Delta x(s_2) = \sqrt{\beta(s_1)\beta(s_2)} \sin(\Delta \Psi) x'(s_1)$$

Effect of angle kick on downstream position

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General phase advance, NOT phase advance/cell $\boldsymbol{\mu}$

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- Sometimes need a local change $\Delta x(s)$ to the design orbit
 - But we really only get changes in angle $\Delta x'$ from magnets
 - e.g. small dipole "corrector": $\Delta x' = B_{corrector} L_{corrector} / (B\rho)$
 - Changes to/corrections of design orbit from dipole correctors
 - Linear errors add up via linear superposition

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$$\begin{pmatrix} \Delta x(s_2) \\ \Delta x'(s_2) \end{pmatrix} = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} \begin{pmatrix} 0 \\ \Delta x'(s_1) \end{pmatrix}$$
$$\Delta x(s_2) = \Delta x'(s_1) \sqrt{\beta(s_1)\beta(s_2)} \sin \Delta \Psi$$
$$\Delta x'(s_2) = \Delta x'(s_1) \sqrt{\frac{\beta(s_1)}{\beta(s_2)}} [\cos \Delta \Psi - \alpha(s_2) \sin \Delta \Psi]$$



Checking Optics with Difference Orbits



- Steering changes through linear elements are linear
 - Compare calculated optical transport of $\Delta x'$ to measured
 - Can localize focusing errors with enough BPMs

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- Above picture is for single-pass AGS to RHIC transfer line
- Strong nonlinearities introduce dependence on initial orbits

Physics of the AGS to RHIC Transfer Line Commissioning, T. Satogata, W.W. MacKay, et al, EPAC'96

Checking Optics with Difference Orbits



- PEP-II vertical orbit during commissioning
 - Projected vertical betatron oscillation in near-FODO lattice
 - Discrepancy starts at s~700m

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- Later found two quadrupole pairs had ~0.1% errors
- Can clearly discriminate systematic optics errors from random BPM errors

Y. Cai, 1998, from M. Minty and F. Zimmermann, "Beam Techniques: Beam Control and Manipulation"

Control: Two-Bump



- A single orbit error changes all later positions and angles
 - Add another dipole corrector at a location where $\Delta \Psi = k\pi$ At this point the distortion from the original dipole corrector is all x' that we can cancel with the second dipole corrector.

$$\Delta x'(s_2) = \Delta x'(s_1) \sqrt{\frac{\beta(s_1)}{\beta(s_2)}} + \text{angle from } s_2 \text{ dipole}$$

- Called a two-bump: localized orbit distortion from two correctors
- But requires $\Delta \Psi = k\pi$ between correctors

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- A general local orbit distortion from three dipole correctors
 - Constraint is that net orbit change from sum of all three kicks must be zero

$$\begin{pmatrix} C_2 & S_2 \\ C'_2 & S'_2 \end{pmatrix} \begin{bmatrix} \begin{pmatrix} C_1 & S_1 \\ C'_1 & S'_1 \end{pmatrix} \begin{pmatrix} 0 \\ \Delta x'_1 \end{pmatrix} + \begin{pmatrix} 0 \\ \Delta x'_2 \end{pmatrix} \end{bmatrix} + \begin{pmatrix} 0 \\ \Delta x'_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$\Delta x'_1 = \frac{x_b}{S_1} \quad \Delta x'_2 = -\frac{C_2 S_1 + S_2 S'_1}{S_1 S_2} x_b \quad \Delta x'_3 = \frac{S_2}{S_1^2} x_b$$

• Bump amplitude $x_b = S_1 \Delta x'_1$

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• Only **three-bump** requirement is that $S_1, S_2 \neq 0$



Steering Error in Synchrotron Ring

- Short steering error $\Delta x'$ in a ring with periodic matrix M
 - Solve for new periodic solution or design orbit (x₀,x'₀)

$$M\begin{pmatrix}x_0\\x'_0\end{pmatrix} + \begin{pmatrix}0\\\Delta x'\end{pmatrix} = \begin{pmatrix}x_0\\x'_0\end{pmatrix}$$

Note that (x₀=0,x'₀=0) is not the periodic solution any more!

$$\begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} = (I - M)^{-1} \begin{pmatrix} 0 \\ \Delta x'_0 \end{pmatrix}$$

$$(I - M)^{-1} = (I - e^{(2\pi Q)J})^{-1} = \left([e^{\pi QJ} (e^{-\pi QJ} - e^{\pi QJ})]^{-1} \\ = -(2J\sin(\pi Q))^{-1} (e^{\pi QJ})^{-1} \\ = \frac{1}{2\sin(\pi Q)} (J\cos(\pi Q) + I\sin(\pi Q))$$
New closed orbit
$$x_0 = \frac{\beta_0 \Delta x'_0}{2} \tan(\pi Q) \implies \infty \text{ if } Q = k\pi \\ \text{ integer resonances} \\ x'_0 = \frac{\Delta x'_0}{2} [1 - \alpha_0 \cot(\pi Q)]$$
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Steering Error in Synchrotron Ring

 We can use the general propagation matrix to find the new closed orbit displacement at all locations around the synchrotron

$$x(s) = \frac{\Delta x'_0 \sqrt{\beta_0 \beta(s)}}{2\sin(\pi Q)} \cos[\Delta \Psi - \pi Q]$$

- This displacement of the closed orbit changes its path length
- If the revolution (RF) frequency is constant, then the beam energy changes, and there is an extra (small) term in the closed orbit displacement

$$x(s) = \frac{\Delta x_0' \sqrt{\beta_0 \beta(s)}}{2\sin(\pi Q)} \cos[\Delta \Psi - \pi Q] + \Delta x_0' \frac{\eta_0 \eta(s)}{\alpha_p C}$$

where $\alpha_p \equiv \left(\frac{dL}{L} / \frac{dp}{p}\right)$ is the momentum compaction and C is the accelerator circumference.

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 Measuring many difference orbits in a ring gives a ton of information about optics, BPM and corrector errors

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- Singular value decomposition (SVD) optimization of error fits
- Has become standard method for correction of synchrotron lattices, particularly synchrotron light sources, led to other methods
 - e.g. model independent analysis (MIA), independent component analysis (ICA)

J. Safranek, "Experimental Determination of Storage Ring Optics using Orbit Response Measurements", 1997

Focusing Error in Synchrotron Ring

- Short focusing error in a ring with periodic matrix M
 - Now solve for Tr M to find effects on tune Q

$$M_{\text{new}} = M \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$
$$\frac{1}{2} \text{Tr } M = \cos(2\pi Q_{\text{new}}) = \cos(2\pi Q_0) - \frac{1}{2} \frac{\beta_0}{f} \sin(2\pi Q_0)$$

• For small errors $Q_{\text{new}} = Q_0 + \Delta Q$ we can expand to find

$$\Delta Q \approx \frac{1}{4\pi} \frac{\beta_0}{f}$$
Can be used for a simple measurement of β_0 at the quadrupole 2°

 Quadrupole errors also cause resonances when Q = k/2: half-integer resonances

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Chromaticity Correction

Natural chromaticity $\xi_N \equiv \left(\frac{\Delta Q}{Q}\right) / \left(\frac{\Delta p}{p_0}\right) = -\frac{1}{4\pi Q} \oint K(s)\beta(s) \, ds$

- How can we control chromaticity in our synchrotron ring?
 - We need a way to connect momentum offset δ to focusing
 - Dispersion (momentum-dependent position) and sextupoles (nonlinear focusing depending on position) come to rescue

 $x(s) = x_{\text{betatron}}(s) + \eta_x(s)\delta$

Sextupole B field $B_y = b_2 x^2$ $B_y(\text{sext}) = b_2 [x_{\text{betatron}}(s) + \eta_x(s)\delta]^2 \approx b_2 x_{\text{betatron}}^2 + 2b_2 x_{\text{betatron}}(s)\eta_x(s)\delta$ Nonlinear! like a guadrupole K(s)!

Total chromaticity from all sources is then

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$$\xi = -\frac{1}{4\pi Q} \oint [K(s) - b_2(s)\eta_x(s)]ds$$

Strong focusing (large K) requires large sextupoles, nonlinearity!





- Displaces beam transversely without changing direction
- What is effect on 6D optics?

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$$\mathbf{M}_{\text{dipole}}(\rho,\theta) = \begin{pmatrix} \cos\theta & \rho\sin\theta & 0 & 0 & 0 & \rho(1-\cos\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & 0 & 0 & 0 & \sin\theta \\ 0 & 0 & 1 & \rho\theta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\sin\theta & -\rho(1-\cos\theta) & 0 & 0 & 1 & L/(\gamma^2\beta^2) - \rho(\theta-\sin\theta) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- Be careful about the coordinate system and signs!!
- If $\rho, \theta > 0$, positive displacement points **out** from dipole curvature
- Be careful about order of matrix multiplication!



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(C&M 3.102)

Reverse Bend Dipole Transport

- What is the correct 6x6 transport matrix of a reverse bend dipole?
- It turns out to be achieved by reversing both ρ and θ
 - $\rho\theta=L$ (which stays positive) so both must change sign

$$\mathbf{M}_{\text{dipole}}(-\rho,-\theta) = \begin{pmatrix} \cos\theta & \rho\sin\theta & 0 & 0 & 0 & -\rho(1-\cos\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \rho\theta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{\sin\theta}{0} & \rho(1-\cos\theta) & 0 & 0 & 1 & L/(\gamma^2\beta^2) - \rho(\theta-\sin\theta) \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \end{pmatrix}$$
$$M_{\text{drift}} = \begin{pmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & L/(\gamma^2\beta^2) \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

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Aside: Longitudinal Phase Space Drift

- Wait, what was that M₅₆ term with the relativistic effects?
 - Recall longitudinal coordinates are (z, δ)
 - This extra term is called "ballistic drift": not in all codes!
 - Important at low to modest energies and for bunch compression
 - Relativistic terms enter converting momentum p to velocity v





 $\mathbf{M}_{\text{dogleg}} = \mathbf{M}_{\text{dipole}}(\rho, \theta) \mathbf{M}_{\text{drift}} \mathbf{M}_{\text{dipole}}(-\rho, -\theta)$

$$\mathbf{M}_{\text{weak dogleg}} = \begin{pmatrix} 1 & \rho\theta & \frac{\rho\theta^2}{2} \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \rho\theta & \frac{\rho\theta^2}{2} \\ 0 & 1 & -\theta \\ 0 & 0 & 1 \end{pmatrix} \\ = \begin{pmatrix} 1 & L + 2\rho\theta & -L\theta \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\eta, \eta')_{\text{in}} = 0 \\ \Rightarrow & (\eta, \eta')_{\text{out}} = (-L\theta, 0)$$

Strong dogleg can also be derived:

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$$D' = \frac{L\sin^2\theta}{\rho}$$

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 $D = -L\cos\theta\sin\theta$

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A small momentum offset of $+\delta$ reduces the dipole kick by a factor of delta, and this is magnified to a transverse offset from design at the end of the dogleg by $-\delta L\theta$.

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Achromatic Dogleg

- How can we make an achromatic dogleg? $(\eta, \eta')_{in} = (0 \text{ m}, 0) \implies (\eta, \eta')_{out} = (0 \text{ m}, 0)$
- Use an I insertion (e.g. four consecutive $\pi/2$ insertions)

$$\mathbf{M}_{\pi/2} = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix} = \mathbf{J} \quad (\text{Recall } \mathbf{J}^4 = \mathbf{I})$$

$$\mathbf{M}_{\text{achromatic dogleg}} = \begin{pmatrix} \cos(2\theta) & \rho \sin(2\theta) & 0\\ -\frac{\sin(2\theta)}{\rho} & \cos(2\theta) & 0\\ 0 & 0 & 1 \end{pmatrix} \quad \text{achromatic!}$$

• Any transport with net phase advance of $2n\pi$ will be achromatic ($n\pi$ if all dipoles bend in same direction)

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• common trick for matching dispersive bending arcs to nondispersive straight sections.





Achromatic Dogleg: Steffen CERN School Notes

Example of nondispersive translating system

- Φ = sector magnet bend. angle
- $\varphi = \ell \sqrt{k} = quadrupole magnet phase angle$
- $d_{\lambda} = drift space lengths.$

The system is nondispersive if the sinelike trajectory (with respect to the central symmetry point) goes through the mid-point of the bending magnets, i.e. if

$$\rho \tan \frac{\Phi}{2} + \lambda = \frac{1}{\sqrt{k}} \frac{d\sqrt{k}\cos\varphi + 2\sin\varphi}{d\sqrt{k}\sin\varphi - 2\cos\varphi}.$$

Focusing also in the other plane may be obtained by adding a third quadrupole of opposite polarity at the symmetry point.

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Fig. 15: Nondispersive translating system.

K. Steffen, CERN-85-19-V-1, 1985, p. 55



First-Order Achromat Theorem

 A lattice of n repetitive cells is achromatic (to first order, or in the linear approximation) iff Mⁿ = I or each cell is achromatic

• Proof:
Consider
$$\mathbf{R} \equiv \begin{pmatrix} \mathbf{M} & \bar{d} \\ 0 & 1 \end{pmatrix}$$
 where $\begin{pmatrix} x \\ x' \\ \delta \end{pmatrix}_2 = \mathbf{R} \begin{pmatrix} x \\ x' \\ \delta \end{pmatrix}_1$ $\bar{d} = \begin{pmatrix} M_{16} \\ M_{26} \end{pmatrix}$
For n cells : $\mathbf{R}^n = \begin{pmatrix} \mathbf{M}^n & (\mathbf{M}^{n-1} + \mathbf{M}^{n-2} + \dots + \mathbf{I})\bar{d} \\ 0 & 1 \end{pmatrix}$
but $(\mathbf{M}^{n-1} + \mathbf{M}^{n-2} + \dots + \mathbf{I}) = (\mathbf{M}^n - I)(\mathbf{M} - I)^{-1}$
So for n cells : $\mathbf{R}^n = \begin{pmatrix} \mathbf{M}^n & (\mathbf{M}^n - \mathbf{I})(\mathbf{M} - \mathbf{I})^{-1}\bar{d} \\ 0 & 1 \end{pmatrix}$
• So the lattice is achromatic only if $\bar{d} = 0$ or $\mathbf{M}^n = \mathbf{I}$
 $\mathbf{M}^n = \mathbf{I} \cos \mu_{\text{tot}} + \mathbf{J} \sin \mu_{\text{tot}} \Rightarrow \mu_{\text{tot}} = 2\pi k$
S.Y. Lee, "Accelerator Physics" (Interpretent of the second seco



- Divert beam around an obstruction
 - e.g. vertical bypass chicane in Fermilab Main Ring
 - e.g. horizontal injection chicane in CEBAF recirculating linac
 - Essentially a design orbit "4-bump" (4 dipoles)
- Usually need some focusing, optics between dipoles
- Usually design optics to be achromatic

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- Operationally null orbit motion at end of chicane vs changes in input beam energy
- Naively expect M₅₆<0 (bunch lengthening or decompression)
 - Higher energy particles (+δ) have shorter path lengths
 - But can compress bunches with introduction of longitudinal correlation







CEBAF Spreaders/Recombiners

- Problem: Separate different energy beams for transport into arcs of CEBAF, and recombine before next linac
 - Achromats: arcs are FODO-like, linacs are dispersion-free
 - "I" insertion: 1 betatron wavelength between dipoles
 - Single dogleg: unacceptably high beta functions
 - Two consecutive "staircase" doglegs with same total phase advance was solution



EAST ARC ELEVATION

Still quite a challenge in physical layout of real magnets!

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D. Douglas, R.C. York, J. Kewisch, "Optical Design of the CEBAF Beam Transport System", 1989





Mobius Insertion

- Fully coupled equal-emittance optics for e⁺e⁻ CESR collisions
 - Symmetrically exchange horizontal/vertical motion in insertion
 - Horizontal/vertical motion are coupled
 - Only one transverse tune degree of freedom!

$$Q_{x,y}$$
: unrotated tunes $Q_{1,2} = \frac{Q_x + Q_y}{2} \pm \frac{1}{4}$ $Q_1 - Q_2 = \frac{1}{2}$

• Match insertion to points where $\beta_x = \beta_y$ and $\alpha_x = \alpha_y$ with phase advances that differ by π between planes

• Normal insertion:
$$\mathbf{M}_{\text{erect}} = \begin{pmatrix} \mathbf{T} & 0 \\ 0 & -\mathbf{T} \end{pmatrix}$$

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- Rotated by 45 degrees around s axis: $\mathbf{M}_{\text{mobius}} = \begin{pmatrix} 0 & \mathbf{T} \\ \mathbf{T} & 0 \end{pmatrix}$
- A purely transverse example of an emittance exchanger

S. Henderson, R. Talman, et al., "Investigation of the Möbius Accelerator at CESR", Proc. of the 1999 Particle Accelerator Conference, New York, NY; R. Talman, "A Proposed Möbius Accelerator", Phys. Rev. Lett **74**, 1590-3 (1995).



Transverse/Longitudinal Emittance Exchange

- X-ray FELs demand ultra-low transverse emittance beam*
- State-of-the art photo-injectors can generate low 6-D emittance. Typically asymmetric emittances. Emittance exchange can swap transverse with the longitudinal emittance.
- Allows one to convert transverse modulations to longitudinal modulations : Beam shaping application
- Can also be used to suppress microbunching instability**

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J.C.T. Thangaraj, Experimental Studies on an Emittance Exchange Beamline at the A0 Photoinjector, 2012

Fermilab A0 Emittance Exchanger



heta: Bending angle η : dogleg dispersion L: dogleg length L_c : RF cell length

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J.C.T. Thangaraj, Experimental Studies on an Emittance Exchange Beamline at the A0 Photoinjector, 2012

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Double Bend Achromat (approximate) quadrupole dipole dipole $L = \rho\theta$ dipole l f $L = \rho\theta$

 You calculated constraints for the double bend achromat in your homework last night

$$M_{\rm dipole} = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho[1-\cos\theta] \\ -\frac{1}{\rho}\sin\theta & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{pmatrix}$$

Keep lowest-order terms in θ , including θ^2 in upper right term since $\rho\theta\text{=}L$

$$M_{\rm dipole} = \left(\begin{array}{ccc} 1 & L & L\theta/2 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{array} \right)$$

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Double Bend Achromat (approximate)

$$\mathbf{M}_{\text{DBA}} = \begin{pmatrix} 1 & L & L\theta/2 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & l & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & l & 0 \\ -\frac{1}{f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & l & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L & L\theta/2 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{M}_{\text{DBA}} = \begin{pmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{pmatrix} \qquad C = 1 - \frac{(L+l)}{f}$$
$$S = \frac{(L+l)(2f - L - l)}{f}$$
$$D = \theta \frac{(L+l)(4f - L - 2l)}{2f}$$
$$C' = \left| -\frac{1}{f} \right|$$
$$S' = 1 - \frac{(L+l)}{f} = C$$
$$D' = \theta \frac{(4f - L - 2l)}{2f} = \frac{D}{L+l}$$

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Double Bend Achromat (approximate)

 The periodic solutions for dispersion for the general M matrix were derived in class and the text

$$\eta(\text{periodic}) = \frac{[1 - S']D + SD'}{2(1 - \cos \mu)} = 0$$
$$\eta'(\text{periodic}) = \frac{[1 - C]D' + C'D}{2(1 - \cos \mu)} = 0$$

- It turns out that the η' equation is satisfied automatically!
 - This is a consequence of the mirror symmetry of the system
- The η equation is satisfied if D=0:

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$$\begin{split} D &= \theta \frac{(L+l)(4f-L-2l)}{2f} = 0 \\ \Rightarrow 4f-L-2l &= 0 \qquad \Rightarrow \qquad f = \frac{L+2l}{4} \qquad \hat{\eta} = \frac{(L+2l)\theta}{2} = 2f\theta \end{split}$$





Triple Bend Achromat Cell (ALS at LBL)

