USPAS Accelerator Physics 2017
University of California, Davis

Chapter 9: RF Cavities and RF Linear Accelerators

Todd Satogata (Jefferson Lab) / satogata@jlab.org
Randika Gamage (ODU) / bgama002@odu.edu
http://www.toddsatogata.net/2017-USPAS

Happy Birthday to Hideki Yukawa, Paul Langevin, Rutger Hauer, and Gertrude Elion!
Happy Rhubarb Pie Day, National Handwriting Day, and Measure Your Feet Day!

Happy Handwriting Day

Vertical Dispersion Correction

- The end of section 6.10 in CM notes how sextupoles at locations of horizontal dispersion compensate **both** horizontal and vertical chromaticity

$$k(s) \equiv \frac{B'}{(B\rho)} \quad \Delta x' = [k(s)ds]\,x = (kL)\,x = \frac{x}{f} \quad \approx [k_0(s)ds]\,x(1-\delta)$$

$$\xi_{\mathrm{x,N}} = -\frac{1}{4\pi Q_H}\oint \beta_x(s)k_0(s)\,ds$$

CM 6.126

- Normal sextupoles: $b_2(s) \equiv \frac{B''}{(B\rho)} \quad \Delta x' \approx [b_2(s)ds](x^2-y^2)(1-\delta)$
- Horizontal dispersion:

$$x \to x + \eta_x(s)\delta$$

- First order in x, δ:

$$\Delta x' \approx b_2\,ds[x^2 + 2\eta_x(s)\delta\,x + \delta^2](1-\delta) \approx 2[b_2\,ds\,\eta_x(s)\delta]x$$

Vertical Dispersion Correction

$$k(s) \equiv \frac{B'}{(B\rho)} \quad \Delta x' = [k(s)ds]\, x = (kL)\, x = \frac{x}{f} \quad \approx [k_0(s)ds]\, x(1-\delta)$$ Quad

$$\Delta x' \approx b_2\, ds[x^2 + 2\eta_x(s)\delta\, x + \delta^2](1-\delta) \approx 2[b_2\, ds\, \eta_x(s)\delta]x$$ Sext

$$\xi_{\mathrm{x,N}} = -\frac{1}{4\pi Q_H} \oint \beta_x(s) k_0(s)\, ds \qquad \xi_x(\mathrm{sext}) = \frac{+1}{4\pi Q_H} \oint \beta_x(s) b_2(s) \eta_x(s)\, ds$$

- Vertically **for quads**, sign flips and *x* goes to *y*

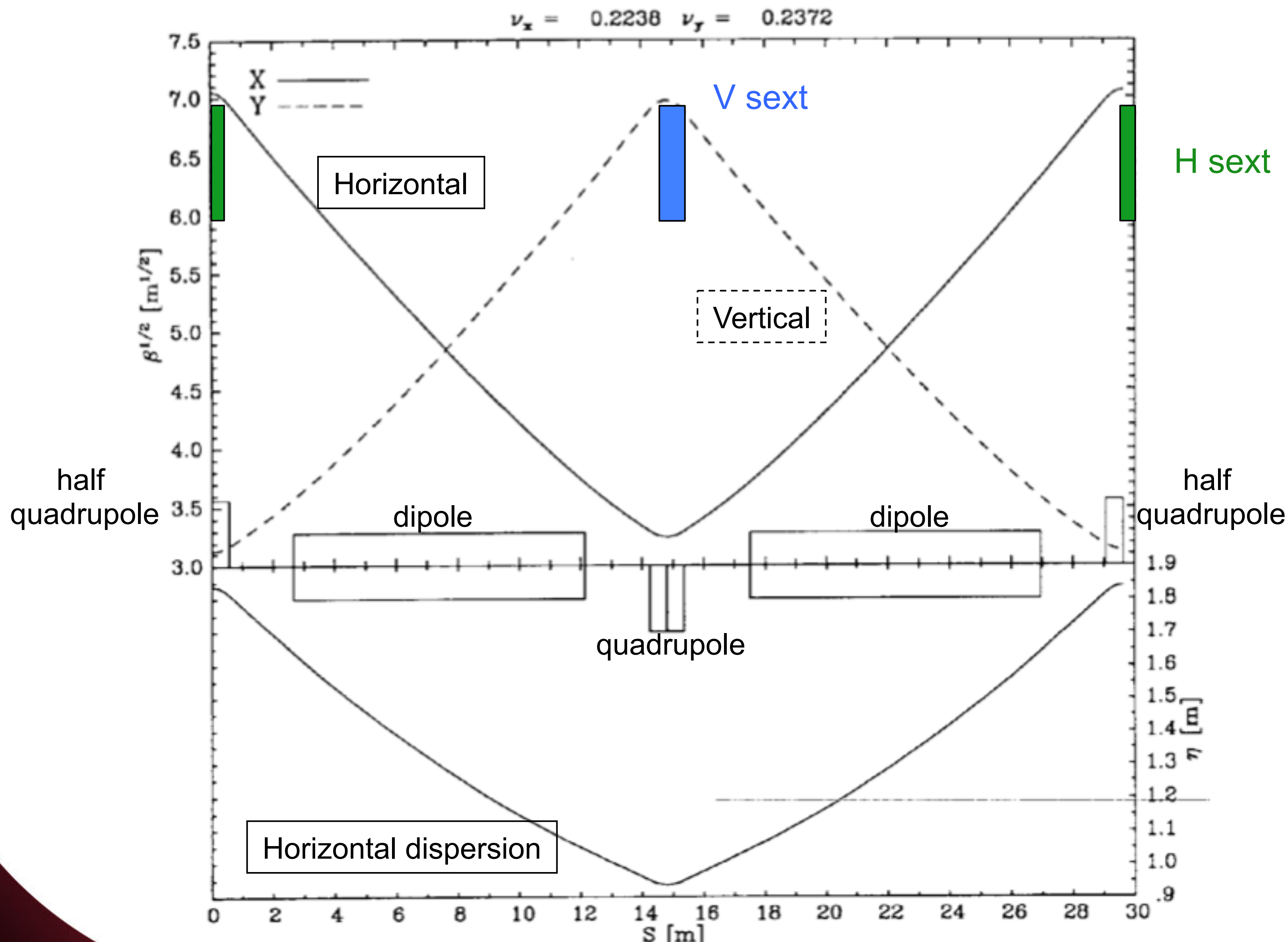
$$\xi_{\mathrm{y,N}} = \frac{+1}{4\pi Q_V} \oint \beta_y(s) k_0(s)\, ds$$ CM 6.127b

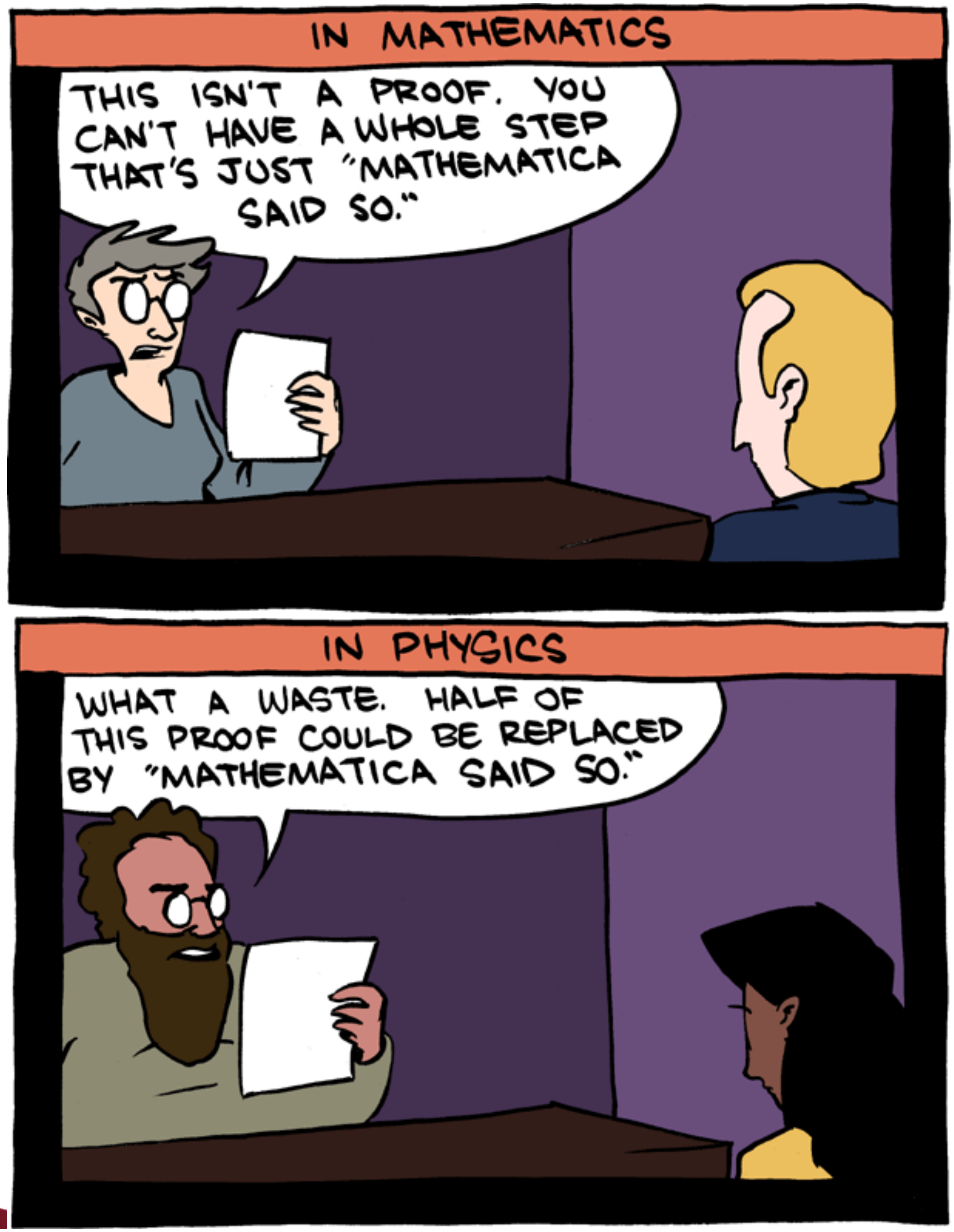
- Sextupole effect is coupled in vertical plane, with horz dispersion:

$$\Delta y' \approx [b_2(s)ds](2xy)(1-\delta) = [b_2(s)ds][2(x+\eta_x(s)\delta)y](1-\delta)$$

$$\xi_y(\mathrm{sext}) = -\frac{1}{4\pi Q_V} \oint \beta_y(s)[2b_2(s)\eta_x(s)]\, ds$$

RHIC FODO Cell




IN MATHEMATICS
THIS ISN'T A PROOF. YOU CAN'T HAVE A WHOLE STEP THAT'S JUST "MATHEMATICA SAID SO."
IN PHYSICS
WHAT A WASTE. HALF OF THIS PROOF COULD BE REPLACED BY "MATHEMATICA SAID SO."

RF Concepts and Design

- Much of RF is really a review of graduate-level E&M
  - See, e.g., J.D. Jackson, "Classical Electrodynamics"
  - The beginning of this lecture is hopefully review
    - But it's still important so we'll go through it
    - Includes some comments about electromagnetic polarization
  - We'll get to interesting applications later in the lecture
    - (or tomorrow)
  - Particularly important are cylindrical waveguides and cylindrical RF cavities
    - Will find transverse boundary conditions are typically roots of Bessel functions
    - TM (transverse magnetic) and TE (transverse electric) modes
    - RF concepts (shunt impedance, quality factor, resistive losses)

9.1: Maxwell's Equations

Electric charge density

$$\vec{\nabla} \cdot \vec{D} = \rho$$

Electric charge generates electric displacement

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Faraday's Law

$$\vec{\nabla} \cdot \vec{B} = 0$$

No magnetic charges

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Magnetic field generated by real or displacement current density

Electric current density

Maxwell's Equations are linear in the source terms ρ and $\vec{J}$. In general will generate linear (partial) differential equations to solve. Superposition valid!

Constitutive Relations and Ohm's Law

$$\boxed{\vec{D} = \varepsilon \vec{E} = \varepsilon_r \varepsilon_0 \vec{E}}$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \frac{\mathrm{C}}{\mathrm{N} \cdot \mathrm{m}^2}$$

$\vec{D}$: Electric flux density

$\vec{E}$: Electric field density

ε : Permittivity

$$\boxed{\varepsilon_0 \mu_0 = \frac{1}{c^2}}$$

$$\boxed{\vec{B} = \mu \vec{H} = \mu_r \mu_0 \vec{H}}$$

$$\mu_0 = 4\pi \times 10^{-7} \frac{\mathrm{N} \cdot \mathrm{s}^2}{\mathrm{C}^2} \approx 1.257 \frac{\mathrm{cm} \cdot \mathrm{G}}{\mathrm{A}}$$

$\vec{H}$: Magnetic field density

$\vec{B}$: Magnetic flux density

μ : Permeability

$$\boxed{\vec{J} = \sigma \vec{E}}$$

σ : conductivity

Boundary Conditions

- Boundary conditions on fields at the surface between two media depend on the surface charge and current densities:
  - $E_{||}$ and $B_{\perp}$ are continuous
  - $D_{\perp}$ changes by the surface charge density ρ_{s} (scalar)
  - $H_{||}$ changes by the surface current density $\vec{J}_{\mathrm{s}}$

$$\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_{\mathrm{s}}$$

- C&M Chapter 9: no dielectric or magnetic materials

$$\mu = \mu_0 \qquad \epsilon = \epsilon_0$$

Wave Equations and Symmetry

- Taking the curl of each curl equation and using the identity

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\nabla^2 \vec{E}$$

then gives us two **identical** wave equations

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \qquad \nabla^2 \vec{H} = \mu\sigma \frac{\partial \vec{H}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

These **linear wave equations** reflect the deep symmetry between electric and magnetic fields. Harmonic solutions:

$$\vec{\Psi}(\vec{r}, t) = \hat{\Psi}(\vec{r}) \mathrm{e}^{-i\omega t} \qquad \text{for} \quad \vec{\Psi} = \vec{E}, \vec{H}$$

$$\Rightarrow \quad \nabla^2 \hat{\Psi} = \Gamma^2 \hat{\Psi} \qquad \text{where} \quad \Gamma^2 = (-\mu\epsilon\omega^2 + i\mu\sigma\omega)$$

Conductivity and Skin Depth

- For high conductivity, $\sigma \gg \omega\epsilon$, charges move freely enough to keep electric field lines perpendicular to surface (RF oscillations are adiabatic vs movement of charges)
  - Copper: $\sigma \approx 6 \times 10^7\ \Omega^{-1}\,\mathrm{m}^{-1}$
  - Conductivity condition holds for very high frequencies (10^{15} Hz)
- For this condition

$$\Gamma \approx \sqrt{\frac{\mu\omega\sigma}{2}}(1+i)$$

  - Inside the conductor, the fields drop off exponentially

$$\hat{\Psi}(z) = \Psi(0)\, \mathrm{e}^{-z/\delta}$$

$$\text{skin depth} \qquad \boxed{\delta \equiv \sqrt{\frac{2}{\mu\omega\sigma}}}$$

e.g. for Copper

Frequency	Skin depth (μm)
60 Hz	8470
10 kHz	660
100 kHz	210
1 MHz	66
10 MHz	21
100 MHz	6.6

Surface Resistance and Power Losses

- There is still non-zero power loss for finite resistivity
  - Surface resistance: resistance to current flow per unit area

$$\text{surface resistance} \qquad R_s \equiv \sqrt{\frac{\mu\omega}{2\sigma}} = \frac{1}{\sigma\delta}$$

$$\text{surface power loss} \qquad \langle P_{\text{loss}} \rangle = \frac{R_s}{2} \int_{\text{S}} |H_{||}|^2 \, dS$$

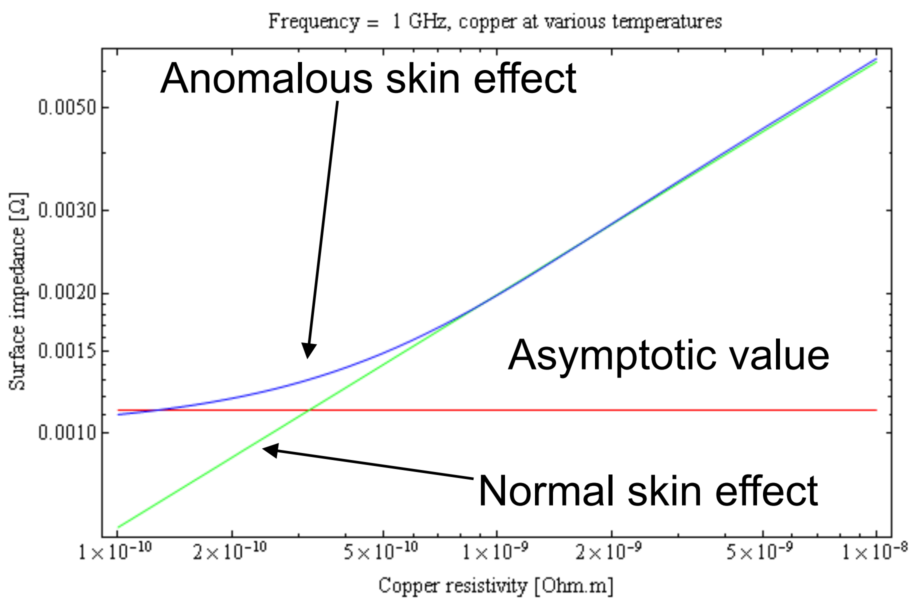
  - This isn't just applicable to RF cavities, but to transmission lines, power lines, waveguides, etc – anywhere that electromagnetic fields are interacting with a resistive media
  - Deriving the power loss is part of your homework!
  - We'll talk more about transmission lines and waveguides after one brief clarification…

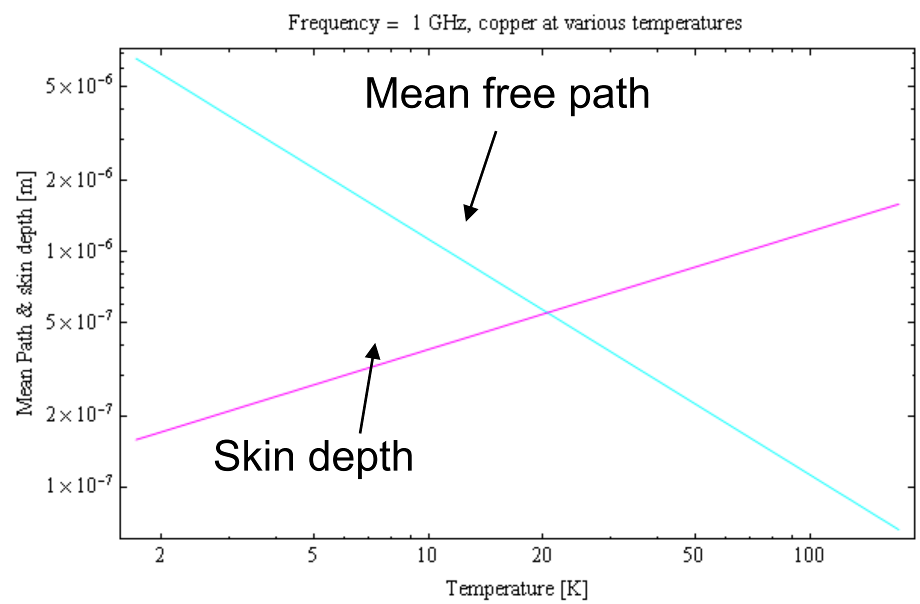
Anomalous Skin Effect

- Conductivity really depends on the frequency ω and mean time between electron interactions τ (or inverse temperature)

$$\sigma(\omega) = \frac{\sigma_0}{(1 + i\omega\tau)}$$

  - Classical limit is $\omega\tau \to 0$, adiabatic field wrt electron interactions
  - For non-classical limit, $\vec{J} = \sigma\vec{E}$ no longer applies since electrons see changing fields between single interactions





Reuter and Sondheimer, Proc. Roy. Soc. A195 (1948), from Calatroni SRF'11 Electrodynamics Tutorial

Plane Wave Properties

$$\vec{E}(\vec{x},t) = \vec{E}_0\, \mathrm{e}^{i\vec{k}\cdot\vec{x}-i\omega t}$$

$$\vec{B}(\vec{x},t) = \vec{B}_0\, \mathrm{e}^{i\vec{k}\cdot\vec{x}-i\omega t}$$

Generally $F(z,t) = \int_{-\infty}^{\infty} A(\omega)\mathrm{e}^{i(\omega t - k(\omega) z)}\, d\omega$

$$k = \frac{2\pi}{\lambda_{\mathrm{rf}}} \quad \text{(wave number)}$$

$$v_{\mathrm{ph}} = \frac{\omega}{k} \quad \text{(phase velocity)}$$

$$v_{\mathrm{g}} = \frac{d\omega}{dk} \quad \text{(group velocity)}$$

The source-free divergence Maxwell equations imply

$$\vec{k}\cdot\vec{E}_0 = \vec{k}\cdot\vec{B}_0 = 0$$

so the fields are both transverse to the direction $\vec{k}$

(This will be the $\hat{z}$ direction in a lot of what's to come)

Faraday's law also implies that both fields are spatially transverse to each other

$$\vec{B}_0 = \frac{\vec{k}\times\vec{E}_0}{\omega} = \frac{n\hat{n}\times\vec{E}_0}{c} \qquad \hat{n} \equiv \frac{\vec{k}}{k} \quad \text{index of refraction}$$

Standing Waves

$$\vec{E}(\vec{x},t) = \vec{E}_0\, \mathrm{e}^{i\vec{k}\cdot\vec{x}-i\omega t}$$

$$\vec{B}(\vec{x},t) = \vec{B}_0\, \mathrm{e}^{i\vec{k}\cdot\vec{x}-i\omega t}$$

Note that Maxwell's equations are linear, so any linear combination of magnetic/electric fields is also a solution.

Thus a **standing wave** solution is also acceptable, where there are two plane waves moving in opposite directions:

$$\vec{E}(\vec{x},t) = \vec{E}_0 \left(\mathrm{e}^{i\vec{k}\cdot\vec{x}-i\omega t} + \mathrm{e}^{-i\vec{k}\cdot\vec{x}-i\omega t}\right)$$

$$\vec{B}(\vec{x},t) = \vec{B}_0 \left(\mathrm{e}^{i\vec{k}\cdot\vec{x}-i\omega t} + \mathrm{e}^{-i\vec{k}\cdot\vec{x}-i\omega t}\right)$$

Polarization

$$\vec{E}(\vec{x},t) = \vec{E}_0\, \mathrm{e}^{i\vec{k}\cdot\vec{x} - i\omega t}$$

$$\vec{B}(\vec{x},t) = \vec{B}_0\, \mathrm{e}^{i\vec{k}\cdot\vec{x} - i\omega t}$$

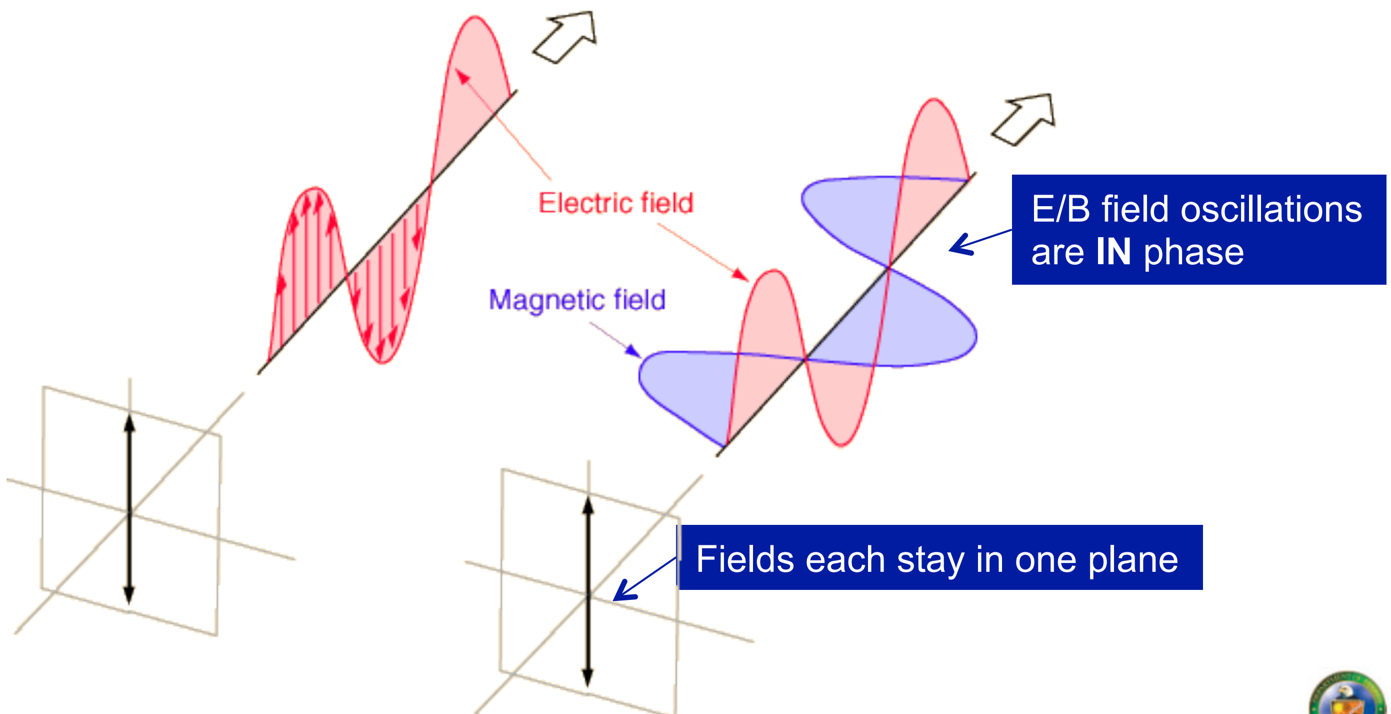
- As long as the $\vec{E}$ and $\vec{B}$ fields are transverse, they can still have different transverse components.
  - So our description of these fields is also incomplete until we specify the transverse components at all locations in space
  - This is equivalent to an uncertainty in phase of rotation of $\vec{E}$ and $\vec{B}$ around the wave vector $\vec{k}$.
  - The identification of this transverse field coordinate basis defines the **polarization** of the field.

Linear Polarization

$$\vec{E}(\vec{x},t) = \vec{E}_0\, \mathrm{e}^{i\vec{k}\cdot\vec{x}-i\omega t}$$

$$\vec{B}(\vec{x},t) = \vec{B}_0\, \mathrm{e}^{i\vec{k}\cdot\vec{x}-i\omega t}$$

- So, for example, if $\vec{E}$ and $\vec{B}$ transverse directions are constant and do not change through the plane wave, the wave is said to be **linearly polarized**.

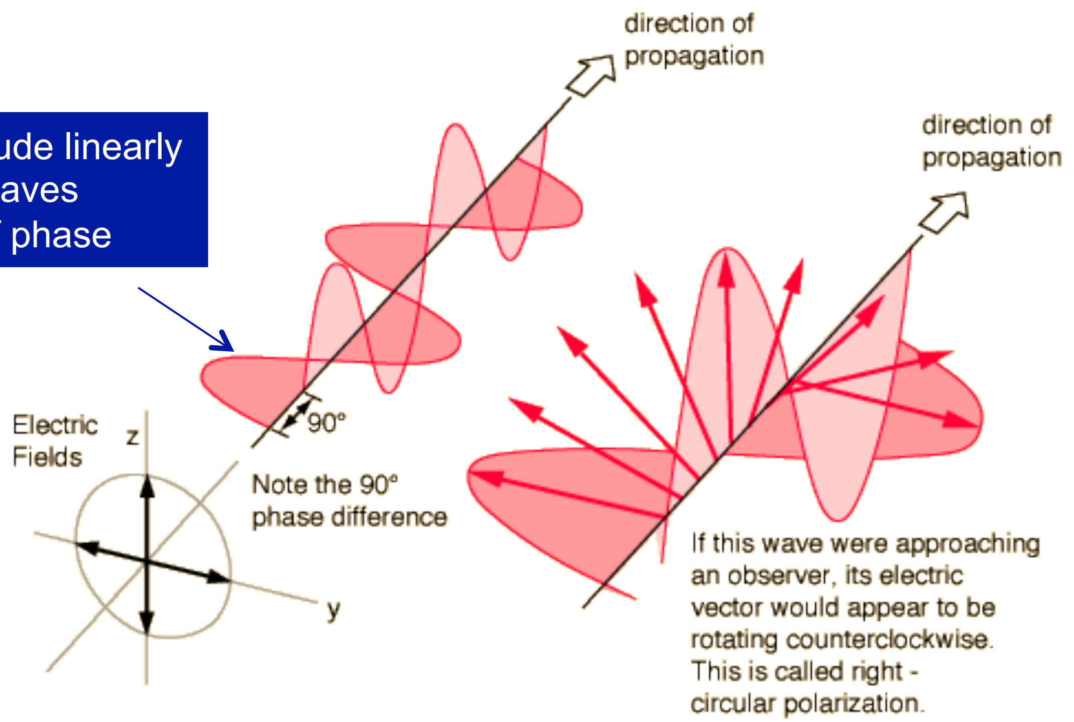


Circular Polarization

$$\vec{E}(\vec{x},t) = \vec{E}_0 \left(\mathrm{e}^{i\vec{k}\cdot\vec{x}-i\omega t} + \mathrm{e}^{i\vec{k}\cdot\vec{x}-i\omega t+\pi/2} \right)$$
$$\vec{B}(\vec{x},t) = \vec{B}_0 \left(\mathrm{e}^{i\vec{k}\cdot\vec{x}-i\omega t} + \mathrm{e}^{i\vec{k}\cdot\vec{x}-i\omega t+\pi/2} \right)$$

- If $\vec{E}$ and $\vec{B}$ transverse directions vary with time, they can appear as two plane waves traveling out of phase. This phase difference is 90 degrees for **circular polarization**.

Two equal-amplitude linearly polarized plane waves 90 degrees out of phase



9.2: Cylindrical Waveguides

- Consider a cylindrical waveguide, radius a, in z direction

$$\vec{E} = \vec{E}(r,\theta)\mathrm{e}^{i(\omega t - k_g z)}$$

  - k_g can be imaginary for attenuation down the guide
  - We'll find constraints on this cutoff wave number
  - Maxwell in cylindrical coordinates (math happens) gives

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 E_z}{\partial \theta^2} = -\,k_c^2 E_z$$

$$\frac{\partial^2 E_z}{\partial z^2} = -\,k_g^2 E_z$$

$$\text{(and the same for } H_z\text{)}$$

$$k \equiv \left(\frac{\omega}{c}\right) \quad \text{(free space wave number)}$$

$$k_c^2 \equiv k^2 - k_g^2$$

$$E_r = -\frac{1}{k_c^2}\left[ik_g\frac{\partial E_z}{\partial r} + \frac{i\omega\mu}{r}\frac{\partial H_z}{\partial \theta}\right],$$

$$E_\theta = \frac{1}{k_c^2}\left[-\frac{ik_g}{r}\frac{\partial E_z}{\partial \theta} + i\omega\mu\frac{\partial H_z}{\partial r}\right],$$

$$H_r = \frac{1}{k_c^2}\left[\frac{i\omega\epsilon}{r}\frac{\partial E_z}{\partial \theta} - ik_g\frac{\partial H_z}{\partial r}\right],$$

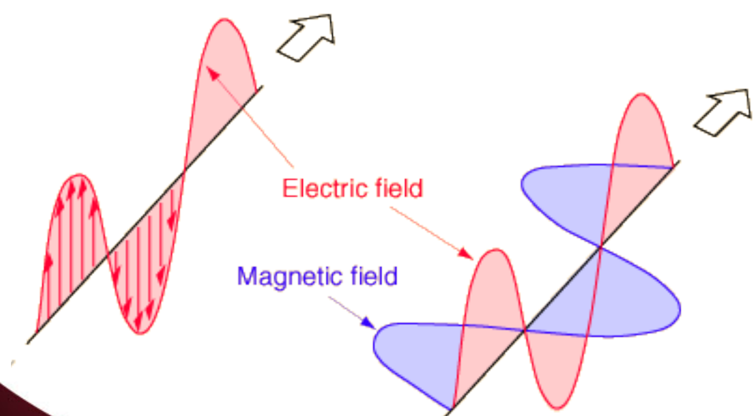
$$H_\theta = -\frac{1}{k_c^2}\left[i\omega\epsilon\frac{\partial E_z}{\partial r} + \frac{ik_g}{r}\frac{\partial H_z}{\partial \theta}\right].$$

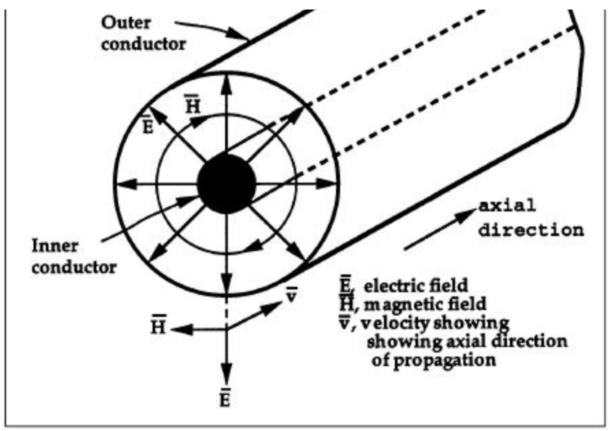
Transverse Electromagnetic (TEM) Modes

- Various subsets of solutions are interesting
  - For example, the $E_z, H_z = 0$ nontrivial solutions require

$$k_c^2 = k^2 - k_g^2 = 0$$

  - The wave number of the guide matches that of free space
  - Wave propagation is similar to that of free space
  - This is a TEM (transverse electromagnetic) mode
    - Requires multiple separate conductors for separate potentials or free space
    - Sometimes similar to polarization pictures we had before





Coaxial cable

Transverse Magnetic (TM) Modes

- Maxwell's equations are linear so superposition applies
  - We can break all fields down to TE and TM modes
  - **TM**: Transverse magnetic (H_z=0, E_z≠0)
  - TE: Transverse electric (E_z=0, H_z≠0)
  - TM provides particle energy gain or loss in z direction!

$$\text{Separate variables } E_z(r,\theta) = R(r)\Theta(\theta)$$

$$\text{Boundary conditions } E_z(r=a) = E_\theta(r=a) = 0$$

  - Maxwell's equations in cylindrical coordinates give

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + \left(k_c^2 - \frac{n^2}{r^2}\right)R = 0$$

Bessel equation

$$\frac{d^2\Theta}{d\theta^2} + n^2\Theta = 0$$

SHO equation
=> n integer, real

TM Mode Solutions

- The radial equation has solutions of Bessel functions of first $J_n(k_c r)$ and second $N_n(k_c r)$ kind
  - Toss N_n since they diverge at r=0
  - Boundary conditions at r=a require $J_n(k_c a)=0$
  - This gives a constraint on k_c

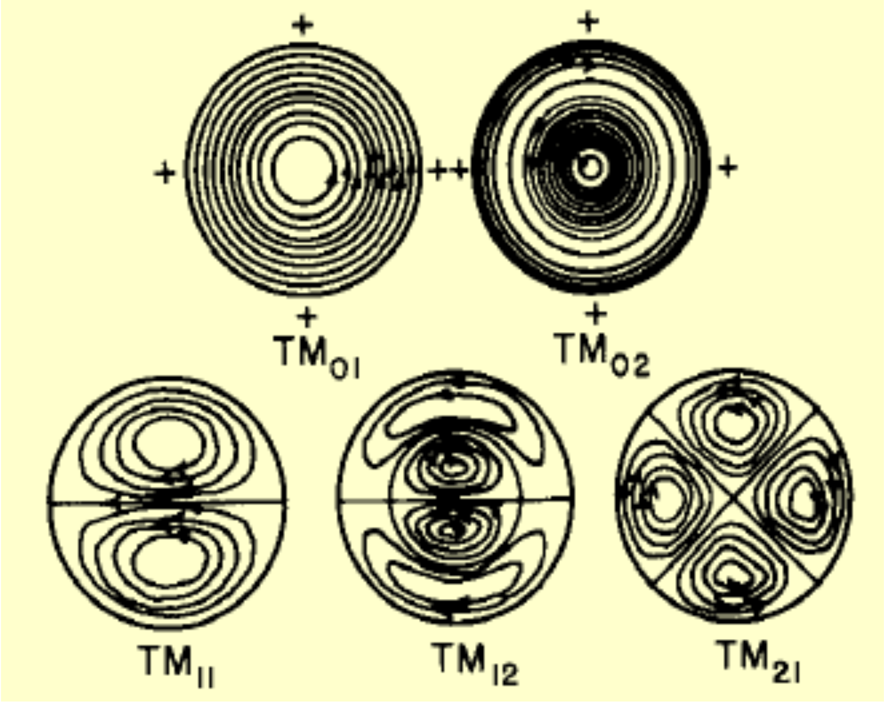
$$k_c = X_{nj}/a \quad \text{where} \quad X_{nj} \text{ is the } \mathrm{j}^{\mathrm{th}} \text{ nonzero root of } J_n$$

$$\Rightarrow E_z(r,\theta,t) = (C_1 \cos n\theta + C_2 \sin n\theta) J_n\left(\frac{X_{nj}}{a} r\right) \mathrm{e}^{\mathrm{i}(\omega \mathrm{t} - \mathrm{k_g z})}$$

  - This mode of this field is commonly known as the TM_{nj} mode
  - The first index corresponds to theta periodicity, while the second corresponds to number of radial Bessel nodes
  - TM_{01} is the usual fundamental accelerating mode

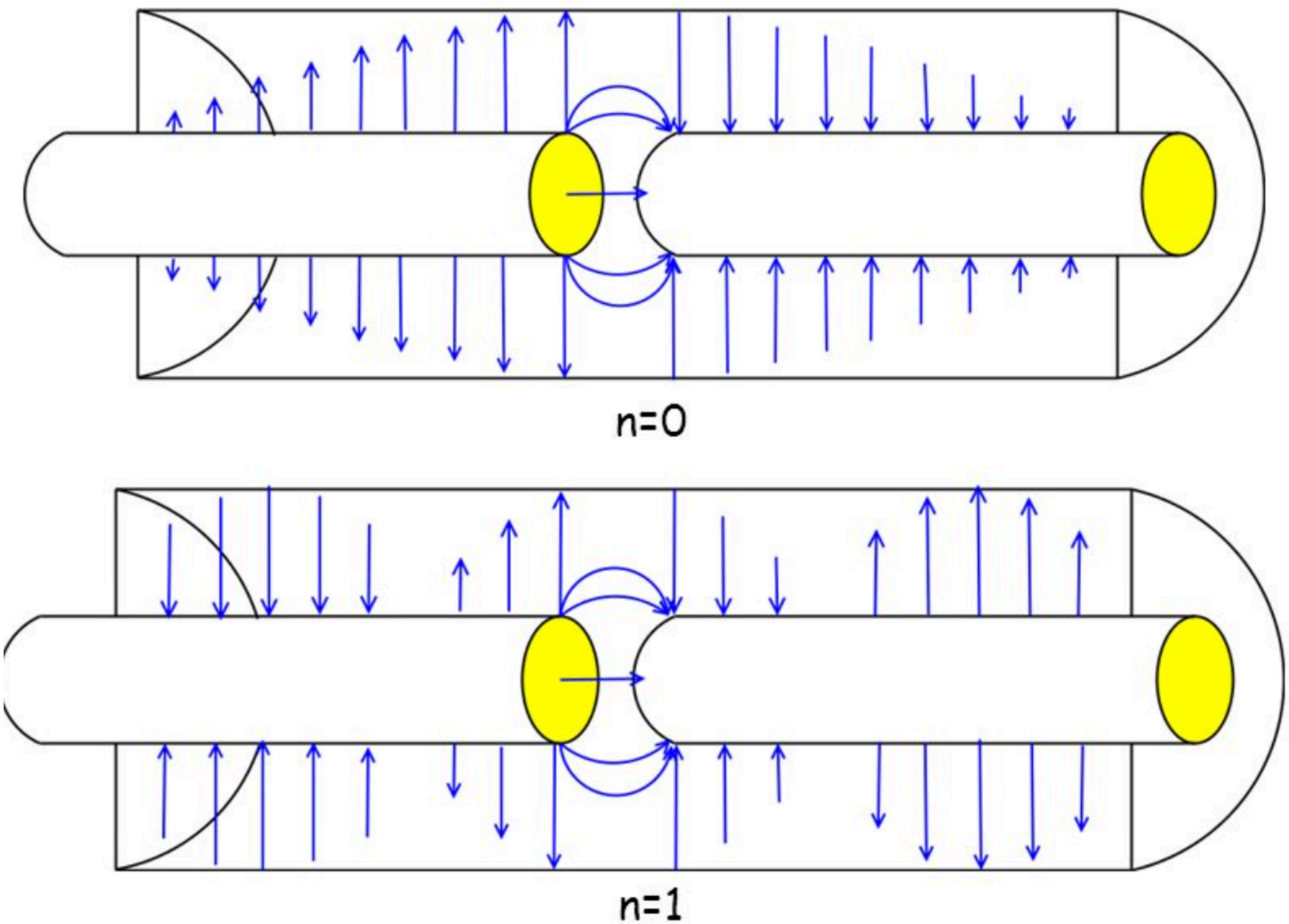
TM Mode Visualizations

$$E_z(r,\theta,t) = (C_1 \cos n\theta + C_2 \sin n\theta) J_n\left(\frac{X_{nj}}{a} r\right) \mathrm{e}^{\mathrm{i}(\omega \mathrm{t} - \mathrm{k_g z})}$$



- Java app visualizations are also linked to the class website
  - For example, http://www.falstad.com/emwave2 (TM visualization)
    - Scroll down to Circular Modes 1 or 2 in first pulldown

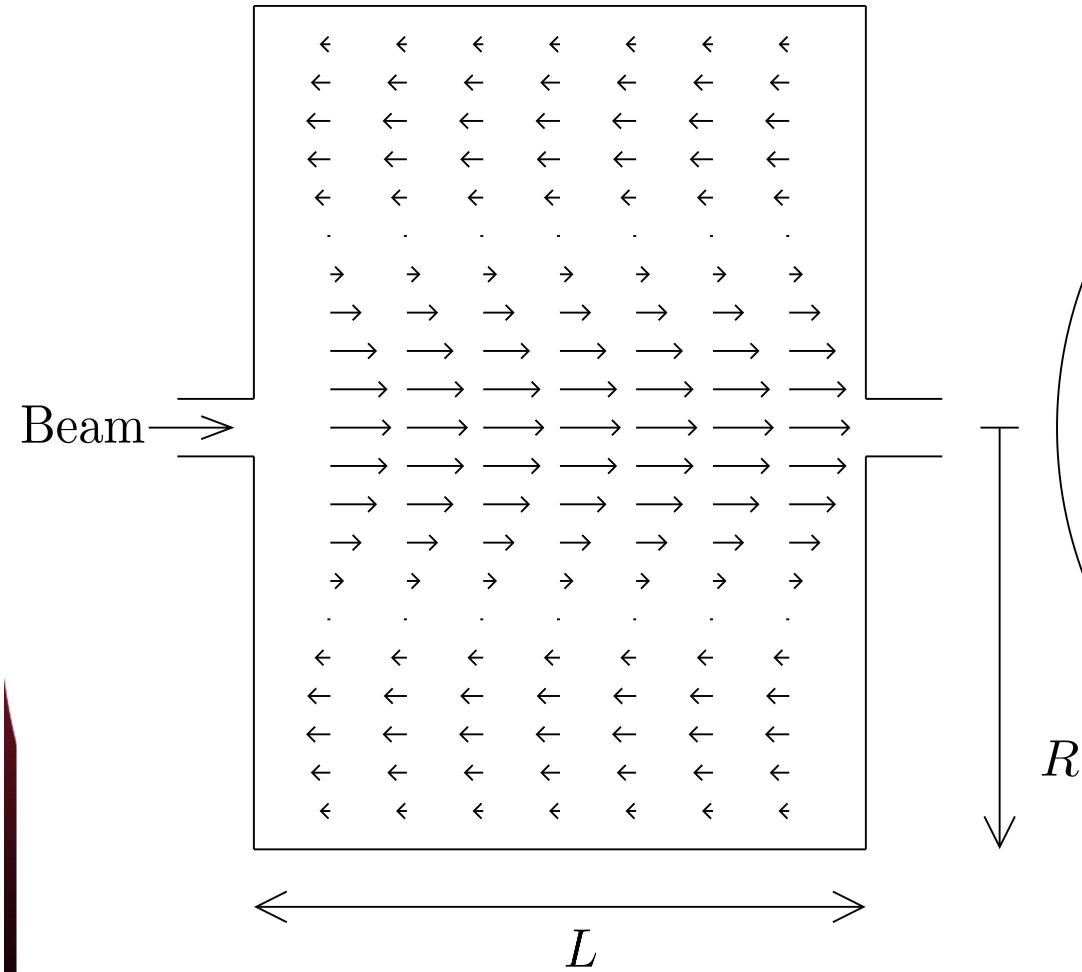
Cavity Modes

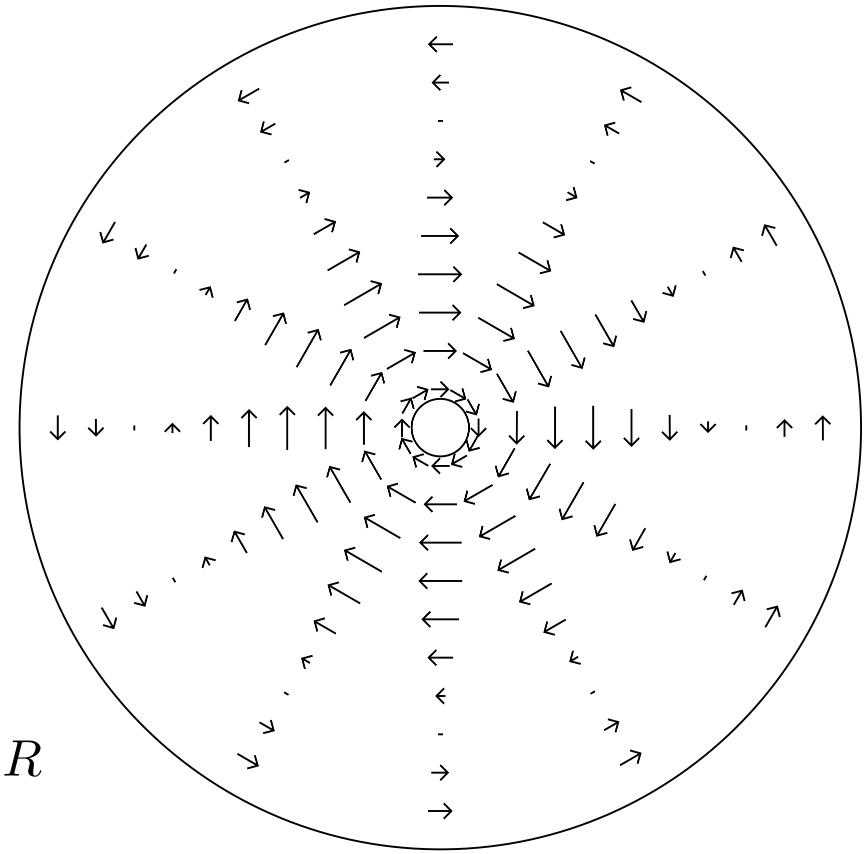


Dave McGinnis, FNAL

TM020 Mode

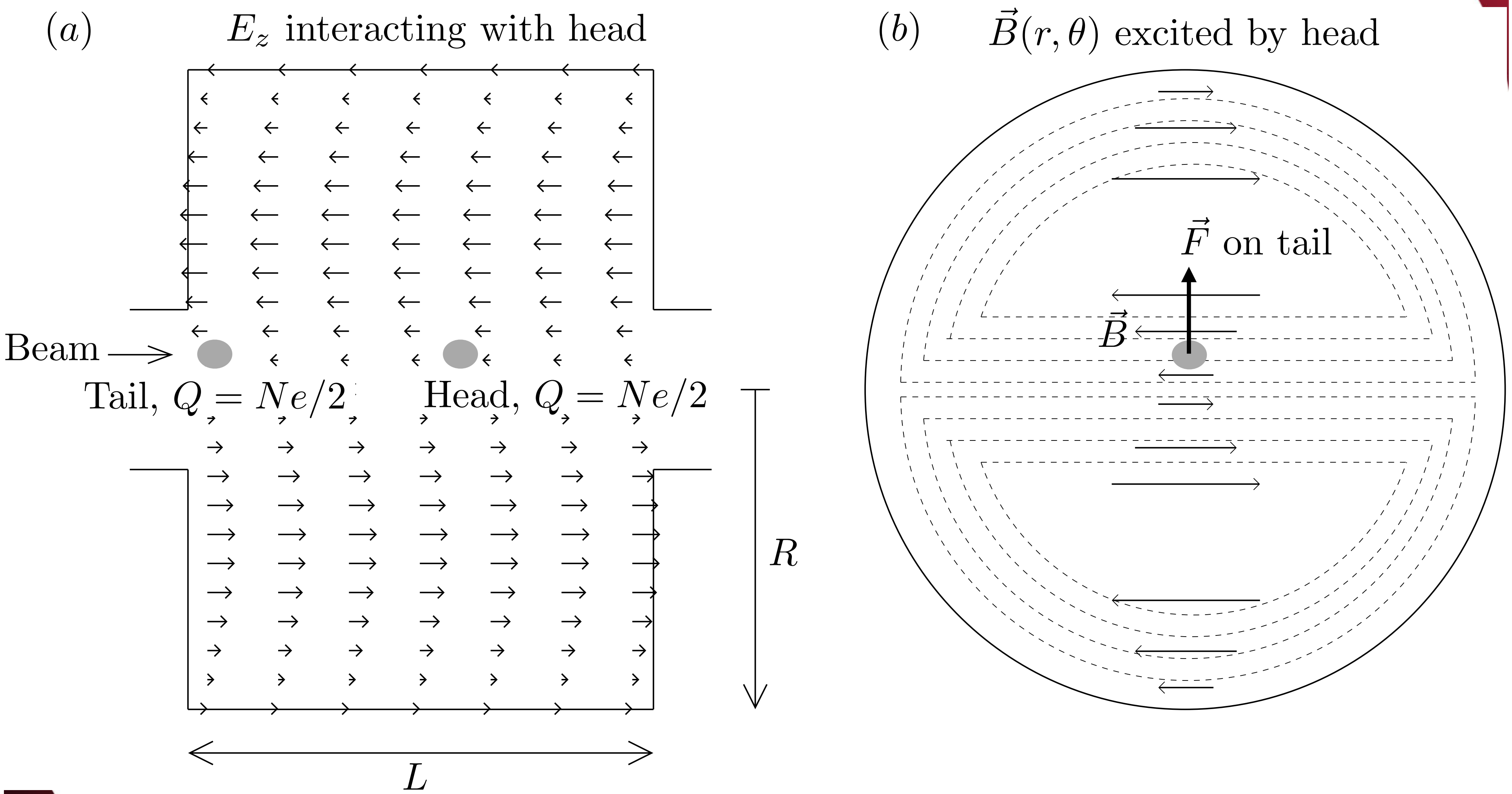
$$E_z \sim J_0\left(u_{02}\frac{r}{R}\right) \qquad B_\theta \sim -J_1\left(u_{02}\frac{r}{R}\right)$$





Peggs/Satogata

TM110 Mode



Peggs/Satogata

Cutoff Frequency

- It's easy to see that there is a minimum frequency for our waveguide that obeys the boundary conditions
  - Modes at lower frequencies will suffer resistance
  - Dispersion relation in terms of radial boundary condition zero

$$k^2 - k_g^2 = \left(\frac{\omega}{c}\right)^2 - k_g^2 = k_c^2 = \left(\frac{X_{nj}}{a}\right)^2$$

  - A propagating wave must have real group velocity $k_g^2 > 0$
  - This gives the expected lower bound on the frequency

$$\frac{\omega}{c} \geq \frac{X_{nj}}{a} \quad \Rightarrow \quad \boxed{\text{cutoff frequency } \omega_c = \frac{cX_{nj}}{a}}$$

  - Wavelength of lowest cutoff frequency for j=1 is

$$\boxed{\lambda_c = \frac{2\pi}{k_c} \approx 2.61a}$$

Dispersion or Brillouin Diagram

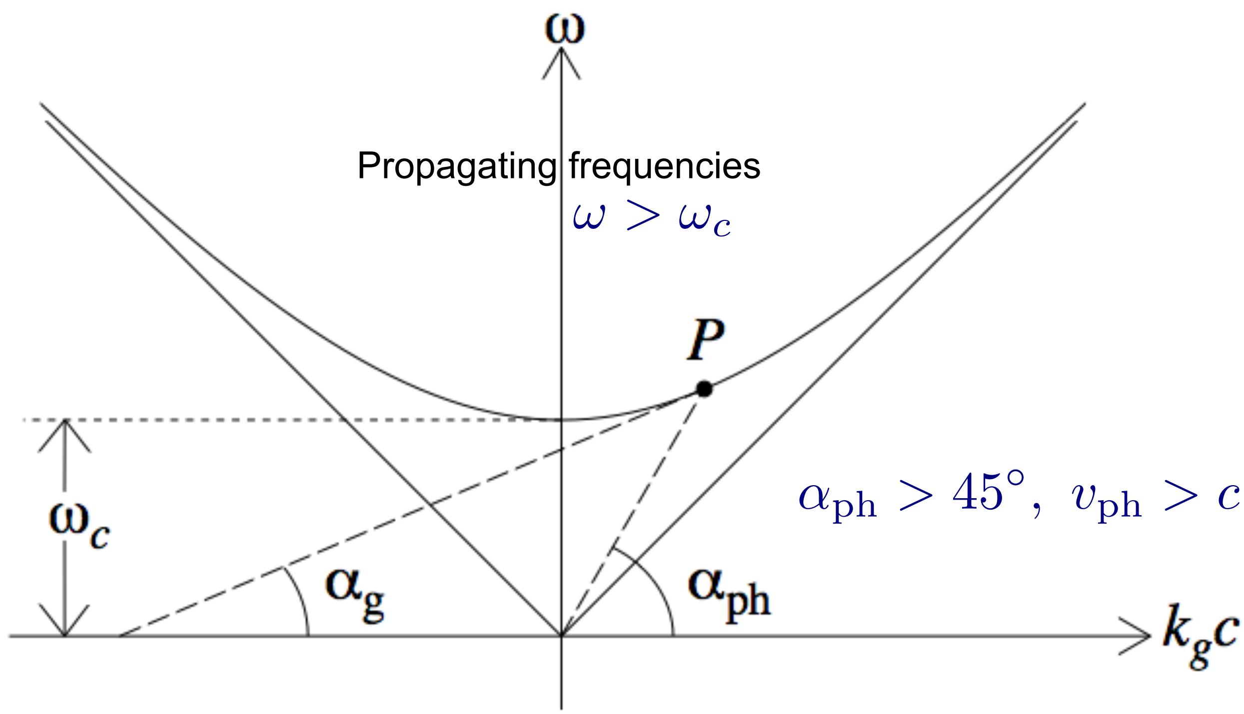


Figure. 9.1 The dispersion or Brillouin diagram for a uniform cylindrical waveguide of radius a. The angles α_{ph} and α_g for the point P are defined by the relations $\tan\alpha_{ph} = v_{ph}/c$, and $\tan\alpha_g = v_g/c$. The asymptotic diagonal lines are the dispersion curve for a wave traveling in free space.

- Circular waveguides above cutoff have phase velocity >c
- Cannot easily be used for particle acceleration over long distances

Iris-Loaded Waveguides

- Solution: Add impedance by varying cylinder radius
  - This changes the dispersion condition by loading the waveguide

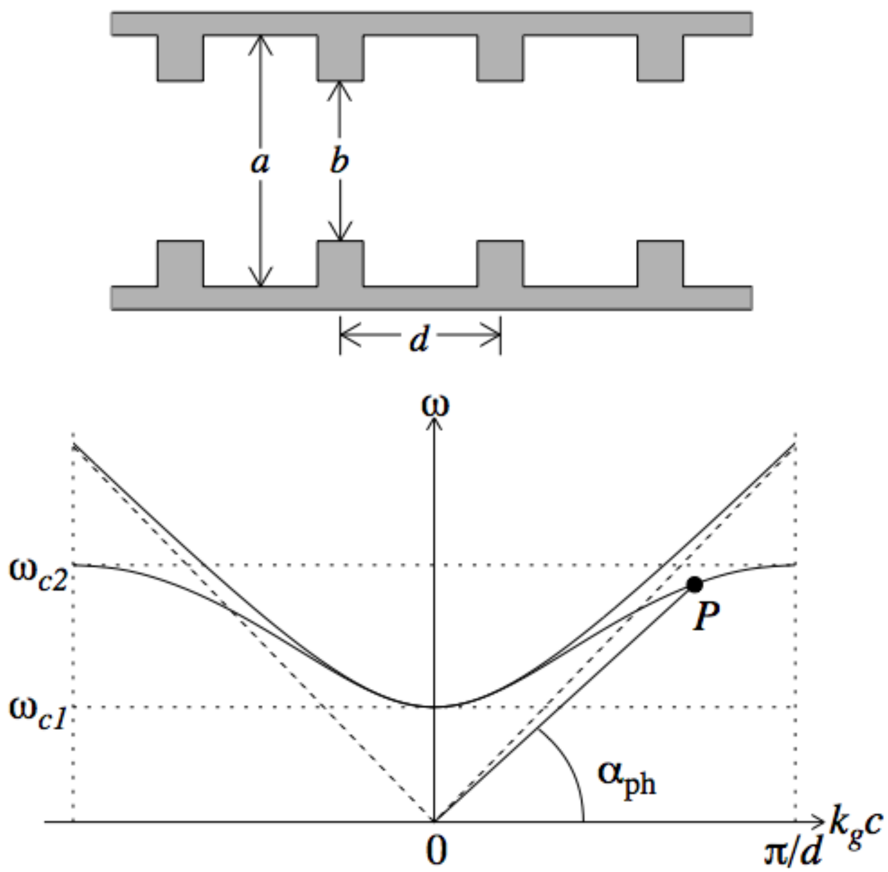
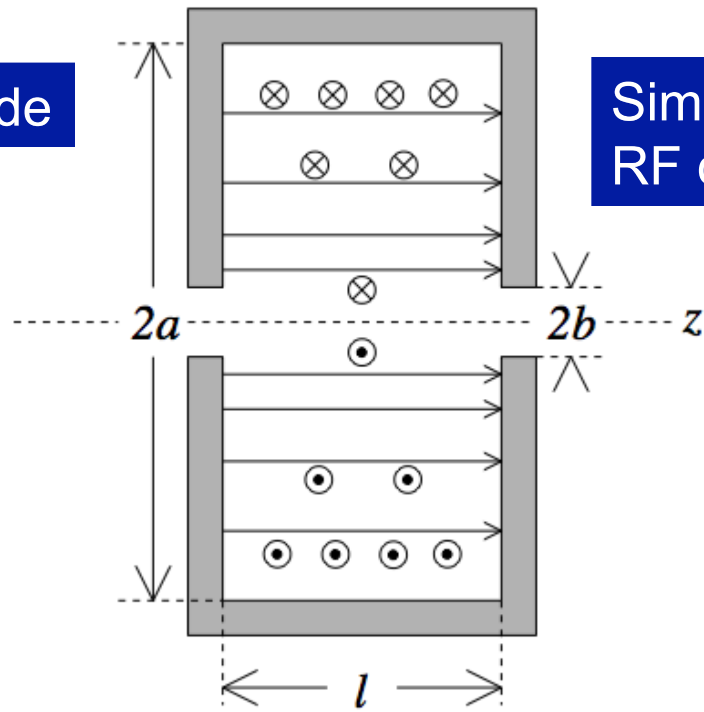


Figure. 9.3 Brillouin diagram for a loaded cylindrical waveguide. The point P has phase velocity less than c, i. e., $\alpha_{\rm ph} < 45°$. As $k_g c$ approaches π/d, the group velocity goes to zero.

Cylindrical RF Cavities

- What happens if we close two ends of a waveguide?
  - With the correct length corresponding to the longitudinal wavelength, we can produce longitudinal standing waves
  - Can produce long-standing waves over a full linac
  - Modes have another index for longitudinal periodicity

TM010 mode



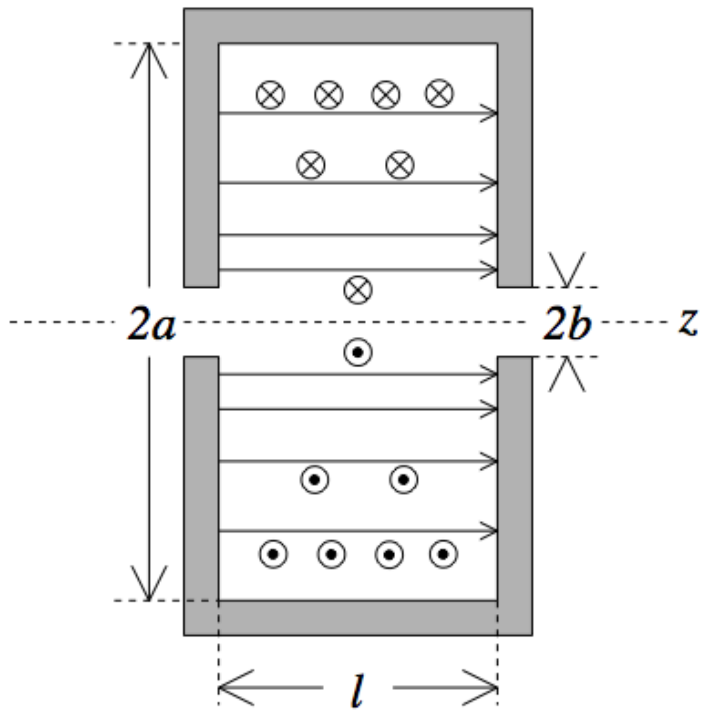
Simplified “tuna fish can” RF cavity model

Boundary Conditions

- Additional boundary conditions at end-cap conductors at z=0 and z=l

$$H_z(r,\theta,0) = H_z(r,\theta,l) = 0$$

$$E_r(r,\theta,0) = E_\theta(r,\theta,0) = E_r(r,\theta,l) = E_\theta(r,\theta,l) = 0$$



TE_{nmj} Modes

$$E_z = 0 \text{ everywhere}$$

$$H_z(r,\theta,z) \sim J_n(k_c r)(C_1 \cos n\theta + C_2 \sin n\theta) \sin\left(\frac{m\pi z}{l}\right)$$

Longitudinal periodicity

$$k_c^2 = \left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{l}\right)^2$$

$$J_n'(k_c a) = 0 \qquad X_{nj}' \text{ is j}^{\text{th}} \text{ root of } J_n'$$

$$f_{nmj} = \frac{c}{2\pi}\sqrt{\left(\frac{X_{nj}'}{a}\right)^2 + \left(\frac{m\pi}{l}\right)^2}$$

TM_{nmj} Modes

$$H_z = 0 \text{ everywhere}$$

$$E_z(r,\theta,z) \sim J_n(k_c r)(C_1 \cos n\theta + C_2 \sin n\theta) \cos\left(\frac{m\pi z}{l}\right)$$

Longitudinal periodicity

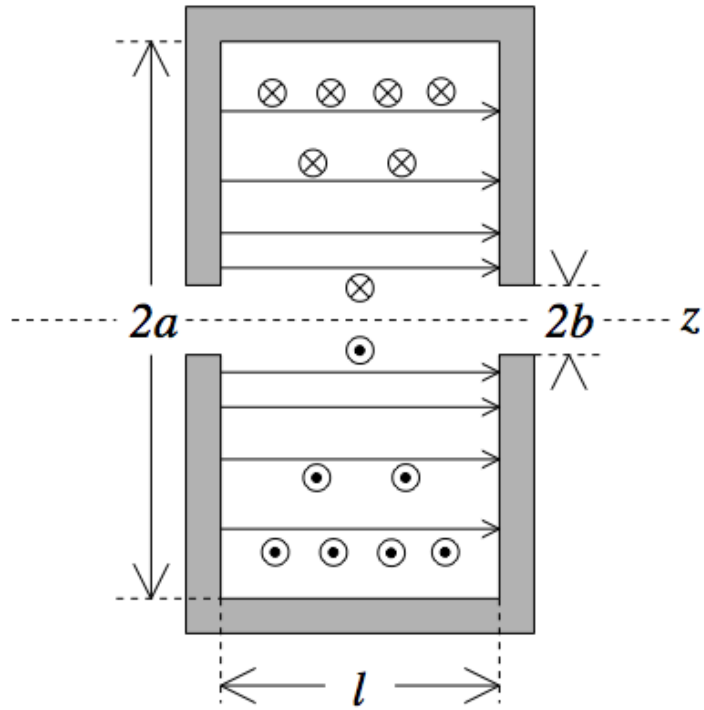
$$J_n(k_c a) = 0 \qquad X_{nj} \text{ is the j}^{\text{th}} \text{ root of } J_n$$

$$f_{nmj} = \frac{c}{2\pi}\sqrt{\left(\frac{X_{nj}}{a}\right)^2 + \left(\frac{m\pi}{l}\right)^2}$$

TM_{010} is (again) the preferred acceleration mode

Equivalent Circuit

- A TM_{010} cavity looks much like a lumped LRC circuit
  - Ends are capacitive
  - Stored magnetic energy is inductive
  - Currents move over resistive walls
- E, H fields 90 degrees out of phase



- Stored energy

$$U = \frac{\epsilon_0}{2} \iiint |\vec{E}(r,\theta,z,t)|^2 d^3r + \frac{\mu_0}{2} \iiint |\vec{H}(r,\theta,z,t)|^2 d^3r$$

$$U = \frac{\epsilon_0}{2} E_0^2 \int_0^l \int_0^{2\pi} \int_0^a \left[J_n \left(\frac{X_{01}}{a} r \right) \right]^2 r\, dr\, d\theta\, dz$$

$$= \frac{\epsilon_0}{2} E_0^2\, \pi a^2 l \, [J_1(X_{01})]^2,$$

C&M 9.82

TM_{010} Average Power Loss

$$\langle P_{\text{loss}} \rangle = \frac{1}{2\sigma\delta} \left\{ 2 \times 2\pi \left(\frac{E_0}{\mu_0 c} \right)^2 \int_0^a \left[J_1 \left(\frac{X_{01}}{a} r \right) \right]^2 r\, dr \right.$$

ends

$$\left. + \pi a l \left(\frac{E_0}{\mu_0 c} \right)^2 [J_1(X_{01})]^2 \right\}$$

sides

$$= \frac{\pi a (a+l)}{\sigma\delta} \left(\frac{E_0}{\mu_0 c} \right)^2 [J_1(X_{01})]^2,$$

- Compare to stored energy
  - Both vary like square of field, square of Bessel root
  - Dimensional area vs dimensional volume in stored energy

$$U = \frac{\epsilon_0}{2} E_0^2 \pi a^2 l [J_1(X_{01})]^2$$

Quality Factor

- Ratio of average stored energy to average power lost (or energy dissipated) during one RF cycle
  - How many cycles does it take to dissipate its energy?
  - High Q: nondissipative resonator
  - Copper RF: Q~10^3 to 10^6
  - SRF: Q up to 10^{11}

$$Q = \frac{U\omega}{\langle P_{\mathrm{loss}} \rangle}$$

$$U(t) = U_0 \mathrm{e}^{-\omega_0 t/Q}$$

$$\text{Pillbox cavity} \quad Q = \frac{al}{\delta(a+l)}$$

Niobium Cavity SRF "Q Slope" Problem