Fundamentals of Accelerators - 2015

Lecture 9 - Synchrotron Radiation

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Photon energy to frequency conversion
Numbers to Remember

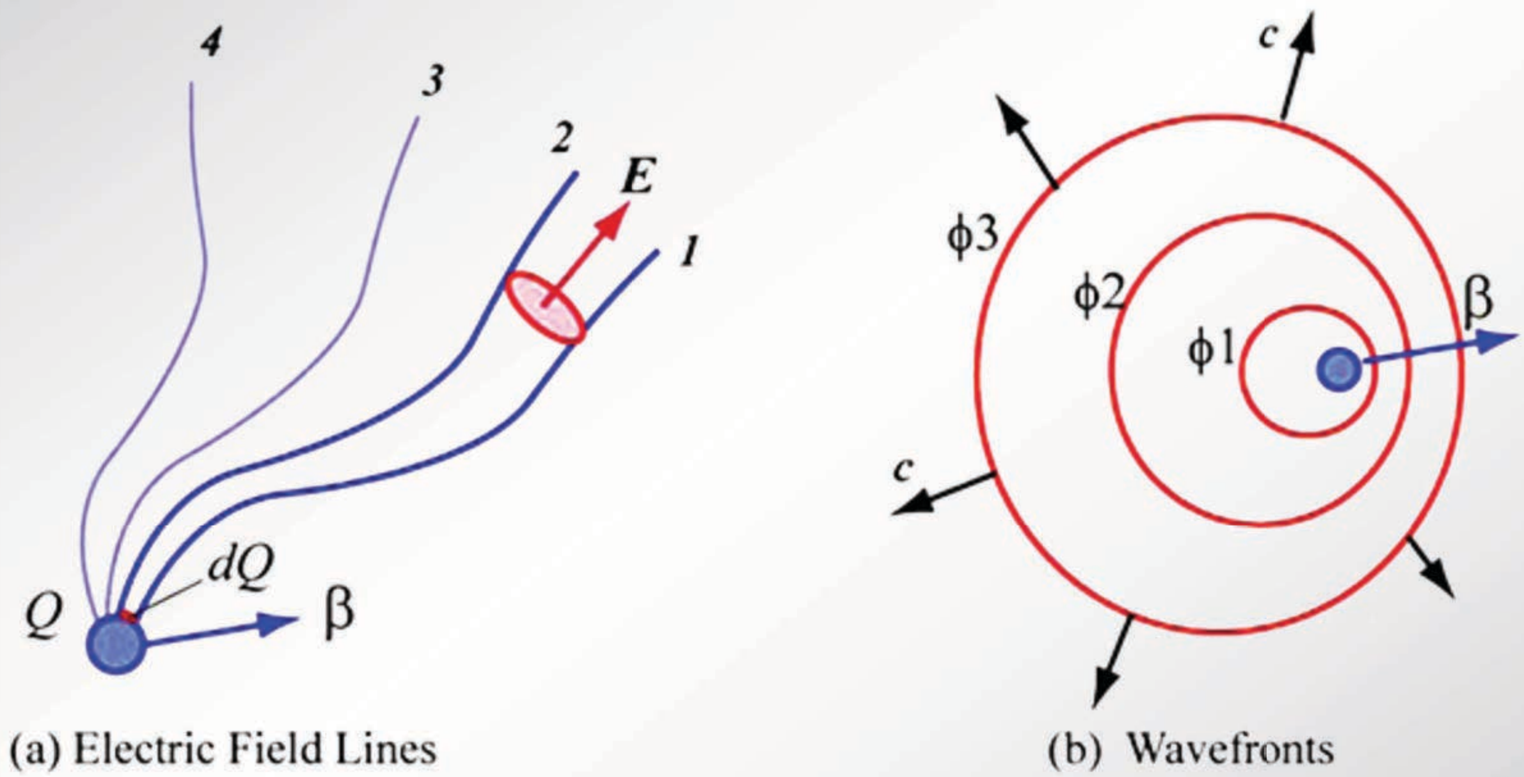
$$E_\gamma = \hbar\omega_\gamma = h\nu_\gamma = h\frac{c}{\lambda_\gamma}$$

$$\boxed{\begin{array}{l} \lambda_\gamma = 1\ \mu\mathrm{m} \quad ==> \quad \nu_\gamma \sim 300\ THz \\ \\ \lambda_\gamma = 1\ \mu\mathrm{m} \quad ==> \quad E_\gamma = 1.2\ \mathrm{eV} \end{array}}$$

What do we mean by radiation?

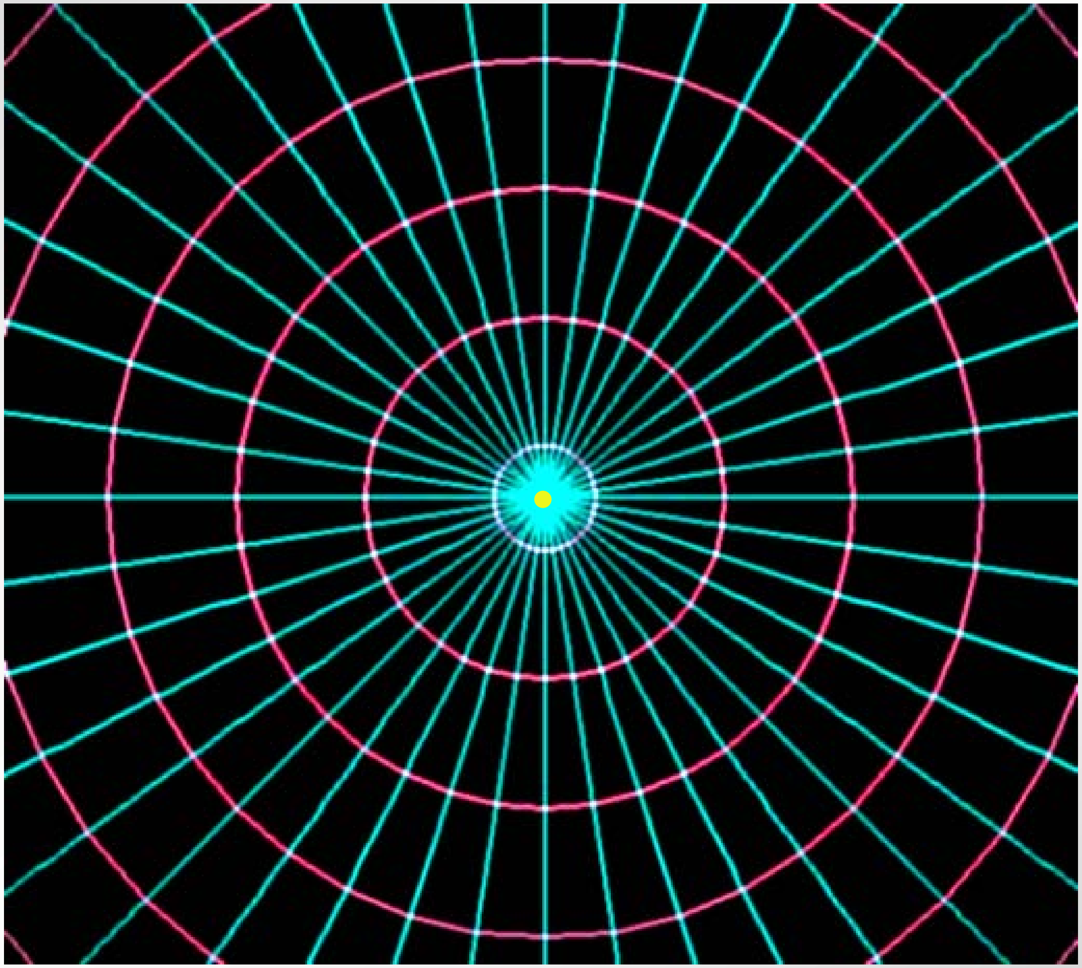
- Energy is transmitted by the electromagnetic field to infinity
  - Applies in all inertial frames
  - Carried by an electromagnetic wave
- Source of the energy
  - Motion of charges

Schematic of electric field



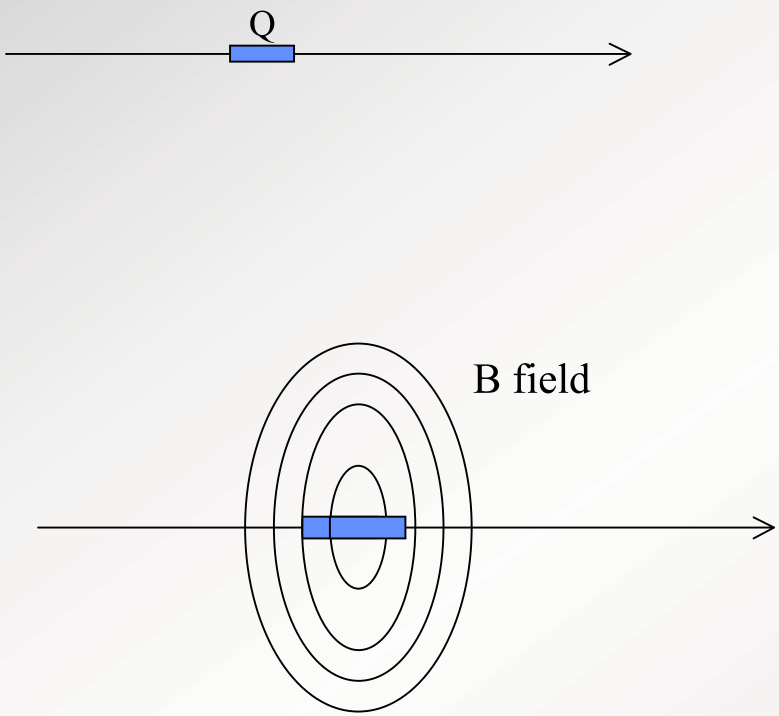
From: T. Shintake, New Real-time Simulation Technique for Synchrotron and Undulator Radiations, Proc. LINAC 2002, Gyeongju, Korea

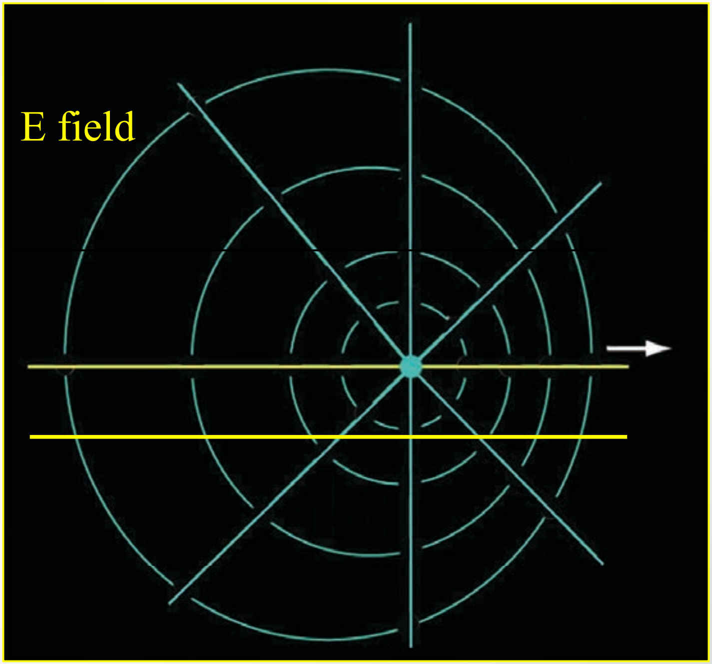
In the rest frame of the particle Electric field is that of a static charge



Field lines in turquoise

Particle moving in a straight line
with constant velocity





We can compute the fields using the Lorentz transformation

Lorentz transformation of E.M. fields

$$E'_{z'} = E_z$$

$$E'_{x'} = \gamma\left(E_x - vB_y\right)$$

$$E'_{y'} = \gamma\left(E_y + vB_x\right)$$

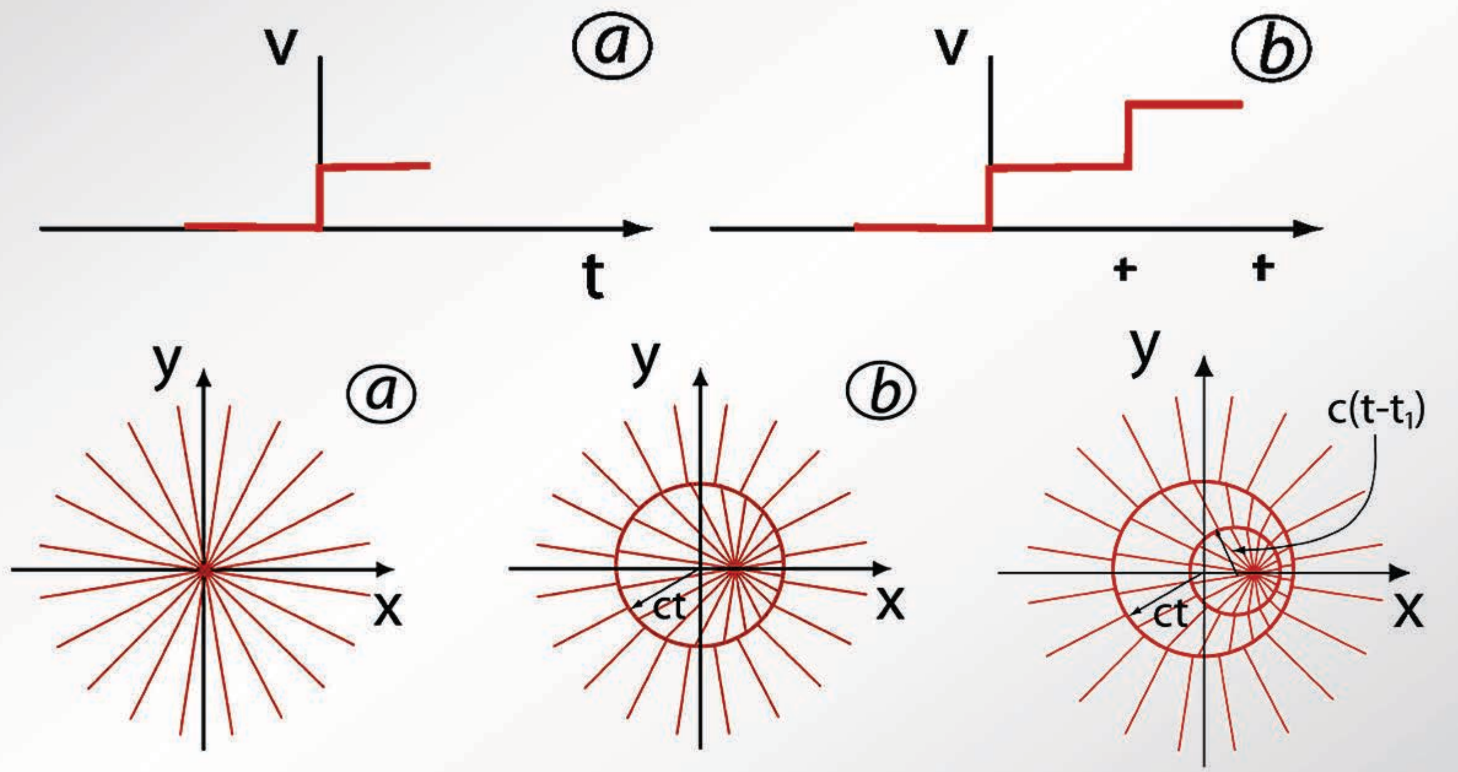
$$B'_{z'} = B_z$$

$$B'_{x'} = \gamma\left(B_x + \frac{v}{c^2}E_y\right)$$

$$B'_{y'} = \gamma\left(B_y - \frac{v}{c^2}E_x\right)$$

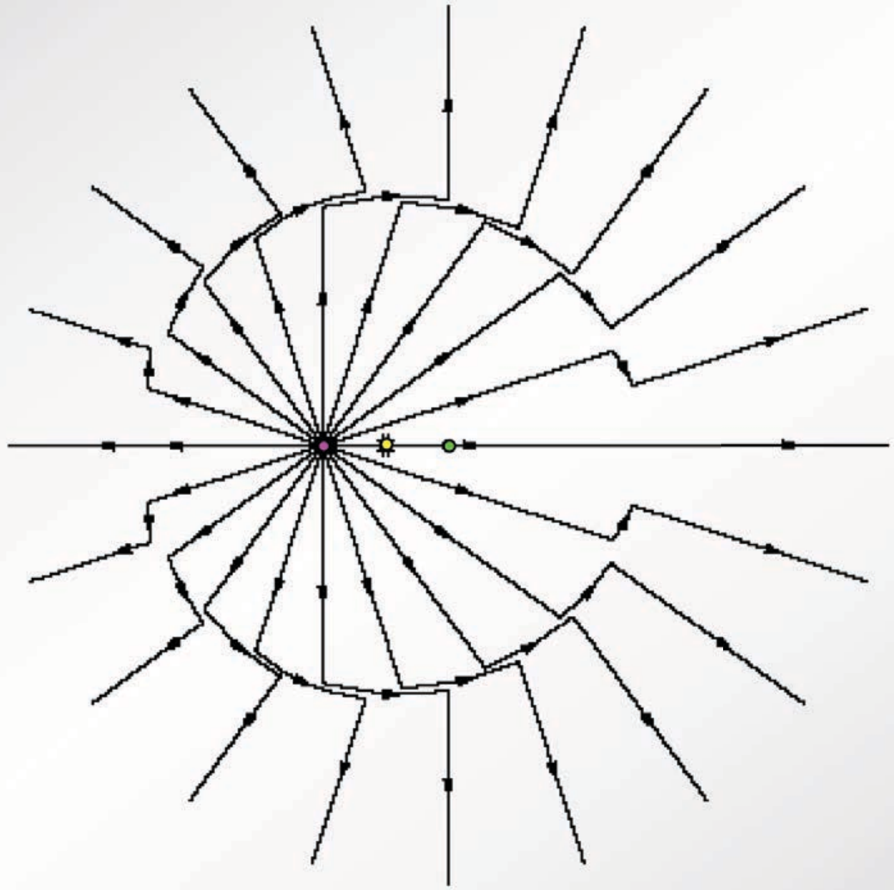
Consider the fields from an electron
with abrupt accelerations

- At r = ct, ∃ a transition region from one field to the other. At large r, the field in this layer becomes the radiation field.



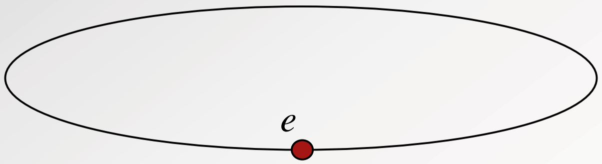
The radiation field (kink) moves outward at the speed of light

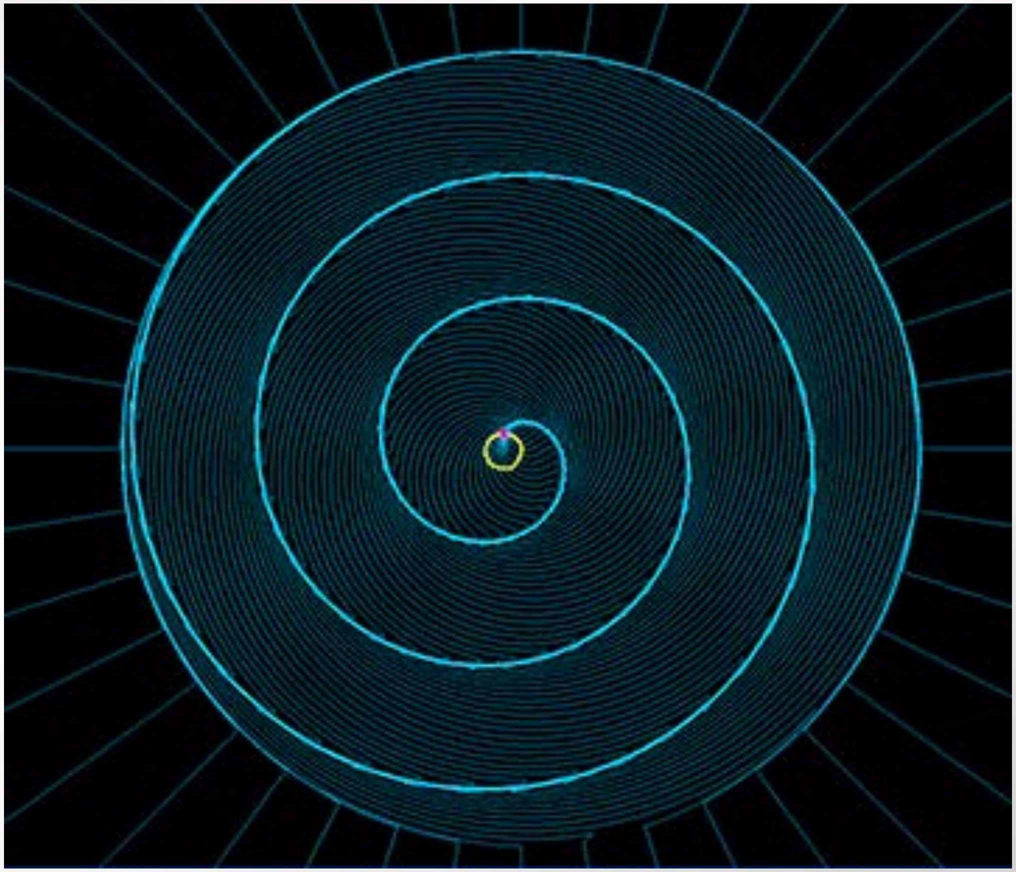
Looking more closely



The physical photons have an E-field transverse to the direction of propagation

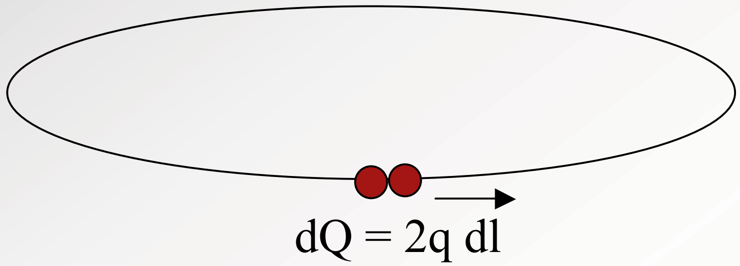
Particle moving in a circle
at constant speed

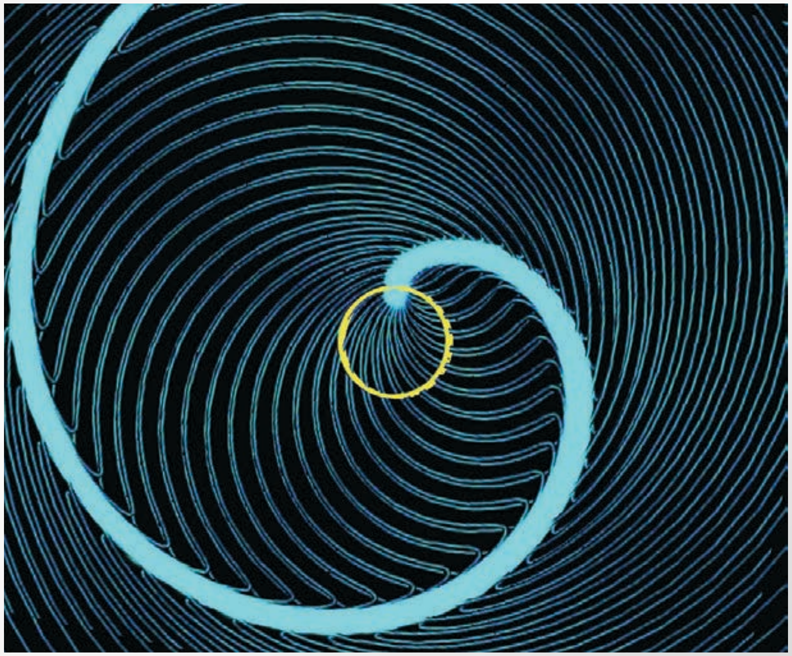




The radiation field energy flows to infinity

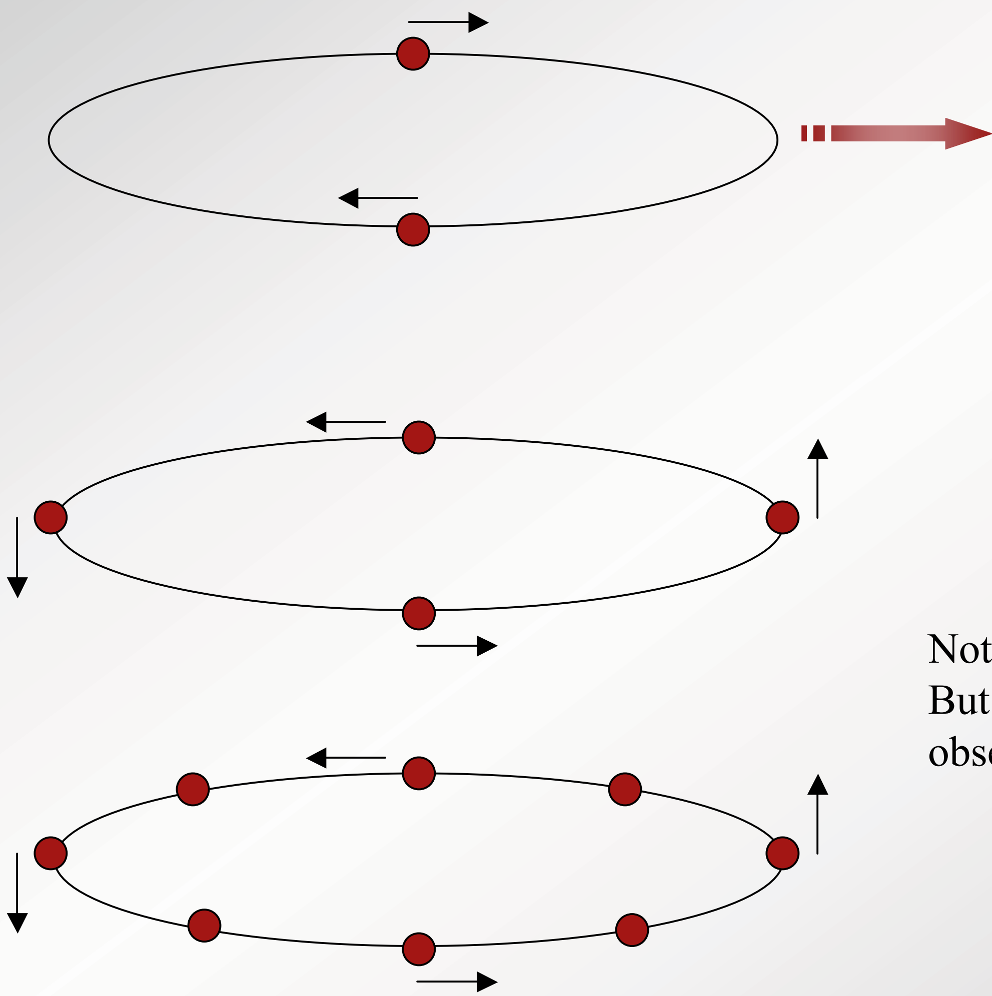
Remember that fields add, we can compute radiation from a charge twice as long

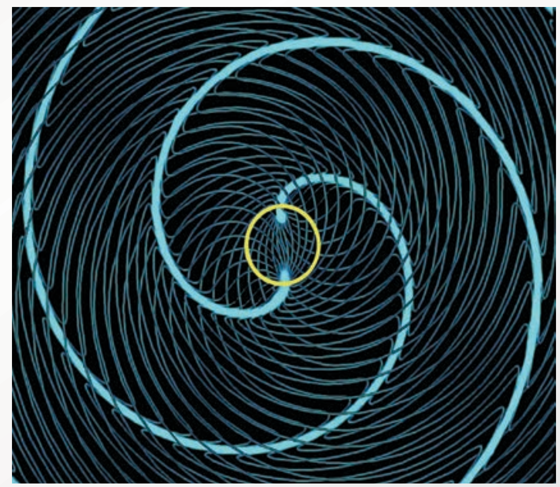




The wavelength of the radiation doubles

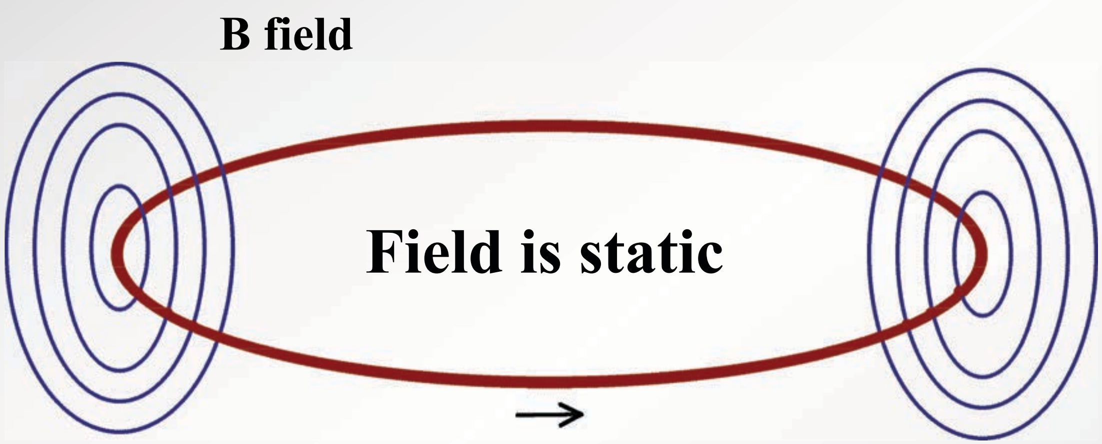
All these configurations radiate





Not quantitatively correct because E is a vector;
But we can see that the peak field hits the observer twice as often

Limiting case of a continuous charge distribution
Current loop: No radiation



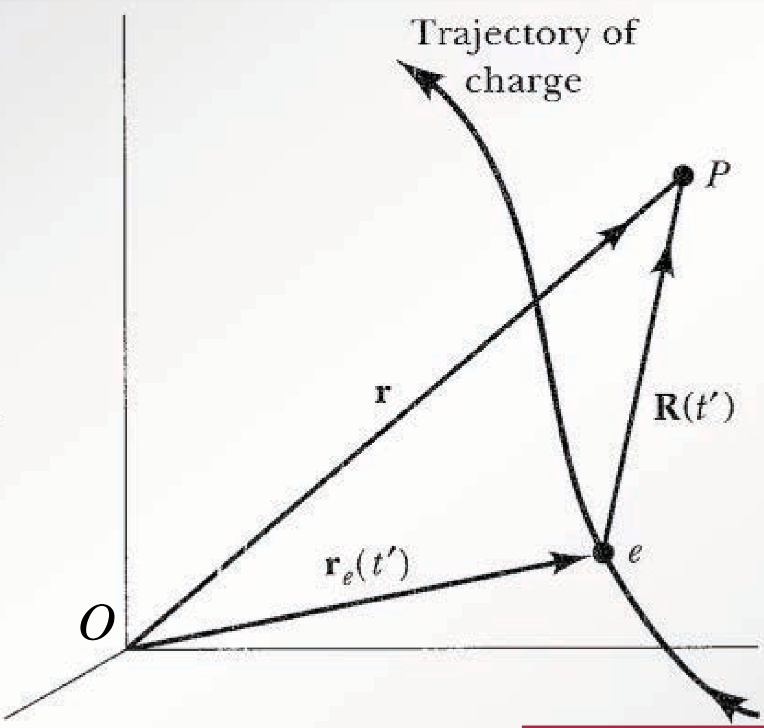
The radiation is due to the discrete nature of the electric charge

$$\frac{dJ}{dt} = 0 \quad \Rightarrow \quad \text{No radiation}$$

Retarded time

- Quantitative description of radiation must relate points on the radiation front to the place & time of emission

- When we measure a field produced by charge **e** at point **r**, at time t = t_1, => the charge sent the signal field at time t = t_r, where



$$t_r = t_1 - \frac{\left\| \mathbf{r} - \mathbf{r}_e(t)_{t=t_r} \right\|}{c}$$

- From the observed fields, we can compute the velocity of **e** *when it emitted the radiation*:

$$\mathbf{V}_e = \frac{d\mathbf{r}_e}{dt_r}$$

Larmor's formula (non-relativistic)
(Gaussian units)

- If $v_{part} << c$ and in the far field, with **n** the unit vector from the particle to the observer (evaluted at the time of emission)

$$\mathbf{E}(\mathbf{x},t) = e\left[\frac{\mathbf{n}-\beta}{\gamma^2(1-\beta\bullet\mathbf{n})^3R^2}\right]_{ret} + \frac{e}{c}\left[\frac{\mathbf{n}\times\left\{(\mathbf{n}-\beta)\times\dot{\beta}\right\}}{(1-\beta\bullet\mathbf{n})^3R}\right]_{ret}$$

- For v<<c,

$$\mathbf{E}(\mathbf{x},t) = e\left[\frac{\mathbf{n}}{R^2}\right]_{ret} + \boxed{\frac{e}{c}\left[\frac{\mathbf{n}\times\mathbf{n}\times\dot{\beta}}{(1-\beta\bullet\mathbf{n})^3R}\right]_{ret} \quad \textit{Radiation field, } E_a}$$

- The Poynting vector, S, is the instantaneous energy flux

$$S = \frac{c}{4\pi}\mathbf{E}\times\mathbf{B} = \frac{c}{4\pi}\mathbf{E}\times\left((\mathbf{n})_{ret}\times\mathbf{E}\right) = \frac{c}{4\pi}\left|\mathbf{E}_a\right|^2\mathbf{n}$$

Radiated power (Gaussian units)

❖ The radiated power per solid angle is

$$\frac{d\mathcal{P}}{d\Omega} = R^2(\mathbf{S} \bullet \mathbf{n}) = \frac{e^2}{4\pi c}\left(\dot{\beta}^2 \sin^2\vartheta\right) = \frac{e^2}{4\pi c^3}\left(\dot{v}^2 \sin^2\vartheta\right)$$

where ϑ is angle between $\mathbf{n}$ and $\dot{\mathbf{v}}$

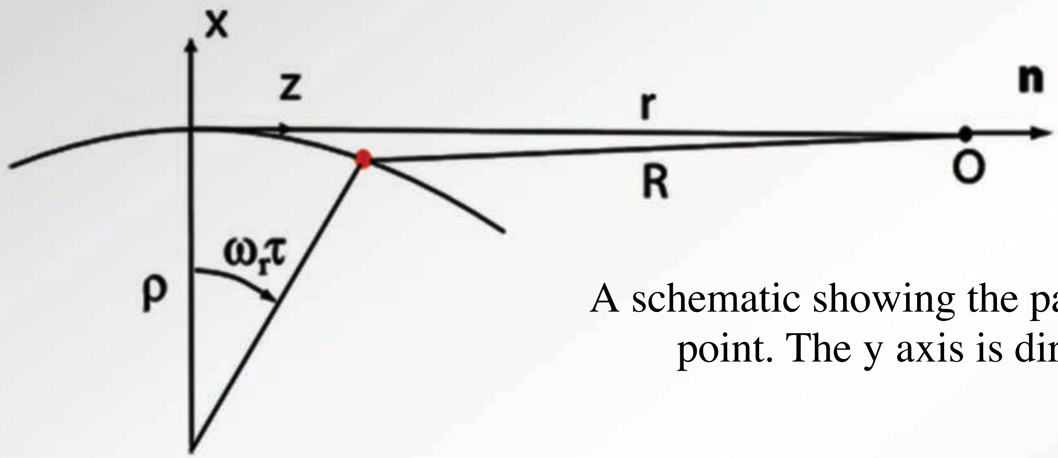
❖ Integrating over all angles we have the Larmor formula

$$\mathcal{P} = \frac{2e^2}{3c^3}\dot{\mathbf{v}}^2 = \frac{2e^2}{3m^2c^3}\left(\frac{d\mathbf{p}}{dt} \bullet \frac{d\mathbf{p}}{dt}\right)$$

❖ The covariant generalization gives the relativistic formula

$$\mathcal{P} = \frac{2e^2}{3m^2c^3}\left(\frac{dp_\mu}{d\tau} \bullet \frac{dp^\mu}{d\tau}\right) = \frac{2e^2}{3m^2c^3}\left\{\frac{1}{c^2}\left(\frac{dE}{d\tau}\right)^2 - \left(\frac{dp}{d\tau}\right)^2\right\}$$

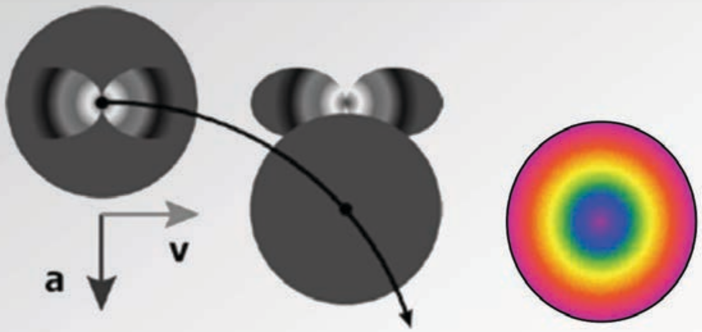
Synchrotron radiation pulses
in the plane of the orbit



A schematic showing the particle's orbit & observation point. The y axis is directed out of the page.

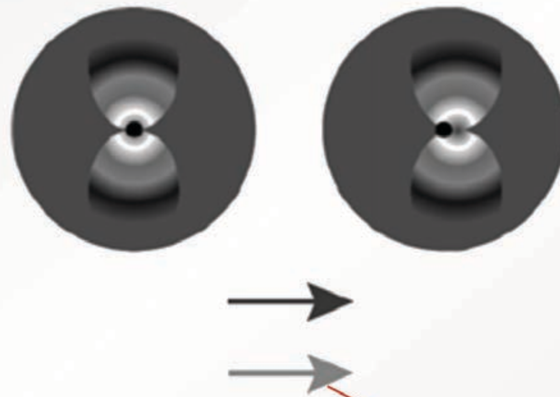
- The observer sees a periodic sequence of pulses of electromagnetic radiation with the period equal to the revolution period of the particle around the ring
- Each pulse is emitted from the region $x \approx z \approx 0$

Longitudinal vs. transverse acceleration (MKS)



Radiation field quickly separates itself from the Coulomb field

$$P_\perp = \frac{q^2}{6\pi\varepsilon_0 m_0^2 c^3}\gamma^2\left(\frac{d\mathbf{p}_\perp}{dt}\right)^2$$



Radiation field cannot separate itself from the Coulomb field

$$P_\parallel = \cancel{\frac{q^2}{6\pi\varepsilon_0 m_0^2 c^3}}\left(\frac{d\mathbf{p}_\parallel}{dt}\right)^2$$

negligible!

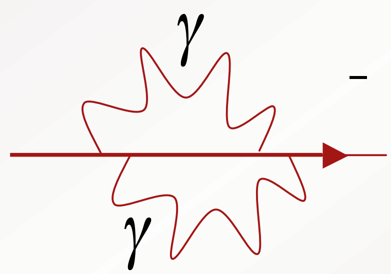
$$\boxed{P_\perp = \frac{c}{6\pi\varepsilon_0} q^2 \frac{(\beta\gamma)^4}{\rho^2} \qquad \rho = curvature \ \ radius}$$

Radiated power for transverse acceleration increases dramatically with energy

Limits the maximum energy obtainable with a storage ring

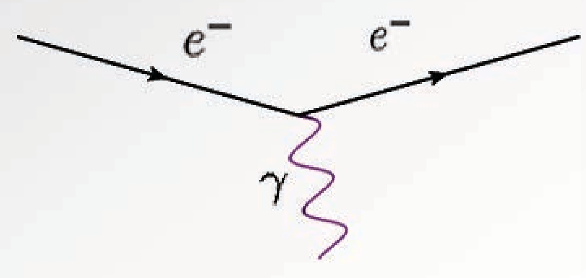
QED approach: Why do particles radiate when accelerated?

- Charged particles in free space are "surrounded" by *virtual photons* with longitudinal polarization.
  - Virtual photons appear & disappear & travel with the particles.



- The emission of a virtual photon of energy E_γ violates energy conservation for a time $\Delta\tau \leq \hbar / E_\gamma$ until the virtual photon is reabsorbed
- A force acting on the particle allows the energy to be conserved

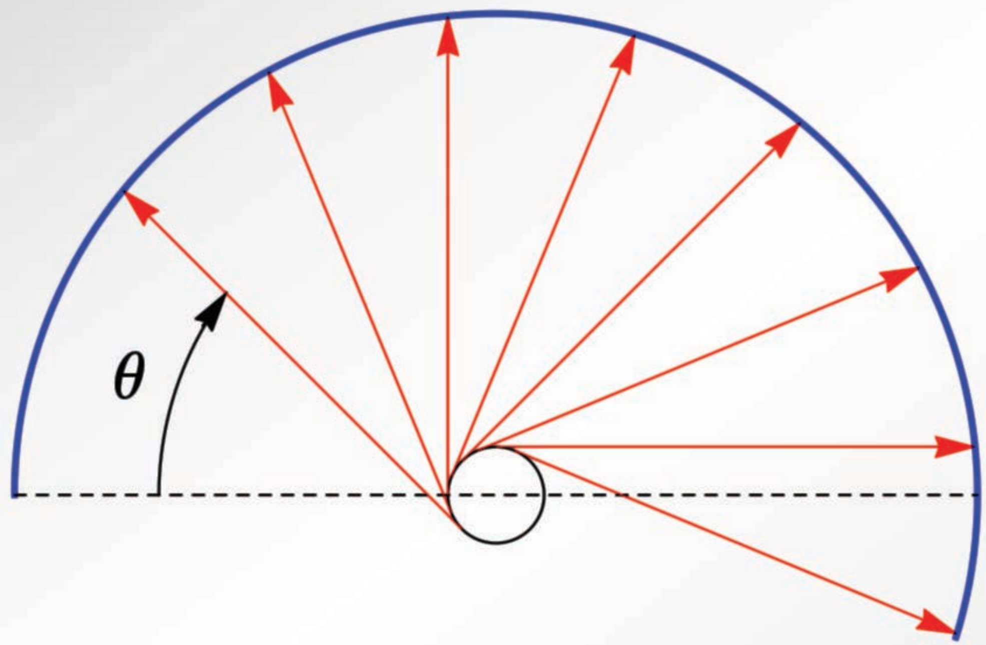
QED approach (cont'd)



- Acceleration separates the charge from the photons & "kicks" photons onto the "mass shell"
  - The electromagnetic coupling constant is α
- Lighter particles have less inertia & radiate photons more efficiently
  - For a given amount of acceleration it is easier to conserve E & p
- In the field of the dipoles in a synchrotron, charged particles move on a curved trajectory
  - Transverse acceleration generates the *synchrotron radiation*

Electrons radiate ~$\alpha\gamma$ photons per radian of turning

Radiation from a circular orbit



Synchrotron radiation fan from a circular orbit shown as a small circle at the center.

Radiation travels tangentially to the orbit & is detected on a remote surface shown in blue

Energy lost per turn by electrons

$$\frac{dU}{dt} = -P_{SR} = -\frac{2c\,r_e}{3\left(m_0c^2\right)^3}\frac{E^4}{\rho^2} \Rightarrow U_0 = \int_{finite\ \rho} P_{SR}\,dt \quad energy\ lost\ per\ turn$$

For relativistic electrons:

$$s = \beta ct \cong ct \Rightarrow dt = \frac{ds}{c}$$

$$U_0 = \frac{1}{c}\int_{finite\ \rho} P_{SR}\,ds = \frac{2r_eE_0^4}{3\left(m_0c^2\right)^3}\int_{finite\ \rho}\frac{ds}{\rho^2}$$

For dipole magnets with constant radius *r* (*iso-magnetic* case):

$$\boxed{U_0 = \frac{4\pi\,r_e}{3\left(m_0c^2\right)^3}\frac{E_0^4}{\rho} = \frac{e^2}{3\varepsilon_o}\frac{\gamma^4}{\rho}}$$

The average radiated power is given by:

$$\boxed{\left\langle P_{SR}\right\rangle = \frac{U_0}{T_0} = \frac{4\pi\,c\,r_e}{3\left(m_0c^2\right)^3}\frac{E_0^4}{\rho L} \qquad where\ L \equiv\ ring\ circumference}$$

Energy loss to synchrotron radiation (practical units)

Energy Loss per turn (per particle)

$$U_{o,electron}(keV) = \frac{e^2\gamma^4}{3\varepsilon_0\rho} = 88.46\frac{E(GeV)^4}{\rho(m)}$$

$$U_{o,proton}(keV) = \frac{e^2\gamma^4}{3\varepsilon_0\rho} = 6.03\frac{E(TeV)^4}{\rho(m)}$$

Power radiated by a beam of average current I_b: to be restored by RF system

$$P_{electron}(kW) = \frac{e\gamma^4}{3\varepsilon_0\rho}I_b = 88.46\frac{E(GeV)^4 I(A)}{\rho(m)}$$

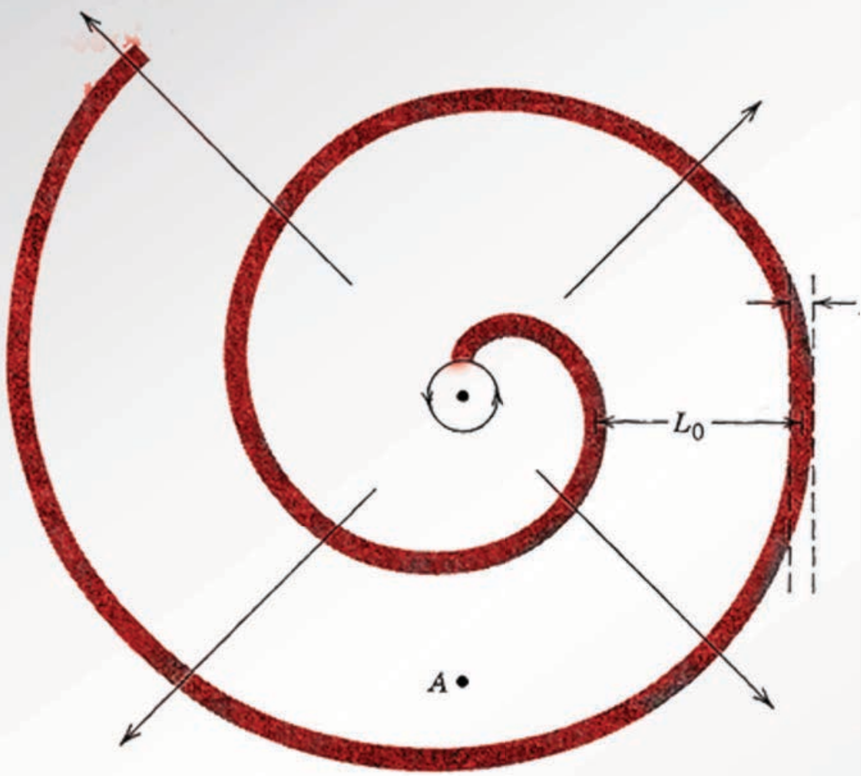
$$P_{proton}(kW) = \frac{e\gamma^4}{3\varepsilon_0\rho}I_b = 6.03\frac{E(TeV)^4 I(A)}{\rho(m)}$$

Power radiated by a beam of average current I_b in a dipole of length L (energy loss per second)

$$P_e(kW) = \frac{e\gamma^4}{6\pi\varepsilon_0\rho^2}LI_b = 14.08\frac{L(m)I(A)E(GeV)^4}{\rho(m)^2}$$

Radiation from an ultra-relativistic particle

Only acceleration ⊥ to the velocity produces appreciable radiation



==> consider radiation by a particle in a circular orbit

1. Radiation pattern spirals outward
2. Radiation is emitted in a cone of angle

$$\boxed{\Delta\theta \approx 1/\gamma}$$

The charge travels

$$d \sim \rho\Delta\theta \sim \rho/\gamma$$

while it illuminates the observer

Frequency spectrum

- ==> Radiation that sweeps by the observer is emitted during the *retarded time period*

$$\Delta t_{ret} \sim \frac{d}{v} \sim \frac{\rho}{\gamma v}$$

- The front edge of the radiation travels

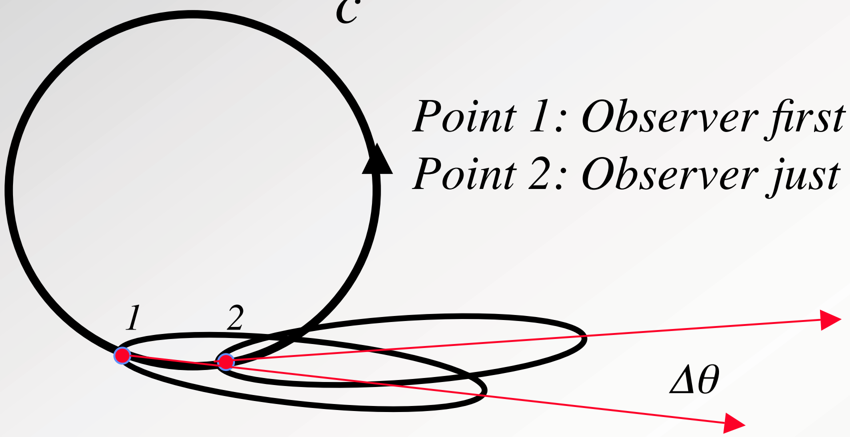
$$D = c\Delta t_{ret} \sim \frac{\rho c}{\gamma v} = \frac{\rho}{\gamma \beta}$$

- The trailing edge is at a distance L = D - d behind the front since the charged moved by *d* in the same period

Convert retarded time to observer's time

- The pulse width for the distant observer is

$$\frac{L}{c} = t_2 - t_1 = \left(t_{r,2} + R_2/c\right) - \left(t_{r,1} + R_1/c\right) = \Delta t_r - \left(R_1 - R_2\right)/c$$



Point 1: Observer first begins to see pulse
Point 2: Observer just stops seeing pulse

Since $R_1 - R_2 \approx v_r \Delta t_r$

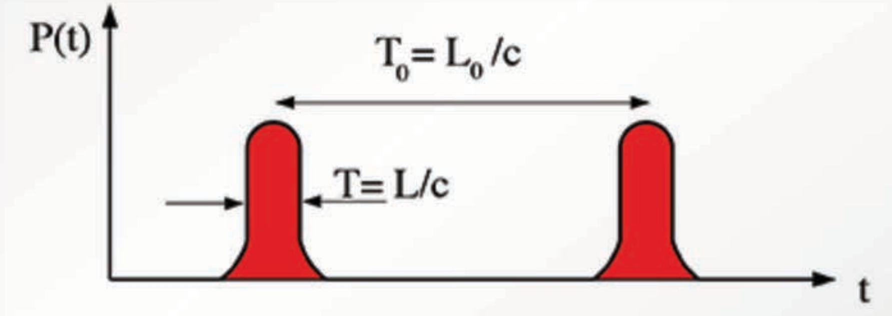
$$\Delta t_{obs} = \frac{L}{c} = t_2 - t_1 = \Delta t_r - \left(R_1 - R_2\right)/c = \Delta t_r\left(1 - v_r/c\right) \approx \frac{\Delta t_r}{2\gamma^2}$$

$$\Delta t_{obs} \approx \frac{\Delta t_r}{2\gamma^2} \approx \frac{1}{2\gamma^3}\frac{\rho}{v}$$

Frequency spectrum is the Fourier transform of the pulse

- Radiation that sweeps by the observer is emitted by the particle during the *retarded time period*

$$\Delta t_{ret} \approx \rho / \gamma c$$



- For the observer

$$\Delta t_{obs} = \Delta t_{ret} \frac{dt_{obs}}{dt_{ret}} \sim \frac{1}{2\gamma^2} \Delta t_{ret}$$

∴ the observer sees $\Delta\omega \sim 1/\Delta t_{obs}$

$$\boxed{\langle \Delta\omega \rangle \sim \frac{c}{2\rho}\gamma^3 \sim \frac{\omega_{rev}\gamma^3}{2}} \quad \text{====>} \quad \boxed{\langle \varepsilon_\gamma \rangle = \hbar \langle \Delta\omega \rangle \sim \frac{\hbar\omega_{rev}\gamma^3}{2}}$$

Number of photons emitted
(very important result)

- Since the energy lost per turn is

$$U_0 \sim \frac{e^2\gamma^4}{\rho}$$

- And average energy per photon is the

$$\langle \varepsilon_\gamma \rangle \approx \frac{1}{3}\varepsilon_c = \frac{\hbar\omega_c}{3} = \frac{1}{2}\frac{\hbar c}{\rho}\gamma^3$$

- The average number of photons emitted per revolution is

$$\boxed{\langle n_\gamma \rangle \approx 2\pi\alpha_{fine}\gamma}$$

This is the direct result in quantum electrodynamics

In class exercise (10 minutes):
The Advanced Light Source @ Berkeley

ALS is a 300 m, ~1.5 GeV storage ring with 1.5 kG dipoles

- ➢ Bend magnets are a small fraction of the circumference

Part 1

- ❖ *Estimate the average photon energy & the critical energy*

Part 2 (5 minutes):

- ❖ The average current = 250 mA

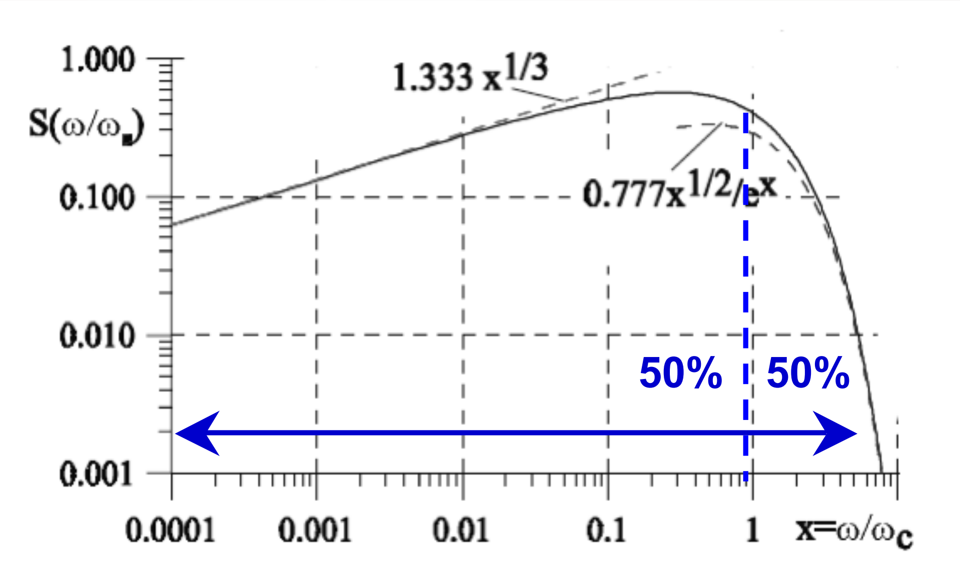
How many photons per second are emitted from a 10 cm long section of dipole magnet

Frequency distribution of radiation

The integrated spectral density (see appendix) up to the critical frequency contains half the total energy radiated; the peak occurs approximately at $0.3\omega_c$

where the critical photon energy is

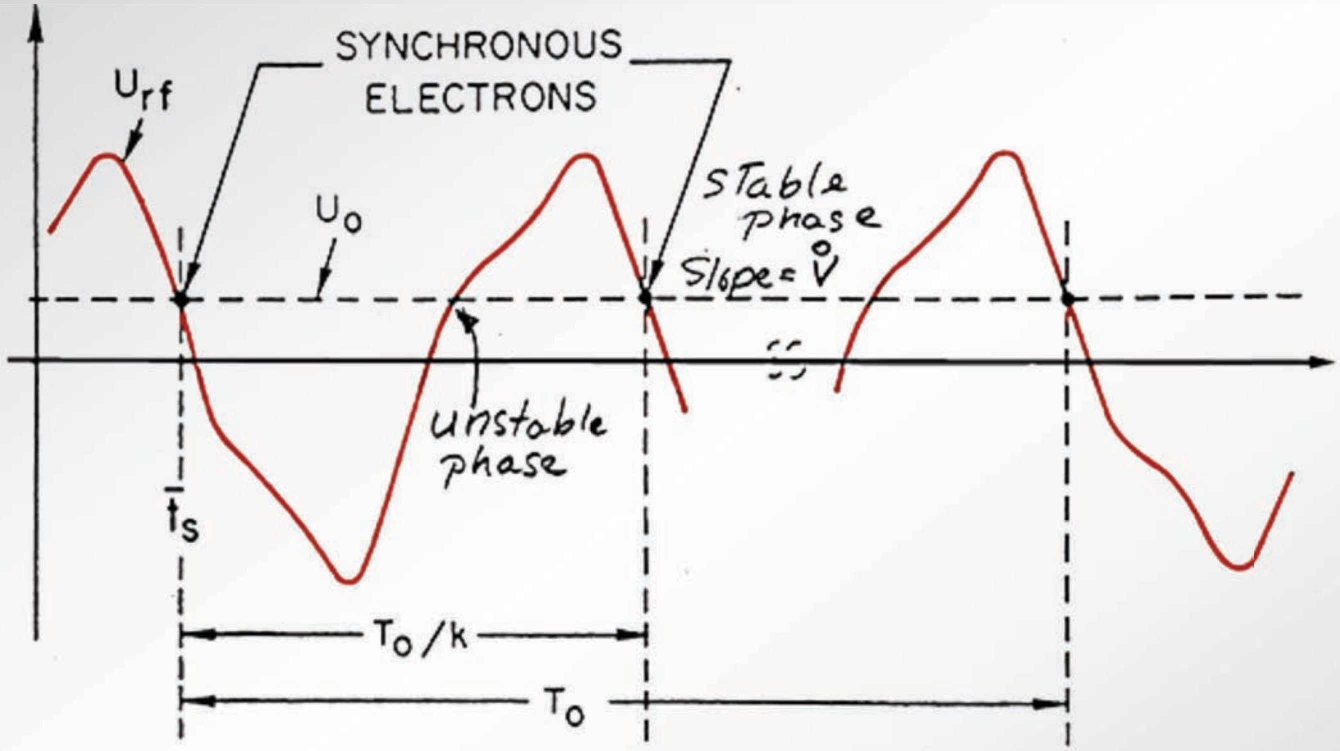
$$\varepsilon_c = \hbar\omega_c = \frac{3}{2}\frac{\hbar c}{\rho}\gamma^3$$



For *electrons*, the critical energy in practical units is

$$\boxed{\varepsilon_c[keV] = 2.218\frac{E[GeV]^3}{\rho[m]} = 0.665 \cdot E[GeV]^2 \cdot B[T]}$$

RF system restores energy loss



Particles change energy according to the phase of the field in the RF cavity

$$\Delta E = eV(t) = eV_o \sin(\omega_{RF} t)$$

For the synchronous particle

$$\boxed{\Delta E = U_0 = eV_0 \sin(\varphi_s)}$$

Comparison of S.R. Characteristics

		LEP200	LHC	SSC	HERA	VLHC
Beam particle		e+ e-	p	p	p	p
Circumference	km	26.7	26.7	82.9	6.45	95
Beam energy	TeV	0.1	7	20	0.82	50
Beam current	A	0.006	0.54	0.072	0.05	0.125
Critical energy of SR	eV	$7\ 10^{5}$	44	284	0.34	3000
SR power (total)	kW	$1.7\ 10^{4}$	7.5	8.8	$3\ 10^{-4}$	800
Linear power density	W/m	882	0.22	0.14	$8\ 10^{-5}$	4
Desorbing photons	$s^{-1}\ m^{-1}$	$2.4\ 10^{16}$	$1\ 10^{17}$	$6.6\ 10^{15}$	none	$3\ 10^{16}$

From: O. Grobner CERN-LHC/VAC VLHC Workshop Sept. 2008

Synchrotron radiation plays a major role in dynamics of electron storage rings

- Charged particles radiate when accelerated
- Transverse acceleration induces significant synchrotron radiation

longitudinal acceleration generates negligible radiation ($1/\gamma^2$).

$$\frac{dU}{dt} = -P_{SR} = -\frac{2 c\, r_e}{3\left(m_0 c^2\right)^3} \frac{E^4}{\rho^2}$$

$$U_0 = \int_{finite\ \rho} P_{SR}\, dt \quad \textit{energy lost per turn}$$

$$\alpha_D = -\frac{1}{2T_0} \frac{dU}{dE}\bigg|_{E_0} = \frac{1}{2T_0} \frac{d}{dE} \left[\oint P_{SR}(E_0)\, dt \right]$$

$$\alpha_{DX}, \alpha_{DY} \quad \textit{damping in all planes}$$

$$\frac{\sigma_p}{p_0} \quad \textit{equilibrium momentum spread and emittances}$$
$$\varepsilon_X, \varepsilon_Y$$

Energy loss + dispersion ==>
Longitudinal (synchrotron) oscillations

Longitudinal dynamics are described by

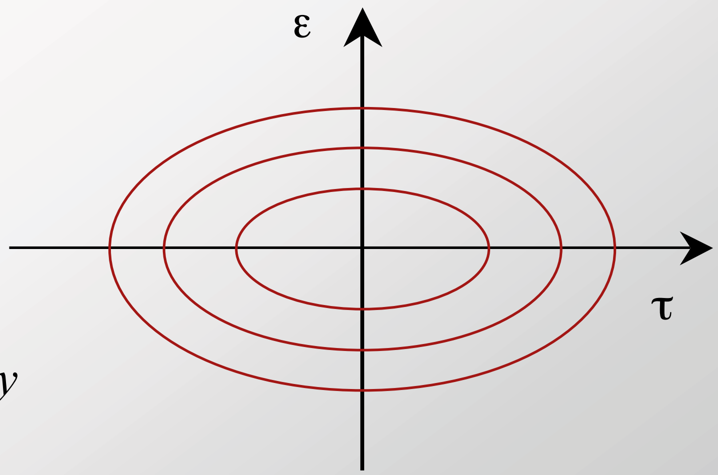
1) **ε**, energy deviation, w.r.t the synchronous particle

2) **τ**, time delay w.r.t. the synchronous particle

$$\varepsilon' = \frac{qV_0}{L}\left[\sin(\phi_s + \omega\tau) - \sin\phi_s\right] \quad \text{and} \quad \tau' = -\alpha$$

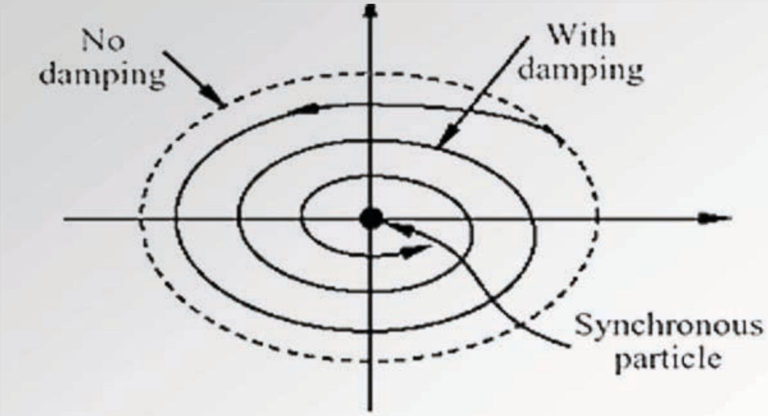
Linearized equations describe elliptical phase space trajectories

$$\varepsilon' = \frac{e}{T_0}\frac{dV}{dt}\tau \qquad \tau' = -\frac{\alpha_c}{E_s}\varepsilon$$



$$\boxed{\omega_s^2 = \frac{\alpha_c e\dot{V}}{T_0 E_0}}$$ *angular synchrotron frequency*

Radiation damping of energy fluctuations



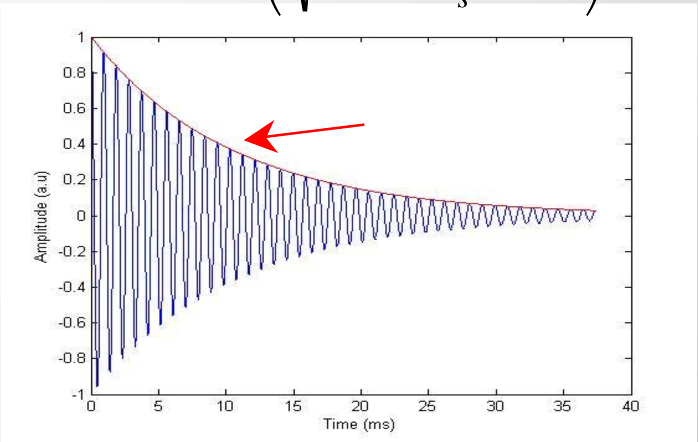
The derivative $\frac{dU_0}{dE}$ (>0)
is responsible for the damping of the longitudinal oscillations

Combine the two equations for (ε, τ) in a single 2nd order differential equation

$$\frac{d^2\varepsilon}{dt^2} + \frac{2}{\tau_s}\frac{d\varepsilon}{dt} + \omega_s^2\varepsilon = 0 \longrightarrow \varepsilon = Ae^{-t/\tau_s}\sin\left(\sqrt{\omega_s^2 - \frac{4}{\tau_s^2}}\,t + \varphi\right)$$

$$\omega_s^2 = \frac{\alpha e\dot{V}}{T_0 E_0}$$ **angular synchrotron frequency**

$$\frac{1}{\tau_s} = \frac{1}{2T_0}\frac{dU_0}{dE}$$ **longitudinal damping time**



Damping times

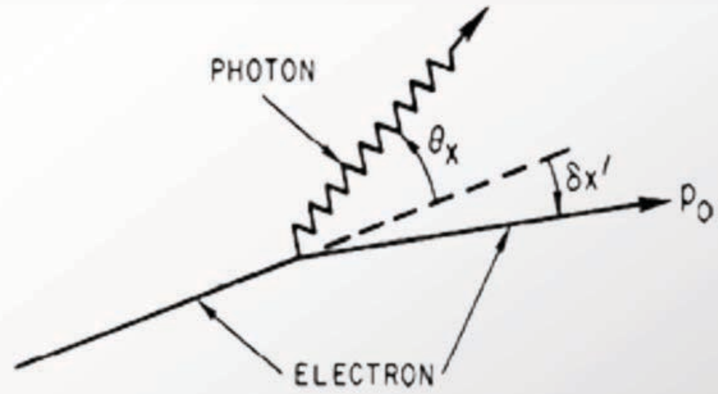
- The energy damping time ~ the time for beam to radiate its original energy
- Typically

$$T_i = \frac{4\pi}{C_\gamma} \frac{R\rho}{J_i E_o^3}$$

- Where $J_e \approx 2$, $J_x \approx 1$, $J_y \approx 1$ and $C_\gamma = 8.9 \times 10^{-5} \, meter - GeV^{-3}$
- Note $\Sigma J_i = 4$ (partition theorem)

Quantum Nature of Synchrotron Radiation

- Synchrotron radiation induces damping in all planes.
  - Collapse of beam to a single point is prevented by the *quantum nature of synchrotron radiation*
- Photons are randomly emitted in quanta of discrete energy
  - Every time a photon is emitted the parent electron "jumps" in energy and angle
- Radiation perturbs excites oscillations in all the planes.
  - Oscillations grow until reaching *equilibrium* balanced by radiation damping.



Energy fluctuations

- Expected $\Delta E_{quantum}$ comes from the deviation of $<\mathscr{N}_\gamma>$ emitted in one damping time, τ_E

- $<\mathscr{N}_\gamma> = n_\gamma \tau_E$

$$\Longrightarrow \Delta<\mathscr{N}_\gamma> = (n_\gamma \tau_E)^{1/2}$$

- The mean energy of each quantum $\sim \varepsilon_{crit}$

- $\Longrightarrow \sigma_\varepsilon = \varepsilon_{crit}(n_\gamma \tau_E)^{1/2}$

- Note that $n_\gamma = P_\gamma / \varepsilon_{crit}$ and $\tau_E = E_o / P_\gamma$

Therefore, …

- The quantum nature of synchrotron radiation emission generates energy fluctuations

$$\frac{\Delta E}{E} \approx \frac{\langle E_{crit} E_o \rangle^{1/2}}{E_o} \approx \frac{C_q \gamma_o^2}{J_\varepsilon \rho_{curv} E_o} \sim \frac{\gamma}{\rho}$$

where C_q is the Compton wavelength of the electron

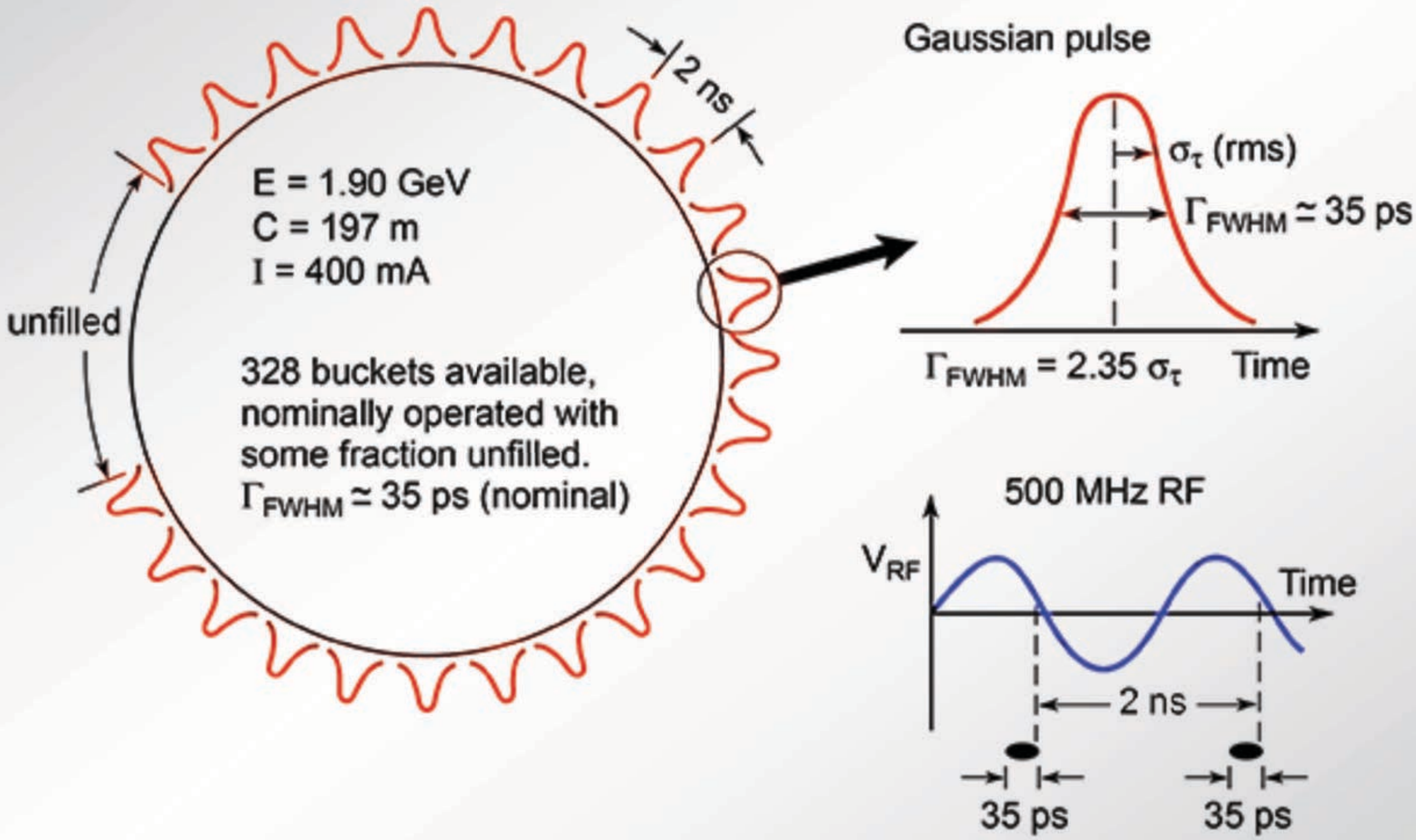
$$C_q = 3.8 \times 10^{-13} \text{ m}$$

- Bunch length is set by the momentum compaction & V_{rf}

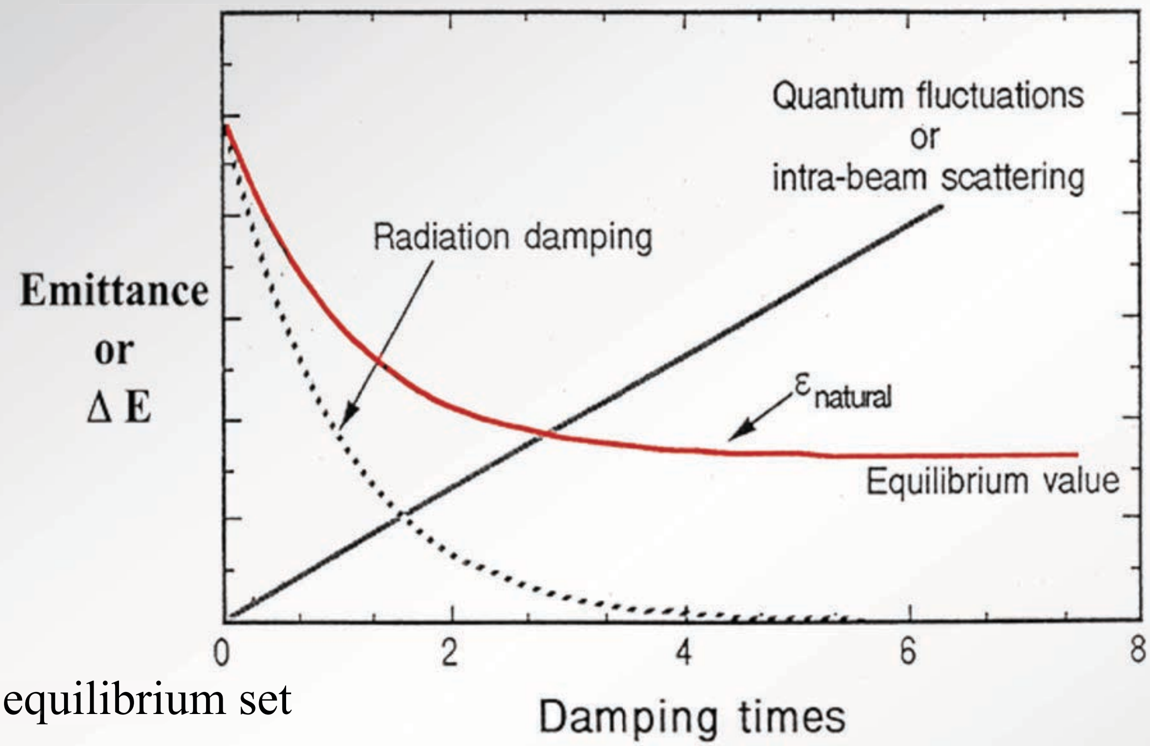
$$\sigma_z^2 = 2\pi \left(\frac{\Delta E}{E} \right) \frac{\alpha_c R E_o}{e \dot{V}}$$

- Using a harmonic rf-cavity can produce shorter bunches

Energy spread & rf system create the time structure of electrons & the SR



Equilibrium emittance & energy spread



- At equilibrium set

Growth rate due to fluctuations (linear) = exponential damping rate due to radiation

==> equilibrium value of emittance or ΔE

$$\varepsilon_{natural} = \varepsilon_1 e^{-2t/\tau_d} + \varepsilon_{eq}\left(1 - e^{-2t/\tau_d}\right)$$

Quantum lifetime

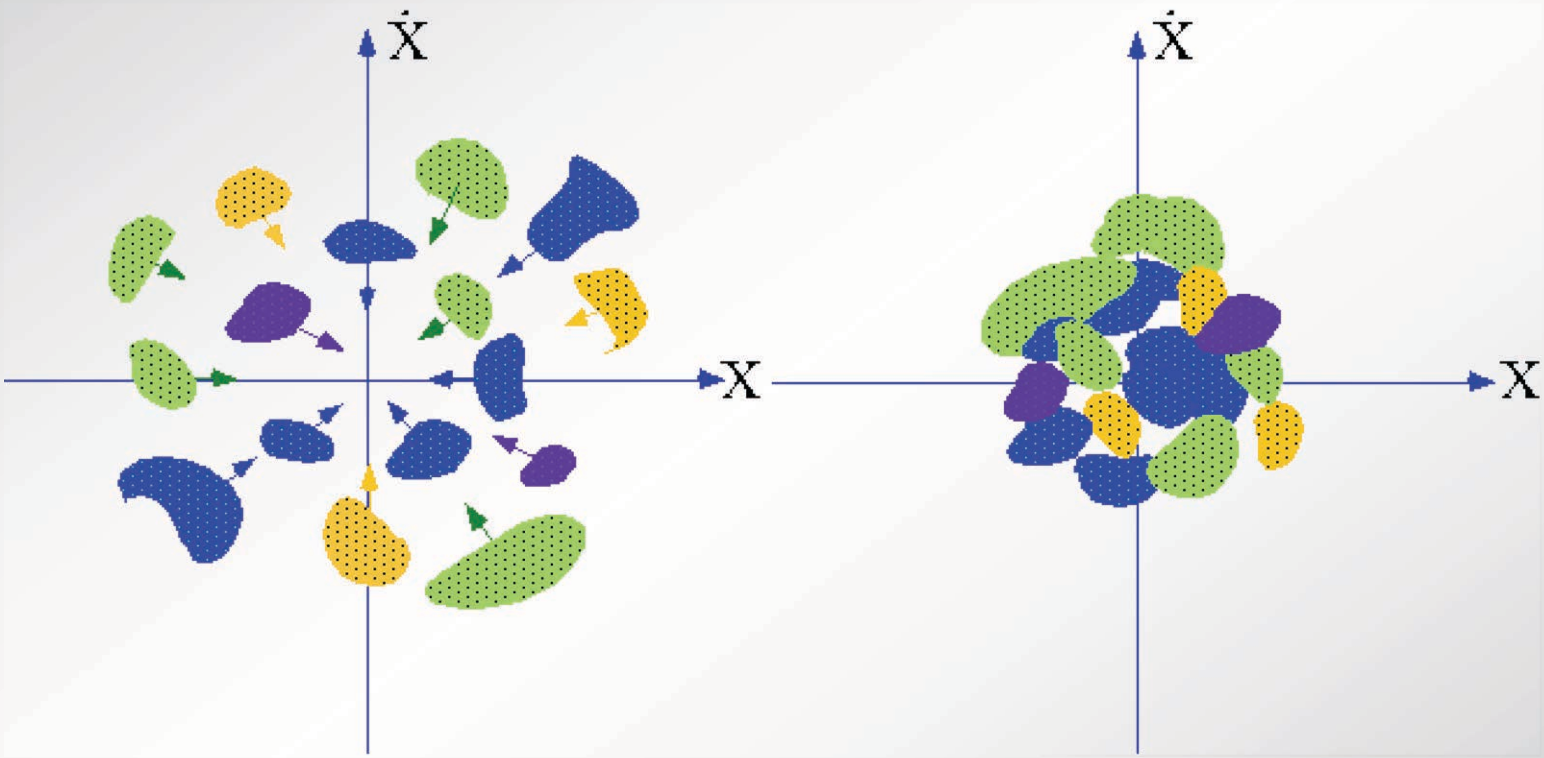
- At a fixed observation point, transverse particle motion looks sinusoidal

$$x_T = a\sqrt{\beta_n}\,\sin\left(\omega_{\beta_n} t + \varphi\right) \qquad T = x, y$$

- Tunes are chosen in order to avoid resonances.
  - At a fixed azimuth, turn-after-turn a particle sweeps all possible positions within the envelope
- Photon emission randomly changes the “invariant” *a*
  - Consequently changes the trajectory envelope as well.
- Cumulative photon emission can bring the envelope beyond acceptance at some azimuth
  - The particle is lost

This mechanism is called the transverse quantum lifetime

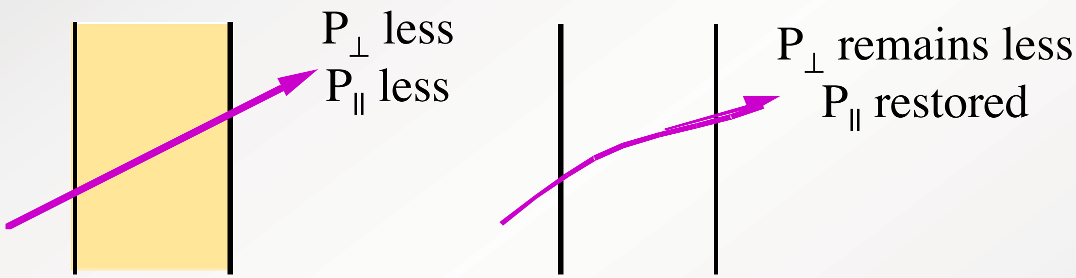
Is it possible to cool the beam?



Phase-Space Cooling in Any One Dimension

Schematic of radiation cooling

Transverse cooling:



Passage through dipoles

Acceleration in RF cavity

Limited by quantum excitation

Why must high luminosity colliders operate at very low pressures ?

1) Background gas causes beam loss via

- Elastic scattering
- Inelastic scattering (bremsstrahlung)
- Ion trapping

❖ Beam lifetime:

$$\frac{1}{\tau_g} = \frac{1}{N_b}\frac{d N_b}{dt} = 3.22 \times 10^{22}\, n_z P_{Torr}\, \beta c\, (\sigma_{el} + \sigma_{Br})$$

where n_z is the number of molecules of species z.

❖ At high energy bremsstrahlung dominates. We expect

$$\tau_g \approx 3 \text{ hr @ } 10 \text{ nTorr}$$

… & near the collision point

2) Hard photons & scattered electrons striking apertures generate backgrounds in the detector (depends on masking and lattice)

- Background µ Pressure in interaction region
- Sources of gas:
  - Thermal out-gassing,
  - Photo-desorption
  - Leaks)

Generic issues of vacuum system design

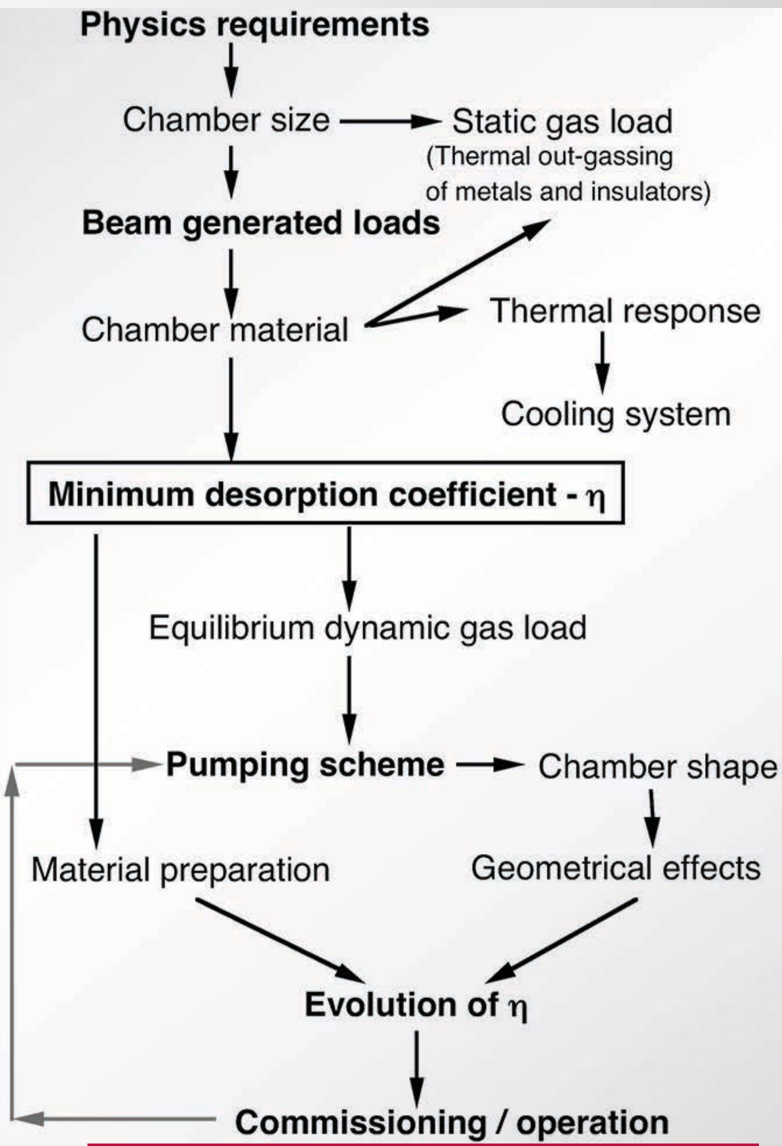
- Thermal loads: 1 - 5 kW/m —> 5 – 40 kW/m
  - Cooling, thermal fatigue, 7 technical risk issues
- Photon flux: 5×10^{17} photons/s/m —> $\approx 10^{19}$
  - Chamber materials & preparation for low design η
  - Commissioning time
- Choice of materials: Stainless steel —> cladded Al & Cu
  - Fabrication & cost issues
- Chamber shape: elliptical ==> Complex (antechambers)
  - affects fabrication complexity, costs, magnet designs
- Pumping speeds: 100 - 300 L/s/m —> up to 3000 L/s/m
  - impacts choice of pumps, chamber design
- High current increases consequences of fault modes

Practical consequences in storage rings of synchrotron radiation

Vacuum System Components

- Beam chamber
  - Provide sufficient beam aperture
  - Provide good vacuum environment
  - Shield magnets, electronics from synchrotron radiation (SR)
- Pumping system
  - Maintain operating vacuum
    - 1 - 10 nTorr in arcs
    - 1 - 3 nTorr in straight sections
    - $\approx$ 0.2 nTorr near IR
  - Provide for rapid commissioning
- Cooling system
  - Waste heat removal at high synchrotron radiation flux
  - Ensure mechanical stability of chamber
- Special components
  - Ports, Bellows, Transitions, Diagnostics
  - Consistent with low impedance

Iterative design of vacuum system



Thermal load from radiation

- Distributed over the dipole arcs the average thermal load is

$$P_L = 1.26 \text{ kW/m } E^2_{GeV} I_A B^2_T$$

- In terms of collider parameters this yields

$$P_L = 19.5 \frac{\text{kW}}{\text{m}} \left(\frac{L}{10^{34}}\right)\left(\frac{\beta_y^*}{1 \text{ cm}}\right)\left(\frac{0.03}{\xi}\right)\left(\frac{B_D}{1 \text{ T}}\right)^2 \frac{E_{GeV}}{1+r}$$

==> 5 kW/m (PEP-II) - 40 kW/m @ 3 x 10^{33} cm^{-2} s^{-1} in B factory designs (9 GeV, 3 A)

- Small beam height raises issue of thermal fatigue, influences choice of alloy; even 10 kW/m --> residual plastic strain
- Cycles to fatigue failure is a strong function of residual strain

Design approach for PEP-II: 1) Keep material in elastic regime;

2) Keep high load regions always in compression

==> Minimized technical risk in engineering realization

Choosing material of vacuum chambers

	Al	Cu	SS
Photo-desorption	+	++	++
Self-shielding	-	++	++
Thermal conductivity	+	+ to ++	-
Strength	+	- to +	++
Ease of fabrication	++	+	++
Experience	++	+	++
Cost	$	$$	$$

- Choice for PEP-II is Class 1 OFE Cu:
  - superior vacuum properties
  - superior thermal properties - eases thermal management
  - superior self-shielding - eliminates Pb cladding

Thermal loads in e^+ & e^- rings

- Each beam generates a synchrotron radiation power, P_{sr},

$$P_{sr} = 88.5 \text{ Watts } E^4_{GeV}\, I_{mA} / \rho_m , \qquad (1)$$

or in terms of the B-field in Tesla,

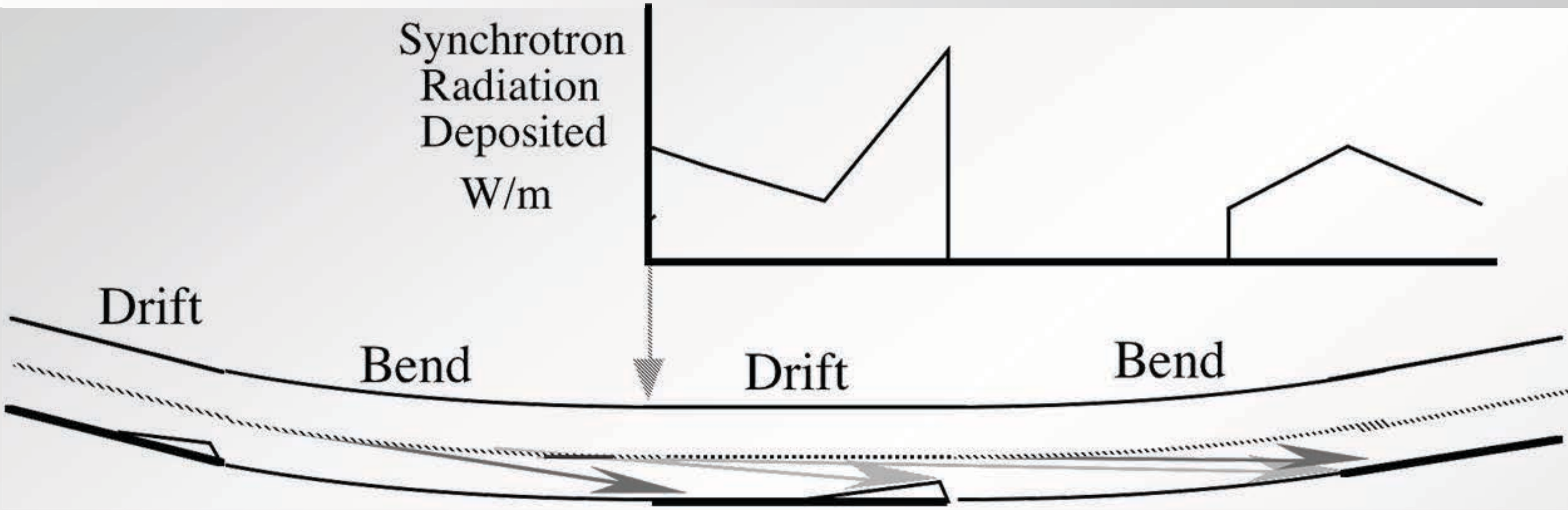
$$P_{SR} = 26.5 \text{ KW } E^3_{GEV}\, I_A\, B_T$$

- Were the radiation is deposited over $2\pi\rho$, the linear power density deposited by each beam on the walls, P_L, would be

$$P_L = 1.26 \text{ kW/m } E^2_{GeV}\, I_A\, B^2_T$$

But, the power is not deposited uniformly along the vacuum chamber

Examples of non-uniform power deposition



- Another example, for a phi factory, E = 0.51, B_T = 4 T, & I = 1.2 A.
  - The average thermal load per beam is ≈ 6 kW/m.
  - Both beams circulate in the same ring ==> radiation fans overlap at the center of the bends ==>
  - local thermal load of ≈12 kW/m for an conventional elliptical vacuum chamber.

Areal power density

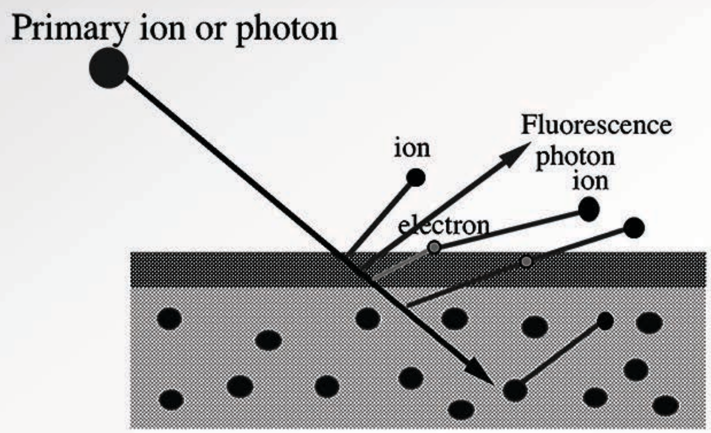
- Power density on the walls depends on the height, h, of the radiation fan at the wall
- h is a function of radiation angle from the beam & on distance, d, from the beam orbit to the wall

$$\theta \approx \frac{m\,c^2}{E} = \gamma^{-1} \qquad (4)$$

- The vertical spread of the fan is

$$h = \pm\left[\sigma_y^2 + d^2\left(\left(\frac{\varepsilon}{\sigma}\right)_y^2 + \theta^2\right)\right]^{1/2}. \qquad (5)$$

Photodesorption ==> large dynamic gas load



Scaling of desorption (for photons):

- Weak energy dependence $< \sqrt{E}$
- Angular dependance $< 1/(\sin \phi)$
- Strong variation with surface treatment
- Strong variation with surface exposure
- Strong variation with desorbed species

For beam scattering limits use CO equivalents

For pump regeneration times use molecular load

PEP-II design is based on experiments conducted as part of our R&D program

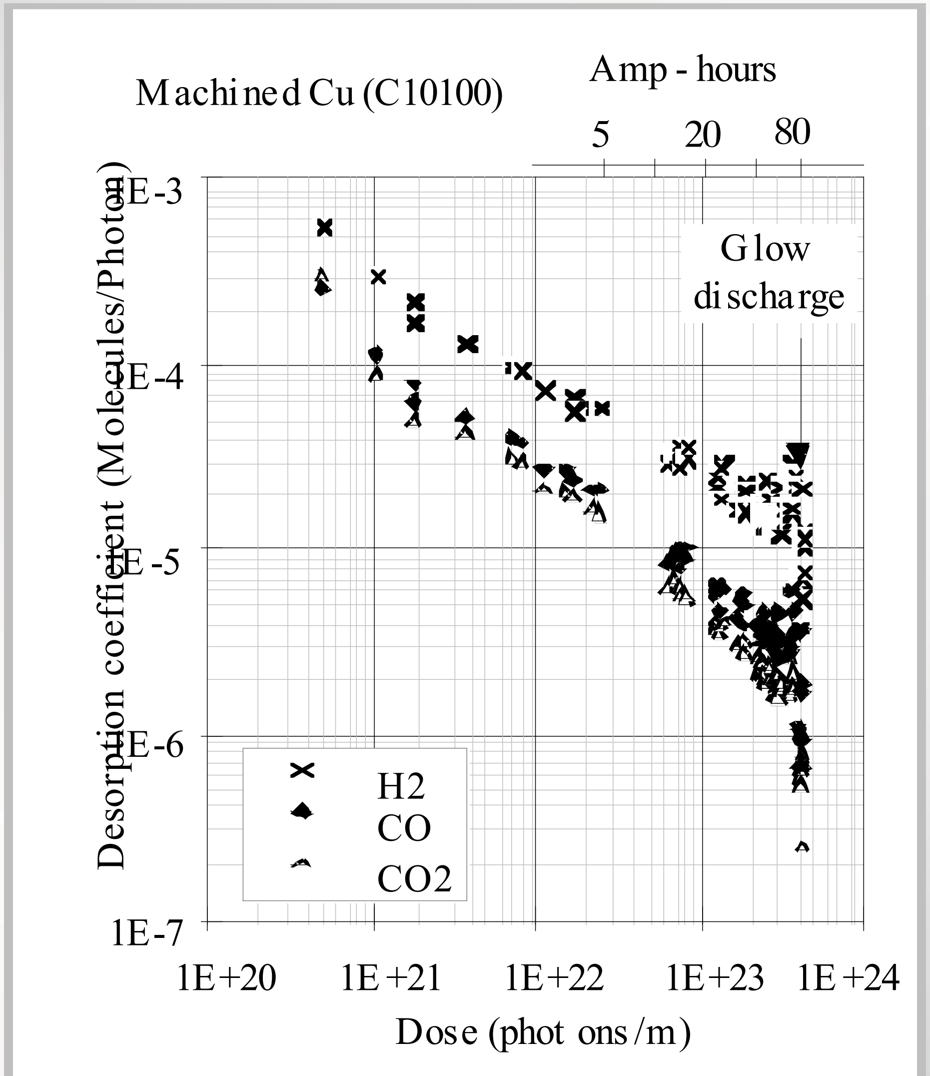
Conservative design accounts for non-uniformity of radiation distribution

- The critical parameter, η_F, is a non-linear function of
  1) photon dose (>10-to-1 variation in LER)
  2) material
  3) fabrication and preparation
  4) incidence angle of photons
  5) photon energy

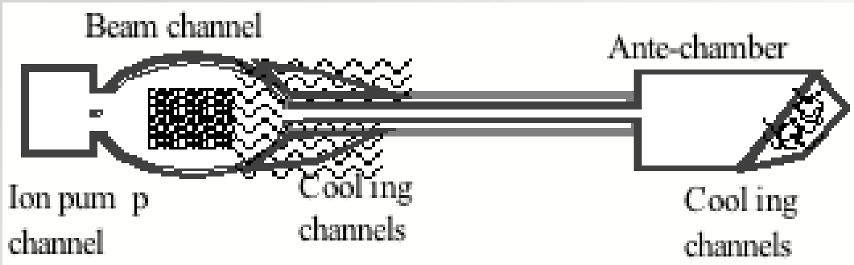
  ==> *Regard η as an average engineering parameter*

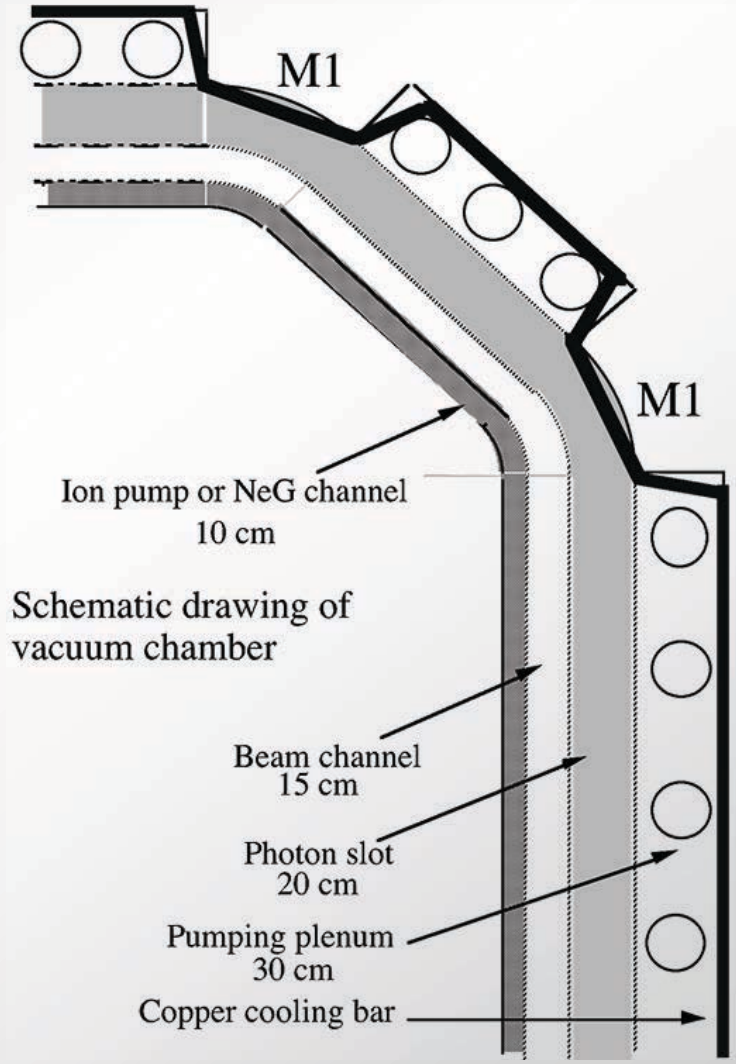
- Accurate modeling of gas load & ring commissioning accounts for the variation in dose to the chamber due to
  1) bend v. straight chamber geometry
  2) illumination by primary v. secondary photons

We measured η for many materials



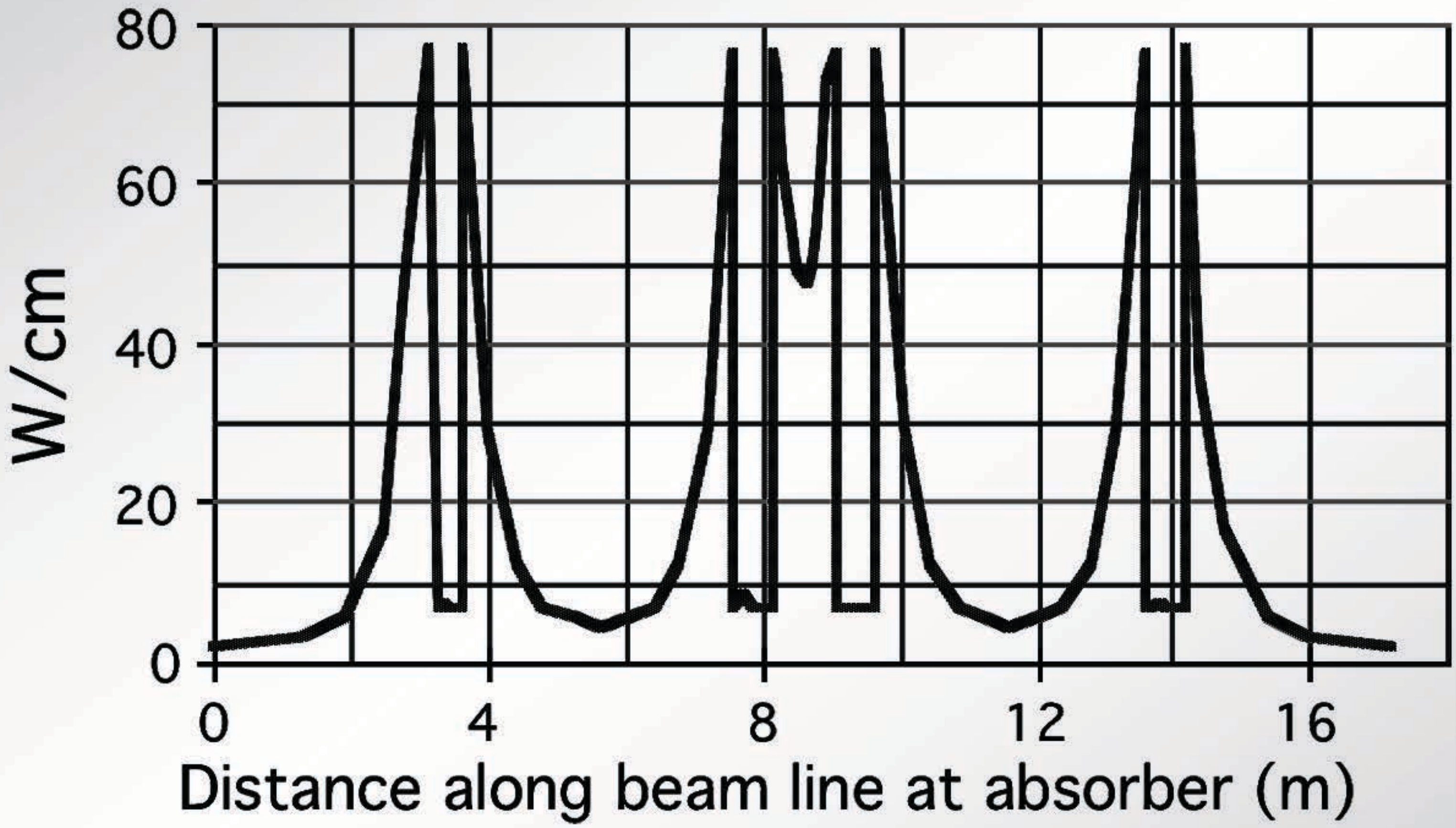
Phi factory example





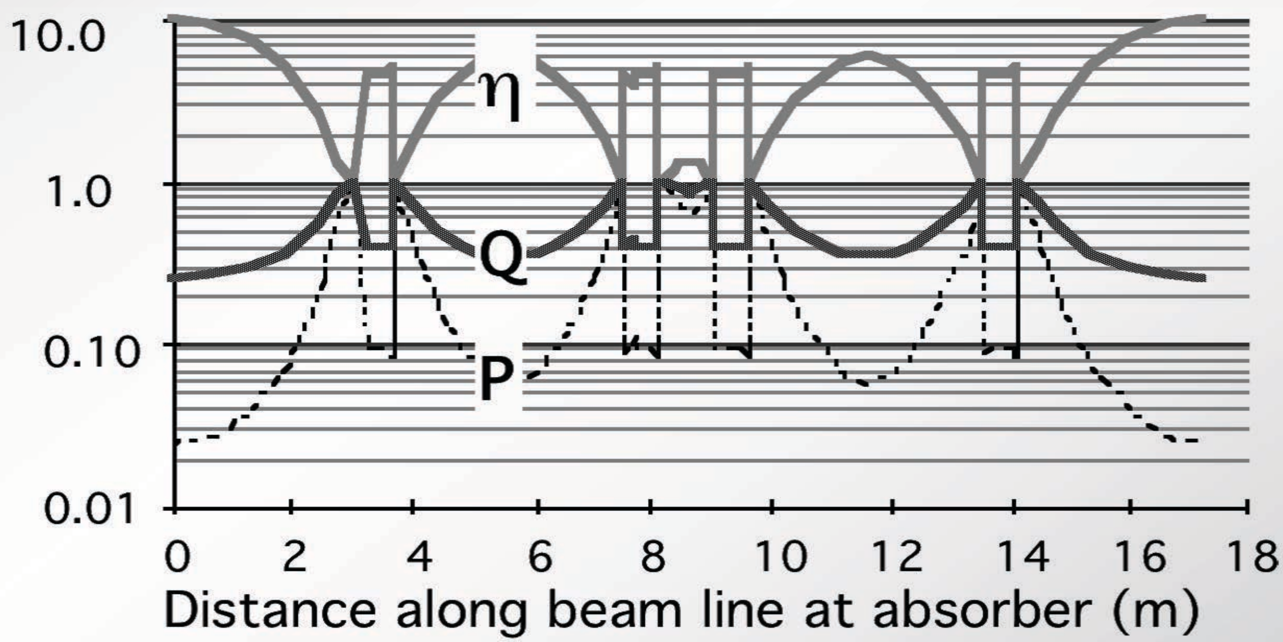
For beam chamber of width $D = 0.27$ m, connected via a thin duct of length, $L = 0.2$ m, to an ante-chamber of width, $l = 0.14$ m with $\rho = 0.42$ m, the distance to the wall, $d = 0.79$ m. Thus, from Eq. (5) $h = \pm 5$ mm in the central dipole of the arcs and $\pm$ 2 mm elsewhere. For the phi factory $P_{sr} \approx 36$ kW, and the maximum power density is ≈ 180 W/cm^2.

Actual distribution along the beamline



Modeling η for a chamber

- Actual rate is governed by the scattered photons that clean the majority of the surface
- Gas load is leveled ==> Much more pumping is required
- Commissioning times are extended



Gas loads set pumping requirements

- Synchrotron radiation produces

$$\dot{N}_{mol} = 8.08 \times 10^{17} E_{GeV}\, I_{mA}\, \eta_F \quad \text{molecules / sec}$$

or

$$Q\ (\text{gas}) = \dot{N}\ (\text{gas}) = 2.4 \times 10^{-2} E_{GeV}\, I_{mA} \eta_F \frac{\text{Torr - l}}{\text{s}} .$$

- In terms of collider parameters we have a pumping requirement of

$$S = 1.2 \times 10^5 \frac{\text{L}}{\text{s}} \left(\frac{\mathsf{L}}{10^{34}}\right)\left(\frac{\beta_y^*}{1\ \text{cm}}\right)\left(\frac{0.03}{\xi}\right)\left(\frac{10\ \text{nTorr}}{P}\right)\left(\frac{\langle \eta_F \rangle}{2 \times 10^{-6}}\right)$$

- For $\eta_{F,min} = 2 \times 10^{-6}$

==> ~ 125 L/s/m (PEP-II) to 3000 L/s/m in some B factory designs (Cornell)

Example from early B-factory design

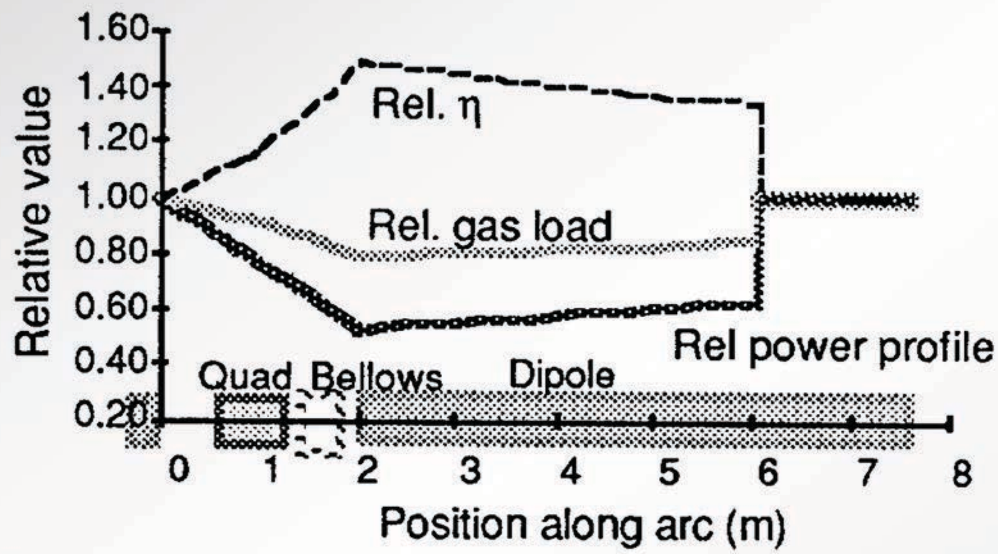


Figure 1. Relative distribution of synchrotron radiation, η and gas load in arcs of 9 GeV APIARY ring.

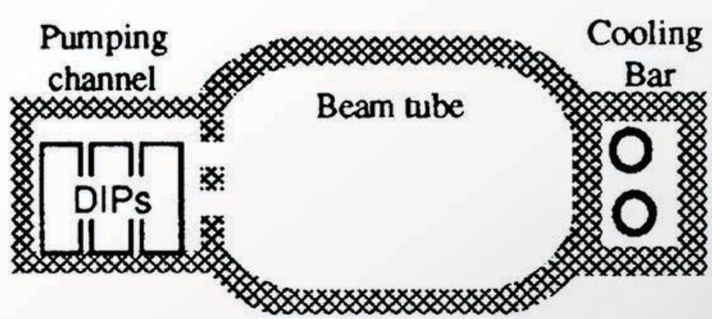
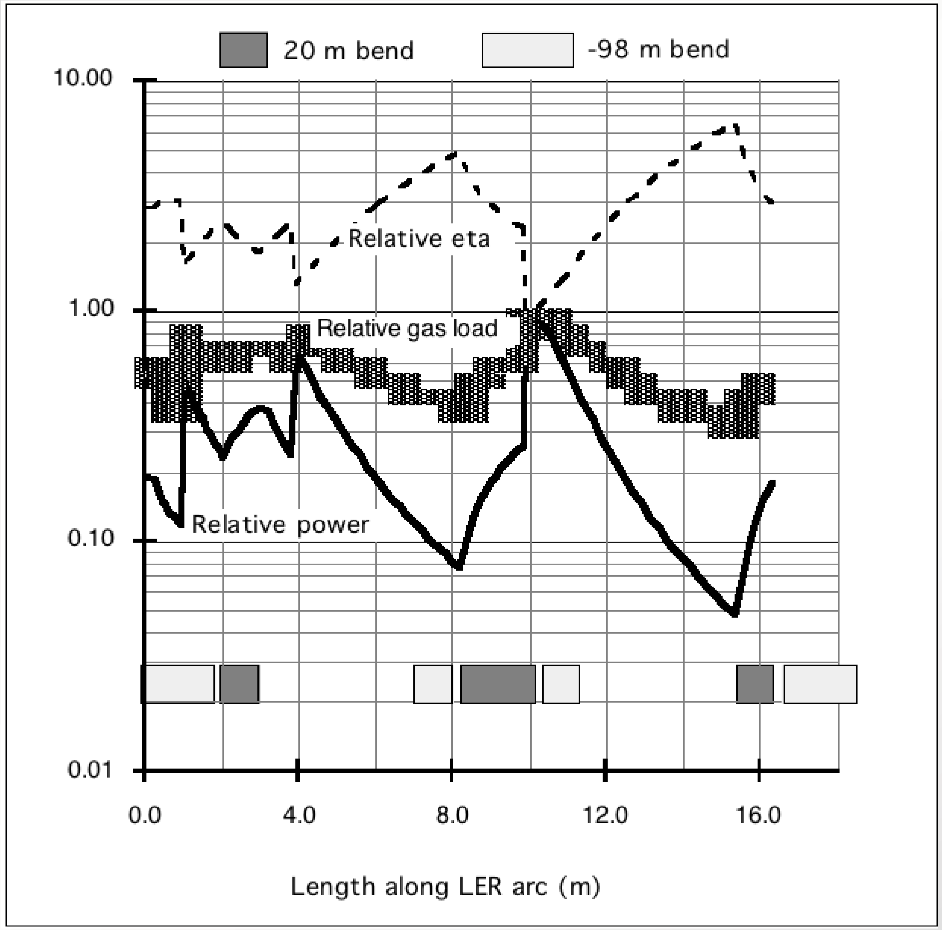


Figure 2. Cross section of the HER arc vacuum chamber.

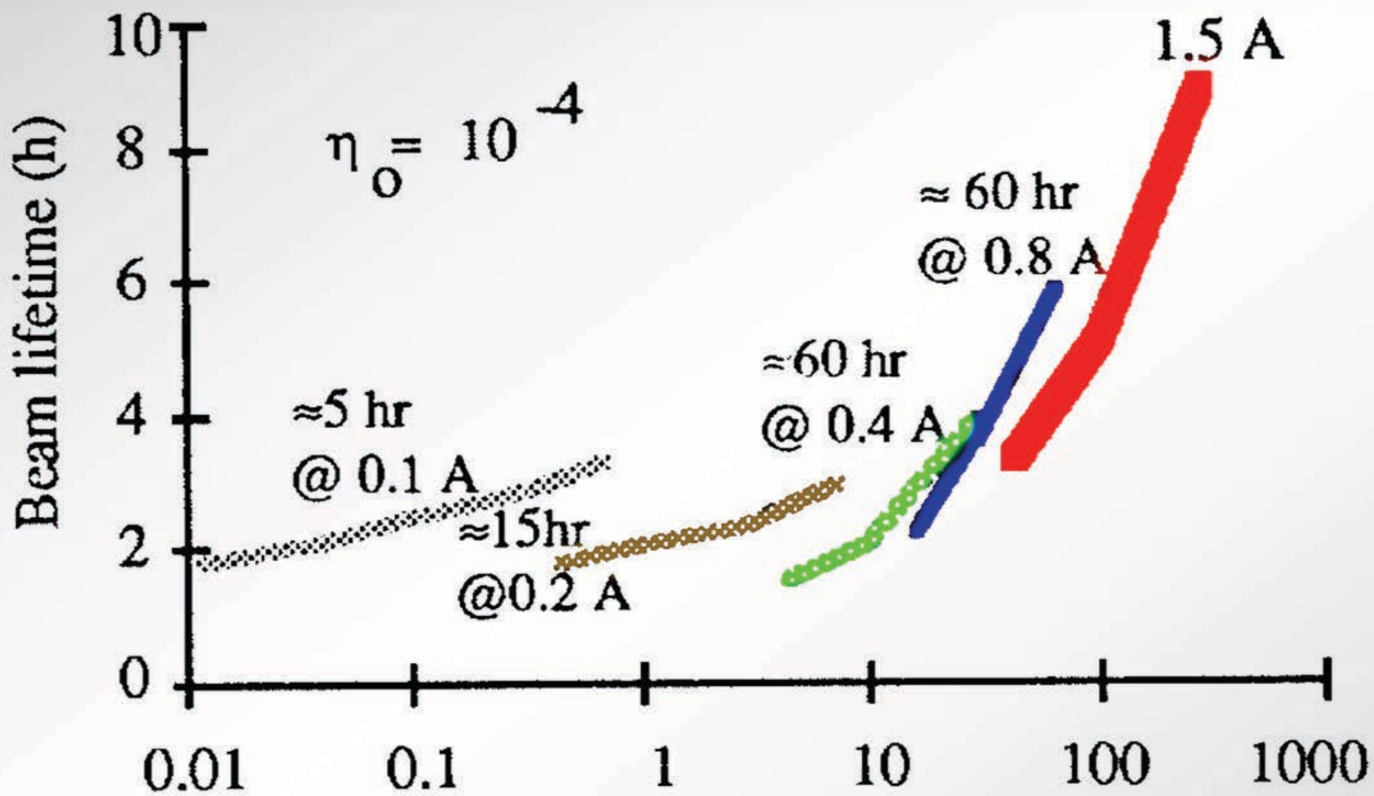
Relative desorption characteristics in tunnel arcs of the CESR-B LER



Relative value = 1.0 corresponds to

$\eta = 1.7 \times 10^{-6}$, $Q = 2.3 \times 10^{-6}$ Torr-l/sec and $P = 10.4$ kW/m

Implication for commissioning PEP-II: Lifetime v. Amp-hours



Commissioning scenario for the APIARY ring with an etched Cu vacuum chamber

Appendix

Synchrotron radiation details

Emittance and Momentum Spread

- At equilibrium the momentum spread is given by:

$$\boxed{\left(\frac{\sigma_p}{p_0}\right)^2 = \frac{C_q \gamma_0^2}{J_S} \frac{\oint 1/\rho^3 \, ds}{\oint 1/\rho^2 \, ds} \qquad where \;\; C_q = 3.84 \times 10^{-13}\, m}$$

$$\boxed{\begin{gathered}\left(\frac{\sigma_p}{p_0}\right)^2 = \frac{C_q \gamma_0^2}{J_S \rho} \\ iso-magnetic \;\; case\end{gathered}}$$

- For the horizontal emittance at equilibrium:

$$\boxed{\varepsilon = C_q \frac{\gamma_0^2}{J_X} \frac{\oint H/\rho^3 ds}{\oint 1/\rho^2 ds}}$$

where: $$H(s) = \beta_T D'^2 + \gamma_T D^2 + 2\alpha_T D D'$$

- In the vertical plane, when no vertical bend is present, the synchrotron radiation contribution to the equilibrium emittance is very small

- Vertical emittance is defined by machine imperfections & nonlinearities that couple the horizontal & vertical planes:

$$\boxed{\varepsilon_Y = \frac{\kappa}{\kappa+1}\varepsilon \quad and \quad \varepsilon_X = \frac{1}{\kappa+1}\varepsilon}$$

$$\boxed{with \quad \kappa \equiv \; coupling \;\; factor}$$

Critical frequency and critical angle

$$\frac{d^3 I}{d\Omega\, d\omega} = \frac{e^2}{16\pi^3 \varepsilon_0 c}\left(\frac{2\omega\rho}{3c\gamma^2}\right)^2 \left(1+\gamma^2\theta^2\right)^2 \left[K_{2/3}^2(\xi) + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2} K_{1/3}^2(\xi)\right]$$

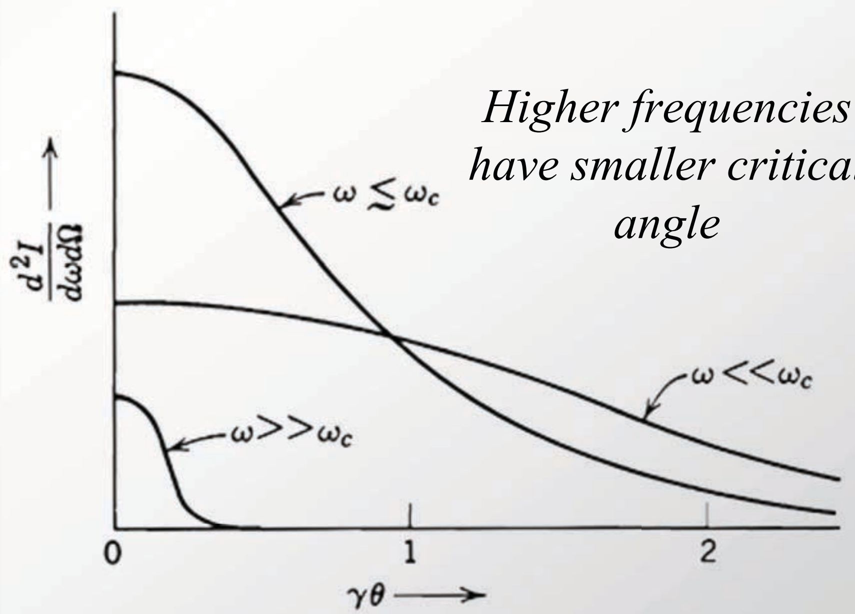
Properties of the modified Bessel function ==> radiation intensity is negligible for x >> 1

$$\xi = \frac{\omega\rho}{3c\gamma^3}\left(1+\gamma^2\theta^2\right)^{3/2} >> 1$$

Critical frequency $\omega_c = \frac{3}{2}\frac{c}{\rho}\gamma^3$

$$\approx \omega_{rev}\gamma^3$$

Critical angle $\theta_c = \frac{1}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3}$



Higher frequencies have smaller critical angle

For frequencies much larger than the critical frequency and angles much larger than the critical angle the synchrotron radiation emission is negligible