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Chapter 11: Space Charge Effects Space Charge, Beam-Beam

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Happy birthday to Polykarp Kusch, Paul Newman, and Ellen DeGeneres! Happy National Peanut Brittle Day, National Spouses Day, and Dental Drill Appreciation Day!

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Debye Length

 Comes from thermodynamic description of large systems of moving charges

$$\lambda_{\rm D} \equiv \left(\frac{\epsilon_0 k_B T}{N e^2}\right)^{1/2}$$

- Collisional regime: dynamics dominated by binary collisions in close encounters (e.g. Touschek)
 - Single particle scattering and single particle effects

$$\sigma_{x,y,z} \ll \lambda_{\rm D}$$

- Self-field or space charge regime: dynamics dominated by self-field over large distances compared to average particle separation
 - Collective effects, single-component cold plasmas

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$$\sigma_{x,y,z} \gg \lambda_{\rm D}$$



11.1: Transverse Space Charge Tune Shift

 In space-charge dominated synchrotrons and low-energy beams (without bunch compression), we can approximate the beam as being cylindrical and effectively infinitely long



- The above figure simply establishes the coordinate system and velocity. We assume that the beam is cylindrically symmetric (for now).
- Only radial forces: $F(r) = q(E_r \beta c B_\theta)$

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- Electric repulsion will be stronger than magnetic attraction
- At hyperrelativistic velocities these nearly balance (~no effect)
- Also seen by imagining electrostatic forces only in bunch frame
- Can compensate by shielding charges/currents (e.g. plasma)



Back to Maxwell



Relativistic β

Maxwell's equations in cylindrical coordinates here are

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{r} \frac{d}{dr} (rE_r) = \frac{q}{\epsilon_0} n(r) \qquad (\vec{\nabla} \times \vec{B})_z = \frac{1}{r} \frac{d}{dr} (rB_\theta) = \frac{q}{c\epsilon_0} \beta n(r)$$

We can integrate these directly

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$$E_r = \frac{q}{\epsilon_0} \frac{1}{r} \int_0^r r n(r) dr \qquad B_\theta = \frac{\beta}{c} \frac{q}{\epsilon_0} \frac{1}{r} \int_0^r r n(r) dr = \frac{\beta}{c} E_r$$

- These are simple if the beam has constant density
- Also simple if we treat these beams as having a radial Gaussian distribution N

$$n(r) = A \exp\left(\frac{-r^2}{2\sigma^2}\right)$$
 $A = \frac{N}{2\pi l\sigma^2}$ Bunch length I Bunch population N



Space Charge Force

Calculating the force from these fields

$$F(r) = q(E_r - \beta c B_\theta)$$

gives

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$$F(r) = \frac{Nq^2}{2\pi\epsilon_0 l} \frac{(1-\beta^2)}{r} \left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right]$$

like beam-beam but opposite sign In B term

This is a defocusing force that is equal in both directions
 -- how does it affect our focusing and tune?

$$\frac{d^2y}{ds^2} = \frac{1}{\beta^2 c^2} \frac{d^2y}{dt^2} = \frac{1}{\beta^2 c^2} \frac{F_y}{m\gamma} \qquad \frac{d^2y}{d\theta^2} = \frac{R^2 F(y)}{\beta^2 \gamma mc^2}$$

R: average radius, $s=R\theta$

$$\frac{d^2y}{d\theta^2} + Q_{\rm V}^2 y = \frac{R^2}{\beta^2 \gamma mc^2} F(y)$$

$$r_0 \equiv q^2/(4\pi\epsilon_0 \ mc^2)$$

Equation of Motion

We get something rather unpleasant for the expansion

$$\frac{d^2y}{d\theta^2} + Q_{\rm V}^2 y = \frac{2Nr_0R^2}{l\beta^2\gamma^3} \left[\frac{1 - \exp\left(-\frac{y^2}{2\sigma_{\rm V}^2}\right)}{y}\right]$$

 But we can expand the unpleasantness and ignore the higher order terms for now (which contribute nonlinearities)

$$\left[\frac{1 - \exp\left(-\frac{y^2}{2\sigma_{\rm V}^2}\right)}{y}\right] = \frac{y}{2\sigma_{\rm V}^2} - \sum_{n=2}^{\infty} \frac{(-1)^n}{2^n n!} \frac{y^{2n-1}}{\sigma_{\rm V}^{2n}}$$

Maclaurin series Taylor series about 0

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$$\frac{d^2y}{d\theta^2} + \left(Q_{\rm V}^2 - \frac{N}{l\sigma_{\rm V}^2}\frac{r_0R^2}{\beta^2\gamma^3}\right)y = 0$$

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Linear Space Charge Tune Shift: Calculated

$$\frac{d^2 y}{d\theta^2} + (Q_{\rm V} + \delta Q_{\rm sc})^2 y \approx 0$$
$$\delta Q_{\rm sc} = -\frac{N r_0 R^2}{2l Q_{\rm V} \, \sigma_{\rm V}^2 \, \beta^2 \gamma^3} = -\frac{N r_0}{4\pi \, B_f \, \epsilon_{\rm V,rms}^\star \, \beta \gamma^2}$$

To get the last we've used a few other conversions

$$\beta_{\rm V} \approx R/Q_{\rm V} \qquad \pi \epsilon_{\rm V,rms}^{\star} = \pi \frac{\sigma_{\rm V}^2}{\beta_{\rm V} \beta \gamma}$$

Bunching factor : $B_f \equiv \frac{l}{2\pi R} = \frac{I_{\rm ave}}{I_{\rm peak}}$

Normalized emittance

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• $\delta Q_{\rm sc}$ is called the (vertical) Laslett tune shift

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- For hadron beams, this is a big effect at low energy, high N
- Dominates high intensity boosters (FNAL, BNL, CERN)
- Electrons escape most effects except for very low γ



Flat/Round Space Charge Tune Shift

- This tune shift is symmetric between H,V for round beams
 - e.g. most hadron beams

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 For elliptical or ribbon beams one can show that the proper calculation gives

$$\delta Q_{\rm sc,V} = -\frac{\beta_{\rm V} N r_0}{2\pi B_f \sigma_{\rm V} (\sigma_{\rm H} + \sigma_{\rm V}) \beta^2 \gamma^3}$$

- The horizontal is just given by reversing H and V
- This is true for most synchrotron electron beams
- This tune shift is different for different parts of the beam
 - Commonly called an incoherent tune shift
 - Compare to coherent tune shift given by, e.g., quadrupoles



Head and Tail Space Charge

 Differential defocusing in head and tail of beam from space charge is a source of some (reversible) emittance growth



$$\frac{\text{Messy Nonlinearities}}{\left[\frac{1 - \exp\left(-\frac{y^2}{2\sigma_V^2}\right)}{y}\right]} = \frac{y}{2\sigma_V^2} - \sum_{n=2}^{\infty} \frac{(-1)^n}{2^n n!} \frac{y^{2n-1}}{\sigma_V^{2n}}$$

- Note that space charge also drives many nonlinearities
 - First order is quadrupole (tune shift) so higher orders are octupole (nonlinear tune shift), dodecapole, ...
 - Beam size is natural scaling parameter
- How high can we run $\delta Q_{\rm sc}$?

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- It spreads beam across resonances, so not more than 1!
- Large amplitude particles shift away from resonance though
- Incoherent tune spread across emittance of beam
- Some facilities (FNAL Booster, AGS) run up to 0.7!
- Note that space charge is distributed through the accelerator
 - This makes it quite computationally expensive to model



Space Charge Compensation

- How to compensate for space charge?
- Create neutral plasma by co-propagating opposite charge beam
 - Studied at e.g. CERN PS Booster (Aiba), FNAL Booster
 - Matching beams compensate tune shift and resonances
 - Ion recombination not expected to be substantial
 - Electron columns with strong solenoidal fields (Shiltsev, FNAL, '07)
- Inject small mix of electronegative gases

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- Very low energy high flux ion beam (Dudnikov², FNAL/BINP)
- Low energy electrons: Use focusing solenoidal field in gun
 - Developed by Carlsten (LANL, '94) for 1.3 GHz photoinjector
 - Now routinely incorporated into high-brightness photoinjector design
 - Also incorporate superconducting RF half-cell: faster acceleration
 - Ion back-bombardment problem on RF photocathodes (Pozdeyev)

11.4: Longitudinal Space Charge Defocusing

- Longitudinal space charge is most easily calculated in the center of mass frame of the beam
 - Then boost fields to lab frame with Lorentz transformation
 - Voltage for uniform beams, in the beam center of mass frame

$$V_{\rm cm} = \frac{qg}{4\pi\epsilon_0} \frac{N}{\gamma l_{\rm beam}} \qquad g \equiv \left(1 + 2\ln\frac{b}{a}\right)$$

a: beam radius b: pipe radius

For a more realistic longitudinal distribution (parabolic)

$$V_{\rm cm} = \frac{qg}{4\pi\epsilon_0} \frac{3N}{2\gamma l_{\rm beam}} \left[1 - \left(\frac{2z_{cm}}{\gamma l_{\rm beam}}\right)^2 \right]$$

Boosting back and finding the longitudinal force

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$$F_{\parallel} = \frac{3}{\pi}g \frac{Nq^2}{\epsilon_0 l_{\rm beam}^2 \gamma^2} z$$

- Modifies synchrotron frequency, important near transition
- Creates bunch lengthening in low energy electron beams



Application of Longitudinal Space Charge

Nonlinear Longitudinal Space Charge Oscillations in Relativistic Electron Beams

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In this Letter we study the evolution of an initial periodic modulation in the temporal profile of a relativistic electron beam under the effect of longitudinal space-charge forces. Linear theory predicts a periodic exchange of the modulation between the density and the energy profiles at the beam plasma frequency. For large enough initial modulations, wave breaking occurs after 1/2 period of plasma oscillation leading to the formation of short current spikes. We confirm this effect by direct measurements on a ps-modulated electron beam from an rf photoinjector. These results are useful for the generation of intense electron pulse trains for advanced accelerator applications.



11.3: Beam-Beam Force

- The beam-beam **force** is very similar to space charge
 - Except now use properties of colliding beam
 - Oppositely charged beams focus, same sign defocus
 - Velocities are opposite so sign of force from B term reverses

$$F(r) = q(E_r + \beta c B_\theta)$$

$$F(r) = \frac{Nq^2}{2\pi\epsilon_0 l} \frac{1+\beta^2}{r} \left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right]$$

like space charge but opposite sign In B term

- We also only integrate the beam-beam kick over the bunch length as a short kick, since the interaction is short
 - We can then use the same short kick approximations as before for multipoles



Beam-Beam Force (cont)

$$F(r) = \frac{Nq^2}{2\pi\epsilon_0 l} \frac{1+\beta^2}{r} \left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right) \right]$$

Assume particles are ultrarelativistic ($\beta \sim 1$) and take the leading term to find linear focusing and thus a tune shift

$$F(y) \approx \frac{Nq^2}{2\pi\epsilon_0 l\sigma_{\rm V}^2} y$$

The counterrotating bunch acts like a short lens of length l/2- - 0

$$\delta p_y = F(y)\delta t = F(y)(l/2\beta c) = \frac{Nq^2}{4\pi\epsilon_0\beta c\sigma_V^2}y\left(\frac{p}{\beta\gamma mc}\right) = \frac{Nr_0p}{\gamma\sigma_V^2}y$$

$$\beta \approx 1$$

$$\Delta y' = \frac{\delta p_y}{p} = \frac{Nr_0}{\gamma\sigma_V^2}y$$
Thin quadrupole!
(SPAS Accelerator Physics 16)

Beam-Beam Force (cont)

$$\Delta y' = \frac{\delta p_y}{p} = \frac{Nr_0}{\gamma \sigma_{\rm V}^2} y$$

- We can use the thin quadrupole formula we derived on Friday to derive the corresponding linear tune shift
 - Reverse sign of tune change for oppositely charged beams!

$$\delta Q = -\frac{\beta_y \Delta k}{4\pi} \qquad \qquad \Delta Q_{\rm bb} = -\frac{N_{\rm IP} N r_0 \beta_{\rm V}^{\star}}{4\pi \gamma \sigma_{\rm V}^2} = -\frac{N_{\rm IP} N r_0}{4\pi \gamma \epsilon_{\rm V,rms}}$$

- N_{IP} is the number of interaction points
- Tune shift independent of beta function at collision point
- N and emittance are properties of opposing beam
- For elliptical beams we can similarly generalize to

$$\Delta Q_{\rm V,bb,elliptical} = -\frac{N_{\rm IP}Nr_0\beta_{\rm V}^{\star}}{2\pi\gamma\sigma_{\rm V}(\sigma_{\rm H}+\sigma_{\rm V})}$$

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Beam-Beam Simulation

- Most beam-beam simulations are done using a weakstrong model
 - Assume the beam creating the kick is static or quasistatic
 - Beam-beam kick is then just a thin (very) nonlinear lens
 - Scales as particle partitioning of weak beam
 - Quick: you did this in the Java lab!
- But we can have two-beam collective effects
 - The two beams are two coupled oscillators
 - Both coherent and incoherent effects (strong nonlinearities)
 - This requires a strong-strong model where both distributions must evolve together consistently over time
 - Can be very computationally expensive

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Similar to large detailed space charge calculations



Beam-Beam Coherent Effects

- Colliding beams are not always aligned on center
 - Offsets of beam centers can easily produce coherent coupled oscillations between the beams
 - Can give rise to π and σ oscillator modes



Long Range Beam-Beam

- Recall that our force is very nonlinear at large amplitudes
 - There are good reasons to separate colliding beams to moderate or large amplitudes
 - e.g. short-spaced bunch trains in long interaction regions with crossing angle (LHC with minimum bunch spacing)



- This introduces beam-beam crossings at angles and nonlinearities from parasitic beam-beam kicks
- Drives designs to large crossing angles for large separation
- Luminosity optimization drives need for crab cavities

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Beam Beam vs Space Charge

- Beam-beam and space charge tune shifts look similar!
 - Their nonlinear terms also look similar

$$\delta Q_{\rm sc} = -\frac{Nr_0R^2}{2lQ_{\rm V}\sigma_{\rm V}^2\beta^2\gamma^3} = -\frac{Nr_0}{4\pi B_f \epsilon_{\rm V,rms}^{\star}\beta\gamma^2}$$
$$\Delta Q_{\rm V,bb} = -\frac{N_{\rm IP}Nr_0\beta_{\rm V}^{\star}}{4\pi\gamma\sigma_{\rm V}^2} = \frac{N_{\rm IP}Nr_0}{4\pi\epsilon_{\rm V,rms}^{\star}\beta\gamma^2}$$

- Naively we would expect their behavior to be similar too
 - Space charge is distributed through the accelerator
 - Nearly phase-averages out many nonlinear behaviors
 - But beam-beam is localized in phase

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- Emphasizes nonlinear behaviors, little phase averaging
- Beam-beam tune shift limits are more like 5-7x10⁻³/IR
 - e.g. RHIC polarized proton collisions are beam-beam limited



Beam-Beam Compensation

- Similar to space charge, it's possible to compensate beambeam behaviors with similar nonlinear magnetic fields
 - Local compensation preferred to compensate nonlinearities near source
 - Electron lens (Tevatron, Shiltsev et al)
 - Wires (RHIC, Fischer et al)
- Hollow electron beam collimation

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- Not exactly beam-beam but doesn't fit many other places
- Requires creation of hollow electron beam at source/gun
- Used to limit halo of high energy hadron beams
- Surround core of beam with circular beam of electrons
- Creates nonlinear forces that eliminate halo, do not disturb core
 - See, e.g., http://arxiv.org/abs/1202.1512 (Stancari et al)

