#11: SYNCHROTRON RADIATION, & DAMPING 1/29/19

- Larmor (19th century) knew how charged particles radiate ... BEFORE special relativity & quantum mechanics.
- Pollock (1947) first see synchrotron radiation.

ELECTRONS

0) Radiate copiously : protons "not" at all
1) H, V, S motion is damped & stabilized CLASSICAL
2) H, S motion is driven. QUANTUM ($\hbar \neq 0$)
3) Dynamical equilibrium delivers "natural" emittances

BUT HERE $\hbar = 0$!!

POWER SPECTRUM – LOCAL

Larmor total power $P = \frac{1}{6\pi\epsilon_0} \cdot \frac{q^2 a^2}{c^3}$

boosted, become $P = \frac{1}{6\pi\epsilon_0} \cdot \frac{q^2 c}{\rho^2} \cdot \gamma^4$

more conveniently: $\boxed{P = \frac{1}{6\pi\epsilon_0} \cdot \frac{e^5}{m^4 c^5} \cdot B^2 E^2}$ local!

dipole field

beam energy

Protons "don't" radiate because:

$$\left(\frac{m_{proton}}{m_{electron}}\right)^4 \simeq 10^{13}$$

Q: WHY speak of muon colliders?

$m_\mu = 106\ MeV/c^2$

$\tau_\mu = 2.2\ \mu s$

Universal spectrum shape $(\omega = 2\pi f)$

$$\boxed{P(\omega) = \frac{P}{\omega_c} S\left(\frac{\omega}{\omega_c}\right)}$$

$$S(\xi) = \frac{9\sqrt{3}}{8\pi}\,\xi \int_{\xi}^{\infty} K_{5/3}(\bar{\xi})\, d\bar{\xi}$$

Modified Bessel function

Characteristic frequency

$$\boxed{\omega_c = \frac{3}{2}\frac{c\gamma^3}{\rho}}$$

$\frac{1}{2}$ power $\omega > \omega_c$

$\frac{1}{2}$ $\quad < \omega_c$

Wavelength

$$\boxed{\lambda_c[\mathrm{m}] = \frac{1.86\times 10^{-9}}{B[\mathrm{T}]\, E^2[\mathrm{GeV}^2/c^2]}}$$

Q: TYPICAL?

$B = 0.5$ T

$E = 3$ GeV

$\lambda_c \approx 4\times 10^{-10}$ m

POWER SHAPE in plane of synchrotron, at angle θ

$$\frac{dP}{d\Omega} = \frac{P_0}{(1-\beta\cos(\theta))^3} \cdot \left[1 - \frac{\sin^2(\theta)}{\gamma^2(1-\beta\cos(\theta))^2}\right]$$

RELATIVISTIC $\gamma \gg 1$

$$\boxed{\frac{dP}{d\Omega} \approx \frac{P_1}{(1+\gamma^2\theta^2)^3} \cdot \left[1 - \frac{4\gamma^2\theta^2}{\underbrace{(1+\gamma^2\theta^2)^2}_{\text{Headlight effect}}}\right]}$$

$\boxed{\beta \ll 1}$ Classical 2 lobe dipole antenna pattern

$\boxed{\gamma \gg 1}$ Forward opening angle $\pm \frac{1}{2\gamma}$

TYPICAL: $\gamma = 6{,}000$ [3 GeV] $\pm$ 0.1 mrad

ENERGY LOSS PER TURN

1 particle, 1 turn: $U = \oint P(s) \frac{ds}{c}$

gives an IDEAL value: $\boxed{U_0 = C_\gamma E_0^4 \cdot \frac{C}{2\pi} \langle G^2 \rangle}$

where angle brackets $\langle \, \rangle$ denote a ring average

and $G = \frac{1}{\rho}$ is local bending strength

and constant $\boxed{C_\gamma = \frac{4\pi}{3} \frac{r_0}{(mc^2)^3}}$ r_0 — classical radius

$= 8.85 \times 10^{-5}$ [m GeV^{-3}] ELECTRONS

$= 7.78 \times 10^{-18}$ PROTONS

MUONS

E.G. ISOMAGNET RING : All dipoles have same ρ

$$\langle G^n \rangle = \frac{\oint G^n \, ds}{\oint ds}$$

$$\boxed{\langle G^n \rangle = \frac{1}{R \rho^{n-1}}}$$

Where $2\pi R = C$

$$U_0 = \frac{C_\gamma E_0^4}{\rho}$$

ELECTRONS $\boxed{U_0 \,[\mathrm{keV/turn}] = 88.5 \, \frac{E_0^4 \, [\mathrm{GeV}/c^2]}{\rho \, [\mathrm{m}]}}$

EG 1: $B = 0.4$ T, $E = 3.0$ GeV $\Rightarrow U_0 = 0.29$ MeV NSLS-II Inj.

2: 50 TeV protons, $\rho = 14$ km $\Rightarrow U_0 = 3.5$ MeV VLHC (FCC)

LONGITUDINAL MOTION

Previously SLIP FACTOR $\eta_S = \frac{1}{\gamma_T^2} - \frac{1}{\gamma^2}$

Now ($\gamma \gg 1$) COMPACTION FACTOR

$$\boxed{\alpha = \langle \eta G \rangle} = \frac{1}{\gamma_T^2}$$

So longitudinal displacement evolves like

$$\boxed{z_{n+1} = z_n - \alpha C \cdot \delta_n} \qquad \text{A}$$

On every turn must replenish energy

$$U_0 = eV \sin(\phi_s)$$

TOTAL RF voltage

SYNCHRONOUS PHASE

Turn-by-turn δ evolution

$$\left(\gamma \gg 1 \quad \Rightarrow \quad \delta = \frac{\Delta p}{p} \simeq \frac{\Delta E}{E_0} \right)$$

(B)

$$\boxed{\delta_{n+1} = \delta_n + \frac{1}{E_0} \left[eV \left(\sin(\phi_n) - \sin(\phi_s) \right) - \left(U(\delta_n) - U_0 \right) \right]}$$

Variation with δ is VITAL

IF longitudinal motion is slow $(Q_s \ll 1)$

THEN combine (A) + (B) to give

$$\boxed{\frac{d^2\delta}{dt^2} + \frac{2}{\tau_s} \cdot \frac{d\delta}{dt} + \left(\frac{2\pi Q_s}{T} \right)^2 \delta = 0}$$

where t is time in seconds

$T = \frac{C}{c}$ is circulation time

DAMPED SOLUTION

$$\boxed{\delta = \delta_0 e^{-\frac{t}{\tau_s}} \cdot \cos\left(2\pi Q_s \cdot \frac{t}{T}\right)}$$

with damping time

$$\boxed{\tau_s = \frac{2T}{\left.\frac{dU}{dE}\right|_0}}$$

Slope matters!
Zero for protons

Variation of U with δ??

$$U(\delta) \sim \oint P(\delta)\, ds \sim \oint B^2 E^2 \, ds$$

where

$$B(\delta) \sim G + K(\eta\delta + x_{co})$$

from errors

$$E(\delta) \sim E_0(1+\delta)$$

So $\left.\frac{dU}{dE}\right|_0$ is USUALLY positive & motion is damped.

$\boxed{\text{TIME OUT}}$ Brief return to CONSERVATIVE motion

$U(\delta) = 0$ No radiation

$\phi_s \neq 0$ Particles are ACCELERATING

Hamiltonian shorthand

$$\boxed{H(\phi, \delta) = \frac{1}{2} \alpha \, \omega_{rf} \, \delta^2 - \frac{eV}{TE_0} \cdot \left(\cos(\phi) + \phi \sin(\phi_s) \right)}$$

reproduces (A) + (B) through canonical eqns:

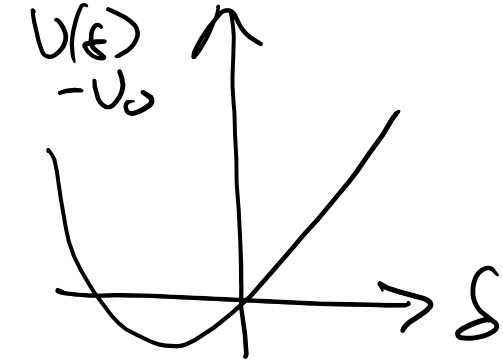
$$\frac{d\delta}{dt} = \frac{\partial H}{\partial \phi}, \quad \frac{d\phi}{dt} = -\frac{\partial H}{\partial \delta}$$

But now can visualize motion:

① Particles follow

② Speed is proportional to steepness!!

$\boxed{\text{TIME IN}}$ Turn radiation back on



- 3 control parameters : (ϕ_s, Q_s, T_s) permit simultaneous radiation + acceleration
- Particles no longe follow contours.
  Some are lost $\{\delta \ll 0\}$
  Others ar trapped in an RF bucket/lake/island
  None escape a bucket, once trapped

- NOTE connection to STRANGE ATTRACTORS

!!

TRANSVERSE DAMPING — VERTICAL

V is easier to understnd — dispersion $\eta_V = 0$

- Consider an electron losing ΔU energy in a single dipole
..... and (in effect) recovers ΔU from RF in that dipole

$$y'_{NEW} = \left(1 - \frac{\Delta U}{E_0}\right) \frac{P_\perp}{P_0} = \left(1 - \frac{\Delta U}{E_0}\right) y'_{OLD}$$

VERTICAL OSCILLATIONS therefore damp:

$$\boxed{y = a_y e^{-\frac{t}{\tau_y}} \cos\left(2\pi Q_y \cdot \frac{t}{T}\right)}$$

where

$$\boxed{\tau_y = 2T \frac{E_0}{U_0}} = \tau_0$$

PARTITION NUMBERS

The H, V & S damping times are connected, e.g. due to horizontal dispersion

$$\boxed{\tau_{x,y,s} = \frac{\tau_0}{J_{x,y,s}}}$$

τ_0 — Characteristic time

where "it can be shown"

$$\boxed{J_x + J_y + J_s = 4}$$

$J_y = 1$ in a flat machine

τ_0 is twice as long as it "would" take to radiate all E_0!!

EG: NSLS-II INJECTOR: $E_0 = 3\,\mathrm{GeV}$, $U_0 = 0.29\,\mathrm{MeV}$

$\Rightarrow \tau_0 \simeq 2{,}000$ turns

3-D STABILITY

Both J_x & J_s must be +ve for stability

$$J_x = 1 - \mathcal{D}$$

$$J_s = 2 + \mathcal{D}$$

$$\boxed{\mathcal{D} = \frac{\langle \eta G^3 \rangle + 2 \langle \eta G K \rangle}{\langle G^2 \rangle}}$$

K: local quad strength

EG: SEPARATED FUNCTION OPTICS

If no combined function magnets $GK = 0$

so $\mathcal{D} = \dfrac{\langle \eta G^3 \rangle}{\langle G^2 \rangle} \sim \dfrac{\eta}{\rho} \ll 1$

and stability is guaranteed in all 3 planes ...

EG COMBINED FUNCTION OPTICS

$\mathcal{D}$ can easily be of order 1 if $G k \neq 0$

DOESN'T MATTER

- in hadron rings : some are CF, some are SF
- if electron storage time is short: eg. CESR injector synchrotron

Q: What prevents electron beam sizes from vanishing?

A: Excitation when "h" is turned on

See next lecture!!!