#1: INTRODUCTION 1/21/19

ANALYTIC TOOLS (pre-computer) make dynamical problems look like DIFFERENTIAL eqns.

NUMERICAL TOOLS (computers) make problems look like DIFFERENCE eqns → MAPS

Q1: Which is right

Q2: Is time continuous?

A: It depends!!

2 EXAMPLES

① PENDULUM

② LOGISTIC MAP

PENDULUM

$$\theta'' = -g \cdot \sin(\theta)$$

$$\theta'' = \frac{d^2\theta}{dt^2}$$

HOW TO SOLVE? SMALL angles ⇒ SHM

SIMULATE?

until finished {

$$\theta = \theta + \theta' \cdot \Delta t$$

$$\theta' = \theta' - g \cdot \sin(\theta) \cdot \Delta t$$

}

DIFFERENCE "STANDARD" map

IS NOT like an 'accelerator" !!

Circular accelerators: inherently DISCRETE
as if "gravity" is pulsed on occasionally

COULD describe as differential eqn.

$$\theta'' = -\sum_{n=1}^{\infty} \delta(t - n.\Delta t).g\sin(\theta).\Delta t$$

But why bother? DIFFERENCE MAPS are as legitimate as DIFFERENTIAL EQNS, and are better suited to ACCELERATORS

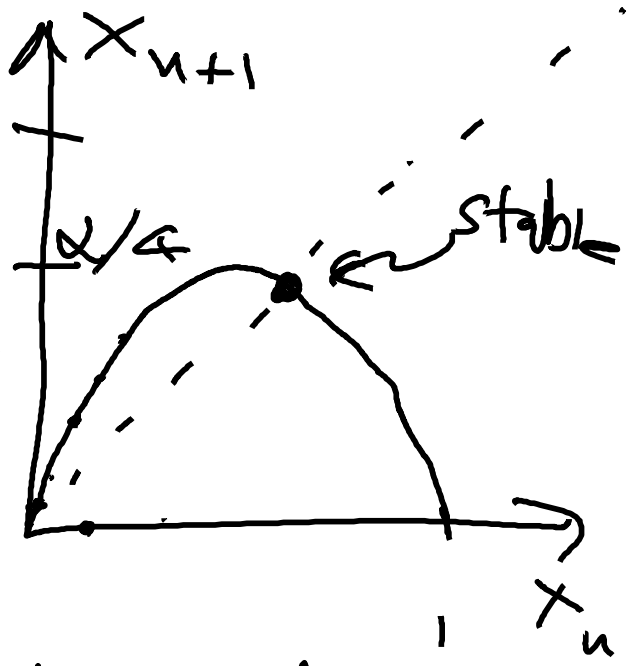
THE PHILOSOPHY OF THE BOOK!

EG 2 LOGISTIC

A population of x {birds, jellyfish, bacteria...} advances from year n to n+1

$$X_{n+1} = \alpha X_n (1 - X_n)$$

$0 < \alpha < 4$ CONTROL PARAMETER



EG

$X_0 << 1$, $\alpha = 1 + \delta$

then $X_n \approx X_0 e^{\alpha n}$ when food is not scarce

DECEPTIVELY SIMPLE! See HW 1-1

PHASE SPACE CO-ORDINATES

EG STORAGE RING

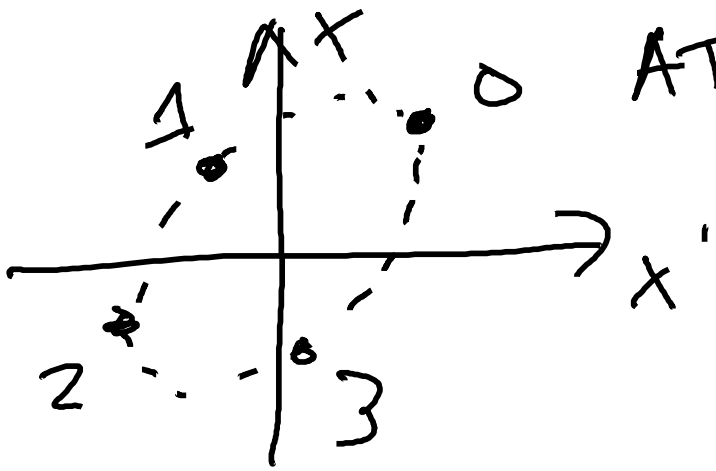
REFERENCE POINT $s = s_0$

$t = 0, 1 \ldots n$

$x \approx 1$ mm, $x' \approx 10^{-4} \ll 1$ $(\gamma \gg 1)$

Plot this motion turn-by-turn

POINCARE SURFACE of SECTION



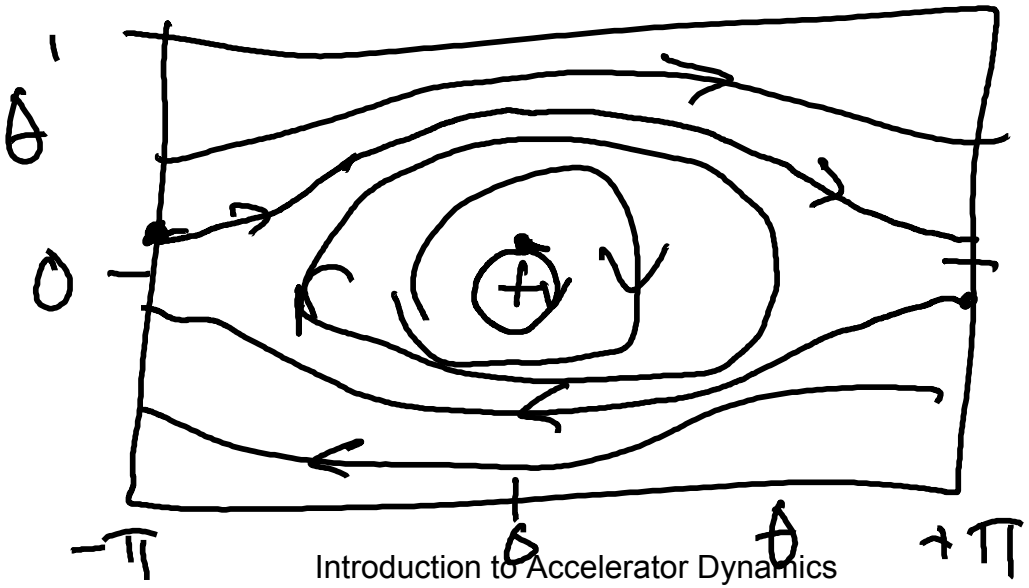
AT $S=S_0$

STABLE?

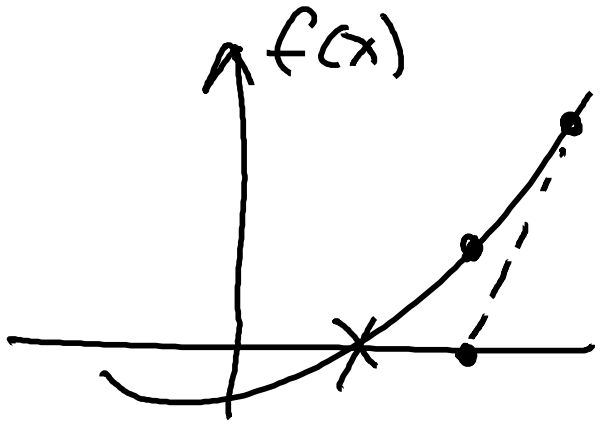
STAYS ON ELLIPSE FOR EVEN?

Poincare discovered chaos in 19th century

EG2: GRAVITY PENDULUM



NEWTON SEARCH



Solve $f(x) = 0$

Know $f'(x)$

x = guess

until converged {

$$x = x - \frac{f(x)}{f'(x)}$$

}

Still used in software libraries

e.g. square root ....

$$y = \sqrt{x}$$

$y = 1$

{

$y = \frac{1}{2}\left(y + \frac{x}{y}\right)$

}

$\sqrt{10}$ is correct to 10 sig. figs in 6 iterations

COLLIDERS: STORAGE RING w. 2 BEAMS

Why not fixed target?

with Lorentz factor γ

FIXED TARGET $E_{TOT} \sim \gamma^2$

STORAGE RING $\sim \gamma$

How?

$$\bar{F} = \frac{d\bar{p}}{dt} = q\left(\bar{E} + \bar{v} \times \bar{B}\right)$$

Consider reversing sign of q and $\bar{v}$

$\bar{F} \Rightarrow -\bar{F}$ (& trajectory is the same)

IF $\bar{E} = 0$: Must use antiparticles

COLLIDER PARAMETERS

If a particle-particle cross-section σ [cm^2] then the rate of events is

$$R = L\sigma \quad \text{per second}$$

R → LUMINOSITY

$$L = f_{REV} \cdot M \frac{N^2}{4\pi\sigma_H^*\sigma_V^*}$$

- N: particles per bunch
- f_{REV}: Revolution frequency $\frac{c}{C}$
- M: # of bunches
- σ_H^*, σ_V^*: H & V beam sizes at collision point

Q1 How to make L large?

Q2 How to have only 1 collision point?

EG RHIC LUMINOSITY

Au ions 10^9

P 10^{11}

$$L = f_{REV} \, M \, \frac{N^2}{4\pi \sigma^{*2}}$$

$f_{REV} \simeq 78\,kHz$

$M = 60$

$\sigma^* \simeq 0.2$ mm

$$L \simeq 10^{27} \left[cm^{-2} \cdot s^{-1}\right] \text{ Au}$$

How to guarantee stability over 10^{10} turns

(lifetime of solar system)

MORE PARAMETERS

AVERAGE Current per beam

$$I = Ze\, f_{REV}\, N M$$

Z = 1 for protons
Z = 79 for gold ions

STORED ENERGY per beam

$$U = N M A \cdot m_0 c^2 \cdot \gamma$$

A = Atomic weight. 1.008 hydrogen
196.97 gold