USPAS Graduate Accelerator Physics Homework 12

Due date: Thursday, February 11, 2021

1 Derivation of β beat formula

(10 points) Using a similar method to that presented in the lecture, show that

$$\frac{\beta^{(1)}(s)}{\beta^{(0)}(s)} = -\frac{1}{2\sin\left(\mu^{(0)}\right)} \int_s^{s+C} K^{(1)}(s')\beta^{(0)}(s')\cos\left(2(\psi(s')-\psi(s))-\mu^{(0)}\right) \mathrm{d}s'$$

You may use Mathematica (or similar) to evaluate the integral (but you don't have to!). Be sure to clearly state your input and output. N.B. There is an error in Eq. (3.31) in the textbook. The bottom left term should have $(\alpha_1-\alpha_2)c_{21}$ in the numerator, NOT $(\alpha_2-\alpha_1)c_{21}$ as written.

What is the physical interpretation of this equation you have derived?

2 `madx` lab: The EIC ESR FODO cell

(20 points) You have already simulated a FODO cell in a RHIC arc. Now you will simulate a group of FODO cells in the EIC Electron Storage Ring (ESR). Download the `madx` file from `http://www.toddsatogata.net/2021-USPAS/lab/esrarc.madx`. There is no fundamental difference between simulating protons or electrons; all you have to do is modify the `BEAM` definition. There are six arcs in the ESR, and there are 16 regular FODO cells in each arc. The lattice has been defined with four families of sextupoles, although at the beginning these are all set to zero.

a) (3 points) Run the initial simulation of the lattice. What are the tunes?
What is the natural (linear) chromaticity?
What is the FODO cell phase advance in radians?

b) (3 points) The linear chromaticity in the ESR needs to be set to +1 in both planes. Why is this target value chosen?
For the rest of this exercise we will suppose we wish to set the linear chromaticity to +1 in just one arc rather than over the whole ring. If you wanted to set the chromaticity to this value using two families of sextupoles, what would be the ratio of the two strengths? Explain your reasoning.

c) (3 points) Using the `MATCH` command you can set up a numerical optimization in `MAD-X`. Run this section of the file (you will need to remove the `STOP;` command). Check to see that the tunes are +1 in both planes.
What are the matched values of the sextupoles (look in the output log)?
Are these values consistent with your answer to your expected ratio in the previous part?

d) (4 points) `MAD-X` can compute W functions. Look at Sec. 1.7.4 in the MAD-X manual. How are the W functions defined? Is this definition consistent with the definition presented in the lecture?
Why are the W functions useful?

e) (3 points) You wish to increase the W function in the horizontal plane to 100 at the end of the arc cells. Why does only one family of sextupoles in the horizontal plane need to be on to achieve this?
Run the next section of code to match the sextupole strengths to achieve this target W_x value. What are the sextupole strengths now? What is the linear chromaticity?
Look at the plot of W function and explain its shape.

f) (4 points) Modify the code to match to $W_x = 100$ at the end of the arc cells while also keeping the horizontal linear chromaticity to +1 using both families of sextupoles, SX1 and SX3.
What are the values of the sextupole strengths? What is the linear chromaticity?
Look at the plot of W function and explain its shape.
Briefly explain how these two families of sextupoles correct both the W function and chromaticity.